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Lecture - 24

We have been talking about numerical integration in the last class and we saw that there are very important concepts that are involved in numerical integration and that there are very interesting things that we saw with respect to what we called as reduced integration. We said that there are two types of integration.

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We said that there is what is called that integration can be complete or reduced. We saw certain advantages and disadvantages of reduced integration. One of the major advantages being that the cost can reduce; the cost of doing analysis can reduce when you do reduced integration and that, but on the other hand it may lead to certain difficulties like what we called as hourglassing and that would result in analysis not being very correct and so on. In fact because of hourglassing, the pressures may be or the loads may be extremely high. Many times people resort to reduced integration for saving time.

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This is especially true for what we call as explicit analysis. If you see many of the softwares, they talk about explicit analysis; may be we will talk a bit later about explicit analysis, but it is very useful for metal forming and other applications. When we talk about it, we will talk about explicit analysis. They save time by doing this kind of reduced integration. We also noted that reduced integration may improve the finite element result. They may not improve the accuracy of integration, but as I told you that when we go to finite element analysis, we make the problem stiffer and when we do reduced integration, we introduced result in the opposite direction or in other words, we make the stiffness to be softer and hence there is some sort of a plus and minus effect that takes place because of this integration and hence we may be at an advantage in certain situations on reduced integration.

That is what we talk, but there is a major problem of hourglassing. The hourglassing comes about because of what we called as zero energy modes and one of the peculiar results of hourglassing is this kind of zig zag positioning of the nodes.

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On a very local scale, the element looks something like this. It looks like an hour glass geometry and hence we call as, we call this as hourglassing.

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As I told you that hourglassing is eliminated by certain techniques called as hourglass controls. These controls rely upon introducing stiffness for those modes which cause hourglassing. This is quite an involved topic. I am not going into the details of it, because this derivation involves lot more things. But on the other hand, I would like to tell you that if you are going to use software where you are going to use reduced integration, have a look at this technique, whether there is hourglass controls; if they have hourglass controls, then what are the assumptions that underline this hourglass controls and so on. In other words, look at the way they introduce stiffness. Please remember that stiffness for a particular mode is going to zero and hence we have what we call as zero energy modes.

These hourglass controls have a particular technique of introducing what we call as the zero energy modes or sorry introducing stiffness in such a fashion that it avoids the zero energy modes. We will stop with this as far as the integration is concerned, but there are very live issues, there are very important issues that we have to see as far as the isoparametric element and other things are concerned. But before we go further, we will look at or we will summarize as to what is the complete, what a complete integration is.



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For example, if you are looking at a 4 noded element, then if you talk about complete integration, then you should have 2 by 2 integration procedures. If you are talking about 8 noded element, then you should have a 3 by 3 scheme and if you are talking about 9 noded elements also, you will be talking about 3 by 3 scheme. That is in summary what we use. But, there is a big but here. We assume in all these cases that the deformations of these elements are small. In other words, the deformations are or in other words, the mesh is quite nice and we do not introduce error due to the

geometry of the mesh. For many of the problems which are linear elastic, we do not have much difficulty. We feel that the error that is introduced by, say, 2 by 2 scheme or 3 by 3 scheme for this mesh is perfectly okay. When the geometries are such that when deformations are large, then even this integration scheme may not be sufficient, but we have to understand quite a bit, before because where the situations are, where the mesh becomes large?

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What are the other difficulties that we may face or in other words, whatever we have been talking about is basically for linear problems or for linear problems and where are the other difficulties that I have been talking about? Where are they going to come, what are the other problems of interest to us? See, what I am going to do is I am going to talk quite a bit about things where applications are to be extended to manufacturing. Hence at this point of time, I just want to digress a bit from finite element analysis and look at certain applications in manufacturing, so that I will come back and when we teach further theory, you will understand what exactly I am talking about or where I am going to apply. For example, as I said when I say that 3 by 3 is okay for regular linear analysis, but when it comes to metal forming this may not be good or sometimes you may end up in difficulty or you may end up in error, if I tell this, you may not understand what exactly I am talking about. Now, I am going to introduce constraints and other thing. Even here you may not be able to understand what the application is, what is the background to which I should apply these things and so on? So, what I am going to do is I am going to digress a bit, go into, say, two or three problems in metal forming, look at these problems much more minutely, much more closely, what we did in the previous class or one of the previous classes, then come back and when I go ahead, you will see, oh this is being done, because the applications are here in this area and so on.

The first thing that we are going to see is why is that we are going to apply this kind of finite element method to metal forming or manufacturing problems? I am sure that by all this, about 20-25 lectures, you are clear as to whatever I have been talking about is directly applicable to a linear elastic problem or in other words, for doing many of the standard stress analysis, in order to prove a component, a machine component or a mechanical component or structure or whatever it is. We have seen about element formulations. We have seen as to how stiffness matrix has to be formed and we said that solution plus stress strain relationship all these things together would make a good package to analyze what we call as an important step in the design of mechanical components. But, whatever we have talked about is very, very preliminary.

Though many of the, I would say, many of the concepts that we have talked about will be applicable to metal forming, but they are absolutely not enough to deal with problems of manufacturing, general problems in manufacturing, specifically problems which are, say which come under what we call as metal forming. Why is that we do analysis for metal forming and what are the peculiarities with which and so what are the algorithms with which we should be equipped, in order that we can solve these kind of problems? That is the thing that I am going to concentrate on in the next few classes and before we come back to wind up certain more elements that I have not covered in this earlier class.

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Now, let us look at the, say requirements of doing finite element analysis, requirements of doing finite elements analysis as well as the need to do finite element analysis for same metal forming problem. Why I am saying same metal forming problem, because very similar things are there or you can do similar analysis for machining of a casting and many, many other welding, for example. All these manufacturing processes also can be analyzed using finite element analysis. But we are going to look at specifically this area, because it is very easy to understand from an application perspective, but very difficult to do it using finite element analysis.

There are two things that are important in doing an analysis using metal forming. One, number one, we are now going to look at post yield behavior. So far, for all our design problems we have been concentrating on the elastic range. Please note that we had looked at what we call as the yield condition or the equivalent stress and then compared it with yield strength and said that if the stress is lower than the yield strength, then there will not be any yielding and we are safe and so on. But as all of you know, if you look at problems of manufacturing, then we obviously cannot, obviously cannot work at the elastic range. We necessarily have to strain the material beyond the elastic range or in other words, you can very confidently say that manufacturing process is one, where the material is abused to a very, very great extent; nowhere else material is abused to such an extent as in manufacturing. So, the first thing is that we should look at post yield behavior.

The second important thing that we should know is what is called as contact, what we should know is what is called as contact. In other words, we should understand that contact say for example, between the die and the stock or the material which you are forming, the punch and the stock, these contact conditions become very important. May be all of you would have studied in your manufacturing process that the friction between the die and the material is very, very important for forming, the friction between the punch and the material is important and so on. Hence the next issue that has to be addressed is how to develop a proper contact algorithm in order to deal with, in order to deal with the contact conditions.

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Thirdly, in many situations, we may have to have algorithms to deal with temperature or thermal problems. So, the post yield behavior requires a nonlinear approach; contact behavior requires a nonlinear approach and many of the manufacturing problems also require a thermal problem to be solved. This can be linear or nonlinear, steady state or transient. In this course, what I am going to do is to extend whatever concept I have talked so far to certain aspects here. That is what I am going to do, not necessarily in this order. For example, next class I may take up as to how to handle thermal problems. But, what I am going to do is to extend these concepts, so that whatever we have done and whatever is applicable only for design can be extended now to the other nonlinear problems which result or which are there during manufacturing. We have to develop algorithms to do this. But before we do that, there are other peculiarities as well, the peculiarity of mesh distortions and so on. Even before we go into these things, let us see why do you want to do these things?



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Now that you are sure that the problem is very difficult, why do people want to do finite element analysis for the manufacturing process? Why not I just look at, just do manufacturing of a component? Just manufacture it, see what happens and if there is anything that I require, change it and so on and so forth. Why not I do that, what people have been doing so far? Why do you want to introduce finite element analysis for manufacturing process, get into all these difficulties and say that these problems are much, much more difficult than the linear elastic problem and try to find solutions for these things?

The world has changed over the past 10 years. The problem is that, today we need or the companies that do manufacturing have very, very small lead time. For example, 6 to 7 years back, in a metal forming or forging industry, the cold forging industry, is given about 7 months, 7 months to develop a product. They can afford to do experiments; they can afford to do a die, break it, look at it and then make some changes and so on and so forth, because they had the time. Of course they were spending money, but nevertheless at least they had that time to do it. All of you may know that it takes a lot of time to make a die and may be a matter of month or two or so on. So in 7 months, at least 2 are 3 trials you can make and then get a reasonable product. But today, this lead time has drastically dropped. Now, we are looking at one month or less. Many of the cold forging industry say that the time required or the time that is given to develop a new product is 1 month. Hence they have to necessarily do at least a suboptimum die design. It is not possible now, to play around with common, what I would say, common sense and say that, look, by common sense approach I can design a die. It is not possible, because the time that is there for them to say make it, is very less. That is the first thing or the major thing, I should not say first, but it is a major thing.

Number two, what are the things or how do we achieve this 1 month target?



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What can happen in developing the die for metal forming? One, we may produce a defective component. The component can be defective. This is because of large, say strains. Number two, Defective component; there the crack in the component, there can be voids in the component and so on. So, I should know in other words, how the components are going to be stressed, what the strain levels are and so on? The second one is that the die may crack; a die or punch or die and punch, one of them may crack or it may fail. Please remember that especially say since we have talked about cold forging, there the force that is required to form these components are extremely high.

May be, many of you may know it that the press capacities that are used in order to form them or form these components are very high, may be even of the order of 600 tonnes; 300, 400, 500, 600 tonnes. The pressure or the force that is required to form is very high, so, the stresses in these dies and the punch may be very high, inspite of the fact that these dies and punches can withstand loads of the order of 200 kg per mm square or 2000 Newton per mm square. Inspite of the fact that the loads are very high, the dies may crack. That is another difficulty that you may face.

Third one is that the material may not be able to fill in completely, they may not be able to fill in completely the die crevices. In other words, there can be defect called under filling; there can be a defect called under filling, because of which again the component may be rejected. So, these are, say some of the difficulties that may be faced during the manufacturing process and if I am able to solve these problems by doing simulation, then definitely I will be able to get or I will be able to do a suboptimum die. I would not say optimum die, because there are lot of things that are required. We will see during the course of these lectures what are the things that are required?

But nevertheless, at least sort of I will have a ballpark figure for the die and say that look, now that this is the die, the failure chances are much less. Before we go further, let us look at some examples to put this in proper perspective. First of all, let us understand why we are doing this before we look at how we are going to do these analyses? That is a long way off, at least let us understand, why we are doing this. Let us now look at our first component.

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This component, which will become clear in the next slide, let us look at the next slide and then see this component slightly more closely.

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Yeah, now look at that component. This component is actually a plane strain problem, some sort of a coining or bending. You see that, actually we are looking at the result; it does not matter, that it is very important that we look at the results. Actually, you can see that the plastic strains developments that take place are shown here. At the center, at the center is the component, where now you see the mark or the cursor. You

can see that that is component and outside, below that component is the die; that is the die and the top part that is the punch.

We see that the component, the die and the punch, are shown and that you see say plastic strains; in the component, you can get stress and so on and so forth. Let us now for a moment understand what are the things that were there? There was what we called as die but interestingly you saw that there is no mesh in the die, the mesh was not present. You saw the punch, again there was no mesh. So you may immediately get a doubt as to how did we do this problem, where is the mesh? On the other hand, in the first figure especially you could have seen that there was mesh for the component. So, the first thing that you should understand is that many times, many times in metal forming, you consider the die to be rigid, the punch to be rigid and the material to be deformable. In other words, what we do is a two step process. Many times, it is not necessary that you have to do like this every time, but we do this as a two step process.

You will understand why we are doing like this later, but right now, to understand these results you should realize that we are doing it as a two step process. In the first step, what is that we do? In the first step, we discretize the material or the stock and then assume that the die is rigid. When we do like this, we are assuming that the dies deformation is extremely small when compared to the deformation of the material. That is obviously true because, the die will still be in the elastic range, so the deformations will be very small. We develop what we call as a contact condition between a rigid die and a deformable body. We will talk about this contact later, but let us first understand that when we do a metal forming problem, we consider a rigid body and a deformable body deformable body being the material that is getting deformed and the rigid body is one where the die and the punch, where the deformations are very small.

Once we complete the deformations, once the deformations are complete, then we get the reactions, the forces that happen to be present in the die and the punch. These forces are then applied to the die and the punch and an elastic analysis is done. This elastic analysis will predict the stress and the strain of the punch. Let us go back and look at that particular model again.

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Let us look at this. This is half way through; the punch is now descending. The punch, it has a different color, compared to, violet color when compared to other things. The punch is descending, the product is half way through, let us look at the next.





In this one the deformations are such that, the stock has entered all the crevices, small gaps that are present. You can see on the right hand corner, right hand corner where small crevices, yeah you can see the cursor going there, all those places, there are small gaps in the die and you can see that the material very well fills up all the

positions. In other words, this component has been analyzed and it is seen that there is no under filling when the punch stroke is complete. This is, as I told you, a very important result. The next slide may be shows that completely.



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Now, you see how beautifully the material after deformation fills up completely all the gaps, all the crevices, all the nice things that are there in the die. But, let us look at the previous slide again; let us just look at the previous slide again.

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Yeah, now see, there is a very important thing that has happened. See that the mesh progressively gets distorted. Especially on the right side, you can see that the mesh is extremely distorted. What do we mean by distortions? Let us see what is meant by distortions?



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Suppose I have a mesh like this, a nice mesh like this, in the beginning of the deformation process. As I deform, the mesh may become like this, something like that. The mesh starts deforming tremendously. This kind of deformation does not happen, does not happen, in a linear elastic problem, but is very much present in the nonlinear case or in the metal forming problem. Hence, what is that we are trying to do here? Though we are trying to fill up this gap, through that process we are deforming the mesh tremendously. When we deform the mesh our results are obviously affected, because all our integration schemes which we said are nice when we do 2 by 2, 3 by 3 and so on, they all go to a toss or they are going to introduce errors. So, as the deformation becomes larger and larger, the mesh becomes worse, bad to worse, it goes from bad to worse and hence it is necessary that we resort to what we call as remeshing. We resort to what is called as remeshing.

What is remeshing? When the deformations are large, the mesh is again redrawn, redrawn on the original mesh. We willlook at the examples of this later, but the mesh is again redrawn and all the variables that were present in the previous mesh are now

transferred to the new mesh. Another important, I would say, requirement of a good metal forming product or a package is to do what we call as remeshing, so that as the mesh becomes distorted during the process of deformation, the mesh is again redrawn or re-meshed or mesh is generated and these things or the variables are transferred to the new mesh. How we do it? All of them are very involved. We will at least first understand what are the requirements? The whole idea now is to tell you what are the requirements?

Let us look at the next product that we can see in the slide.

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This is what is called as a backward extrusion. See, there is a punch at the top. Now, you clearly see that there is a punch and a die. There is a punch, see that the cursor points out the punch and there is a die all around; that is the die. As the punch descends, as the punch comes down, the material enters the gaps on either side. That gaps what is now beginning shown, the material enters those gaps. This is called as backward extrusion, because the material gets extruded through those gaps against the punch.

Let us look at the next slide.

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This shows actually the mesh. You can see that the mesh is highly distorted at the places where it is being shown by the cursor. In fact, the next slide shows the results.



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See that the plastic strains are extremely large in those regions; where the cursor shows, plastic strains are more than 1.55, see, it is very, very large. In fact, this problem requires, this problem requires, about 30 remeshings; before we finish it, we require to, we have to do about 30 remeshings, then only the problem is complete. What does it mean or what does it show? It shows that the remeshing is an important

part of the whole analysis for metal forming, basically because the remeshing if you do not remesh, mesh becomes distorted and it introduces errors during integration procedures and we will not be able to get what we call as convergence and so on. That is the next thing.

Just to summarize what are the things that are required, one is that we should be able to handle post yield behavior; we should be able to handle contact or friction condition between the punch, die and the material, we should able to handle temperature in certain cases, we should be able to handle what we call as remeshing. These are some of the things that we should be able to handle in a package, in a finite element package which does metal forming. If you do not do it, we will introduce errors and it will not be possible to predict many of the things.

Let us now look at the next slide.



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This is what we call as forward backward extrusion. One of the things that I want you to see here is that again there is a punch; there is a punch at the top. There is punch, just to the left; that is the punch and then you see the die at the bottom. See, though these are axisymmetric problem, there is an axis of symmetry at the center. There is an axis of symmetry, where now the cursor moves. That is the axis of symmetry.

Though there is an axis of symmetry, We have mirrored it, so that it is clear for you when you look at it, but these are axisymmetric elements and axisymmetric problems. But, look at one very interesting thing that there is no boundary condition per say, for example in the y direction that is given. The complete, The boundary condition is only through contact.

In other words, the singularity is removed by a proper, what you call as a proper contact algorithm. Hence we need a very robust contact algorithm because if you do not have it, because of the fact that there is no boundary condition, we may end up in singularity. Hence many problems, many metal forming problems are difficult to start, because if the conditions of contact are not good, then it may result in what we call as singularity.

Let us now look at the next slide. That is the first slide showed that it had just started; it is now just starting.



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May be we will look at the next one and look at the result.

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Now you see that the punch has descended and the forward extrusion is almost complete and the backward extrusion has started. You can see that material is going up as the punch descends and that is again, this again requires lot of what I would say remeshing. In fact this whole problem and the next slide shows, I think the completion of problem.

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This whole problem from start to finish this finish requires more than 40 remeshings. Again that requires a very nice, good remeshing, automatic remeshing procedures. Apart from all these requirements, apart from all these requirements, we need other things as well in order to do it. Though they are independent of the preview of finite element analysis, you need certain other things. What are the things that you require? You need the material properties in order to do them. We need to know what the stress strain data is, beyond yielding. What is the relationship between stress and strain? How do you write it? May be, you can write it something like this.

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Sigma_y or sigma flow stress is equal sigma_{y0} plus k epsilon power n. May be after 2 or 3 classes we will realize the importance of this, but nevertheless, I want to point out that the stress strain behavior beyond yielding is very important. We should know the properties of the material where the sigma_{y0} indicates the initial yield, epsilon is the strain; people call this as epsilon or epsilon_p. However it is, whatever be the way it is introduced, we should know k, we should know n. These are material parameters or strain hardening parameters. The other thing that we know, we should have before we start doing analysis to simulate or simulate the manufacturing process are the material properties. We should know all these material properties. We have to do experiments. There are specific experiments to be done in order to obtain these values and we should know these things as well.

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The other one that is the other factor that is very important that we should know also is what we call as friction coefficient. In fact, in many situations, friction plays a very important role. Friction coefficient, what is that? It is going to have a major role in say for example, under filling, in say calculating the total load and so on. Hence we should also have an effect or we should also know what would be, in order to know the effect of coefficient of friction or in order know the friction we should also know what the type of friction law is that we have to use. Most of us usually use coulomb law, but for some problems this may not be sufficient. But that is one thing you should know, what is the type of law that you use?

For example, rolling problems; we say that coulomb friction law is very good, for forging some other slightly different laws and so on. The first thing you should know is how to express and number two is what is the parameters that govern this law? For example, if you say that coulomb friction is the one which is useful, then you should know what the friction coefficient is? What is friction coefficient mu that you have to use? In a nutshell, what we mean to say is that there are lots of things, in order to, there lot of things that you should know, you should appreciate and understand if you have to do this analysis or finite element analysis for metal forming problems. They include contact, yielding and so on.

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One of things which we have not seen here, may be we will see it in the next class, is the importance of thermal problems. Thermal problems are not restricted to manufacturing, obviously not; there are lots of problems. Other places where thermal problems are extremely important are heat exchangers and so on, but since this course concentrates on computational methods in design and manufacturing, it is important that we also know how to do thermal problems, because as you already know, for example, for welding problems, for casting problems and all this, we have to know how to solve thermal equation or in other words, we may have to find out how to predict temperatures or we may have to know how to predict temperatures, given the heat input and so on. So, apart from about five or six factors, what is also important is that we know how to solve the thermal problems.

What I am going to do in the next class is to write down the procedure for thermal problems, how to solve thermal problems? Because once we know that, both steady state and transient problems, once we know how to solve the steady state and transient problems, we can solve problems that arise in, say for example, castings and weldings. But, remember that there is one important fact. What is that? Many problems require not only one type of analysis, it is not that they require, they require the thermal problem to be solved. If you look at, for example, welding, then we require thermal problem as a first step. As a next step, then we have to solve the distortion problems or deformation problems.

Thermal problems plus deformation problem has to be solved simultaneously, in order that we can get the complete picture of the welding process, both temperature as well as distortions and hence in many problems we have to have a combined or coupled analysis that need to be done in order to analyze the manufacturing problems. In the same fashion, for example if you are looking at hot forging, then again we may have to do initially a thermal problem, then followed by a mechanics problem or deformation analysis. Here again, there are thermal contact and so on, but this course may not be sufficient to handle thermal contact, because that needs lot of time. But nevertheless, we have to handle both thermal problem as well as the mechanics problems.

If you look at for example even machining, we have to we may it may be the other way. Like for example, machining may give rise to plastic strains, large plastic strains and hence plastic work. The plastic work may result in heat. It may be converted or it will be converted into heat. We assume a very conservative figure that 90 to 95% of plastic work gets converted into heat and again we need to know what the effect of this heat is on, say, the components, on the fixture and so on and so forth. This again is a factor in a manufacturing problem, where it is just the other way, where say welding we give the heat, we determine the deformations. Here in machining for example, we deform the material. That results in large plastic strains or plastic work and this result in or this result in increase in temperature. This gives rise to heat input into the material, that gives rise to a large temperature and hence the problem again has to be solved.

I think it is quite a long list, but the list is not complete unless we add one more very peculiar thing that goes on in post yield behavior. What is that? It is what we call as volume conservation.

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We know that volumes do not change in the plastic region, except for the small change due to the elastic aspect of the plastic region. There is a constraint due to or in other words we have to introduce an equation for constraint due to volume conservancy as well. If at all you want to solve the problem for say metal forming, then you also have to look at issues or to algorithms that can be used to save or to ensure conservancy of volume.

These are broadly the issues that we have to look at in the next few classes. We will start with the thermal problem, we will proceed to look at a general nonlinear procedure, then look at the formulation, one of the formulations, for solving metal forming problems. That will sort of complete the picture as far as the computational methods application to manufacturing. After doing that, I am going to come back, hopefully we will have time, I am going to come back and look at two or three elements which I have left right now. For example, we have not dealt with beam element. I will deal with beam element, just indicate, we will not be able to do much more than that, indicate the peculiarities of plate shell and axisymmetric shell.

I think that would complete the picture, so that I will not leave out. I do not want to take it at the end these things, because they are quite difficult topics. So, we will start with this now. We will have time to understand it, may be come back later to look at other elements and may be one or two classes we will spend on how to interpret the

results, the errors that may happen in the results and so on. So, next class we will look at thermal problems.