Introduction to Finite Element Method
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Lecture - 20

Before we go further, let us summarize what all we did in the last class about isoparametric formulation. Remember that we talked about isoparametric formulation as a very general methodology for a variety of elements; the elements may be plane stress, plane strain, solid elements and so on. What we are going to do is to quickly summarize all the steps that were involved in the last class and then go ahead to evaluate the stiffness matrix as well as the right hand side that is the load. If you remember that about two or three classes or even before, we had derived the stiffness matrix terms. We said that the stiffness matrix can be written in terms of B transpose DB dv.

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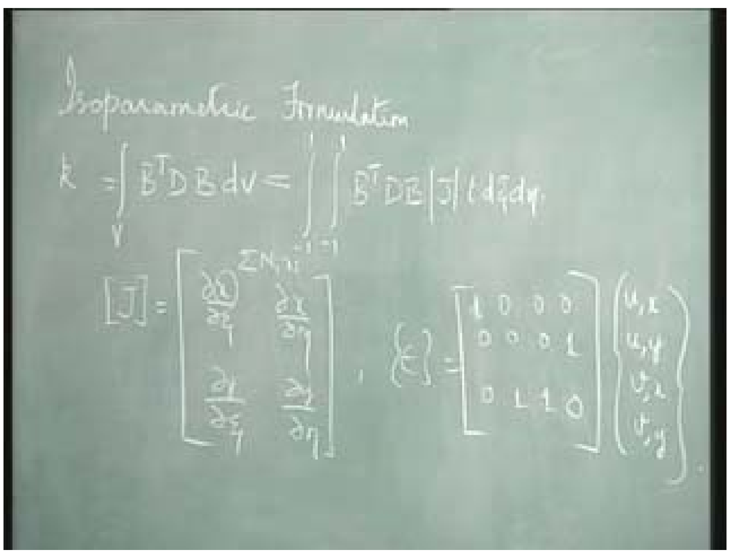

We started, in the last class actually we started, there and we introduced the concept of the natural coordinates and how mapping could be done. Now we transform this equation into an equation of this form, where psi and eta if you remember, we introduced it as natural coordinates. Please note that this J we are talking about, is a Jacobian about which we will see in a minute and that this symbol indicates that it is a determinant of Jacobian. There should not be any confusion. It is not a matrix and it is just a symbol for determinant of the Jacobian. We wrote down Jacobian as dow x by dow psi and so on and then we went ahead to derive the strain matrix. Have a look at the strain matrix. The strain matrix has u comma x u comma y v comma x and v comma y. In order to transform this into a strain matrix, we introduced a 1000 0001 0110 and so on

We defined the Jacobian such that u comma x and u comma y can be related to u comma psi and u comma eta through a Jacobian. Just have a look at that and it will be clear that we introduced the Jacobian to, I think there is a small mistake.

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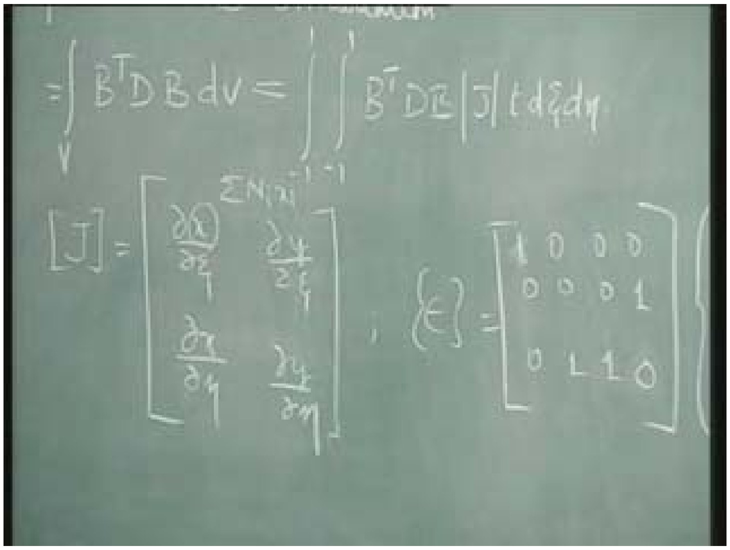

I will just get back here dow x by dow psi dow y by dow psi and sorry about that; I think dow x dow eta and dow y by dow eta. From this, we derive that u comma x and u comma y could be related to u comma psi and u comma eta through an inverse of Jacobian. We did that in the last class and called that as gamma s.

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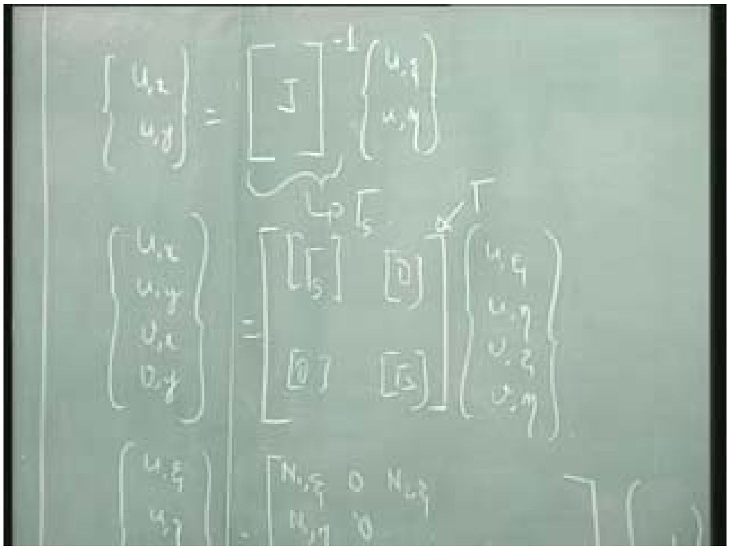

Ultimately, when we stopped in the last class, we said that u comma x u comma y v comma x and v comma y could be related to u comma psi u comma eta v comma psi and v comma eta through a gamma matrix. So, we call this as a gamma matrix. What is the whole idea of doing this? The whole idea of doing this is to replace the dependent of x in the B matrix to the dependent of psi and eta. In other words B, a function of x and y should be replaced in terms of psi and eta. That is what we are trying to do. We have come to a stage now, where I can write out u comma x u comma y v comma x and v comma y in terms of my psi and eta. What is that I am going to do here? At the next stage, I am going to plug that into this particular matrix. You can now see this. So, u comma x, this matrix will be replaced by this gamma matrix into u comma psi u comma eta matrix; this matrix, this vector. How do I replace this?

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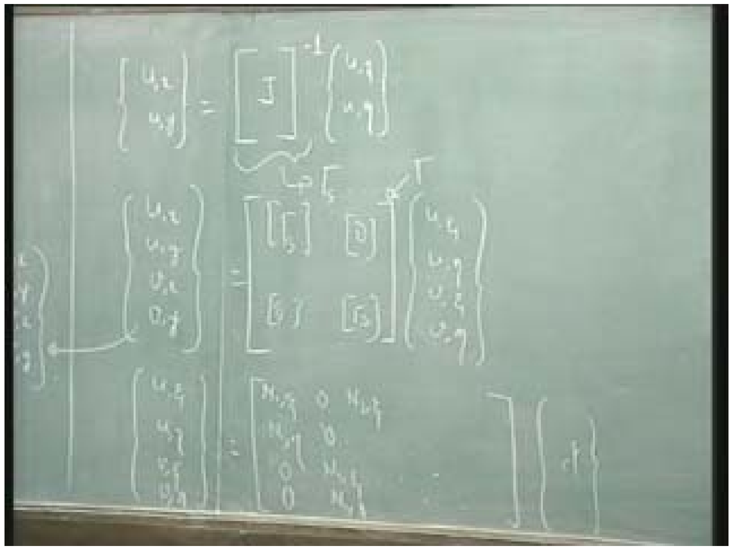

This is now replaced in terms of the N's, because remember u is equal to sigma N_i u_i and so on. u comma psi u comma eta and so on will be replaced by means of N's and D's.

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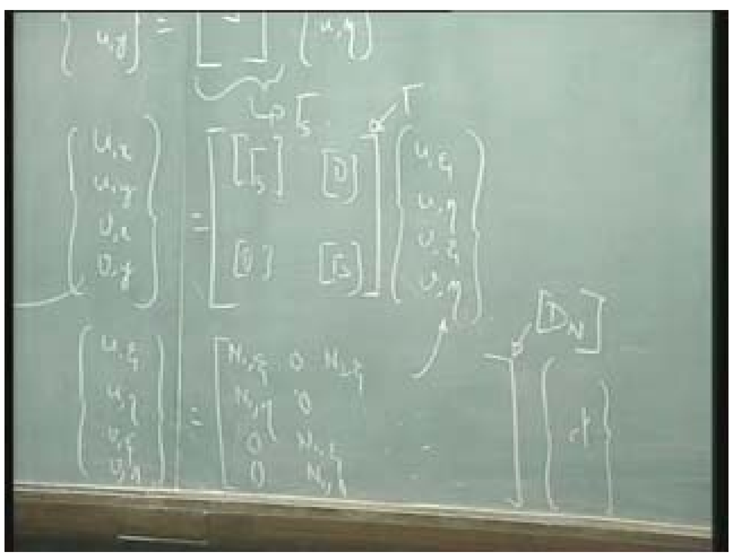

This goes here, so, you will see that let us call this matrix as D_N, part of it we called it as D_N, because we are completing it let us call that as D_N. Then you would see that this is gamma; gamma into D_N into D is equal to this vector, which goes here. That complete thing multiplied by this 1's and zeroes would now give me B.

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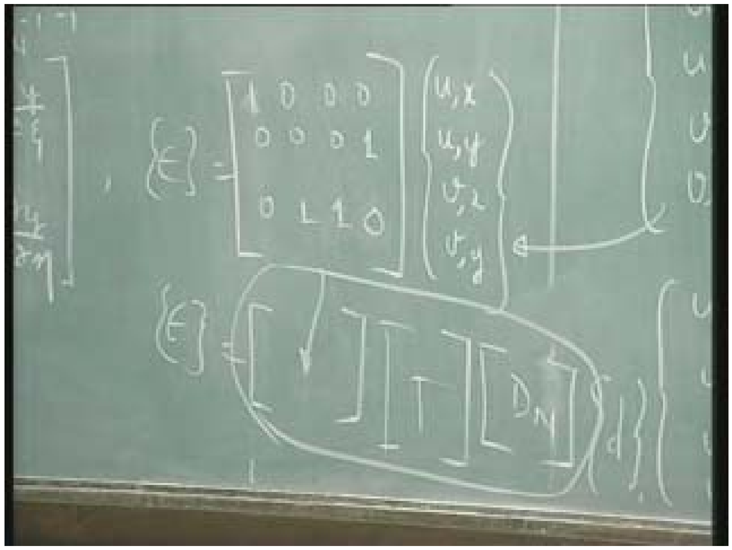

In other words, epsilon is equal to the 1's and zeroes matrix multiplied by this gamma matrix, what we just now saw, multiplied by this D_N matrix multiplied by d. Those are the three matrices that we would have in order to define d. We know that epsilon is expressed as B into D. So, obviously B now becomes, all these terms put together, all these terms put together now becomes B. Please note the difference between the D_N, which I have defined here and the same D_N, name which I have given in the previous class.

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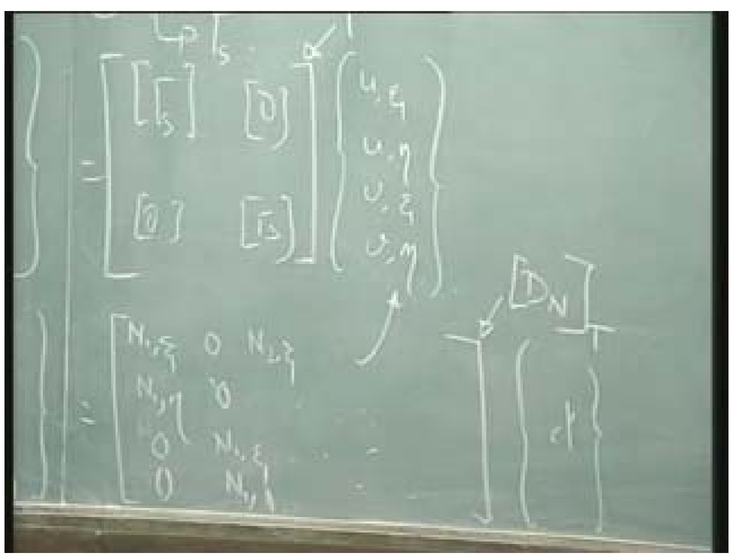

May be if you want to avoid confusion you can call this as D_N total, because the difference is that it has four rows and previous one had two rows. May be, if you are bit confused about it, you can give another name to it, but any way D_N is nothing but dow N's by dow psi and dow eta; that is the matrix.

Having done this, what is the next step? Just substitute this B into my first equation here, first equation here, B transpose DB. What is D? D is nothing but the stress strain matrix; you know about that already. For plane stress, we know what it is? So, we substitute this B into this equation. We know all the terms here.

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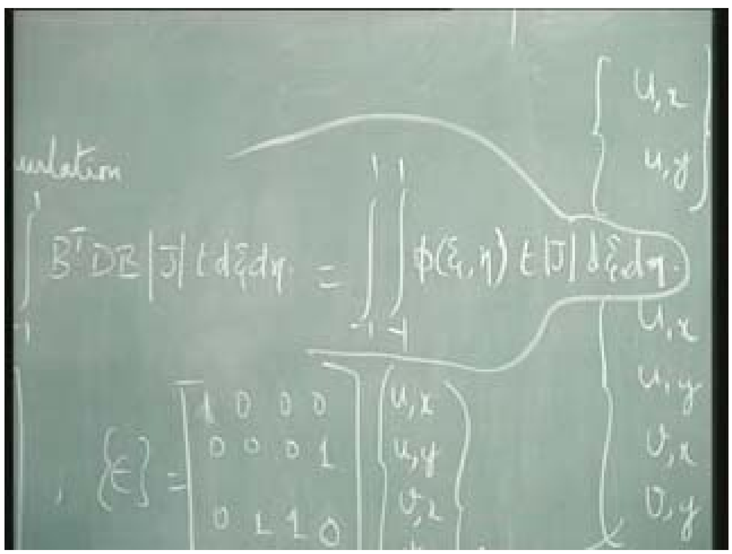

In other words, we can write down that k as phi say psi comma eta t J d psi sorry d eta. We can write down k like that. Our next step, our next step is to evaluate this particular integral. This is a very happy situation, because the integral was surface integral, volume integral which was bit difficult for you to derive, especially when the shape is quite bad. What is that we have done? We have replaced that by means of psi eta coordinate system and the coordinate varies now only from minus 1 to plus 1. What is the advantage now? The advantage is that we can resort to what is called as numerical integration. We are going to see what numerical integration means. I hope things are clear now. We are now in a position to calculate stiffness matrix.

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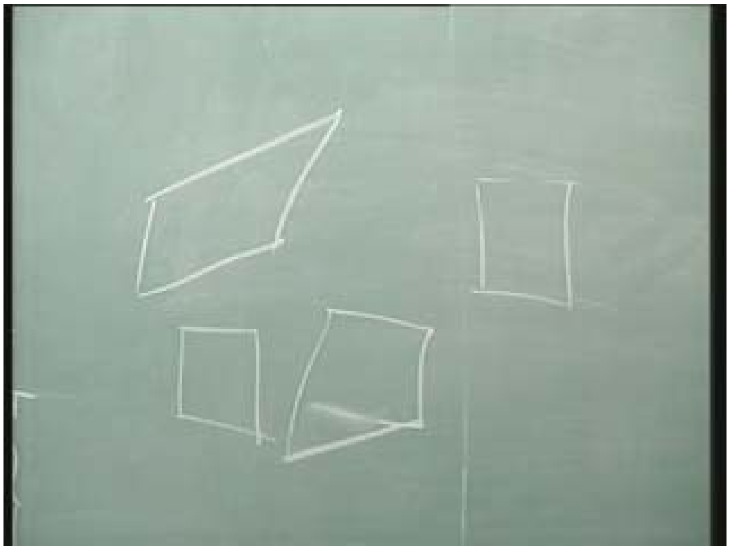

Please note again, a reminder that whatever be the shape, whatever be the quadrilateral, we had just derived it for quadrilateral, whatever be the quadrilateral, even whether the quadrilateral is regular or whether it is sorry, irregular, whatever be the shape, the procedure is the same, because we map all of them into a lovely square in the psi eta coordinate system. The procedure becomes quite well established and it is very easy now to write down computer program. Let us look at quickly summarize what we are going to do with respect to this numerical integration. What is numerical integration?

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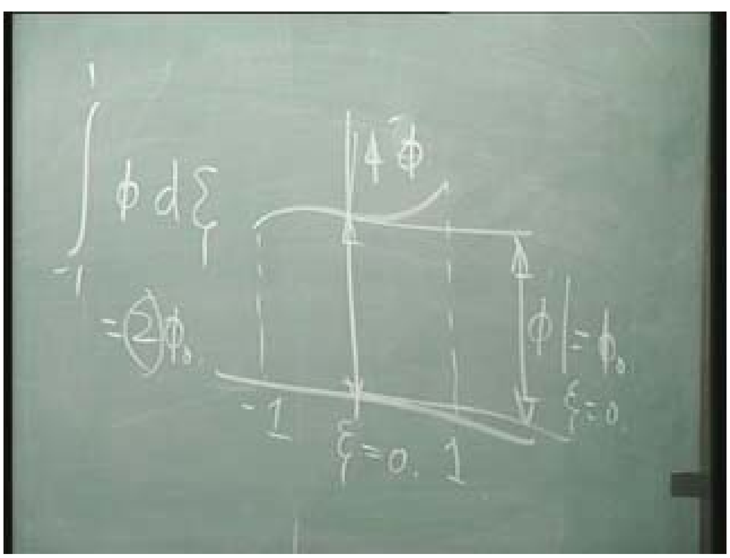

Let us look at an equation of the form, say integral minus 1 to plus 1 phi d psi. We can evaluate this by different schemes, numerical integration schemes. What are these schemes? These schemes are called as Gauss quadrature scheme and then Simpson's rule, for example is one of the numerical integration schemes and so on. Usually in finite element analysis, it is the Gauss quadrature rule that is extensively followed. There are other rules like Newton-Cotes and so on. Thus Gauss quadrature rule is what is usually followed.

What does this rule say? The rule states that if you evaluate phi, what is phi? Phi is now a function of psi. If you now evaluate phi, which is a function of psi at certain designated positions and multiply it by weight and sum them up and sum them up then that result whatever you have, would be very accurate depending upon how many positions of this designated places you include. Let us look at the simplest one dimensional case. Let us say that we are trying to evaluate that kind of curve. Let us say that we trying to integrate this between minus 1 to plus 1.

In a one point quadrature rule, the position at which we are going to evaluate this is given by that position where psi is equal to zero. Obviously, this is our I and that is the psi; sorry that is the phi and that is the psi. The designated position here is psi is equal to zero and what is it that you get here and yeah, that is nothing but phi at psi is equal to zero. Let us call this as say phi is equal to zero. If I now I ask what is the integral value of this curve by evaluating that function phi at a position called psi is equal to zero, your immediate answer would be that look that is only the area under the curve and so you can write this down as 2 phi zero. In other words what does it mean? It means that this two is the, what is called as the weight function. This is one point quadrature rule. You can have more than one points as the quadrature rule and let us see what the other rules say.

Now, let us look at the two-point quadrature rule.

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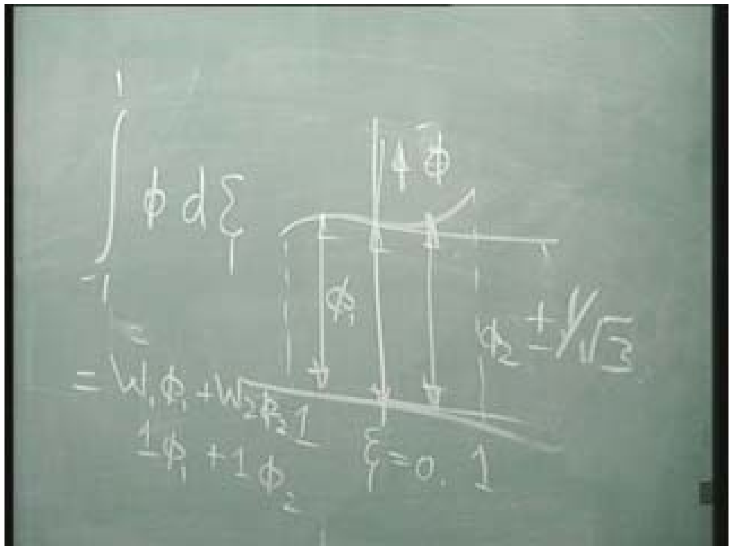

Two-point quadrature rule says that you evaluate the same function at two positions, two positions; no more at this centre, no more at this position, I said two positions. These two positions are given by plus or minus 1 by root 3. So, you evaluate this function at two positions. What do you mean by evaluate the function? Phi is a function of psi. What is that you do? You substitute the value of psi to be plus 1 by root 3 and then again minus 1 by root 3. So, you will get at two positions, the value of phi. Let us call the positions as say phi_1 and phi_2.

The numerical integration rules states that this integral can be written as W_1phi_1 plus W_2phi_2, where W_1 and W_2 are the weights and they happen to be 1 in this particular problem or in this particular rule. So, this is called as two point quadrature rule. You have also three point quadrature rules.

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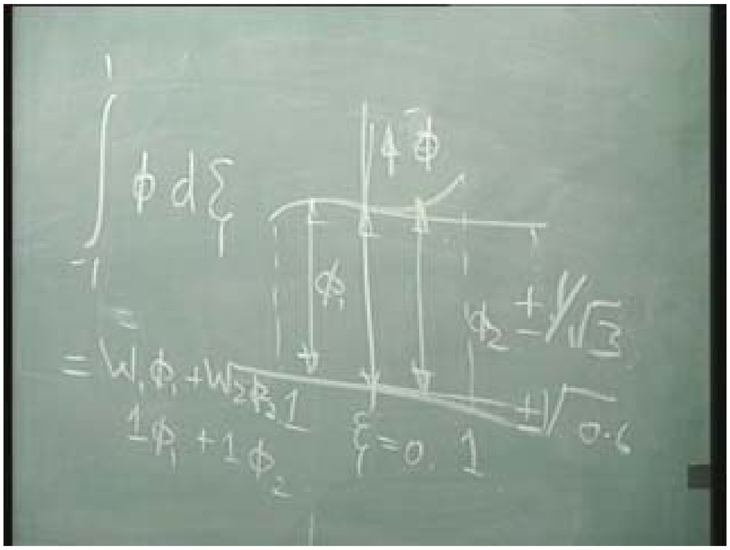

The three points are spread at three different locations and again these locations are given in terms of psi and eta. The locations happen to be zero, plus and minus root of 0.6 plus and minus of root of 0.6.

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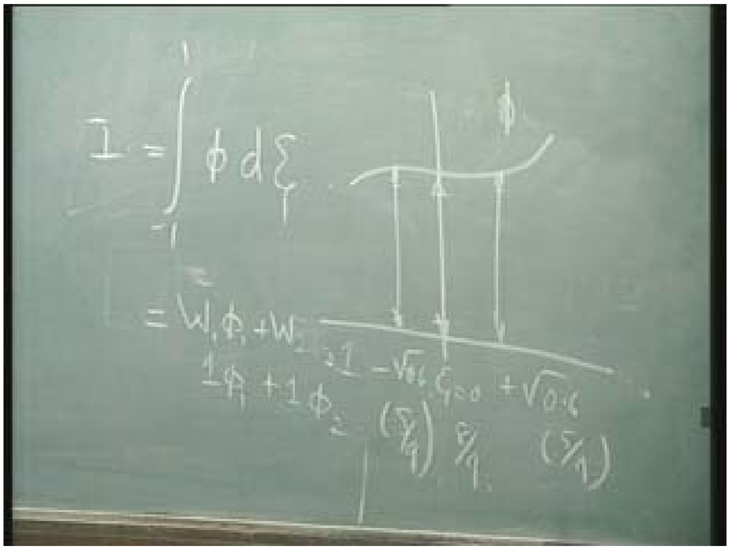

In other words, the three point integration rule states that, one point here, one point here and one point here. These are the three points at which the value of phi is evaluated and the value of phi at psi is equal to zero plus root of 0.6 minus root of 0.6. What are the weights here? The weight happens to be 5 by 9 for this position, 8 by 9 this position and of course 5 by 9 for this position. In other words, now if I use three point quadrature rule, then this, say this integral say I, now becomes W_1 which is say 5 by 9 into phi_1. Phi_1 is the value of phi at minus root of 0.6 plus 8 by 9 into phi_2. Phi_2 is the value of phi at psi is equal to zero plus 5 by 9 into 5 by 9 into phi evaluated at root of 0.6. W_1 phi_1 plus W_2 phi_2 plus W_3 phi_3 is what we get as a three point quadrature rule. You can keep on extending. In fact there are tables that are available. I am not going into the details of it; you can have four point quadrature rule and so on. I am not going to go into the details of it. Nevertheless, it is important that the accuracy of it is well understood in the sense that whenever you have root of 0.6 and you do not want to calculate it or you want to calculate and put it, you do not want the computer to calculate it, then this root of 0.6 should be given to very large number of significant digits, because that will affect our results.

The question now is what is the quadrature rule that you are going to use or you should use? Before we come to that, let us now extend our concept to 2D aspect, because my K matrix, which I had and which I transformed is now in 2D; may be we have to extend to it 3D as well. Let us look at 2D, because if you understand 2D, how to do 2D, then 3D extension is quite simple. In other words, what is that we are going to look at?

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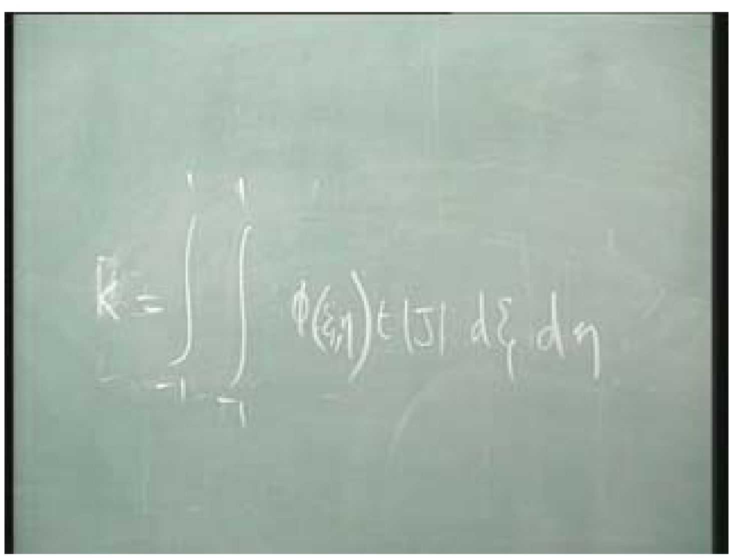

Instead of what i had instead so Now, K is minus 1 to plus 1, minus 1 to plus 1, some phi t J d psi d eta and phi is a function of psi comma eta. What is phi, psi comma eta is always confusion for students. Phi psi comma eta is nothing but B transpose DB calculated at psi and eta.

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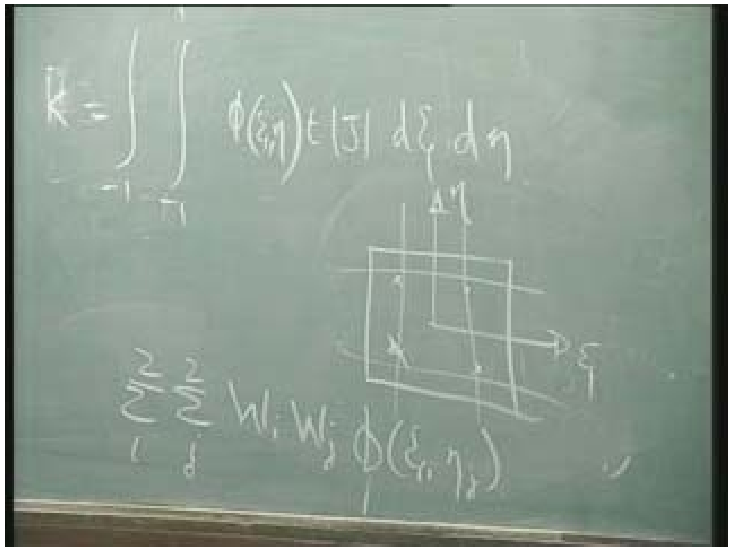

Let us say that that is the element after the transformation into the natural coordinate system and so on. What is the rule that we are going to use? Say for example, we use a 2 by 2 rule. We use two integration point for one-dimensional case. Now, we are going to use 2 by 2 rule, which means that I will have totally 4 points. How do I get four points? By extending that psi plus or minus 1 by root 3 and eta plus or minus 1 by root 3. In other words that, this line, this line and this line are such that, this is equal to plus 1 by root 3 minus 1 by root 3 and minus 1 by root 3 and so on. In other words, the coordinates of this is minus 1 by root 3, minus 1 by root 3, 1 by root 3 minus 1 by root 3, 1 by root 3 1 by root 3 and so on, so, I get 4 points. Then how do I now evaluate this stiffness matrix? That is evaluated as two sigma's; two sigma's ij both of them vary from 1 to 2. Phi calculated at these positions, four positions, let me call at phi i j w i. Two sigma's, let me call W_i W_j and phi calculated at psi_i eta_j, i and j i and j varies from 1 to 2.

I will have now have 4 points or in other words, W_1 into W_1 phi into, phi calculated at not into, phi calculated at psi_1 eta_1, then W_1 W_2 phi calculated at psi_1 eta_2; what is psi_1

eta$_1$ and so on. These are points; that is psi$_1$ eta$_1$ which is minus 1 by root 3, minus 1 by root 3. What is psi$_1$ eta$_2$? That is this point and so on. You calculated these four points, multiplied by the weights then or rather this point and so on. You get or in other words, to calculate the psis at all the 4 points and the weights happen to be for this problem 1 and so W$_i$ into W$_j$ into phis and these four points give you the value of K. In other words, what is that we do? We calculate K at each of these points and then sum them up. That is what we mean by stiffness matrix. Is that clear? May be you can have a look at this whole thing again, what all we did, so this stiffness matrix calculation becomes quite nice and easy.

There are two other things that we have to notice or we have to do. That is quite involved. Let us see what are the other things that we have to be involved? I hope that I have followed all the symbols what we did in the last class. Now, what are the other things that we have to do? We have to calculate the right hand side. In the right hand side, we have basically two types of loads, apart from the concentrated load; the body force and the surface force. Body force is not a problem, because the procedure is exactly similar, but surface forces creates certain troubles. What is the trouble that we are going to have with the surface forces?

Let us look at this element.

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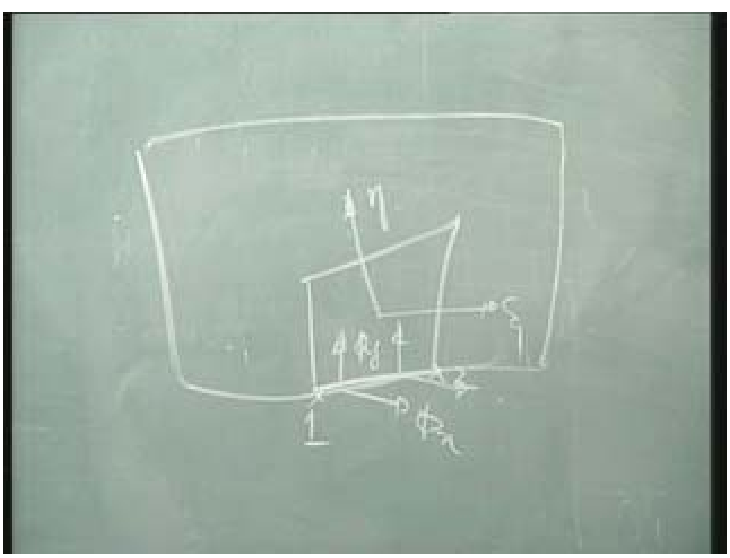

Let us say that that is the psi eta coordinate system. Let us say that this element sits on the boundary of a component, say for example, that is the boundary of the component and so on. In that component or in that element, we have defined a surface force or surface traction, pressure whatever we call it. Let us say that we have given a pressure of say phi_x and phi_y in this particular surface. What is it that I have to do? I have to now calculate the right hand side.

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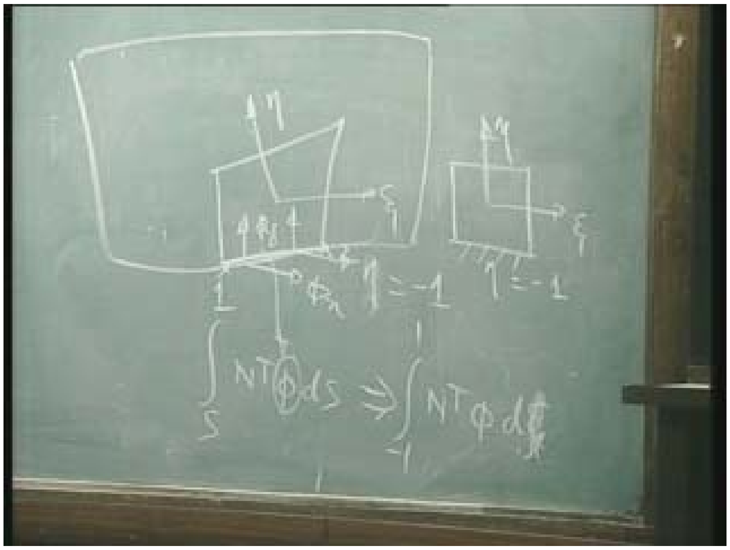

What is the right hand side? If you remember, the right hand side is N transpose phi ds at s. In other words, we have to evaluate a surface integral which is N transpose phi ds. This phi is given as phi_x and phi_y. We know what is N? We have been through it for quite some time and hence we know what N is. The whole idea now, is to calculate what N transpose phi ds is. How are we going to calculate? Because, we have already we are into what are called as natural coordinate system, we have already transformed the element into a natural coordinate system and hence my job is to now transform this equation as well to the natural coordinate system.

Or in other words, my first job is to recognize that the phase in which I have given the load happens to be what? Eta is equal to minus 1 in the natural coordinate system. In other words, symbolically writing it, if I write this as the natural coordinate system, no, sorry; psi eta eta is equal to minus 1 and hence eta is equal to minus 1. That is the phase in which I have given, I have given the load or in other words here is the phase in which I have mentioned phi$_x$ and phi$_y$. Now, how am I going to evaluate this? In other words, my equation now actually becomes, I am sorry, evaluating it as d psi; this implies I should not say equivalent. But there are lots of things I have to do in order to get to this.

What are the things that I have left out? I have left out quite a few things. They are implied; no, they are not equal, because I have left out certain things deliberately. What are the things that I have left out? I cannot straight away convert it like this. Though actually I want to have equation of this sort, I cannot straight away do it. First of all, let me convert this surface into line. Why is that we have to convert the surface into line, because we are now talking only about a two dimensional situation, a plane stress example.

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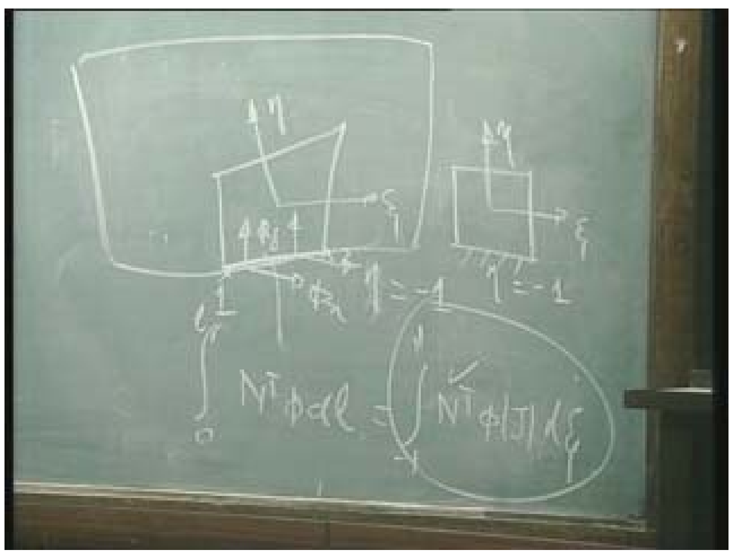

My first step is not to write like this, though ultimately I want to do it. First step is say that look integrate it say along the length of the element. Let me call this as zero to l N transpose phi dl. That is my first step. What is N transpose phi dl? How do I convert it? In other words, I can do that as minus 1 1 N transpose phi J into d psi. That is exactly how I do it. What is J? Determinant of Jacobian or the Jacobian; in this case, what is J and how do I calculate N? Usually these are questions which are confusing to the students, the answer is very simple.

How do I calculate N now, because you may wonder N had psi and eta. How do I calculate N now for one dimensional case, from my previous N's which I have? How do I calculate that? The answer is very simple. In the previous N's, what are the N's you had? 1 by 4 into 1 minus psi into 1 minus eta plus that is for N_1; N_2 you had 1 by 4 into 1 plus psi 1 minus eta and so on. In those N's just substitute eta is equal to minus 1. In other words, what we mean is this N or N transpose is calculated at eta equal to minus 1. So, that problem is solved.

When I substitute eta is equal to minus 1 by N, purely become a function of psi. No problem, so, straight away I can calculate. Phi is a value, phi_x and phi_y that is in other words, the x and y direction they are given. That is not an issue, because let us for the moment assume that they are constants or else we may have to do something else. Let us not confuse the issue. Let us say that they are constants that are acting in the x and the y direction or else we have to interpolate them as well. Let us not worry about that or in other words phi_x is constant through out this phase, phi_y is constant through out that phase. They are just written there in terms of numbers.

The first term is known. Second term is a vector, it has two entries or two rows, one column and next is J. Once I do that, once I calculate the J, my calculation of this integral becomes very simple, because I can use numerical integration. Question is, now what is that J? Let us take a pause and see how to calculate that J matrix? Yeah, so have a look at this and see for a minute as to how we can now calculate J? In other words, we have to very carefully define what J is, the Jacobian. After all what is that we have done here in this equation? We have replaced dl by J d psi. So, what should be J? J should be dl by d psi.

We have to find out now dl and write it in terms of d psi. That is all we have to do. Once we do that, in other words, once we calculate dl, write down dl, then it is very simple for us to calculate J. What is dl or how do you calculate dl, is the question.

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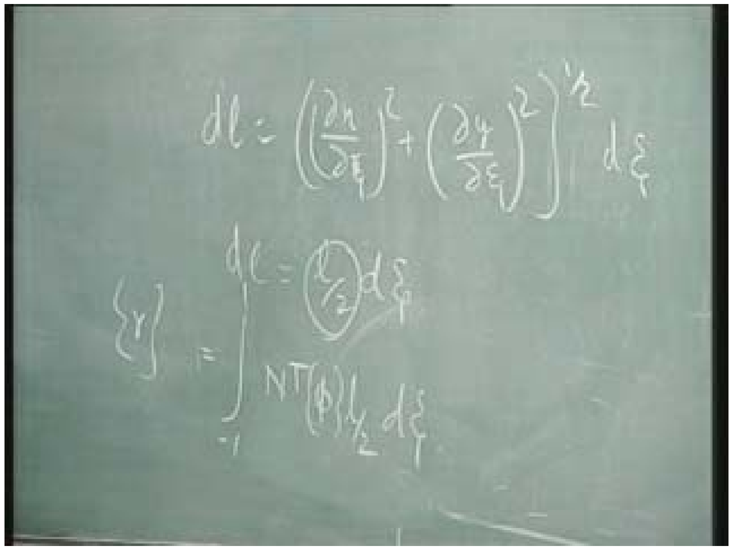

dl can be written as, let us look at that figure. Have a look at this figure and see how we can write down. Have a look that figure and see how we can write down dl. We can write down dl in terms of dx by d psi whole squared or dow x by dow psi whole squared plus dow y by dow psi whole squared whole power half d psi. How do I get dow x by dow psi and dow y by dow psi? That is very simple because, I know that this can be written in terms of N's.

When I now substitute dow x by dow psi and dow y by dow psi, where x is equal to N_i x_i and then simplify this, I get that dl is equal to l by 2 d psi. I am not going to do that exercise; that is very straight forward, just substitute it and you will see that dl and d psi are related by l by 2 and now what is this l by 2? l by 2 is nothing but the Jacobian. With that result, we can write down our previous expression for N transpose phi J d psi as minus 1 to plus 1 N transpose, phi remains the same; now, J becomes l by 2 and we get d psi.

What is l by 2? What is l? l is the length of that element, length of this particular edge we are talking about. So that is the l and hence this expression is useful to determine r_e, or due to pressure. Please note that phi is actually a vector, phi_x and phi_y. Correspondingly this is also a vector and it has two entries. I need not tell you what N has? We can very easily find out, because I have already given you a clue that this N has to be calculated with eta is equal to minus 1. In other words, what is essentially or what is the crux of our approach?

Our approach has been that we not only calculated the stiffness matrix by transforming it to a natural coordinate system and then resorting to what we call as the numerical integration.

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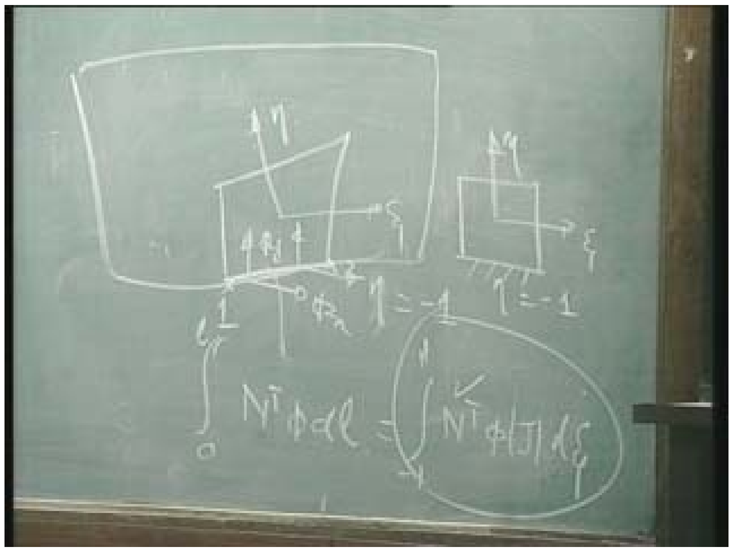

We also calculated or we can also calculate the other r_e's or the r's or the right hand side, to be more precise or in other words the load due to the surface traction or pressure can also be calculated by means of this natural coordinate system by transforming the equation. What is this equation, how did we get? Please note this equation we got it from our virtual work principle. By transforming the equation which we got from the virtual work principle on to say the natural systems or natural coordinate system, where psi varies from minus 1 to plus 1. That in a nut shell, shows that all the aspects whether it is the stiffness matrix or whether it is the force terms, all these things can be calculated in a very consistent fashion. There is always a tendency to replace this phi by means of dividing that or multiplying this by total length and then dividing it by 2 and putting it at 1 and 2.

One of the things we should notice, there are two things I should not say one; two things that you have to notice. One is that, you should always have a look at how phi is defined. Please note that in this particular definition we have assumed that phi is defined per unit length.

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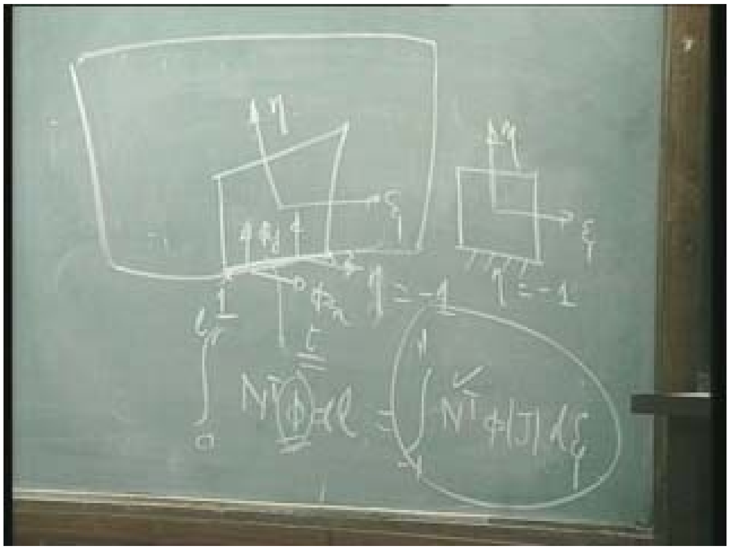

If phi is not defined per unit length, then you will have to now look at t, thickness as well so that, you convert it into unit length. Why is that we are emphasizing on unit length is because we are now in a plane. So, it is important that you look at how phi is defined. If phi is defined anywhere else, then your units on the left hand side and the right hand side should be consistent. Say for example, we have defined a t in the left hand side for K. Then you should also have a t, if it so happens that you have defined phi as well as t. In that case, you have to multiply this by t as well. Please note another

This is a very simple thing that people make a mistake; please note how phi is defined, so that the units what has been given there is consistent and thickness is taken care of properly by the units that are given for phi. That is a very important thing and the other one is that this is only an example for eta is equal to minus 1. It so happens that the same element may be in the other end, at the other side of the boundary and so that we may have to apply pressure not at eta is equal to minus 1, but say psi is equal to 1 and so on. You have to be properly accounting for the surface where we give this pressure. What is the surface that we are interested in, where is that we are giving pressure and accordingly you have to do this derivation.

Thirdly if the pressure does not happen to be constant, if it varies, say varies, from one end to the other, then you have to introduce linear interpolation even for phi's as well. That makes the problem slightly more complicated, but not difficult. It is very straightforward. Only thing is that in that case, you have to now introduce interpolation for the phi's as well. There is always a tendency to lump these loads. We will see later in the course that lumping may be very dangerous; it may not be consistent with the result we have got.

In fact these loads which we have calculated by such a very rigorous procedure of integration are called as consistent and nodal forces. They are consistent, they are consistent with our other definitions. So, they are called consistent nodal forces. You have to be very careful not to miss out this and just to divide say for example length is equal some phi and if the load say phi_y acting is say 5 per unit length, so you cannot multiply 5 into 5 which is 25 and then divide it by 2 is 12.5, 12.5 and so on. Yes, it will work for linear elements, but for other higher order elements this kind of procedure, will not work. Even when you write a program, even if it works for some case, it does not mean that you should not do this integration. You should go ahead and do this kind of integration and if you want to write a code, then this is what should be implemented and that is very important.

A very detailed account of how to write the code for this has already been given by Owen; I think Hinton and Owen, in a book called Finite Element Programming, where you will see that a very detailed exposition of how this code can be written is given. Anyway, for our course it may not be right now necessary to go into the details of how programming is done, but nevertheless, it is nice to know that these things are important.

What is the second issue? We have now looked at what is called as a quadrilateral element. What are the other types of elements which are very popular and which are important?

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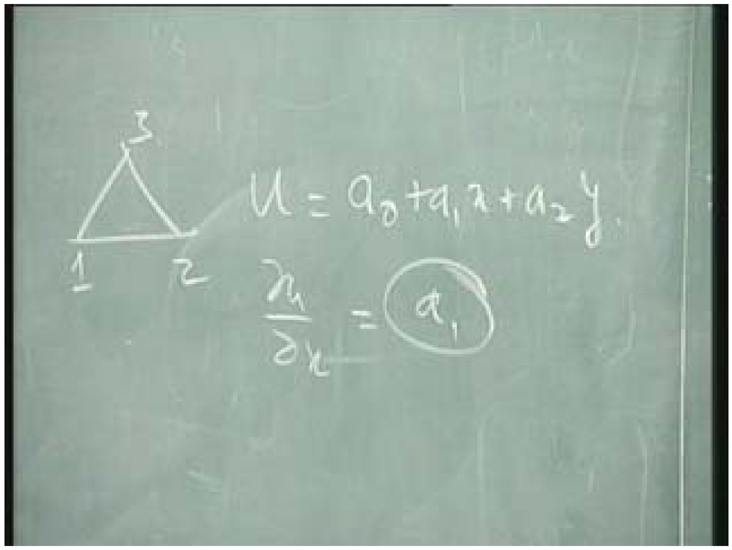

Triangular elements; triangular elements become very important. Why is that triangular elements are important? Because, most of the mesh generation codes or mesh generation packages have triangular or tetrahedron …. three dimensional counter part as a mesh generation facility. Today there are lot of mesh generators, which can do quadrilateral elements. So, there is no problem, but hardly there are any mesh generators which can do what we call as a hexahedron element and hence it becomes important that we know how to handle, how to handle triangular elements.

Basically there are two ways in which triangular elements stiffness matrix are determined. But, by the way, we already noted one thing important about triangular element. If you remember that, we said that triangular elements are constant strain triangular elements or in other words we said that the strain happens to be a constant in a triangle. We illustrated this with a very simple example. We said that, what is that we said? We said that let us say that we call these three nodes as 1 2 and 3. What did we say? We said displacement can be a_0 plus a_1x and a_2y can be written like this.

We said, if we want to calculate dow u by dow x, what is that we get? a_1. What does it mean? It means that, when we express the displacement like that as an interpolated value, then dow u by dow x becomes a constant. It does not matter whether we replace these a’s by the degree of freedoms at the three nodes 1 2 and 3; does not matter, because the basis is only a linear interpolation formula like that and hence dow u by dow x becomes a_1 and so on. So, the strain within that element becomes constant.

What is the trouble with this? We have already seen this with an example. We saw that because of this, we have to have lot of elements, especially when there is bending that is involved. There are lots of elements that we have to use in order that the bending exercise or when an element or when the component bends, bending is properly taken care of. I am reemphasizing it, basically because I have found lot of mistakes during modeling. Why is that there are lot of mistakes? Because it is very easy to do what we call as free meshing. Take a three dimensional component, mesh it with a tetrahedron and just go ahead and finish the problem. But, if you do not do proper mesh sensitivity, as we did on the other day, then you are going to be in trouble.

With all these drawbacks, still we have to learn about triangular element and it is three dimensional generalization which is the tetrahedron element. We will concentrate only the triangular element, because what all we are talking about in the triangular element is very similar to the tetrahedron elements. There are two procedures, the two ways in which you can calculate the stiffness matrix. One, procedure number one, is to collapse two nodes in a quadrilateral element. What do we mean by that?

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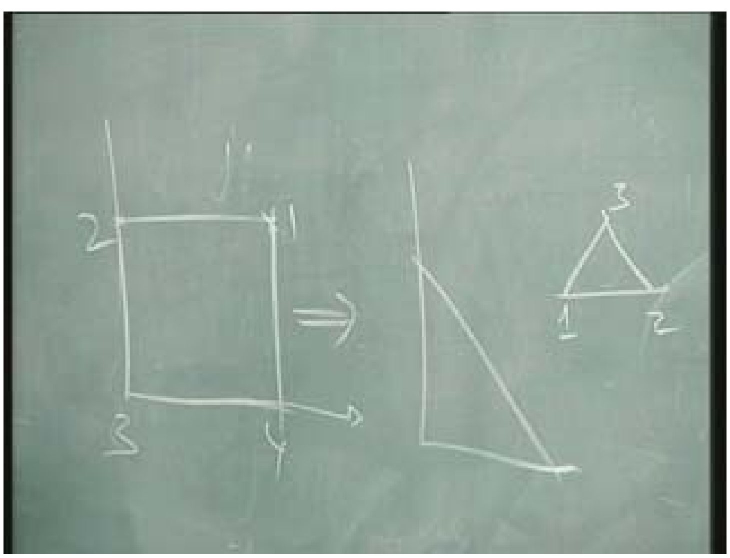

Let us say that I have an element like this, simple element, just illustrate how we collapse. Let me call this as 1 2 3 and 4, anticlockwise sets. Fine; now I can develop a triangular element following exactly the same procedure, whatever I have done so far, exactly the same procedure, by collapsing 1 and 2 and which results in an element something like this. Whatever be the shape, it does not matter. What do I do? I collapse x_1 to x_2.

What do you mean by collapsing? I simply mean that I give the coordinates of 1 and 2 to be the same, x_1 is equal to x_2, y_1 is equal to y_2. Where do I do that? I do that in my expression for x and y, first step.

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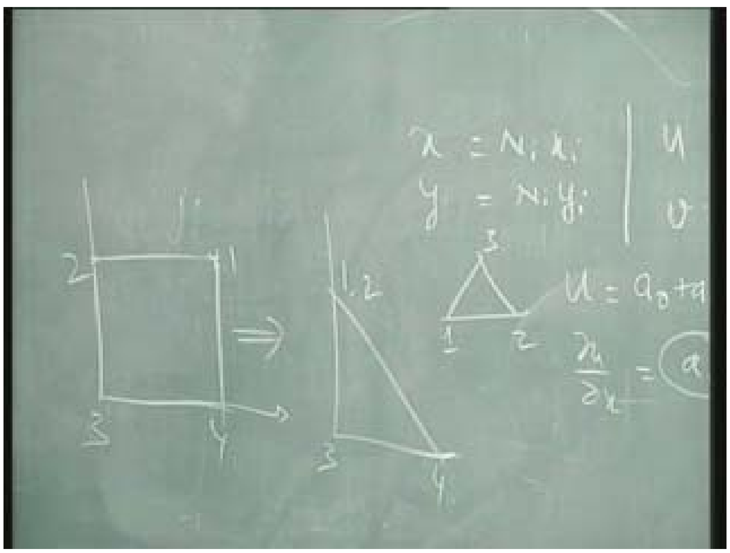

What is that I mean by collapsing? By substituting that x_1 is equal to x_2, y_1 is equal to y_2 in these two equations; is that all? No, because we are using isoparametric formulations and hence we have similar values for u and v as well; for u and v as well. So, u and v are again written and this collapsing exercise is done. So, I can say that it is 1 comma 2 3 and 4. The collapsing exercise is done there as well, so, u_1 is now the same as u_2. In other words, when I do that what essentially I get as degrees of freedom? Simple; u_2 v_2 u_3 v_3 and u_4 v_4. These are the degrees of freedom that I get. One way of calculating the stiffness matrix is by doing this collapsing business. This is very simple and I will do that in the next class by taking an example, simple example and following all the steps that I have done in this class and the previous class.

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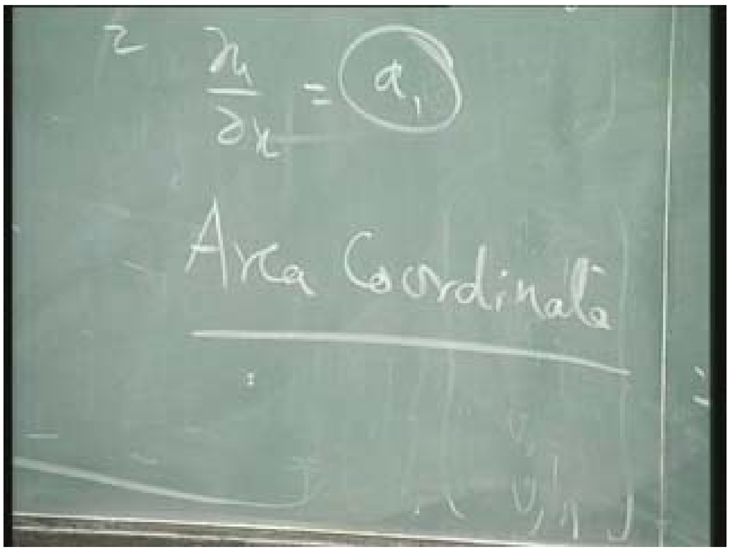

The other way of doing it, there is another way of doing it, is by following what is called as area coordinate systems. The area coordinate system is some sort of a generalization or an extension of the natural coordinate system, which we have in place right now. So, the second way of doing it is by area coordinate system. Whatever we have been talking about so far has been for linear element. But that is not sufficient. I said that the interpolation values can be of a higher order polynomial. It can be quadratic, can be cubic and so on. Whatever we have been taking about, whether it is the quadrilateral element or whether it is a triangular element, it is possible to raise the polynomial level or polynomial degree, from a linear exercise to quadratic, cubic and so on.

Our exercise, next exercise is to complete the triangular element by collapsing and get into this area coordinates and then extend this whole exercise to a higher order element which is not very difficult. I am not going into each of the element. For example, continuum elements of 3D, I am not going to the details, because it is exactly similar. I leave that out as an exercise. We will meet in the next class and see how we tackle these triangular elements.