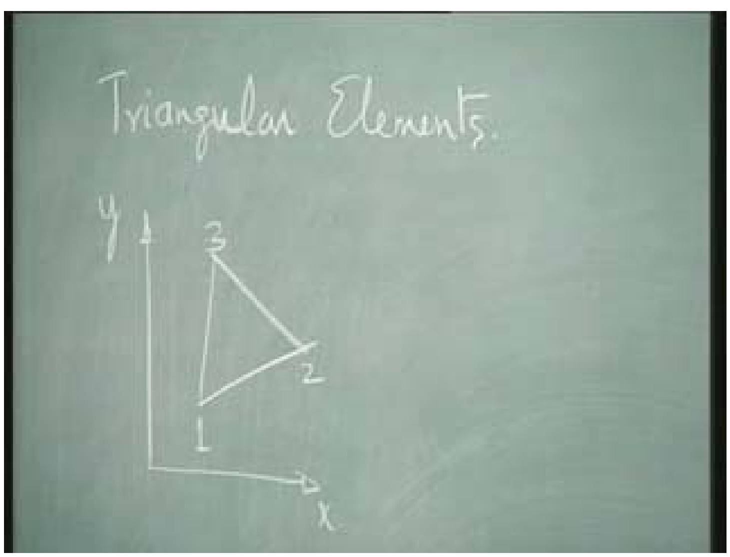
Introduction to Finite Element Method
Dr. R. Krishnakumar
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture - 22

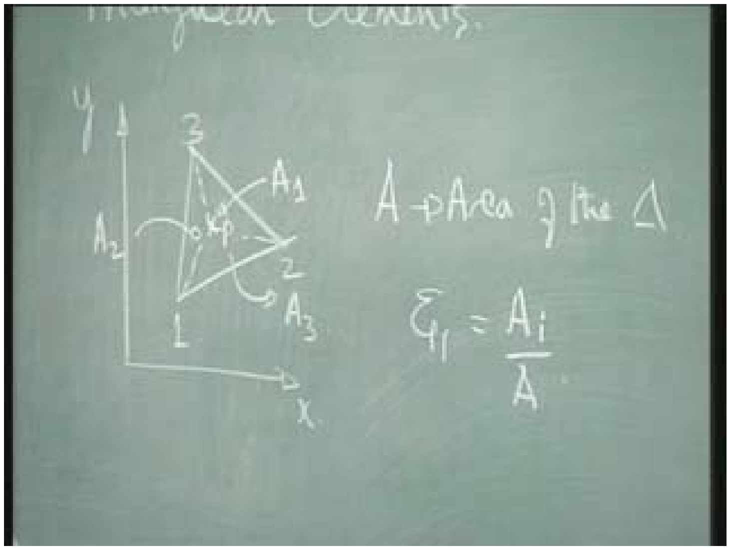
We had started our discussions on an alternate formulation for stiffness matrix in a triangular element. Remember, we had already emphasized the importance of triangular element and hence it is important that we understand what we are in for, when we do triangular elements. Unfortunately in the industry, again I am just emphasizing the importance; unfortunately in the industry if you really look at how people do finite element analysis, most of them do not have the time to put a very proper mesh. They do not have the time to put, for example, a hexahedron element. What they usually tend to do is to rely on this kind of what they call as automatic mesh generation. In fact, I have seen from my experience about 70 to 80% of the people, why it may be more than that also, use only automatic mesh generation. That is what is given in the software, but there are lots of pitfalls in it. It is nice, it is good to use it, because you can finish your job very fast, but there are lots of pitfalls to it and that is the reason why we are going through this kind of derivations.

(Refer Slide Time: 2:14)



Let us now look at, concentrate on this triangular element. We are now going to develop or rather look at what we call as a simple coordinate system. What is the coordinate system that we are going to give or on alternate coordinate system?

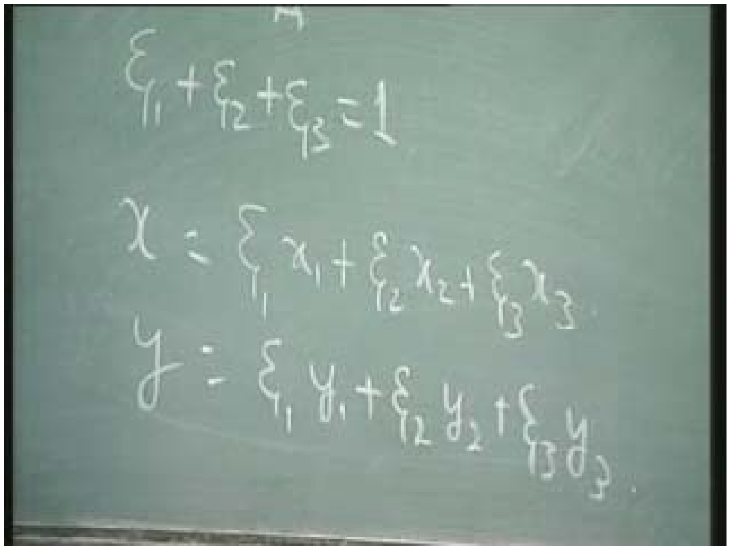
(Refer Slide Time: 2:31)



Let us say that I am looking at a point P. Let me join. P can be at any point whether here, here, here or whatever it is. Let me join the three vertices of the triangle and call as A, the area of this triangle and A_1 and A_1 to be the area of that piece that is area of the triangle P_{23} and so on. This is called as A_1, this is called as A_2 and this is called as A_3. Is that clear? So, A_1, A_2 and A_3. Let me say that any point P, at this point or any point P, can be expressed by means of a coordinate system, say, psi_i; psi_i, which is equal to A_i by A. Is that clear? That means that this point P can expressed by means of coordinate systems psi_1 psi_2 and psi_3.

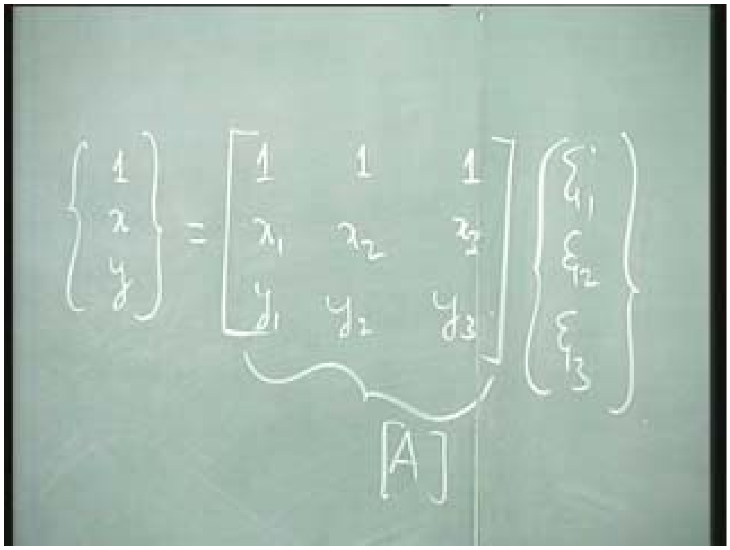
Yes, I can see in your face. Immediately there is a question that we are in a plane and we need only two coordinate systems, independent; the basis all of you know some linear algebra. The bases in a plane are only two, what we call as i and j are what are required. Now, you are talking about three coordinates. Where is the lacuna? Yeah, I can understand. Immediately that is the question that comes to your mind. Exactly, it is correct. In other words, psi_1 psi_2 and psi_3 which are A_1 by A, A_2 by A, A_3 by A are not independent or in other words, psi_1 plus psi_2 plus psi_3 is equal to 1.

(Refer Slide Time: 5:00)



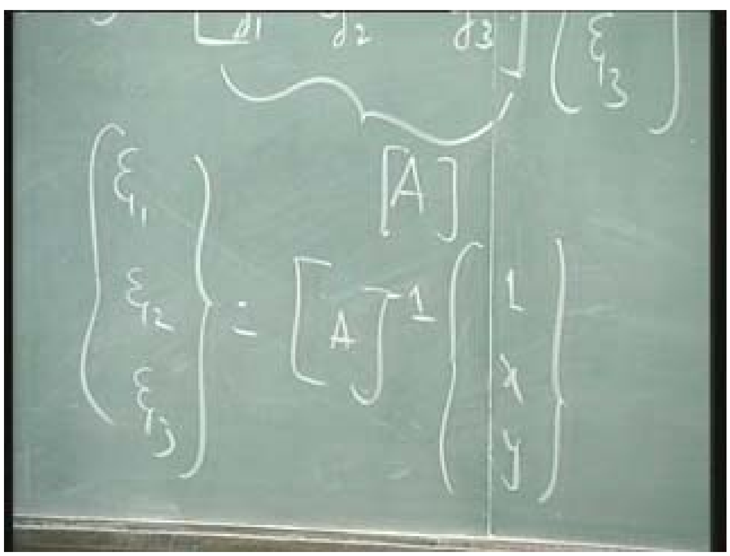
In other words, this brings us to an important concept that at psi$_1$ psi$_2$ psi$_3$ are related and that they can be used as a first set of equation to solve or to express x and y in terms of psi$_1$ psi$_2$ and psi$_3$, to express in terms of psi$_1$ psi$_2$ and psi$_3$. In other words, I can write down, say for example, x to be psi$_1$ into x_1 plus psi$_2$ x_2 plus psi$_3$ x_3. My whole idea immediately, remember what we did? We tried to develop a mapping between a new coordinate system which we had put few days back and x and y. I am going to develop the same thing. I am going to say that, let us say that x is written as psi$_1$$x_1$ plus psi$_2$$x_2$ and plus psi$_3$$x_3$. Similarly, I can write down y to be psi$_1$$y_1$ plus psi$_2$$y_2$ plus psi$_3$$y_3$, so that, taking these three equations, I can write them in the matrix form and say 1 x y, 1 x y, is equal to psi$_1$ psi$_2$ psi$_3$.

(Refer Slide Time: 6:46)



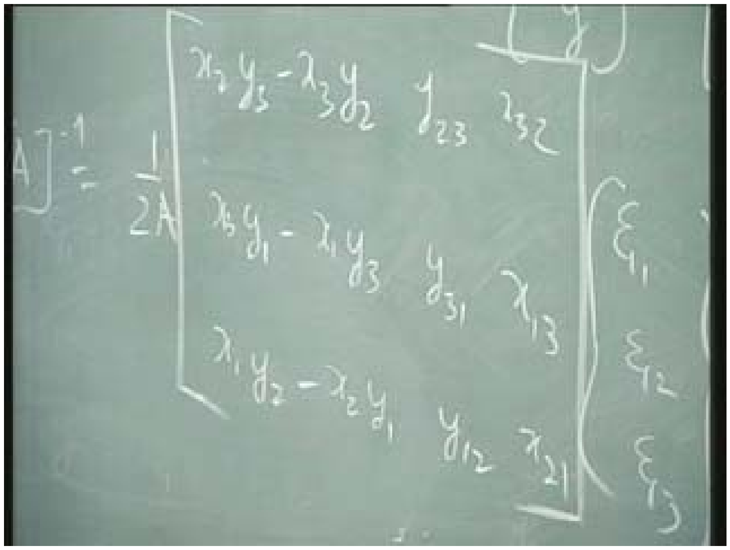
What are these things? Simple. 1 1 1; that is the first equation. Second equation is x_1 x_2 and x_3. Third is y_1 y_2 and y_3. Let me call this matrix, say A matrix. If you like another name you can give it, but be careful, you should not confuse with other matrices that you know.

(Refer Slide Time: 7:25)



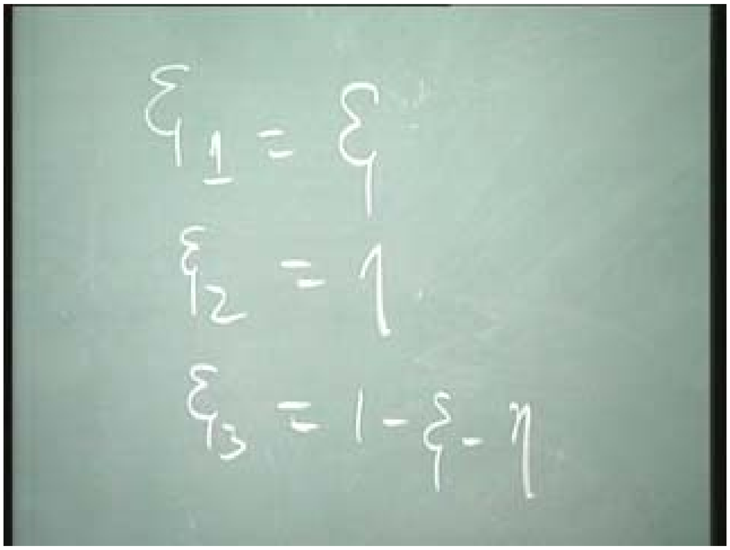
So, that psi_1 psi_2 psi_3 now can be expressed as A inverse. A inverse can actually be expressed in a very explicit fashion. A inverse can be expressed in terms of an explicit fashion. Let us see how we express A inverse, let me do that.

(Refer Slide Time: 8:13)



I mean, just for, is equal to 1 by 2A into x_2y_3 minus x_3y_2, x_3y_2, y_{23} and x_{32}. That is the first row. x_3y_1 minus x_1y_3; what is y_{23}? y_{23} is y_2 minus y_3; y_{23} is y_2 minus y_3. People who know it by heart, this is not a problem for, if you know how to do an inverse of a 2 by a 3 by 3 matrix. y_{31} and x_{13}; this is available in any standard text book. Next is x_1y_2 minus x_2y_1 y_{12} and x_{21}. Not very difficult to remember, because 23 32 31 13 12 21 and similarly 23 32 31 13 12 21; 12 21 12 21. Now, what is that we have achieved? The first step is that we have achieved or we have determined or we have put down, one what we call as a coordinate system. I am not happy with only this, because I want to convert this into a natural coordinate system and follow the procedures which I know. In other words, I do not what to stick to 1 2 3, but can I convert it into a form which is very similar to the form which I had for my quadrilateral? How do I do? How am I going to do it?

(Refer Slide Time: 10:45)

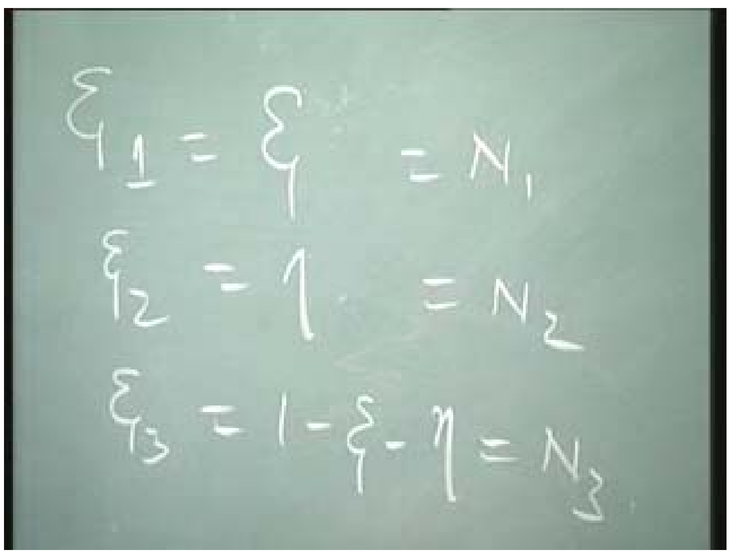


Simple; what I am going to do is, I am going to say that let me call psi_1 is equal to a psi coordinate system; let me call psi_1 is equal to psi. Just wait a minute, till I write that and then psi_2 is equal to eta, so that psi_3 becomes 1 minus psi minus eta. Let me write like this, so that I still stick to my psi eta coordinate system. I am defining, what is that I am doing? I am defining psi and eta to be like this. Psi_1 is equal to psi, psi_2 is equal to eta, so that psi_3 automatically becomes 1 minus psi minus eta. That is my first step, I have defined. What all I have done, I am very simple; whatever I have done is only took a what is called as a area coordinate system and defined my natural coordinates psi and eta in terms of this area coordinate system. But, my job actually begins there. What is my job?

Let us trace, let us keep always in mind what we are trying to do, so that you do not get confused. What is that we are doing in isoparametric element? I am recapitulating for the nth time, but does not matter, because this has to get into your mind when you follow this derivation. First is, I put one coordinate system, one natural coordinate system. This is the first step I am doing. The second step is what? I am going to do, I am going to write, your determinant, it comes later you know. I think, what I should do is, at the end of the class, what I am going to do is I am going to give or I am going to ask you to give me a framework to code this whole thing. That is why I told you in one of the classes that coding is important, because you will understand all the steps very clearly.

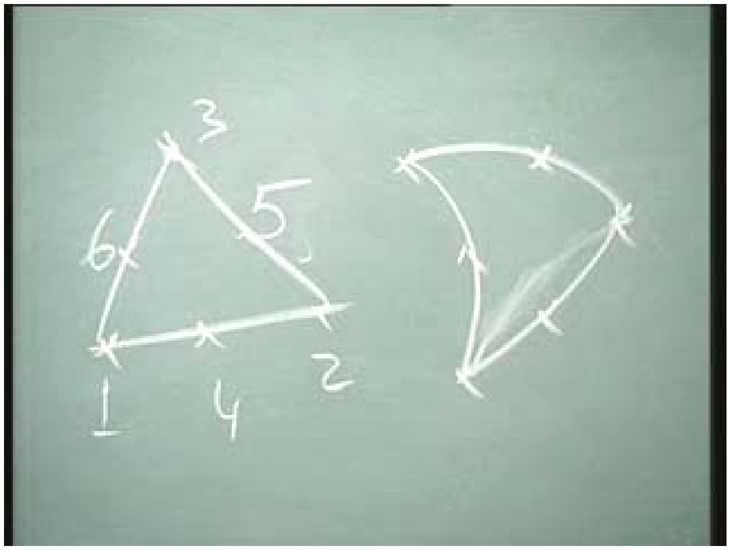
The next step is determination or writing down the shape functions. That is important. I cannot use obviously my 1 by 4 into 1 minus eta into 1minus psi and those shape functions, which I had used in the earlier classes, I cannot use it. Now, I have to write down shape functions for this. Once I write shape functions, then I determine what is called as the Jacobian through the determination of D_N. Is it clear? Then, I determine the D_N value, then I determine Jacobian, then I determine B matrix and then I determine stiffness matrix. So, that is how we proceed. This is common whatever be or however I do it, these steps proceed. In fact, what I am going to do is to illustrate this for a six noded triangular element, so that you will understand that the procedure is exactly the same.

(Refer Slide Time: 13:45)



For a linear triangular element of what we have given, I can straight away use this to be N_1, this to be N_2 and this to be N_3. N_1 N_2 and N_3, shape functions, but let us now move; that is not very difficult and you can work it out yourself. But, let us move to a slightly more difficult or not difficult actually, but a different type of element. So far, we have been looking at what are called as linear element or first order element. Whether I put these three nodes in this element or the four nodes that I had put for a quadrilateral element, the quadrilateral element becomes what is called as a bilinear element. Here it is a linear element. However I put it, it is very obvious that we are looking at only linear interpolations. On the other hand, let us look at an element which looks like this.

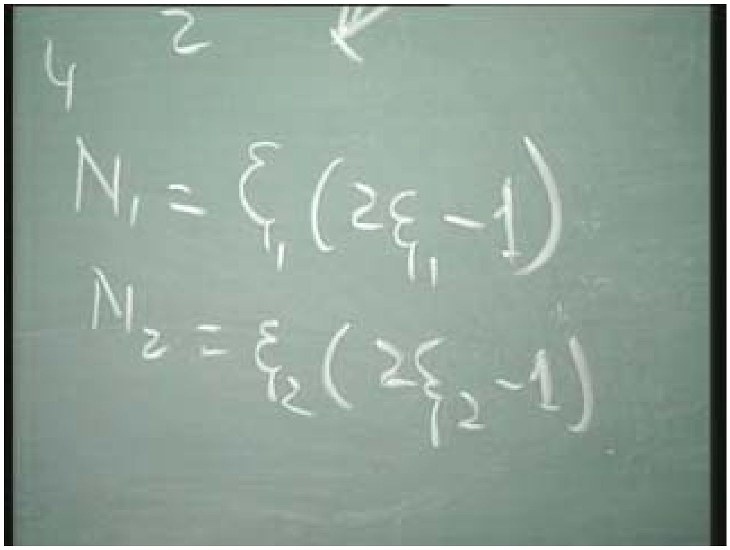
(Refer Slide Time: 14:50)



A 6 noded triangular element, 6 noded triangular element, whose nodes, note this carefully, how I am defining 1 2 3 4 5 and 6, whose nodes I am defining it as 1 2 3 4 5 6. What is the peculiarity of this element? Though I have written it as straight sided, in the x y coordinate system can someone guess, first of all how many nodes are there? 6 nodes are there. So, I have to necessarily go above my linear interpolation. What is that we are doing in isoparametric element? Shapes are also linearly interpolated for that previous element, because x and y, remember I had written it in terms of psi_1 and psi_2. In fact, u and v also I am going to write it in the same fashion. x is equal to N_1x_1 plus N_2x_2 plus N_3x_3. That is what I will write.

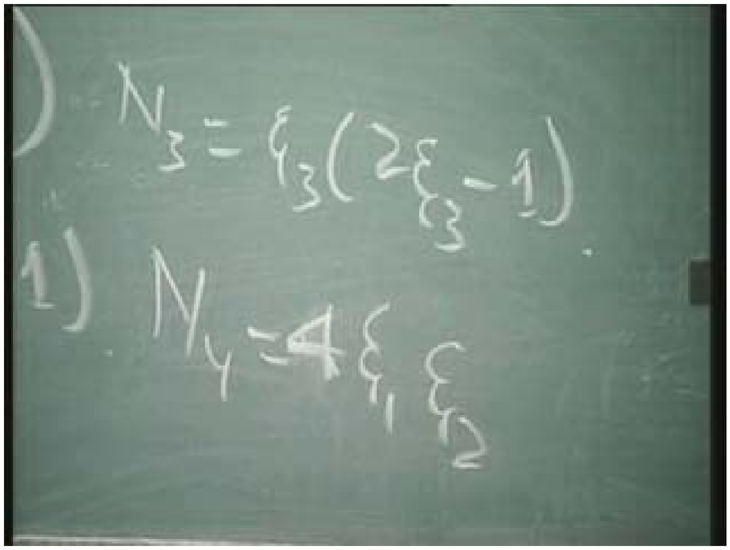
In this case also, both x as well as u, or both x and y as well as u and v, I will now write down by a polynomial which is more than, the orders of which is more than, linear. It would become in this case, quadratic. What does this physically mean? What does this mean? It means that in my x y coordinate system, in my x y coordinate system, this triangle, this triangle what I had put can take what shape? It takes a shape something like this; something like this, because these chaps are defined by quadratic interpolation. What is the quadratic interpolation that we are going to use or in other words, see, please note that I am going to use the same shape functions, sorry, same coordinate system psi and eta or psi_1 psi_2 and psi_3, either way. You can use psi_1 psi_2 and psi_3 or psi and eta; the relationships are already given. How am I going to express the, what I call as the shape functions, because I am interested in the shape functions to express my shape of the element as well as u and v.

(Refer Slide Time: 17:53)



I write this as, say for example, how many shape functions are going to be there? Obviously 6; so, I am going to write this as N_1 is equal to say psi$_1$ into 2psi$_1$ minus 1. Just look at that for a moment and tell me whether it is okay with you. Just check whether it is okay with you, in the sense that does it, I mean I have just put it down by inspection. By experience you can write it down like that. You can see that, what is that I should get? I should have N_1 to be equal to 1 at node 1 and at all other nodes, it should be equal to zero. It happens quite nicely there and N_2 is now given by psi$_2$ into 2psi$_2$ minus 1.

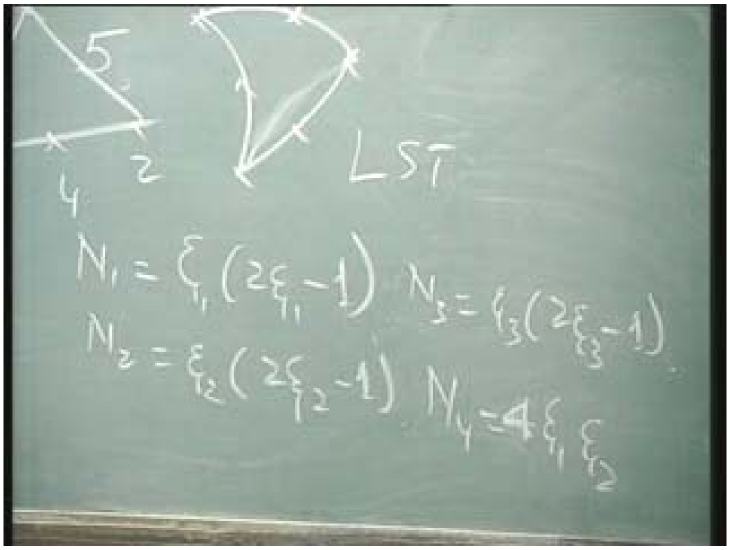
(Refer Slide Time: 18:59)



N_3 is equal to psi_3 into $2psi_3$ minus 1 and N_4 is equal to $4psi_1$ psi_2 and so on. N_5 is similarly $4psi_2$ psi_3 and N_6 is equal to $4psi_3$ psi_1 and so on. What is that I have done? I have essentially put down, what I would call as a natural coordinate system by means of areas and express them either as psi eta or you can keep it as psi_1 psi_2 psi_3. Either way you can substitute, in terms of psi and eta in this as well and written down for a six noded quadrilateral a shape function, which is, sorry, six noded triangle, a shape function which happens to be quadratic. Is that clear, a shape function which is quadratic.

What does this mean in terms of or what is the improvement I have done? What is the improvement I have done here? Pardon? Strain; beautiful. One of my major drawbacks in my previous, say, derivation or our previous discussion, one of the things which we said is that that triangle which I had put down had a problem basically because it was a constant strain triangle. What will happen to this? What will happen to this? Now, it is quadratic. You need not even workout equations; you can have a feel and just tell me what will happen? The displacements are now expressed in terms of psi squared and so on, psi_i squared and so on. So, it will become linear; beautiful.

(Refer Slide Time: 21:18)

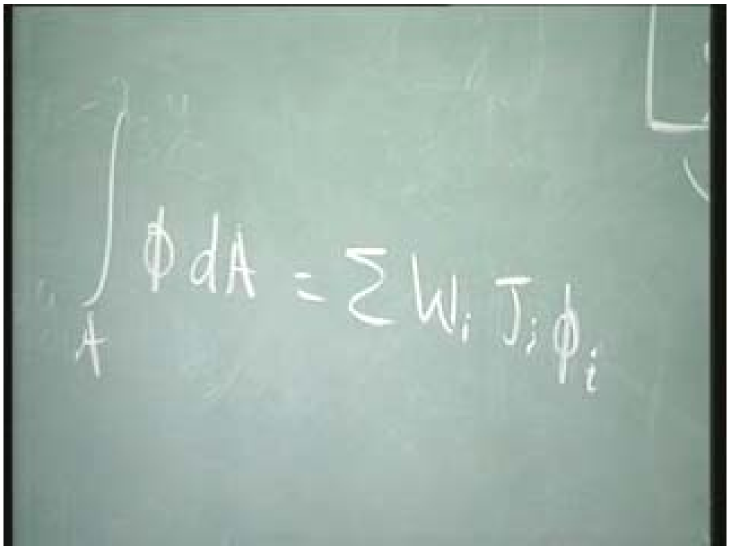


If we call the previous triangle which we had as constant strain triangles, CST as people call it, then this triangle, we call this as a linear strain triangle or LST. Is it clear? What is my next step? What is my next step? Once I have written down shape functions, what is that I am going to do with these shape functions? Yeah, D_N; correct. Why not you try and write down D_N for this? Next step is, because I am moving towards Jacobian, moving towards Jacobian. I have to calculate or I have to write down D_N. By the way what would be the size of D_N? I mean, I am talking about the complete D_N. 6 by, 6 by 6; 6 by 2, you can write down the coordinates as x_1y_1 x_2y_2 x_3y_3 x_4y_4 or if you want to write down as x_1y_1 x_2y_2 completely, either way you can write it down, but you have to be careful as to how you write down. After all Jacobian is given by dow x by dow psi and so on. That is the way you write down for this as well.

We can calculate D_N, put down a D_N and then you can calculate. You can write it down in terms of psi and eta as well, then you can calculate D_N. I am not going to repeat all that, because I am sure all of you know it and once I calculate D_N, then what I am going to do is to calculate J, J inverse and then substitute it into my stiffness matrix expression, from which I can calculate stiffness matrix. Till that point, I leave it to you. I am sure you can all work it out. But, I am going to stop there at that point and look at the stiffness matrix. How am I going to estimate the stiffness matrix?

What is the peculiarity in estimating the stiffness matrix? I cannot use just like that minus 1 to plus 1, minus 1 to plus 1 I have to keep them as area coordinates. The type of integration that I am going to use is different there. It is to be area coordinates. I am not going to again go into the details of it, lot of text books have this information. That is in other words, the exact say Gauss point quadrature and other things that are used, I am not going into the details of this for the triangular element, because it is a big chart. Any standard text books, like for example R.D.Cook, he has listed down what are the Gauss points and the corresponding weightages and so on. I would like you to look at that book for weightages and so on.

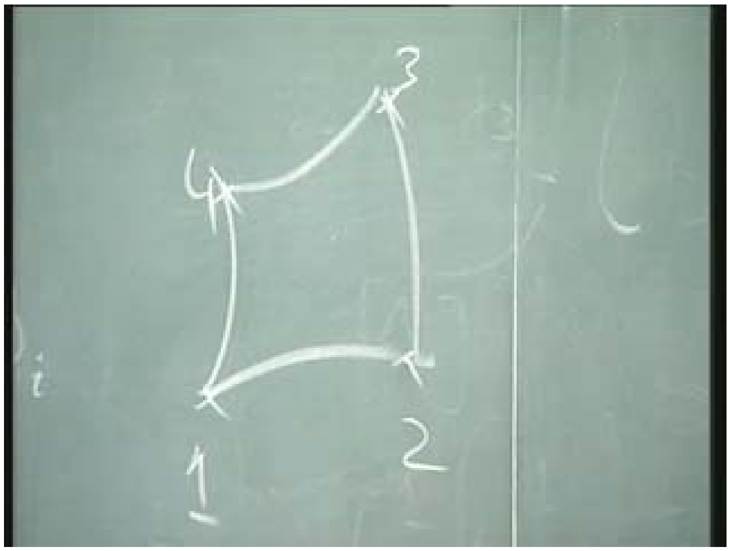
(Refer Slide Time: 25:05)



But, the only thing I want to mention is that if it is a phi dA, then this can be written as sigma W_i J_i phi_i, where the values of the W's, weights and the corresponding positions what are called as the Gauss point positions, the point positions, can be obtained from these tables. They are not just multipli….; it is not 2 by 2 so on. They are, I mean, they are quite involved. I am going to leave that because that is going to consume lot of time to look at them. But, it is anyway available in standard text books or standard even tables that are available for it; you can work it out from that.

In a nut shell, what it really means is that it is possible to express or possible to calculate the stiffness matrices of any type of element through the same procedure. What does that mean? It means that, whether I am going to, what I looked at as a constant, sorry, linear strain triangle or higher order elements, then also it will be possible for me to express or to calculate stiffness matrix in the same fashion. Just as a small exercise, let us now go into what we call as a higher order element for a four sided element. Let us look at this for a four sided element.

(Refer Slide Time: 26:59)

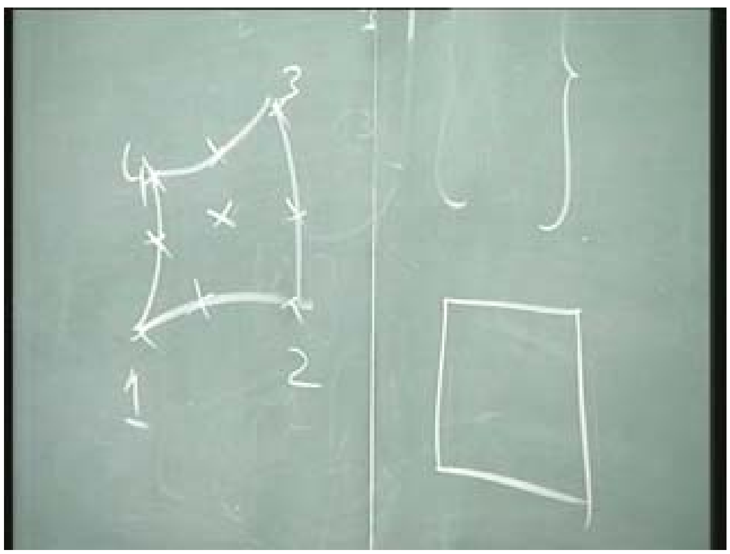


In other words, let us look at an element which looks something like this. Let us now look at a higher order element of this shape. How many nodes will there be? Why 8? I specifically asked this question, basically because all of us have a tendency to say initially, that there are nodes only on the edges. Pardon? Yeah, you are prompting what I am going to teach before hand. But nevertheless, this is 1 2 3 4, which we have, which we know. Why I ask this question though he prompted before hand a type of element that I am going to teach, but what is important or what should have come from you is a different question. What is it? I am going to give you a minute to think about it.

How did we arrive at the shape functions for this element, 4 noded elements? Forget about these curves, just 4 noded elements. Just look at that element and how did we arrive at the shape functions for this? There are two ways in which if you remember, we arrived at that. There are two, in fact three ways. One of the very mathematical way or mathematically correct way, I do not mean others are wrong, but rigorous way which we arrived at is by Lagrange's interpolation. In fact, we interpolated it from a one dimensional case we interpolated it in two dimensions. Remember that, we interpolated it in 1D and 2D. This is for a linear; go back and look at your Lagrange's interpolation. Suppose I want to go to quadratic, next step. How many nodes should be there? How many nodes should be there?

Three nodes. Now, three nodes, if I put three nodes like this on one side, is this complete and can I write down Lagrange's interpolation from this? Because, what did we do? What did we do in the linear case?

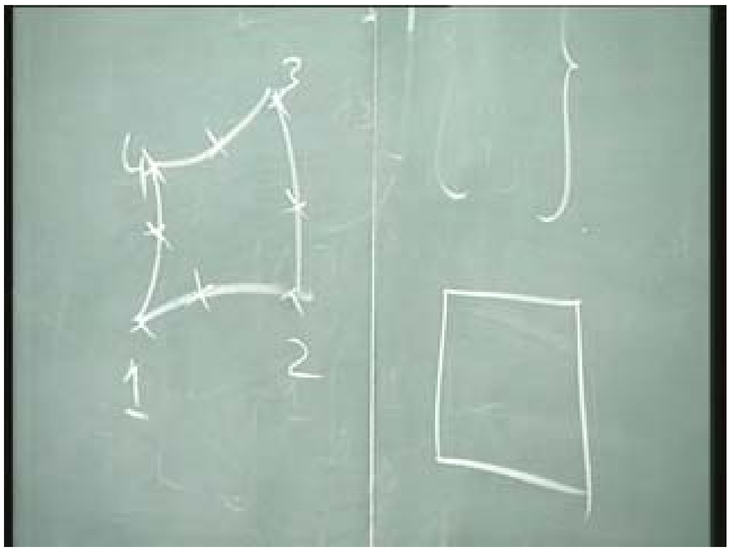
(Refer Slide Time: 29:25)



We wrote, in this case we wrote interpolation in one side, we wrote interpolation in the other side and then we combined them. No, no, no, no, wait a minute, wait a minute. I want, see, I want linear in this direction and linear in this direction. That is how we got in this fashion. Here, if I have to do it, it has to be quadratic throughout if you say from a Lagrangian interpolation point of view, then actually I should have one more node in the center; one more node in the center, then only there will be, you know, quadratic in all the lines that I take.

That is why we call this as a Lagrange element, a 9 noded Lagrange element, whose interpolation function can be determined from Lagrangian formula, Lagrangian interpolation formula. The first thing that you should keep or I mean take away from your mind is that there should be elements only on the side, there can be elements also, sorry, nodes only on the sides of the elements. There can be nodes also inside and such an element becomes Lagrange element. It is a very good element, you know. It behaves very nicely and so on but only thing is cost becomes very expensive when you keep using more nodes and so on. So, there is a variation to Lagrange element, which are called as serendipity elements.

(Refer Slide Time: 31:24)

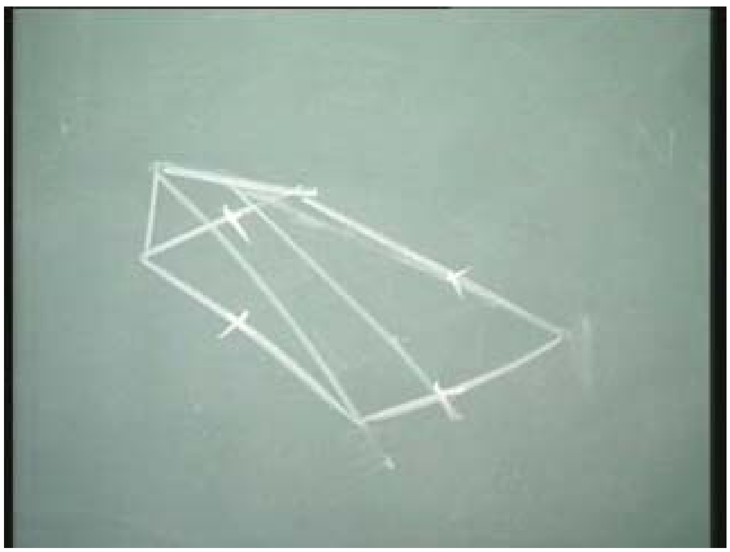


The difference is that I do not have one node at the center. There is no node at the center of a serendipity element and many times the shape function for a serendipity element is written by inspection; by inspection, by intuition. Is it clear? What is the difference between a Lagrange element and a serendipity element? A Lagrange element has a node at the center and its interpolation functions can be determined straight away from the Lagrangian interpolation function in a similar fashion as we did for the one dimensional case. Serendipity elements do not have a node at the center and the shape functions are written by inspection. Is that clear? That is the first thing.

I mean, of course, it is not very difficult to write it. May be for a minute you can see how do you write it, or I will give you a minute to think as to how whether you can give a procedure. First of all can you give me a procedure as to how I can write the shape functions for such an element or in other words, the clue is, can we proceed from the shape functions of the 4 noded elements? Can we proceed from the shape functions of the 4 noded elements? Let me give you, why this serendipity elements become important later, but just look at it and see whether it is possible to proceed from the shape functions of the 4 noded element? Just look at that element, I will give you a second to see how this can be done. I just want a procedure from you.

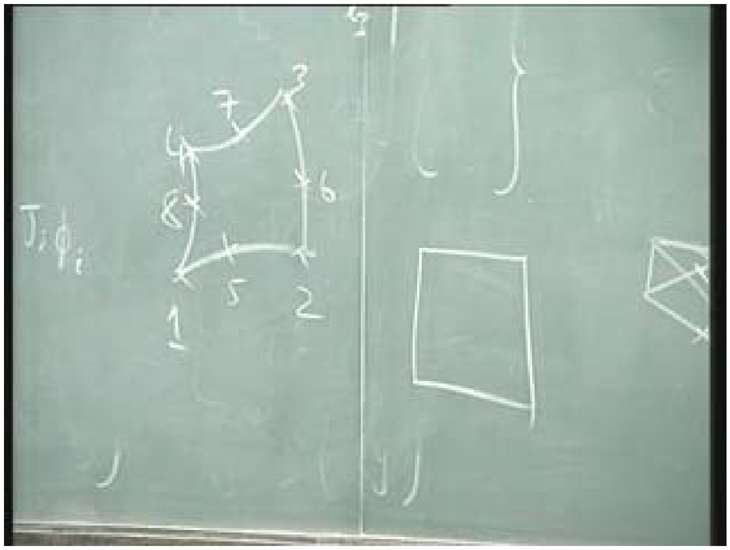
Yeah, the first thing is that let us see how this shape function looks like, looks like for a 4 noded element. See, that can be done something like this.

(Refer Slide Time: 34:05)



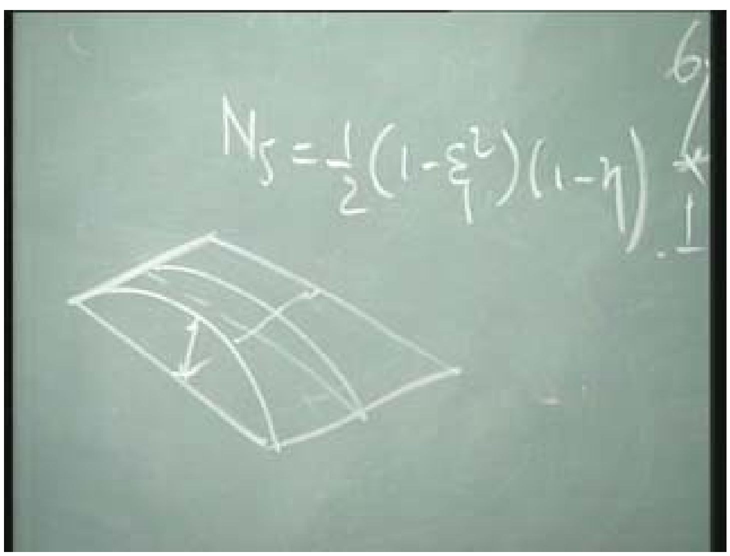
Let us say that that is an element. I just make it lie down and just plot the shape functions. How will it look like? Something like this; this is a, do you understand? It is a N_1 would be a linear surface, such that what is this value? That is this value. 1 and shape functions are zero at all other places. Now, if I want to adopt this, what should I do? There are two things I should do. I have put down nodes at these places. I hope you understand that this is a surface like this, sloping surface. In fact, I think it would be better if I put it like this may be, the picture becomes better, sorry, something like that. What is that I should do now? I should remember that there are nodes at these places and make these shape function go to zero at these places. That is my first thing. How do I do it? Usually how it is done, is something like this.

(Refer Slide Time: 35:51)



I will put down a shape function, say for example, 5 6 7 and 8. Let me call them as 5 6 7 and 8. Let me call them as 5 6 7 and 8. How does my shape function N_5 look like in the same mode?

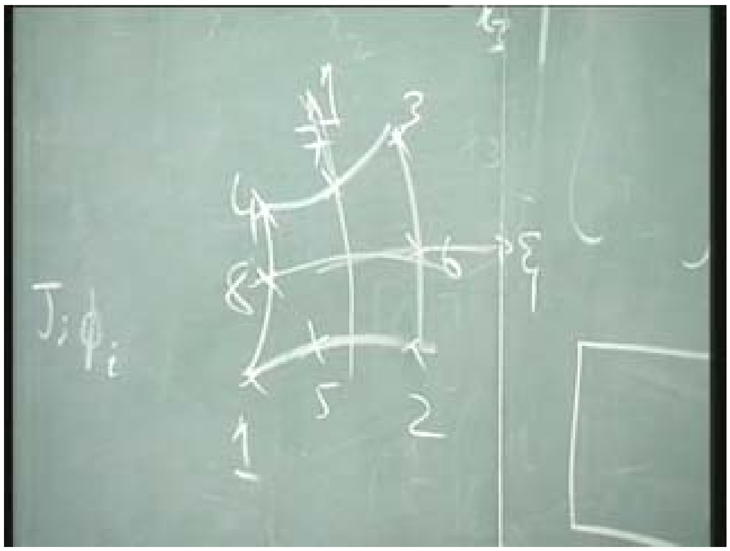
(Refer Slide Time: 36:21)



It looks something like this. That is the one; that is the shape function for this and it would be something like this at the center and goes to zero at other nodes. It is quadratic in one direction for this and linear and goes like that in the other direction or in other words, N_5 can be written as half into 1 minus psi squared into 1 minus eta.

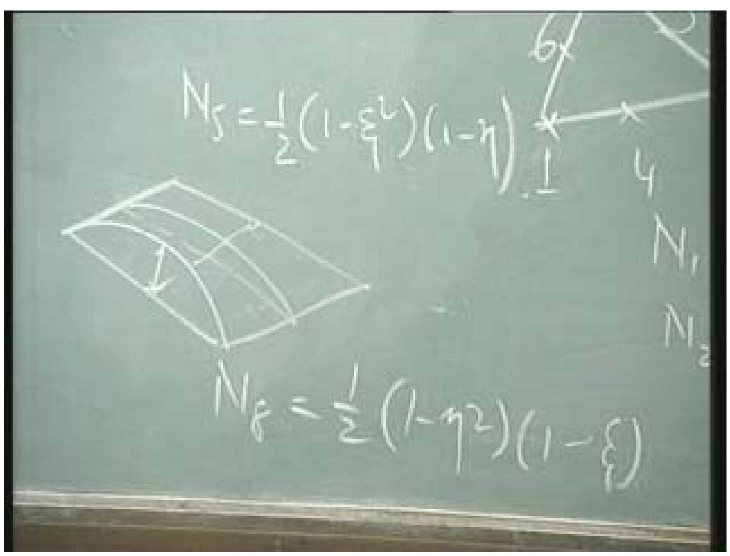
Half into 1 minus psi squared into 1 minus eta is a good candidate for shape function for N_5. Is it clear? When I put down this candidate, just check whether it satisfies all my requirements. N_5, see that they are zeros at all positions. Do they become zeros at all other nodes? Obviously. What is for N_8? What is for N_8? Similar thing for N_8, how do I write down for N_8?

(Refer Slide Time: 38:09)



This is the psi eta coordinate system. I am going to come to that again. I think, I forgot one of the things that I said psi eta coordinate system and how do I write this?

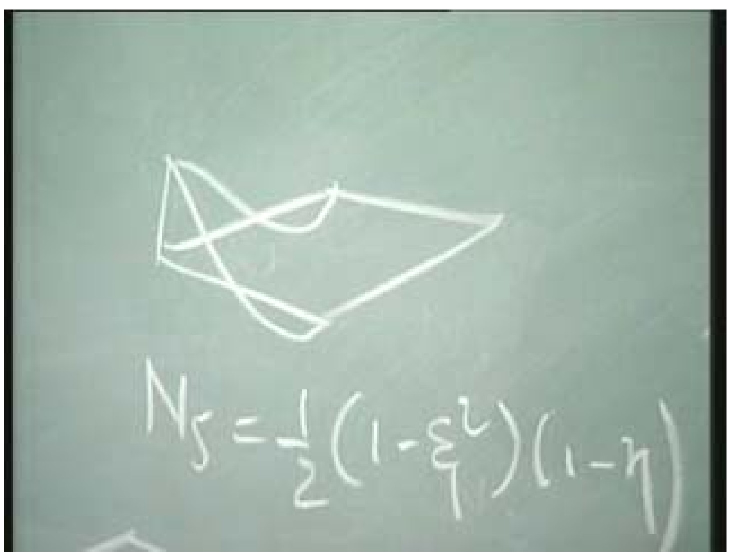
(Refer Slide Time: 38:16)



1 minus eta squared into 1 minus psi; 1 minus psi. Before I proceed, I think that one of the comments which I left out is that please note psi and eta coordinate itself is a curved coordinate system. They are not straight lines, obviously they are curved coordinates, because after all psi and eta we chose as one of the curves in the element and hence psi and eta itself are curved coordinate system. Is that clear? What I did was, yes, you can say how did I write? I wrote it out of a considerable experience; looking at it, saying that this can be quadratic, so 1 minus eta psi squared. So, by lot of things, may be you would have taken one hour to write it, I just like that wrote, because I knew what the thing is. But, this was written purely by inspection, by looking at lot of candidates and saying look, this looks like one candidate which can satisfy whatever I want. N_8 can also be written like this.

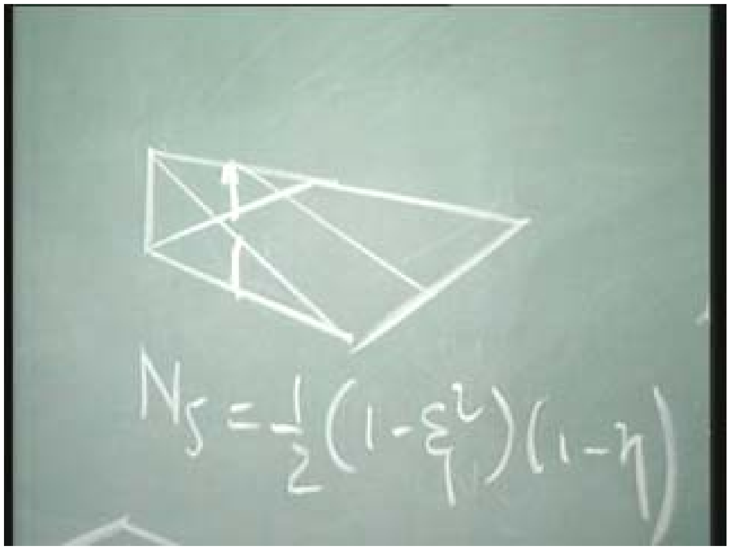
What is that I want to do? I want to now put down one new N_1 in such a fashion that this N_1 can force the values of N's at these nodes here go to zero. Is that clear?

(Refer Slide Time: 39:53)



What does it mean? It means that if I can make it go something like this, the shape go to zero, then, I can get a new N_1.

(Refer Slide Time: 40:10)



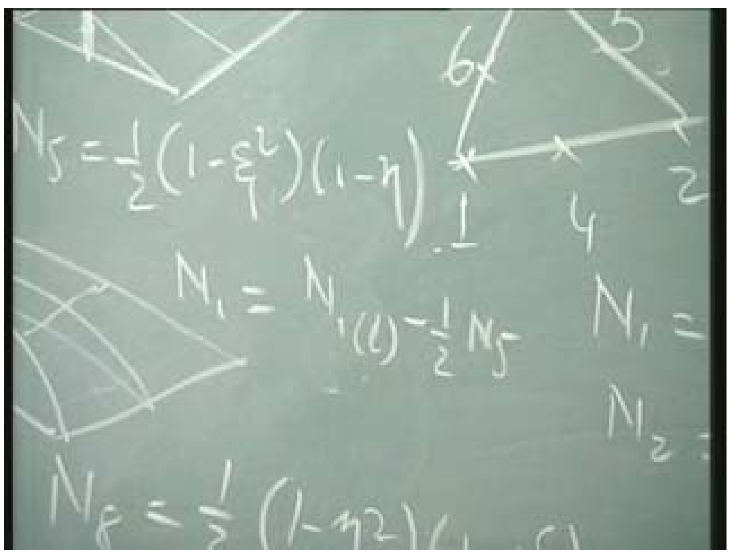
or in other words what it means is, my previous N for a quadrilateral element should go to zero at this point and that point; is that clear, should go to zero at these two points and should not, two things; it should go to zero at these two points and should not now get to non zero value at all other nodes. Wherever those fellows are zero, they should remain the same. I mean, look at that very carefully, I have written down N_5 and N_8 by inspection, by saying that 5 and 8, it should be, you know N should be equal to 1 at 5 and 8 respectively; should go to zero at all other positions. By inspection, by considerable experience, I have written N_5 and N_8 like that.

What I want to now do is to force this N_1. I will tell you, why? What is important later, why we are writing it like this later? I want now this N's to go to zero at these two places and retain zeroes wherever it is going, same level. What does it mean or is there any clue? You understand what I am saying? Is there any clue from what I have done or what I have said? In other words, can you now play around with N_1 N_5 and N_8, because N_5 and N_8 are not zeroes at the places where I want it to go to zero, 5 and 8. Is that clear?

Can I now play with N_5 and N_8, so that I will make this N_1 go to zero at 5 and 8 as well and not make it raise above zero at 2 3 4 and so on. No, no, no, wait, wait, wait; have a look at it. Do not rush, do not rush. Do you understand? Now, I have this much value. I have to make this fellow, multiply it or by something subtract it. How do I do? Is that clear? I have to, I have to make this fellow, this length here, go to zero. The way I can do it is to do something with the shape function. Yeah, how do you or what do I do? So, that is equal to 1 according to this, N_5. Say for example, take only 5, N_5; so, one place we will make it go to zero. What is it that I do? What is the value here? 0.5. $2N_5$ if I put, what will happen? If I put $2N_1$, then what will happen? It is good, you know, I am happy with these answers. If I put $2N_1$, what will happen to N_1 at 1? It will become 2. It will become 2, so, I cannot use $2N_1$.

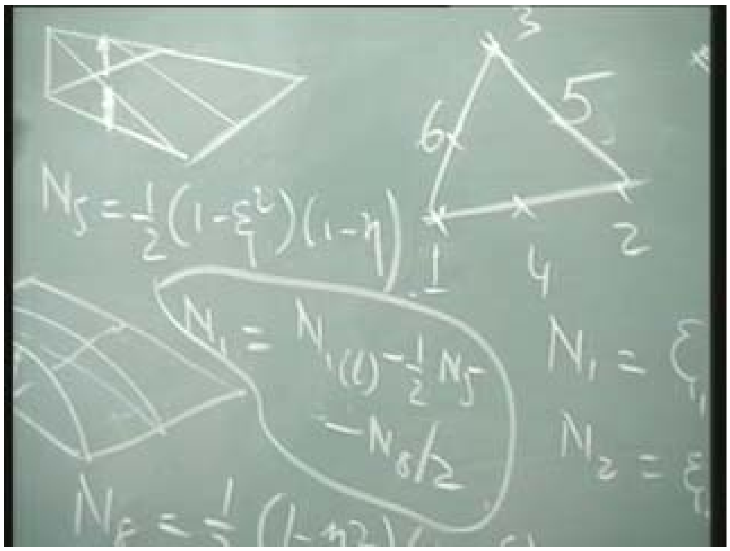
Yeah, come on, someone said that answer. N_5 by, if we put N_5 by 2 alone, correct; that is it.

(Refer Slide Time: 43:57)



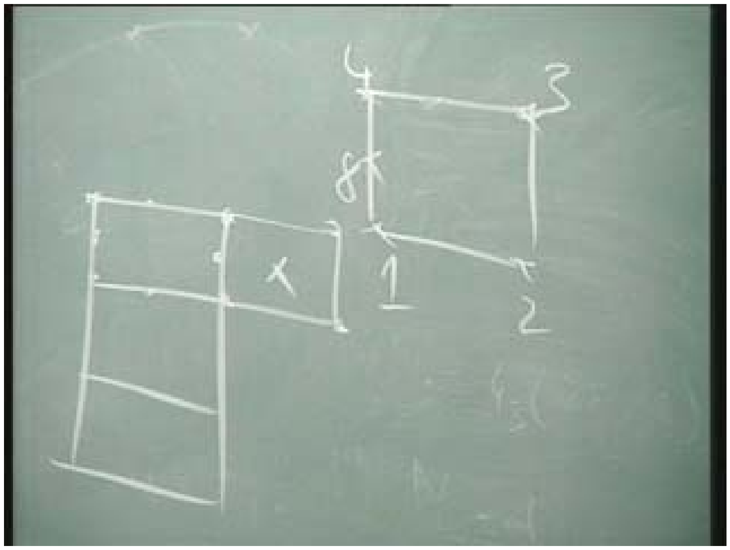
N_1, my linear fellow, minus half N_5. I am spending time, because purely by inspection, by intuition; that is why it is called serendipity element. Serendipity is what? What is serendipity? By accident, by chance; so, this element came about by chance and shape functions are written purely by inspection. Yes, there is one more way of writing it, by our what? By straight lines. We will do that later, because this gives an insight as to how the shape functions look like. That is why I am going through this. Is it complete? Is it complete? You have not completed it. Yeah, but you have made one fellow go to zero. This fellow goes to zero, but you have not touched this fellow. What? What do I do? Because, what happens to N_5 at this position? It is zero. Is it plus? Why, why should I have a plus there? Minus N_8 by 2, period; so minus N_8 by 2.

(Refer Slide Time: 45:24)



N_1 is equal to that linear $N_{1(l)}$ minus N_1, sorry, N_5 by 2 minus N_8 by 2 will be my new shape function. Similarly, look at this. This is quadratic one side and linear on the other side. Similarly, I can write down. I am not going to do it now, but you can say that similarly you can write down for N_2 N_3 and N_4 N_6 and N_7. Is it clear? What is the advantage of writing like this? First of all coding becomes very simple. See, for example you may decide, you may decide to use higher order elements at some places and stick to your linear elements at some other places. So, there will be a transition element. What will be the transition element or how would it look like? How would it look like, a transition element?

(Refer Slide Time: 46:40)



Say, I am going to have 8 noded elements here, all these 8 loaded elements. Then, I want to now go to a linear element in the next step. So, I will have an element like this. Look at this element, look at that element. That element has 5 nodes, 5 nodes; 3 one side, other sides have 2 each; 5 elements. If I develop my shape function like this, by inspection, by writing down N_5, knocking off from N_1 such that making it zero at other positions and so on. What will happen if I have an element like this? Is it first valid or not? Sorry, is it valid? Can you write down now shape function for this? Say for example, 1 2 3 4; 5 is absent, 6 is absent, 7 is absent. So, I have only 8. Can you handle this element? Of course, you can handle it. How do you now write the shape functions for this?

You write down first the shape function for N_8 in the same fashion as we did right now. Half into 1 minus eta squared into 1 minus psi and then what do you do? Knock off from N_1, half N_8, period; knock off from N_1 half N_8 and from what else do you knock off? N_4; N_4 knock them off, so that you will now get a new element itself, 5 noded element. Shape functions can be very nicely written. Is that clear? What we have done essentially right now, is to write shape functions for quadratic elements. I have written down by inspection. If you take some time to sink, my suggestion is go and think about this, this particular thing and now we will extend this concept. By the way, what do I do now with J, I mean, next step? Write D_N, write J B transpose DB T J the psi d eta. Steps are the same; they are no more different. We will comment, is this clear, any question on this? Only thing is, if you know how to write the shape function, period; the whole thing follows. I am not going to emphasize more on this. The only thing, major emphasis that is required is on our numerical integration. There are lots of issues on numerical integration. We will talk about numerical integration, the issues involved in numerical integration in the next class.