Introduction to Finite Element Method

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Lecture - 19

In this class, we are going to talk about isoparametric formulations.

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As I told you in the last class itself, towards the end of last class, that isoparametric formulations form the basis of finite element analysis itself. In other words, it is a very generic formulation which is applicable to a variety of elements. It is applicable to plane stress, plane strain, solid elements, shell elements, plate elements and so on. So, this isoparametric formulation has to be looked at as a concept. What is the key or what is the most important thing, most important difficulty that has given rise to isoparametric formulation?

One of the most important points that you would have noticed when we developed this subject over the past 20 classes is that, we have to evaluate that integral, B transpose dB dv. This integral, that volume integral or surface integral depending upon whether you are in 3D, 2D and so on, becomes difficult when the element shapes become quite complicated. It is not possible for us to stick to nice square or rectangular elements and say that we will use only this kind of elements. Number 1, that it may not be possible to do it even for a nice geometry.

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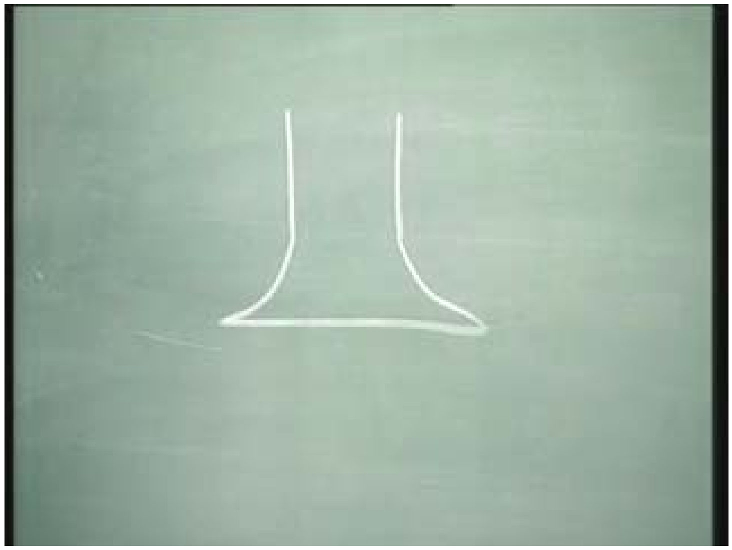

Number 2, when the geometry becomes complex and it has, say for example, radiuses like this, then definition of an element to cover this kind of radiuses may become difficult. We will see that we can use higher order elements quite nicely in this kind of regions and hence the isoparametric formulation is resorted to as a very general philosophy. Now, what is isoparametric formulation?

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Isoparametric formulation depends upon the concept of natural coordinates, natural coordinates. What are natural coordinates, how are they developed is going to be the first part of the lecture. This concept itself is very interesting and very nice and actually, this is the concept which has made possible finite element to develop to this stage and 99% of elements that are used in commercial softwares are based on isoparametric formulation. Let us see first of all, how we develop natural coordinate system?

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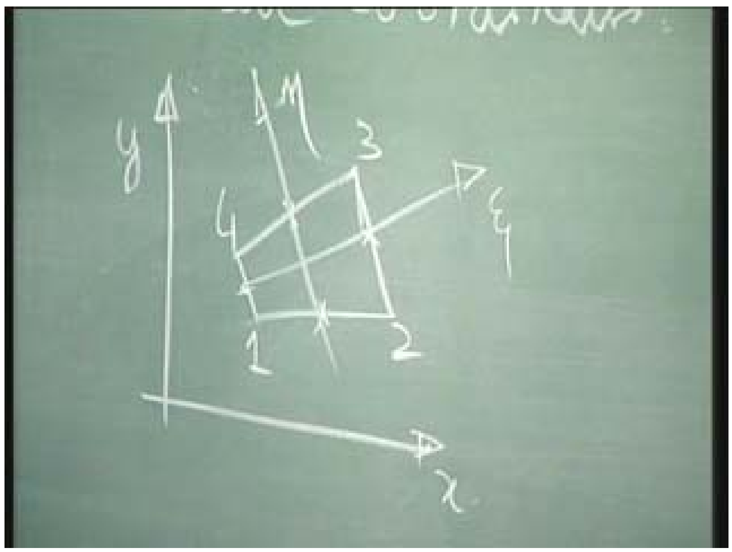

Let us now take a very general, say element. This, let me say as x and y. I am not using x_1 and x_2, because I know there is some confusion. So, I am just using x and y. There should not be any problem. Let us say that 1 2 3 4, these are the nodes. This is the global coordinate system, xy coordinate system.

I am going to develop a natural coordinate system for this. Just watch how I am going to develop? Let me take the midpoint of 1 2, midpoint of 3 4, just join them, draw a line. Let me take midpoint of 1 4 and 2 3 and then join them. Let me call this line as a psi eta coordinate system or in other words psi eta coordinate system. So, that is the psi eta coordinate system. How did it come about is by joining the midpoints. What am I going to do, next step? I am going to normalize this whole quadrilateral by means of this psi eta coordinate system. In other words it is possible to specify a value of psi and eta for every point in this quadrilateral. How am I going to do that?

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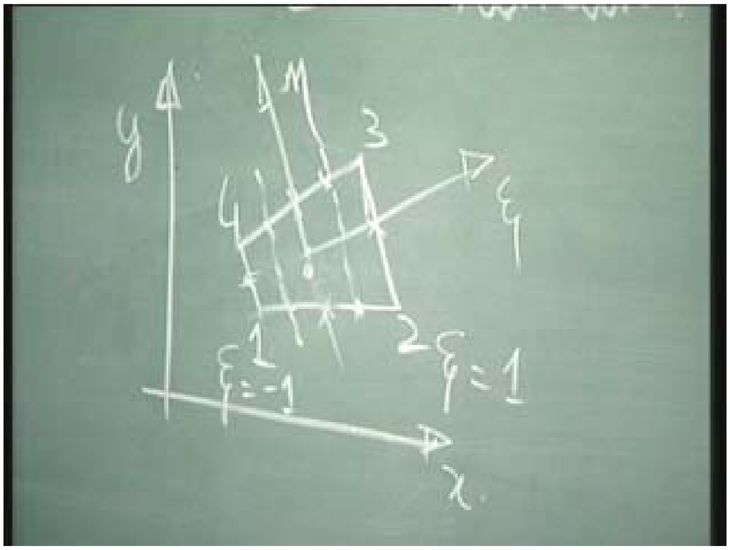

Now, I am going to first of all put a grid which covers this element. How am I going to put this grid? Let me take, let me take the midpoint of this to this. This is the midpoint of these two points. Let me take a midpoint to this and this, join them, then these points midpoint in the other side. Let us just consider only 1 2 3 4 5, these 5 lines. I am going to call this line as normalize that line by saying that psi is equal to 1. Then this line becomes or this is psi is equal to zero and this one becomes psi is equal to minus 1. That is beautiful and this line becomes psi is equal to 0.5. This line becomes psi is equal to minus 0.5. So, if I want 0.25, psi is equal to 0.25, what is that line? That line is, that is the psi, is equal to 0.25 line.

Yes; no, please note they are not parallel lines. I am taking the midpoint and midpoint and connecting them. It so happens, now it is parallel. If you want, we need not make it parallel.

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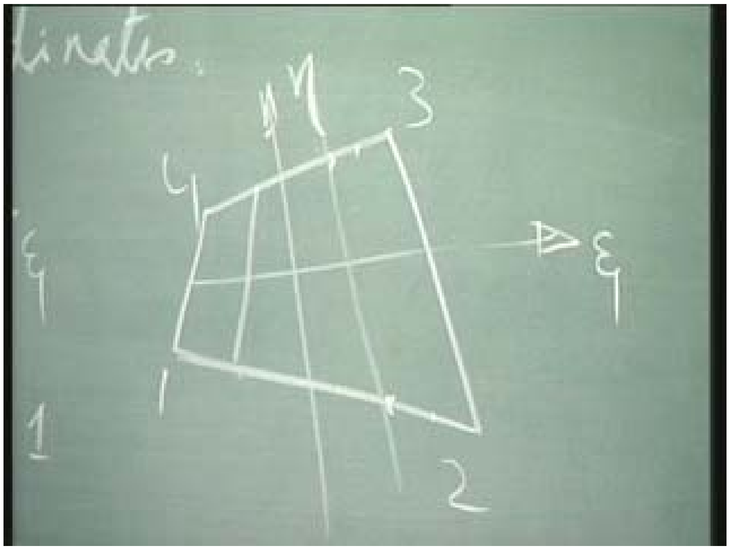

For example, I can have a quadrilateral which runs like this. I can make it like that and then still I can get, joining the midpoints, I can get still a psi eta coordinates system. Is that clear? That is 1 2 3 4 and when I want to now develop psi is equal to 0.5, what do I do? I go and take the midpoint here, midpoint here. Wait a second, wait a second; just develop the grids, so, that is one grid. I am now normalizing. Please note it, I am normalizing and saying that this line 2 3, psi is equal to 1. Please for a moment forget the rectangular or the Cartesian coordinate system. This is not a coordinate system where psi and eta need to be perpendicular. The whole confusion for you is because, and not only for you, for most students when we deal with this natural coordinate system, is that they are so used to xy coordinate system, they are not in a position to accept any other coordinate system. What is that we are trying to do? We are trying to normalize it. So, this 2 3 becomes psi is equal to 1. Any point there has a value of psi equal to 1.

How do I develop from zero to 1? I keep on I draw these grids. How do I draw these grids? Say for example, simple 0.5, 0.5 or half of or midpoint of this, midpoint of this; join it that becomes the say psi is equal to 0.5 line. 0.75, midpoint of this, midpoint of this; it need not be parallel to 2 3, any way it is just midpoints. That is all. In fact as I go to this place, it may become like this. Is that clear? I am normalizing it, so that psi varies from minus 1 to plus 1. Is that clear? That is the first point, minus 1 to plus 1 and eta now varies from, similarly eta varies from minus 1 to plus 1.

What does it mean? It means that you give me a point; you give me a point, it will be possible for me to tell what is the psi eta coordinate system? It is possible, right now we will see, it is possible to say that if you give me psi eta, what is xy? In other words, what does it lead us to or why is that we did like this? Whatever quadrilateral you take, like for example, I took this quadrilateral first, since you had a doubt I took this quadrilateral. Whatever quadrilateral I take, ultimately what is that I get out of this mapping?

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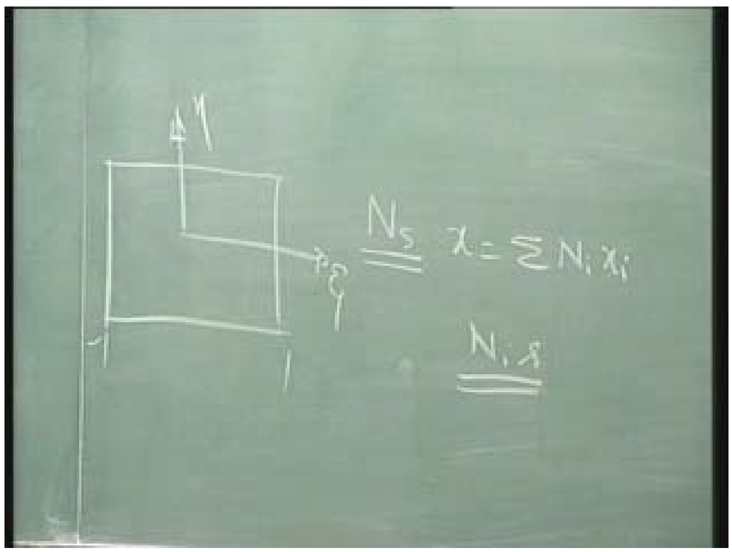

I can get a square or in other words all these points, whatever be the quadrilateral, can be mapped to a square in a psi eta coordinate system, where psi and eta varies from minus 1 to plus 1; psi and eta vary from minus 1 to plus 1. Is there any questions? Yeah; because psi and eta, in the xy coordinate system, it need not be perpendicular. But, here psi varies from minus 1 to plus 1, eta varies from minus 1 to plus 1. This is a natural coordinate system, so, every point can be mapped inside a square so that psi varies from minus 1 to plus 1, eta varies from minus 1 to plus 1. So, it can be mapped into this square. That will be the quadrilateral, I can map it to this place. Is that clear?

Let us look at it the other way. Given psi and eta, I can always get back x and y, because, what is that I have done? Just think about it for a minute, what is that I have done? I have simply linearly interpolated psi, eta to x's and y's. Is that clear? It is a linear interpolation. In other words, if I write shape functions N's, shape functions

N's, in the psi eta coordinate system it is possible for me to get x and y from or by invoking my interpolations, because what I have essentially done is to linearly interpolate. Is that clear or in other words x can be written as sigma N_i x_i, where N_i's are, what are these? N_i's are the shape functions for this square.

What is this new concept that we are introducing? We are introducing that the shape or x's and y's, the geometry itself can be now used as a, or can be determined by interpolations. What are N's, say for example N_1? Remember, yesterday we did that; for a similar thing we did.

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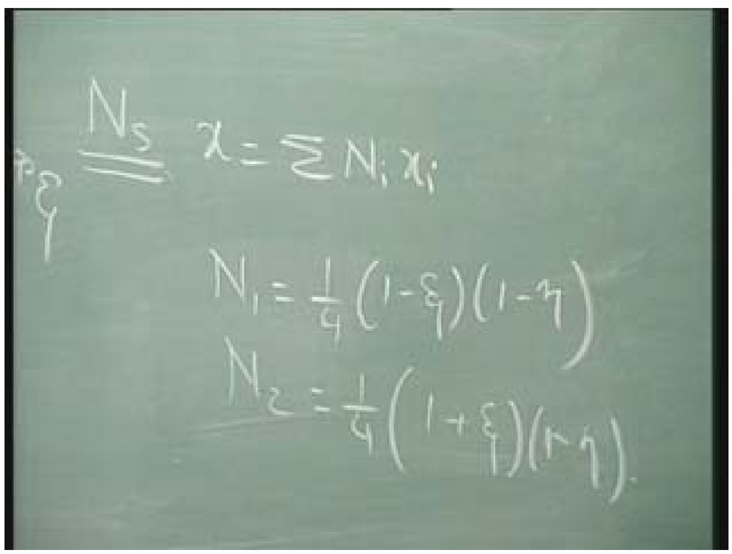

No, N_1 is equal to 1 by 4 1 minus psi into 1 minus eta. N_1; so, what is N_2? What is N_2? Yesterday we did that; 1 minus 4 into 1 by 4 into 1 plus psi into 1 minus eta.

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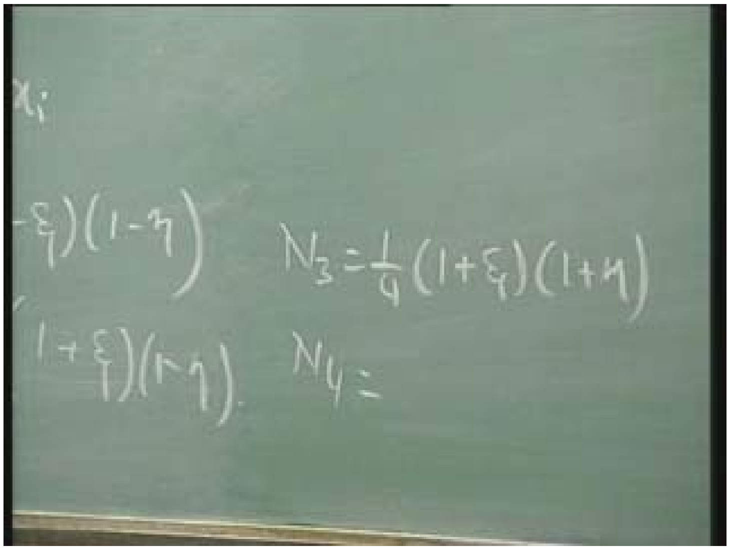

N_3 is 1 by 4 into 1 plus psi into 1 plus eta and N_4, you can write down N_4 that is equal to 1 by 4 into 1 minus psi into 1 plus eta. Just to check that we have linearly interpolated, just to check what is that we have got out of it, let us take one line, say eta is equal to this line, eta is equal to minus 1 line, say for example, eta is equal to minus 1 line.

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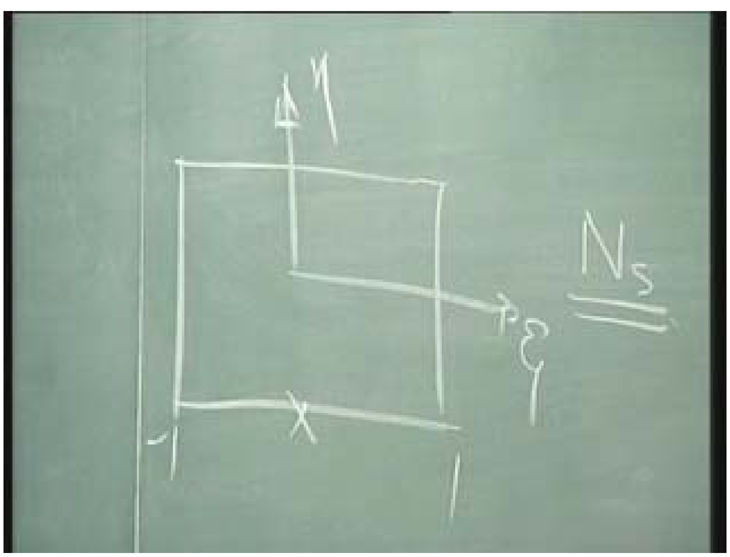

What is this point in my case? No, no what is the coordinates of that? Psi is equal to, psi is equal to zero, eta is equal to minus 1. This point should lead me to which point here according to the physically, how we have developed however we developed, this point that means it should be x_1 plus x_2 divided by 2.

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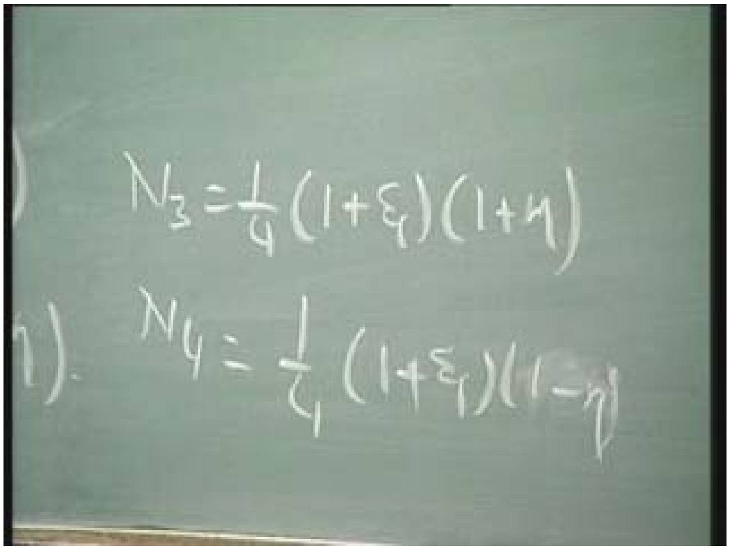

Can you please now check, the next one minute whether what you are getting, just check, have this and then just check what is that you are getting out of this interpolation? What happens to N_1? Eta is equal to minus 1; so eta is equal to minus 1, psi is equal to zero, so N_1 becomes 1 by 2; beautiful. What happens to N_3? Zero; what happens to N_2? N_2 is 1 by 2 and N_4, zero. So, what is x? Yes, that is what x_1 plus x_2 divided by 2. So, I get back to my midpoint. So, what is the point that I get from here? When I map it by this linear interpolation, I get to this point.

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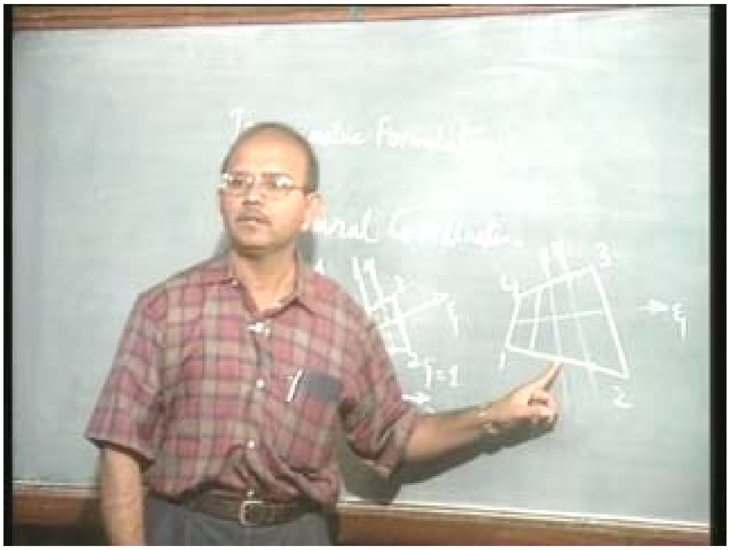

So, I can linearly interpolate everywhere this point. If you have a doubt, please go ahead and do for, say for example, midpoint. These are the points which you can easily understand. Psi is equal to zero, eta is equal to zero. What is that you get? Straight away you can see it and tell me from looking at it. What is it that I would get? 1 by 4, correct; 1 by 4 into x_1 plus x_2 plus x_3 plus x_4. That is the interpolation at the midpoint. So, essentially what does this particular equation give us? This or this shape functions give us, the shape functions tell us that it is possible to linearly interpolate psi and eta to get x and y, that is the central concept, concept number 1. I have mapped. I will not bother or I will not worry about whatever be the shape of the quadrilateral as long as I can map it to a square, nice square like this and keep working on this square, keep using this shape function, I have no problems. But this is the very first step, lot of things that are now attached to this. What are the things?

Immediately our tendency, very good tendency, is to keep working in this particular coordinate system, because it is nice, shape functions are known and I will later do a transformation of that big integral into nice little integral which can be integrated and so on. That is the whole idea of now developing. In other words, I can say that look, I can interpolate the displacements or any field for that matter, using the shape functions given here and keep working in this region or in other words, I can say that or you can immediately look at this and say that why not I write, instead of writing it in the global coordinate system, let me write u as N_1u_1 plus N_2u_2 plus N_3u_3 plus N_4u_4 in this particular coordinate system. You are perfectly right.

In other words, you are now linearly going to interpolate or you are going to use the bilinear, nice bilinear law that you did before, which, please remember resulted in the same equations, same shape functions. I can write down u to be N_1u_1 plus N_2u_2 plus N_3u_3 plus N_4u_4, N's defined like this. Agreed? I am going to interpolate. Whatever I do, whether I do it in the xy coordinate system or in this coordinate system, both of them are the same, because I am linearly interpolating them between the nodes, but I have a major difficulty. What is that difficulty?

The difficulty is that, though I can linearly interpolate it very nicely, when I go to epsilons, I need to calculate dow u by dow x, dow u by dow y, dow v by dow x and so on. In other words, I will have to get also a relationship between u comma x and the corresponding values in psi and eta. Is that clear? I have to get u comma x and the corresponding eta, is that clear, because psis and etas are now function of x or x is a function of psi and eta, however you look at it. That is not a very straightforward calculation.

What are we going to do to get that equation or in other words B transpose dB dv remains, but the B’s when I want to do a shift of the coordinates from x y z or x y z to psi eta, say gamma or dow coordinate system, then I may have to properly transfer these variables. I cannot just say since u comma x is what I require, I will replace it by u comma psi. Obviously, you know it is not possible, because if I want to calculate it, x is a function of psi. In other words, you know that chain rules have to be used and what is called as Jacobian has to be used. Simply, if you are familiar, whatever I am saying you need not worry about it, because you know that when I shift from one coordinate system to another coordinate system, Jacobian plays a role. What is this Jacobian and how is it going to play a role and how is it going to alter my stiffness matrix are the issues which we are going to see.

To just summarize what we did, we transferred or we defined what is called as a natural coordinate system. We said that the geometry can be interpolated in the same fashion as we interpolate the field quantities that is u and v and that becomes very straight forward and now let us go and attack our B transpose DB dv, so that I can do all my operations in this nice mapped region coordinate system, instead of the original coordinate system. Is that clear?

Now, let us see how to do that? What do you think will be the crux of this whole issue? What do you think or what is it that you think would you have to use? Obviously you have to use a chain rule. Now, before we go there, let me write down, let me write down for a specific case, for example let us take plane stress; plane stress and try to develop an isoparametric formulation for it. It is the same for anything, any other element, but this would be easier for you to understand. If it is 3D, it is going to be slightly, you have to introduce one more dimension and so on. But, let us take say for example a plane problem, say a plane stress. It does not matter whether plane stress or plane strain, D would vary; a simple say plane stress example. So, I have to now write down epsilon for this. What is my goal? My goal is, let me put my goal clearly.

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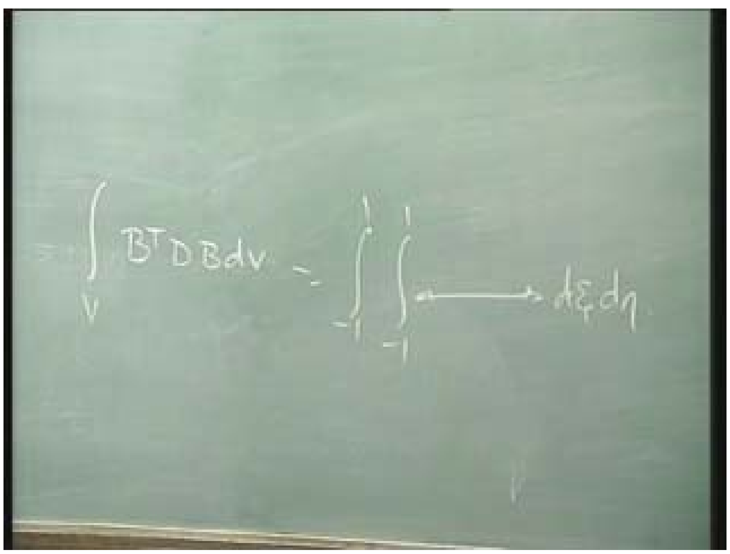

My goal is B transpose DB dv has to be now transferred to minus 1 minus 1 and get some function there say d psi d eta. How is that I am going to do that, is the question. Is that clear, is the goal clear? That is what I want to do. I am going to first of all I mean my D is not a problem here, my D is not a problem here. My problem is B, my problem is B.

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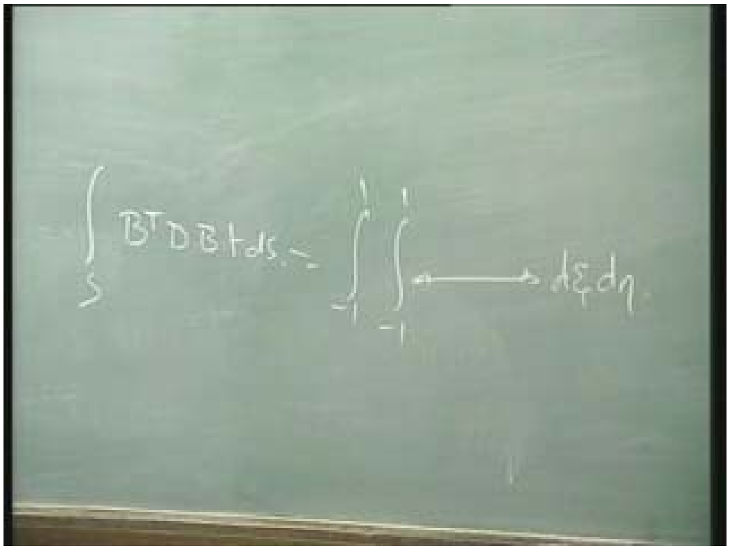

Already you know, from your earlier classes, that this can be, say for example, replaced by say surface as tds. What is tds? tdx dy.

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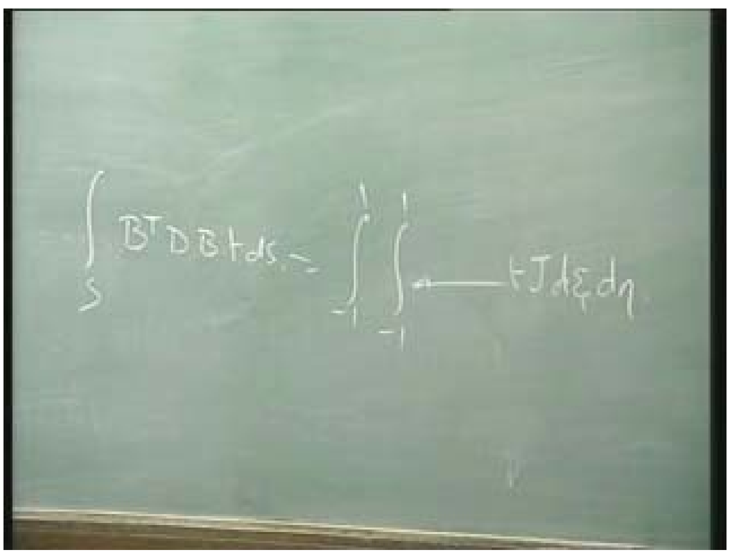

When I go to another coordinate system, you know from very elementary mathematics, calculus that this has to be replaced by a Jacobian which is J. This part is simple. Of course t is there, because I am looking at a plane problem. t is the thickness of the element. dx dy or ds now becomes j d psi d eta. Let us look at how we get Jacobian and how we are going to transfer B. I cannot just put this B here, because it is the function of x's. Let us go back to a class or one or two classes before and see how we defined B. What was B, by the way you remember? Yeah, epsilon; no, please note that epsilon is equal to B into d, small d. So, the relationship between strain and displacement is what we called as B and B involved u comma x, u comma y and so on. Is that clear? So, that is the first issue. Let us see how we are going to tackle this problem. Any question so far? Let me write down epsilon.

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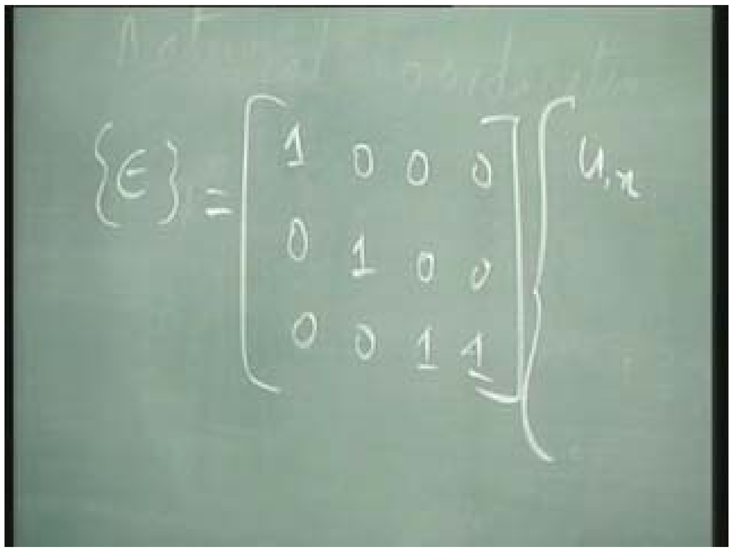

How am I going to write down epsilon, because what is that I want? I want this u comma x and u comma y and so on. Let me write down epsilon as 100 say 0, 0100 0011 into what is that I would get? u comma x; this is dow u by dow x. Please note that u comma x is dow u by dow x. What is epsilon, by the way? Remember, what is epsilon? Epsilon is epsilon_{11} epsilon_{22} and gamma_{12}, say, we are now sticking to gamma_{12}, because this is a plane problem. Epsilon_{11} is dow u by dow x; so I get 1, so, I get u there and sorry, we will just change this, so that you know we will write that down first.

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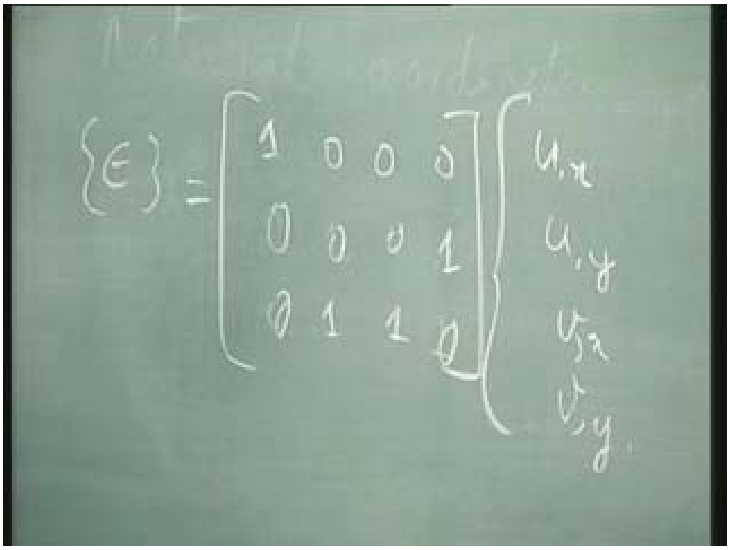

So we will change u comma y, v comma x and v comma y; we will put it like this, so that it is easier to write it. So, that zeros and 1, I will write it down accordingly. So epsilon$_{11}$ is equal to dow u by dow x, epsilon$_{22}$ is zero into this zero, zero into this zero, zero into this zero, 1 dow v by dow y. It is very important how you write it? Anyway you can adjust it either way, but I think it is easier. Please note how I am writing it; further derivations would depend upon how I write it. So, the last line will become 0110, that is all. I will write first u's followed by v's.

It is very simple for you to say that look u comma x is, what is there in u comma x? I can just do a chain rule. What is a chain rule? You may come and tell me that u comma x is what dow u by so dow psi into dow psi by dow x and so on. Just I will write it down in a minute, you will realize it is not very easy.

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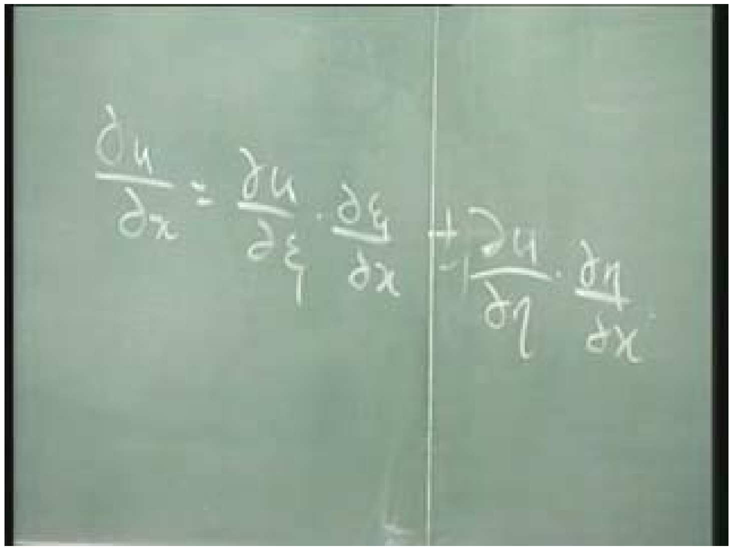

What is that dow u by say dow x? You may say that what is this into dow psi by dow x plus dow u by dow eta into dow eta by dow x. That is all. But, there is a problem. What is the problem? Dow psi by dow x and dow eta by dow x, we do not have. We have dow x by dow psi, because x is equal to N. What is that? x is equal to N_i x_i and what are N's? N's are functions of psi and eta. So, dow x by dow psi if you ask me, very simple. I can go and differentiate, very straight forward. But if you ask me dow psi by dow x, then I have to get an inverse map and that is not going to be easy. Is the point clear? Just have a look at this. Just assimilate that it is not possible for you to get dow psi by dow x that easily. On the other hand dow x by dow psi is available. So, I have to beat this, I have to beat this. How am I going to beat this?

Simple; why not I write the other way? What is it? Why not I write u comma psi? Instead of this, why not I write u comma psi? What is u comma psi? That is dow u by dow psi. What happens when I write dow u by dow psi?

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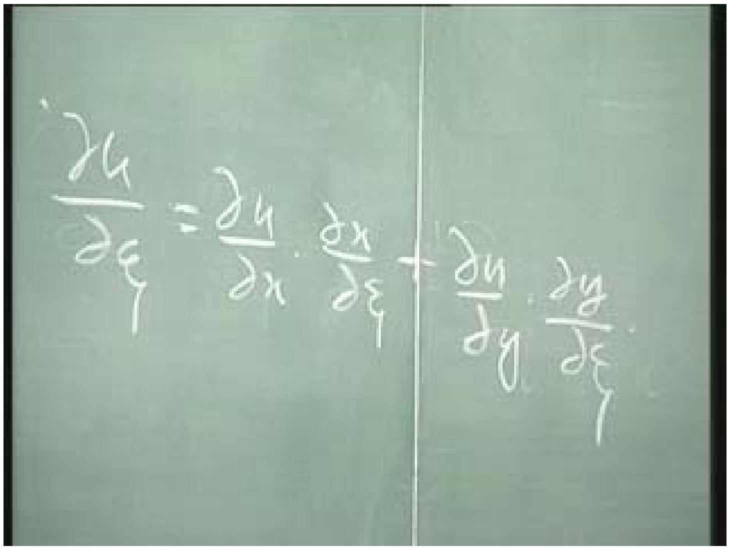

You understand this? No? Any question? Yes; very good. So, dow u by dow psi is what? Dow u by dow x into dow x by dow psi plus dow u by dow y into dow y by dow psi. I will give you one minute to write the other one and put it in the matrix form. Because students usually find it very difficult to, when I say put it in a matrix form, they get bit confused; though they know the multiplication of matrixes very, very well, they find it difficult. Let me say that you put this in the matrix form.

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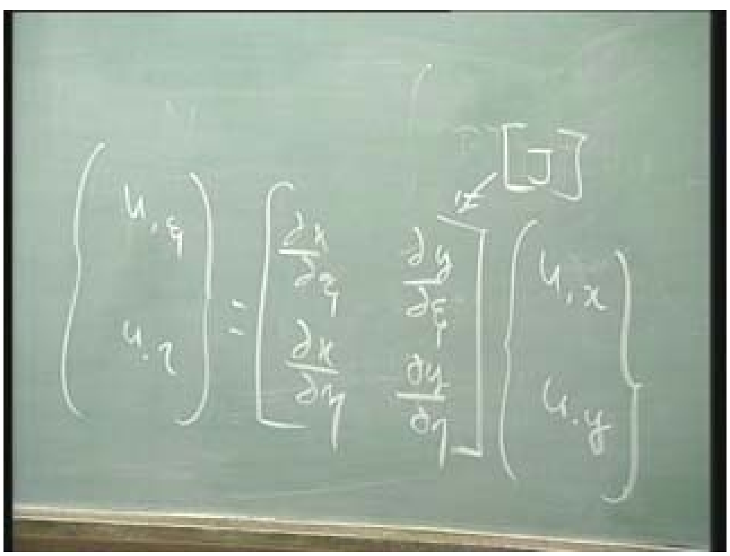

In other words, what I am trying to say is put it down as u comma psi u comma eta is equal to something is equal to some 2 by 2 matrix into u comma x and u comma y. Is it clear? What am I trying to do now? I am trying to get what I know; I am trying to get the other chap through what I know. Now, what is this, what is this equation? dow x by dow psi dow y by dow psi dow x by dow eta and dow y by dow eta. What is this matrix? Jacobian; beautiful. So, that is the Jacobian matrix, which is very, very fundamental when you change the coordinate system. That is what is called the Jacobian matrix.

Can you get Jacobian now for us? Simple, what is it? Because I know x. x is N_i $x_{i.}$

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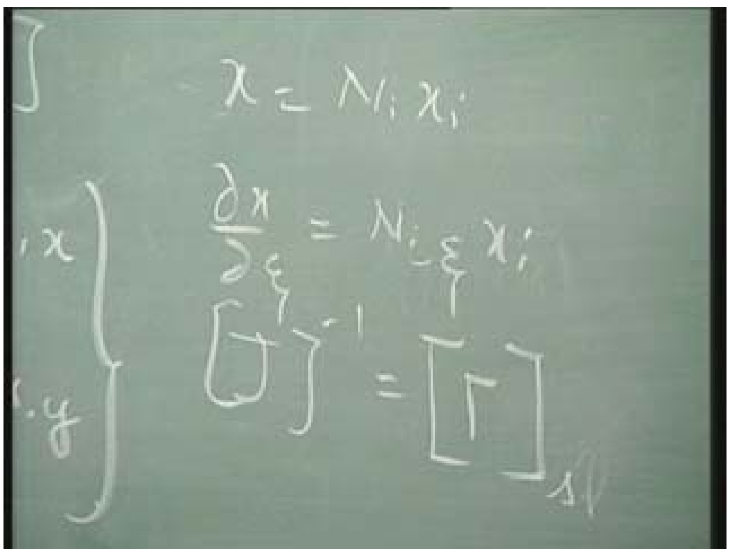

So, I can say that x is N_i x_i where I say that i has to be summed up and dow x by dow psi is equal to N_i comma psi x_i. Can I write like that? N_i comma psi x_i. Now, I can do the same thing with respect to y, with respect to eta and dow y by dow eta and so on. Any question so far? All these terms can very easily be calculated; after all dow N_1 by dow psi dow N_2 by dow psi and so on can be straight away differentiated from what you have. Is that clear? Once I know J, what is that I am going to do? I want to find out, what is that I need? Please have that in mind, what is that I need? I need u comma x and u comma y, v comma x and v comma y; same thing can be extended. I do not want to put too many things, because you will get confused. That is why I am putting u comma psi and u comma x and u comma y. The same thing can be done for v comma psi and v comma eta. Similarly here I will have v comma x and v comma y.

What is that I should get now? Say J; so I know how to get J straight away. Inverse, J inverse; I have to get J inverse. Let me call J inverse as, say gamma, gamma s. I will say just gamma s because I am going to expand this gamma later. Just note this carefully that I am going to say J inverse is equal to gamma s. So, gamma s right now, is a 2 by 2 matrix. What will be $gamma_{11}$? Can someone, do you know what $gamma_{11}$ is? No, no, no, no, no, no, wait, wait, wait, wait, wait. From J, if I inverse it, say suppose this I call this as J_{11} J_{12} J_{21} and J_{22}; 1 by J, J_{22}. So, that is gamma; I mean, straight away if you know the inverse of a matrix you can say that $gamma_{11}$ is 1 by J, J_{22}. $Gamma_{12}$ is minus J 12 or 21? 12, then again the $gamma_{21}$ become J_{21}; of course all of them divided by J that is determinant J. The last one becomes or $gamma_{22}$ becomes J_{11} by J. In other words, I need not go and invert it. I can invoke a known formula and write down this j and so on. Once I do that, what is my next step? What is my next step?

I am going to now, is it clear, what is that I did? u_i u comma x and all those things nicely I have found out. Now, my whole crux of the thing, I am repeating, is to get that B, that chap B, who is now actually a function of x, I am trying to get it in terms of psi. What is it that I am going to do now? u comma x, u comma y and all these things with which I started here, I am going to now substitute it by those chaps whom I know. Is that clear? How do I now write? I will just give you a minute to assimilate what all we did. Just have a look at this so that you will just assimilate what all we did now, because this I mean, though we are logically proceeding with the lot of steps here so just a minute, you just take a minute to assimilate what all we did. Just see that. Yes, any doubts?

Please note x_i is not a function of psi. x_1 x_2 x_3 x_4 are the coordinates of the quadrilateral; they are not functions of psi and eta. They are the coordinate systems from which we are interpolating x through a bilinear law, psi and eta. Yeah, what? Yes, you are jumping one class ahead; one or two classes ahead that. Yes, we have to talk about triangular elements. This class I am not going to talk about them. We will develop a procedure to do triangular element. You have to wait for another may be two classes before we look at triangular element. There the natural coordinate system, that is a nice question, because it is not very easy to expand whatever we have done now for natural coordinate system for triangle.

But again, guys who have worked in finite element are very intelligent guys. They have done, you know, look at this concept it is so simple, but the concept has come out so well that the whole lot of industry has benefited by this single, simple concept. Like that for triangular element also they have beaten the system by means of a very simple, natural coordinate system. We will see that later in the course. You know right now, I do not want to confuse putting both of them in one class. We will take another class to do that, but you understand this, what is happening.

The concept is very simple and it has beaten all this geometry and in fact if you look at the difference between finite difference and finite element method, a finite element methods course is basically because of this concept. It is not correct, say, lot of people ask questions like this, which is better finite element or finite difference? This is a standard question, which is better. It is not a question of which is better, you know. When you ask them back what you mean by better, they would say which will give you better result? This is the question they will ask, which will give better result? It is not a question of giving better result. That is not the way to compare. It is the question of versatility verses rigidity.

Finite element is much more a versatile tool than a finite difference. Finite element can handle very complex geometries very simply, whereas finite difference, you will find it very difficult to handle. Yes, there are techniques to handle it, handle using finite difference, but that becomes difficult in finite difference. So, that is why finite element has scored and there are lots of softwares that exist in finite element analysis. So, we will come back to your question on triangular element later in the course. Right now, let us concentrate on the quad element, quadrilateral element. I hope now you understand how to extend this to a 3D, say just expand your question. Now, you know how to extend it to 3D? Simple; what is that you are going to get? Instead of this nice square, you are going to get a cube. That is all.

If you have a brick with x y and z coordinates, the brick which is not very nice brick, rectangular brick, you can always go and map it down to a nice cube. The procedure is exactly very similar. Is that clear? What is it that I am going to do? Coming back, looking at this, I am going to now substitute u comma x or whatever we have done in this equation. How is that I am going to do that?

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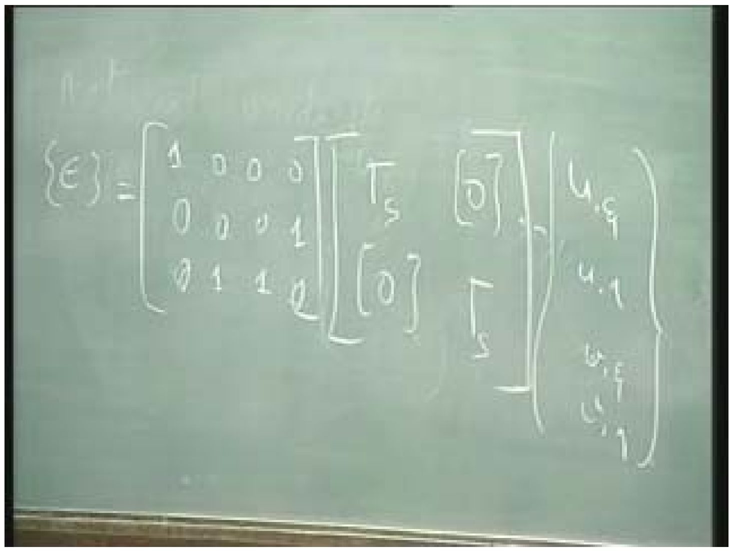

Gamma 0 0 gamma s and then write it down in terms of u x u y and so on, because what is gamma? Gamma is the fellow who changes u x comma u i or y?. So dow psi u x comma u i or y? is changed by this. So, I will get instead of u x u y and so on, I will get u psi; u comma psi u comma eta v comma psi v comma eta. No, no, no, no. What is this? No, zero, this is zero, I mean, correspondingly; what is this? 2 by 2. So, 2 2 another 2 2 here; this is very obvious.

This is the style of writing; no, you do not write. I mean, then I have to require the whole board to write. You will immediately understand that this is nothing but 2 by 2, 2 by 2, 2 by 2, 2 by 2, so, 4 by 4; multiplying that with this. In other words, if you want you can, write down completely. It does not matter, you can write it down. Ultimately I will get that I will multiply by this 1 zero zeroes and 1's here to get epsilons. My job does not stop there. What is it that I want? I want in order to get B, I have not come to B yet.

Why, because this chaps here are not the guys who will define my B. My B has to be defined ultimately by D. How do I get that? Yeah, remember that when we started this whole derivation, when we started this whole derivation, we stared by saying that u will be interpolated, u at any point will be interpolated by means of the use that I get in these nodes by using N's in a natural coordinate system, because that is also a linear interpolation. In other words, we said that u is equal to N_i u_i. The N's are a function of psi and eta, obviously.

What does it mean? It means that I can replace this chap by a big matrix of N's, which is very similar to what we did before. It is very similar to what we did for the rectangular case and then replace that, then d.

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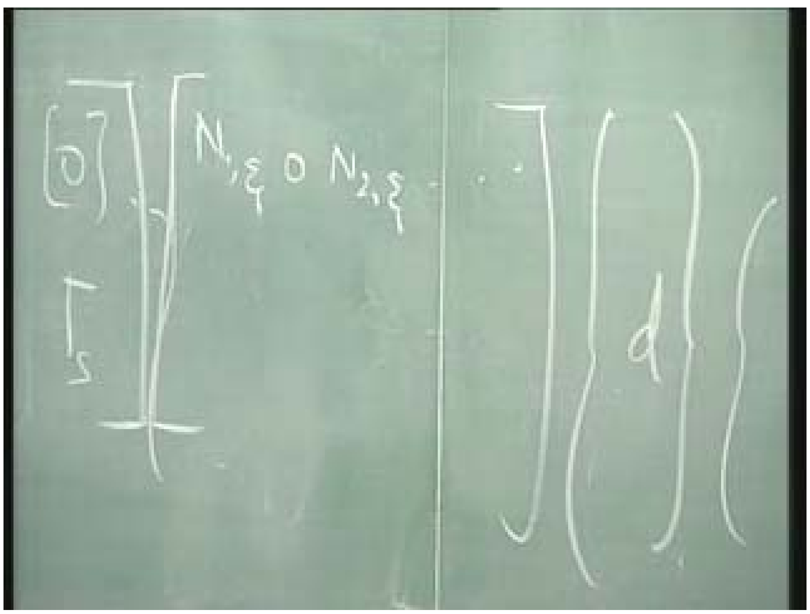

What is this? This is d_1 or u_1 u_2; u_1 v_1 u_2 v_2 u_3 v_3 u_4 v_4 that is that d. What is this big matrix here? Not shape function, derivates of the shape function. Note that carefully, because u comma psi is what we are replacing. They are the derivatives of the shape function. This would give me, say for example, N_1 comma psi 0 N_2 comma psi and so on. Is that clear, yes?

The next one, similarly I have to write it down for psi eta and all these things. We will do that, we will continue this derivation; derivation is not yet complete. We will continue from this step in the next class. I want you, I am just leaving it blank, because I want you to write it down, because that is a good exercise to understand this. I will write down in the next class completely and another thing that is important for you to keep on looking; see, one of the mistakes you can make, I am very happy nicely you have caught me saying that this 2 by 2 cannot be multiplied. Yes, but what I implied is 4 by 4, but it is always very important that you keep a track on what is the size of the matrix. What you are multiplying? What is the ultimate size you are going to get? Does it coincide with the concept that you have?

All these things should always be looked at. Is it clear? Then only things would become clear, would fall in place. We are now what is that? We are proceeding towards region where we know all these expressions. Next class, we will define B, put it back into that equation and then we have to have two or three more things that we have to learn before we master isoparametric elements. Are there any questions? Then, we will meet in the next class.