Introduction to Finite Element Method
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Lecture - 18

By now we have understood that the crux of finite element analysis is interpolation. It is that concept that helps us to reduce an infinite degree of freedom system to a finite degree of freedom system and make us focus on the so-called nodes. So, it is very important that we understand what is it there for us to look at on this interpolation or what are the things that we should be focusing on in case of writing down this interpolation. I think as a passing mention I already did that, when we put down the interpolation function. But let me just summarize it and put this whole concept of interpolation in proper perspective. This becomes especially important, because if you remember that when we did that example we had what we called as a jump or a discontinuity in the stress as we moved from one element to another element.

This I am sure would have surprised you quite a bit, because people do not understand this kind of stress jump. But it is very clear that the stress jump happened again because of certain things we did in the interpolation. It is not that the results are wrong, but it is important that we understand what we are in for. Hence it is important that we look at what the requirements are for an interpolation function. One of the things that I want you to realize right in the beginning is that many of the concepts we are talking about now, is not only for structural analysis or displacement based finite element analysis; many of them are applicable even if you look at other problems, say for example, heat transfer. If you want to analyze heat transfer again interpolation is a concept which we will be using. Is that right? If you are looking at say electro-magnetics, totally different area, then again this concept of interpolation and many of the other things that we are going to talk about in this class and in the next class are important. Hence usually many people and textbooks do not just say u and v but putdown what is called as a field variable, say many people use for example a field variable, phi.

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What does it mean? It means that this variable phi can take on values like u or can take on other variables like u, v, w, temperature t, whatever it is, velocities, we are going to see that later in the course, velocities all those things are generically mentioned by means of a quantity called phi. So, many people express the concepts of finite element analysis through a field variable phi, which is a generic name for u, v and so on. Is that clear? Whenever I use phi, then immediately you should understand that that phi can be replaced by displacement in the x direction and y direction and so on. Let us now look at what are the requirements of the interpolation function?

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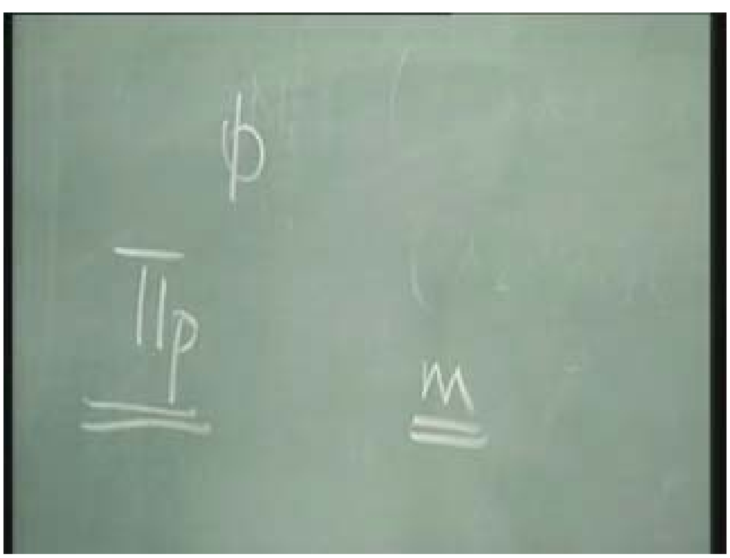

In order to look at the interpolation functions requirement, we have to concentrate on what we called as the functional pi_P. What we have to do first is to go and look at this expression for pi_P and then look at the derivatives of the field quantities or variables that exist in pi_P. There may not be any derivate, which rarely happens; may be a derivative of the first order, there can be a derivative of the second order and so on. The first thing that we have to do is to go and look at the derivatives that exist in pi_P. Let us say that the derivative exists till the order say m. This varies from one problem to the other. That is why I gave right in the beginning of the class I said that we are going to talk about a generic procedure.

In other words what I am going to tell you or what I intend to convey is that this pi_P what we talked about is not only applicable or the functional concept is not only applicable for this displacement problem, but it is applicable to a range of other problems as well. As long as you are able to express this functional, whatever we are going to talk about are applicable. Hence tomorrow I am going to do or later in the course, I am going to do temperature or heat transfer analysis through finite element. I will not explain all the things what I am doing now. I will just pick up these concepts and apply. Let us say that pi_P has derivatives up to an order m. Now, what is m in our case, what we did? Can you please check up that? What is this m in our case? In our case in the sense that what we did the last class.

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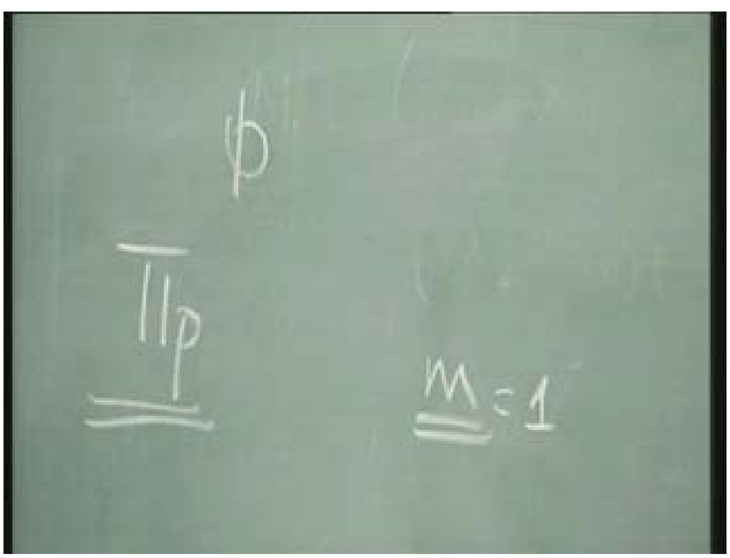

m is equal to, look at that, du by dx is what we had. 1; correct. m is equal to 1. It may vary. We are going to see later that it may increase; m is equal to 2 and so on. Let us look at that; as the problem comes I will point out what that m is. I do not want to confuse you with more examples on this. Just we will take whatever we have done so far. One of my first rules is that the interpolation function should be complete to this degree m; it should be complete to this degree m. What do I mean by completeness? You remember I had said that it should lower a, correct; so, the lower order polynomials should be present. That is very important. So, the first thing is that it should be complete up to the degree m.

The next thing that we are going to talk about why is that we said this, …. physically. There are other mathematical reasons, but why is that we said that it should be complete up to m? Because, if I start with say x cubed, there is good chance that we will miss. This is physically to understand that mathematically it is much more rigorous than this. Here if I start with say x cubed term I will as well miss lot of things. The solution itself I would miss. So, it is important that it is complete up to this m is equal to 1.

The next condition that I am going to put forward comes from my jumps. Next condition is that the discontinuity across elements or it should be at least continuous, let me put it like that.

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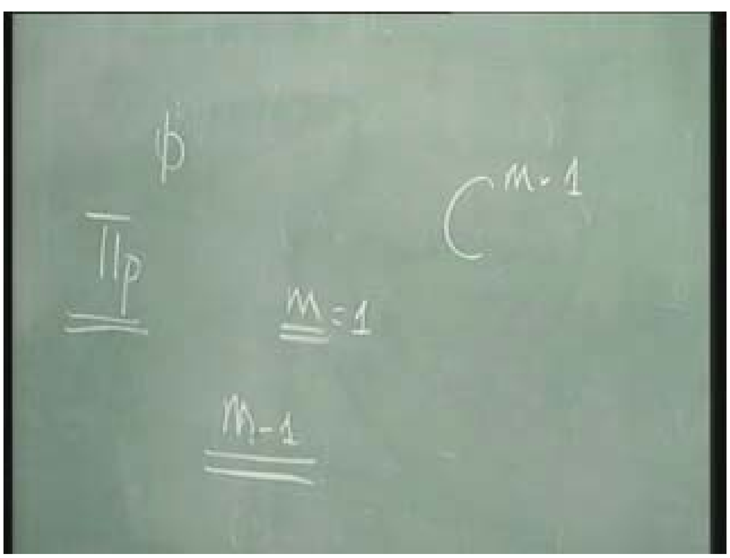

The interpolation function, which I am going to choose, should be continuous at least up to m minus 1; continuous at least up to m minus 1. Is that clear? In our case, what does this mean? This means that, what is m? m is equal to 1. So it is continuous up to m minus 1. The first derivative need not be continuous. Usually it is a practice to put down continuity by means of a symbol say C and then put it like this, m minus 1. In other words when I say that my continuity is now not there for the first differential, then the type of element that I am using is equal to or is called as C zero element.

What we have been talking about now, or C zero element, which does not possess continuity of the first deferential, they are C zero element. Since there is no continuity of the first differential, we landed up with that kind of jumps. Is that clear? So this depends upon, So you are theoretically correct to use this element as long as my π_p has the order up to m. What is that we are using? We are using C zero continuity elements. If the slope happens to be continuous, which is the first differential, then what is the element that I will be using? C 1 element. So when someone says I am using C zero element now you understand what it means. That is the second condition. I am just pointing out four important conditions that should be met by the interpolation function.

The third condition, which becomes very important is that when I define an element using an interpolation function and put this element in a mesh, I define an element with a particular interpolation function and put it in a mesh, then and apply the boundary condition, apply a boundary condition such that I specify the differential up to m minus 1. For example in this case, suppose I specify constant strains or constant displacement or constant strain, then the element should be capable of taking that differential. In other words, when I say that I specify constant strain, the element should have the ability to respond to my boundary conditions by assuming a constant strain throughout. Usually this is achieved even with few numbers of elements, for well behaved ones, but for some other elements you may have to keep increasing the numbers. But nevertheless, the constants ability to take constant strain should be present in the element so that, in the light of I reducing the size of the element, ultimately I will be able to get to a constant strain state.

There are more advanced concepts like patch test which is based on constant strain elements and so on. But it is important at this stage to understand that constant strain state should be there. What does is it mean for us? Can you just think for a minute and tell me what does it mean for us or from the point view of writing down the interpolation function. Yes that is correct. So, if I say u is equal a_0 plus a_1x plus a_2y and so on, you should at least have u is equal to a_0 plus a_1x plus a_2y so that, I will get a constant strain to be depicted or to be shown by this element. Is that clear? Fourthly I should not introduce any geometric anisotropy inside the element.

What do I mean by geometric anisotropy?

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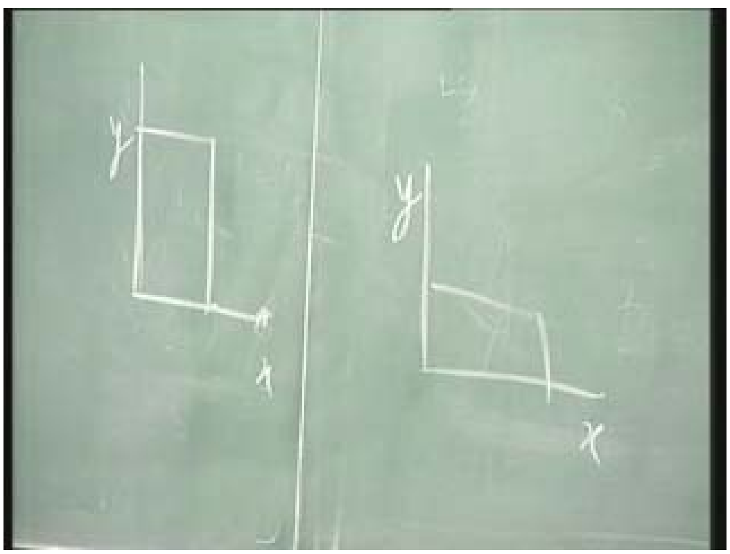

Suppose I have an element which is like this and I have an another element which is like this and I have another like this, where this is x and y and x and y. What do I mean by geometric anisotropy? Just think for a minute because concepts which comes very naturally. Just look at this and then tell me what do you understand by geometric anisotropy? It is very simple. What is anisotropy? They are different; correct. They are different. Isotropy means it is the same; so, there should not be geometric anisotropy. Correct, so, the properties of the element should not depend upon the type of, no, no type of coordinates. The property will depend upon the shape, but it should not depend upon the coordinates or in other words what does it mean?

Whatever be the type of coordinate system that I choose, whether I choose this coordinate system or this coordinate system, I should have the element to behave in the same fashion. Is that clear? What does it mean from our interpolation function point of view? What does this give me? What does it mean? Is it clear that whether I have an element like this or like this so it should behave in same fashion.

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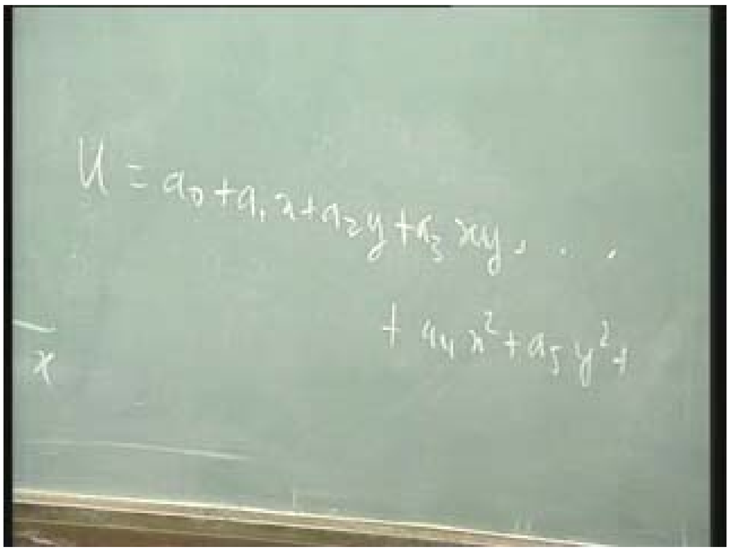

Yes; so in other words, suppose I say u is equal to a_0 plus a_1x plus a_2y plus a_3xy plus and so on, then or in fact I should go further then; a_4 say x squared plus a_5y squared plus and so on. If I introduce x squared, I should introduce y squared. So, it should be symmetrical about, say, a centre.

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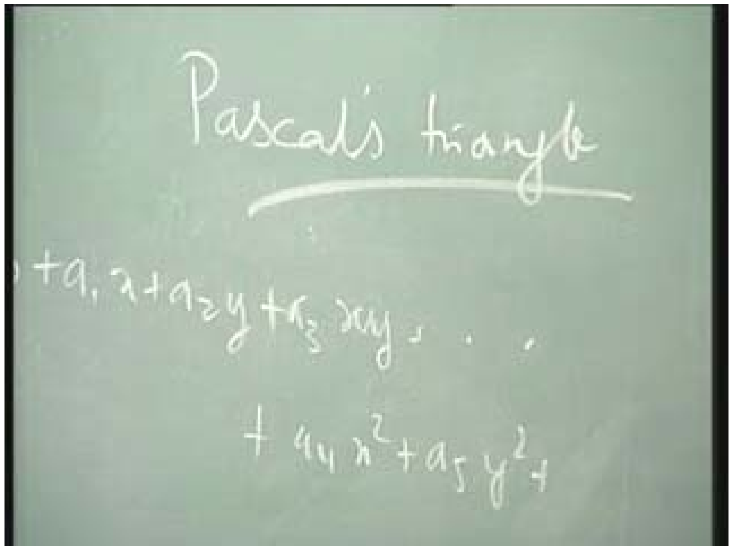

That is brought about by writing down this expansion by means of or in a term called Pascal's triangle, Pascal's triangle. What I am going to do is to write down one term and give you one minute for you to fill up. Let us see; just think about it. So, Pascal's triangle I am going to write like this.

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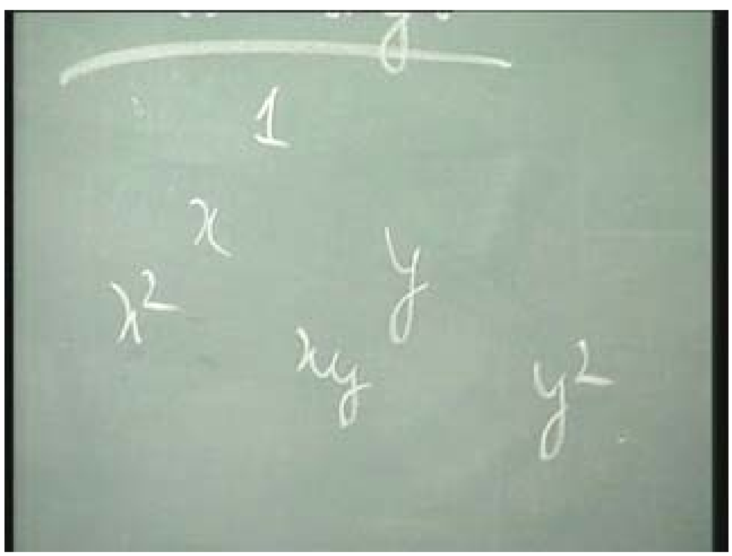

1 x y; next is x squared xy y squared. That is the quadratic term. Then the next line, just think about it and write it about the centre. What will be the next term? x cube x squared y xy squared and y cube. If I introduce next term, as x square in the line if I

introduce x squared y, I should introduce also xy squared. So, that is another condition. Whenever I am in this line for example, I should have included what all are there above and also about the centre it should be symmetrical. We are going to talk a lot more about Pascal’s triangle or some more about Pascal’s triangle later in the course, but right now, it is important that we understand that we cannot introduce this geometric anisotropy by including only terms which may bias my whole interpolation. Is that clear, any questions?

We have already written down the easiest way to write down interpolation; the easiest way to find out interpolation which we called or easiest way to go to shape function. What was the easiest way? Remember that we wrote down u is equal to say a_0 plus a_1x plus a_2y plus a_3xy which means that I included up to that.

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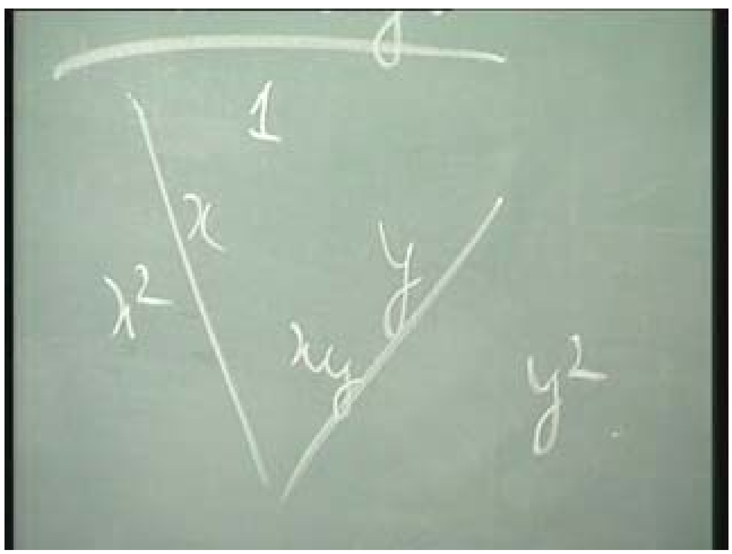

For example, we did this for a rectangular element. Then, what did we do? We then swapped a_0 a_1 a_2 a_3 for u_1 u_2 u_3 u_4 or v_1 v_2 v_3 v_4. In that process, we got what are called as shape functions. Is it clear? Now, the question is should I do that every time? Yes; you can do it every time and get it, get the shape functions. But, only thing is that it should fulfill all these conditions whatever we have been talking about. But, sometimes it becomes difficult to keep on inverting this A matrix. It becomes difficult to invert. If you can do it, well and good, go ahead. That is the more scientific way of doing it. But, it may become sometime cumbersome. So, I would like to get a much easier way of doing things.

One is by inspection. We are going to see how by inspection you can write down shape functions. The other is by using Lagrange's interpolation. By the way, what are properties of the shape function which we already know? Sigma N is equal to 1 and n_i or n_1 is equal to 1 at node 1 and zero at other nodes and so on. So, we are going to use Lagrange's interpolation and we will use another type of interpolation later. But right now, stick on to Lagrange's interpolation or Lagrangial interpolation, which probably all of you are familiar with. Is it clear? Now, let us look at how to write that.

How many of you remember Lagrange's interpolation? All of you? Yeah, very good; so, why not you write it?

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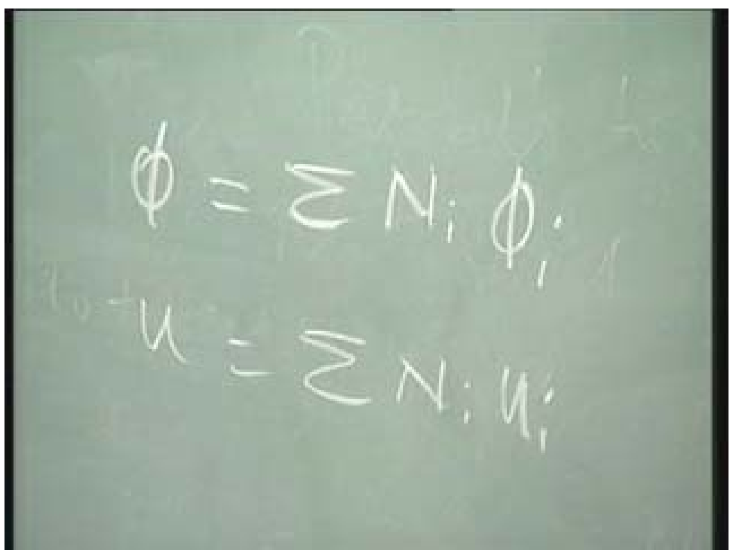

Say for example, I write phi equation is equal to sigma N_i phi_i. Deliberately I have written like this, because that phi can now be replaced by means of u v and so on, so, remember that. I know that there is lot of confusion; students get confused on looking at so many a to z, psi, eta, phi and all these things. I think you have to get used to it. In other words, when I write like this, I also mean that the displacement u can be written as N_i u_i and so on. What are these N's and what does this Lagrange's interpolation yeah, question? Is it clear? No, what I am trying say is, we are trying to write down the interpolation function without going through my earlier route that u is equal a_0 plus a_1x plus and a_2y and so on; no, I am not go through that route. I am trying to get this interpolation; after all I am interpolating the values. So, I am trying to get to this interpolation through another well known technique like Lagrange's interpolation. Is that clear?

In other words what does Lagrange's interpolation does? It passes a curve along the well known or indicated points without bothering about the, what is that? What does it not bother? It passes through these points, but does not bother about the slope to be satisfied at these points. It would coincide with the actual curve at these points as far the values of phi is concerned, but will not satisfy the gradient or slope of the curve at these points.

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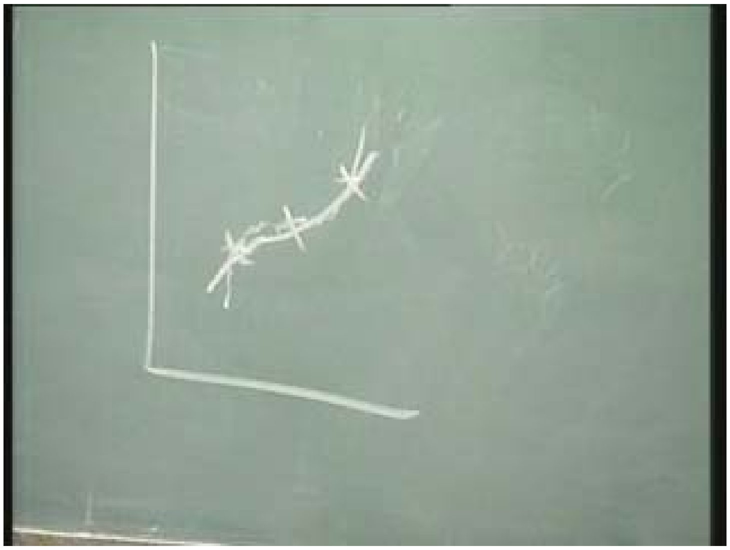

In other words, if I now plot, say for example, this is the actual curve that I have, Lagrangian interpolation, suppose these are the points which I am taking. The Lagrangian interpolation will follow, go like that. It will follow these points and it will satisfy the values at these points, but they will not make the slope of the curve that has been predicted or has been interpolated by the Lagrangian interpolation to be the same as that of the original curve. Is that clear? So, we can very clearly see that it passes through all these points, but the slopes are different.

There is another type of interpolation, which does this, where the slope continuity is also satisfied, Hermitian interpolation or Hermite interpolation, which we are going to use next case. But by the way, why am I interested right now in the Lagrange's interpolation? You would link up what all I said till now to this. No, it is not that we do not know the derivatives. C zero; beautiful, so, we are interested in C zero elements. I am not interested right now to look at, you know, continuity of the first derivatives for another problem. For example, when I go to beam problems, then I have to put continuity of the slope as well. I will give that as one of the degrees of freedom, but right now what is that I am looking at? I am looking at only C zero type of element and hence, is it clear, and hence I am only looking at a Lagrange's interpolation.

How do I now write down Lagrange's interpolation?

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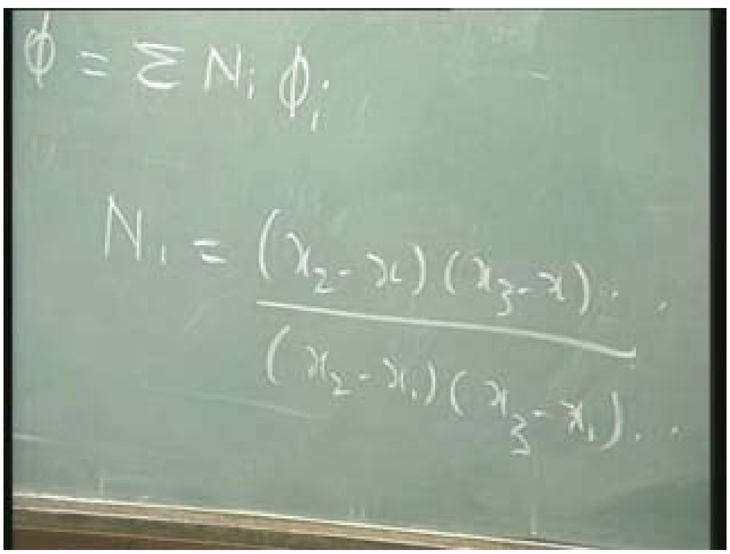

Do you remember how you write down say N_1 is equal to what? x_2 minus correct, that is it. So x_2 minus x into say x_3 minus x and so on divided by say x_2 minus x_1 into x_3 minus x_1 and so on. Is that clear? That is N_1. N_2 is what? x_1 minus x into x_3 minus x and so on divided by, in this case instead of x_1 it will become, x_2 x_1 minus x_2 and so on. Is that clear? x_1 yeah, right x_3 minus x_2 and so on. Now, I am going to talk about a much more difficult concept later in the course, but right now, let us see how we can write down an interpolation function for a rectangle.

This is simple, this is one dimension; you can write down. Suppose now I am going to talk about a rectangle, how do you combine? I am going to give two minutes, just think about it and see whether it is possible for you to combine the x and the y direction. The rectangle looks something like this.

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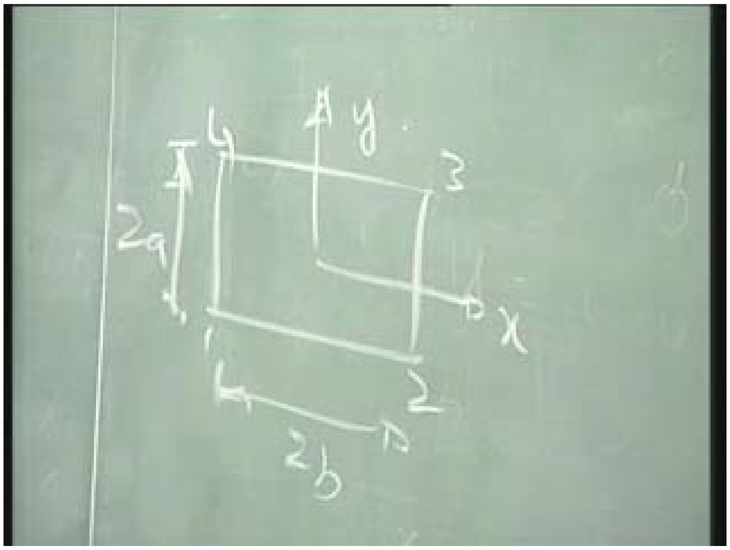

Let us say that, that is 2b, that is 2a, that is x and y; 1 2 3 and 4, those are the nodes. Just take two minutes to realize what it is? because this is so Though it is not very practical element, but there are a few concepts that we are going to get out of it. Just look at this and see how we can use Lagrange's interpolation to write down. I will give a clue. First take the side 1 4; that is this side 1 4. Write down a linear interpolation in that side; same way 2 3. Then do another linear interpolation from left hand side to the right hand side or in other words, we to a go a bilinear interpolation, linear in the x as well as y direction. Is that clear?

Let us just look at this and let me give you two minutes to do that. Slowly I will give you clue one by one, so that you will understand what we are trying to do.

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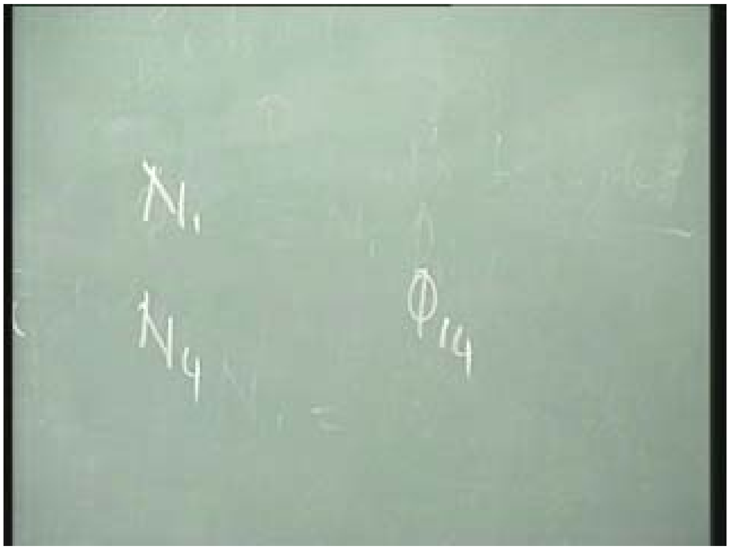

In other words, first step that I want you to do is to write down what is N_1 and what is N_4? Once I have N_1 and N_4, any field variable phi can be interpolated between 1 and 4 along y. Is that clear? Is that clear? Same way, 2 3; once I do that, let me call the interpolation along 1 4 to be say phi_{14}, along 2 3 to be phi_{23}. What does this remind you, people who have taken CAD course? Bilinear interpolation, which you would have used probably in patch; parametric patch, bilinear patch as you would have called. So phi_{14} and write down phi_{23}, then interpolate between phi_{14} and phi_{23}.

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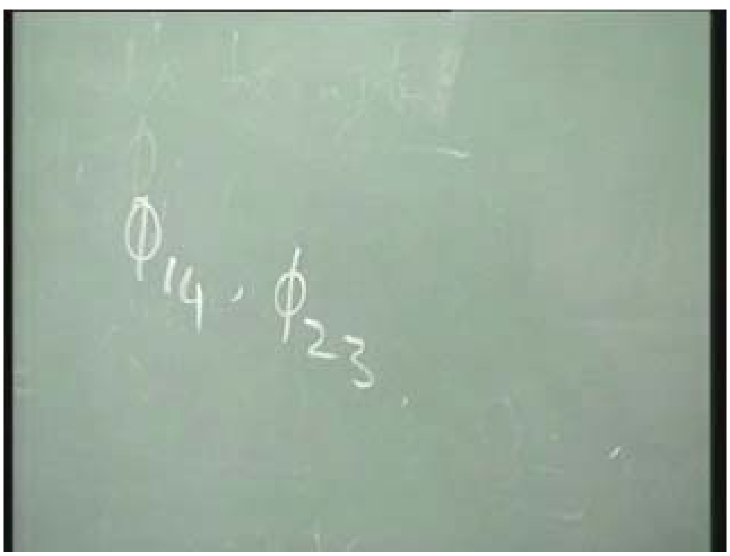

Is that clear? Now, what is N_1? Pardon; no, no. I want it in terms of b and a, ab; b minus y divided by 2b. Fine; this is 2b and I have given this as 2a and fine, let me stick to this.

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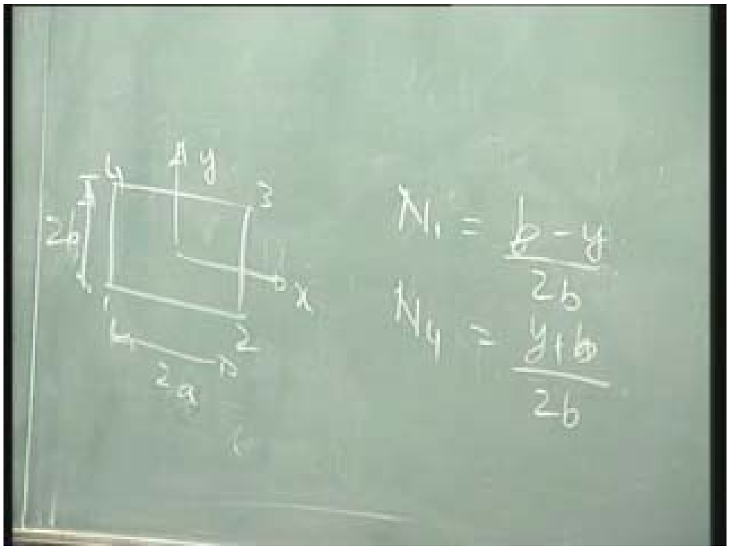

Let us say that 2a and 2b, so that I will be consistent later; y minus b divided by 2b. This comes out straight away from substituting, mere substitution in the Lagrange's interpolation, N_1. This is y plus b divided by 2b. This is 2a and this is 2b; so, y minus b by 2b and that is y minus a, sorry, y plus b divided by 2b; sorry, b minus y, sorry because minus y and b minus y by 2b, b minus y by 2b and y plus b by 2b.

What did I do? y_4 minus y; actually it is very simple y_4 minus y divided by y_4 minus y_1, y_4 minus y_1 in this case is 2b. y_4 happens to b; so b minus y divided by 2b. That is all, nothing else to it. In the other case, what happens? It is just the other way. Both of them become minus and hence minus in the numerator denominator canceled, so you get y plus b divided by 2b. So, you can write down phi_{14}. What is that you are doing? You are linearly interpolating in the 1 4 direction. Do the same thing; for 2 and 3 also you can do. Now interpolate in the x direction, interpolate in the x direction. How do I do that? How do I do that? Just look at that, just two minutes. Just see it, you will be able to understand how to do it. Is this clear? Till now it is clear that you are interpolating linearly in the y direction, is it clear? First thing you have finished that. Then, I want you to linearly interpolate in the x direction.

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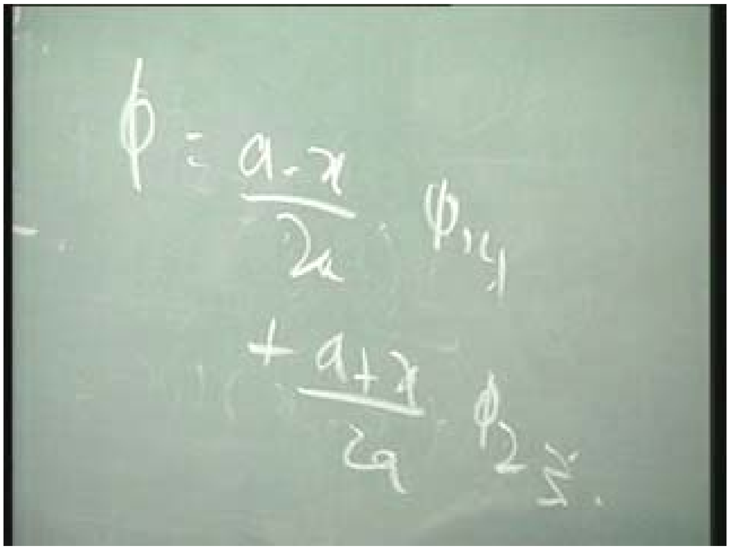

In other words, what I want to say is that I can write down phi as a minus x by 2a into phi_{14} plus a plus x by 2a into phi_{23}; that is all. It is something like linearly interpolating it here and then, again …. this. Is that clear? There will be lines which will go from, linear lines which will go from this end to that end. Is it clear, linear lines which will go from this end to that end. Ultimately what is that you are going to do? Phi_{14} you are going to substitute, phi_{23} you are going to substitute and get N. The first concept that comes out is that for this rectangular element we have used bilinear interpolation, linear in both cases, which means that it is equivalent to using up to xy.

Ultimately, what is that I want you to do? I want you to write down phi in terms of N_1 phi_1 plus N_2 phi_2 plus N_3 phi_3 plus N_4 phi_4. So, this is for local N_1 N_2, only in that direction. Is it clear, any questions? Is it clear, every one comfortable with it? It is just interpolation by Lagrange's method. What is that you get ultimately? I have not yet got your answers for N_1 N_2. So, what is N_1?

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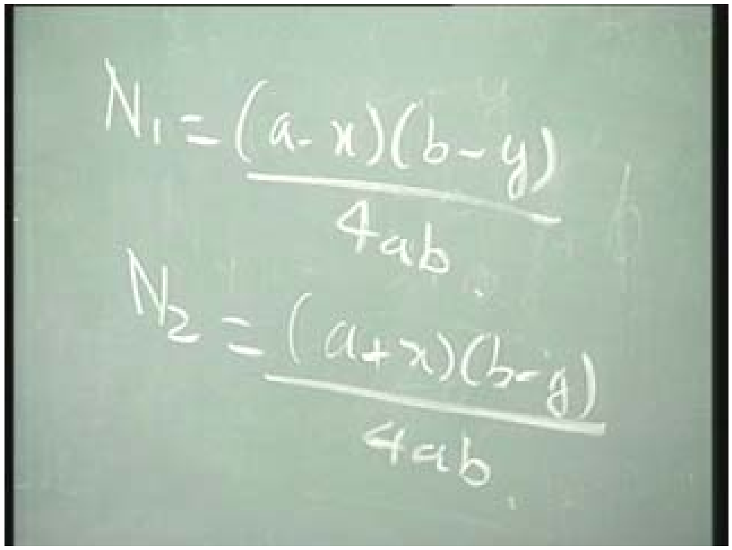

Yes, a minus x into b minus y divided by 4ab. How do I check whether this answer is right? First check, straight away check; how do I check this? Yes, very good; so, substitute for node 1, x and y values. You substitute this node 1 x and y values and what should you get? N_1 should be equal to 1. See whether you getting 1. What is this minus a comma minus b? Substitute it, you are getting 1. That is the first check. What is the other check that you can do at this stage for the shape function? It should become zero at all other positions, at all other nodes rather; not positions, all other nodes, it should become zero. Does it become zero? Yeah, straight away it becomes zero. Now, what is N_2? a; no a plus x into b minus y; correct, a plus x into, that is why I am telling these clues, just check before you answer whether you are right on target. What is N_3?

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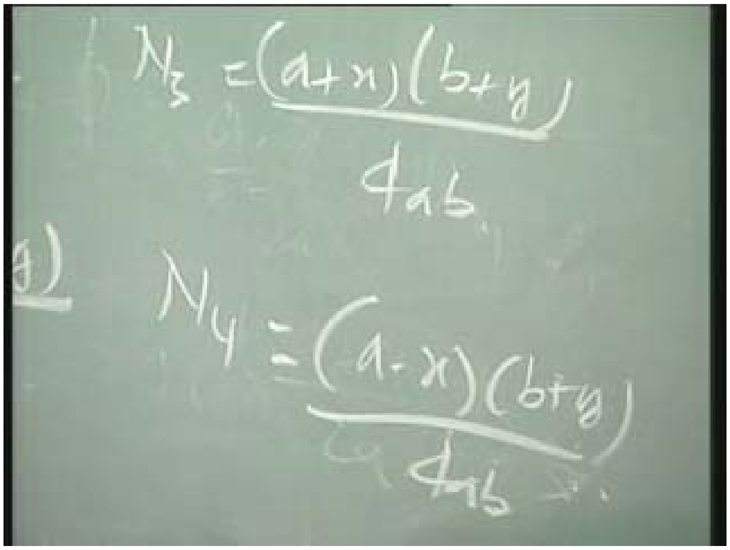

a plus x into b plus y divided by 4ab and N_4 is what? Just the other way, a minus x into b plus y divided by 4ab. So, that becomes a minus x into b plus y divided by 4ab. What is the other check that you can do now? Add them; sigma of N's should be equal to 1 and so on. This itself I am not, Yeah it is a nice way, it is a much better way than doing what is that we did? We interpolate, Lagrangian interpolation in the x and y direction I did and so on. I am happy because, this you now know that it is much easier than writing down in terms of a's, because it works faster than this for than what we did in the previous case. But I am still not very, say, happy when I want to work out. I feel that I can get a much easier method. Can someone suggest? Look at this.

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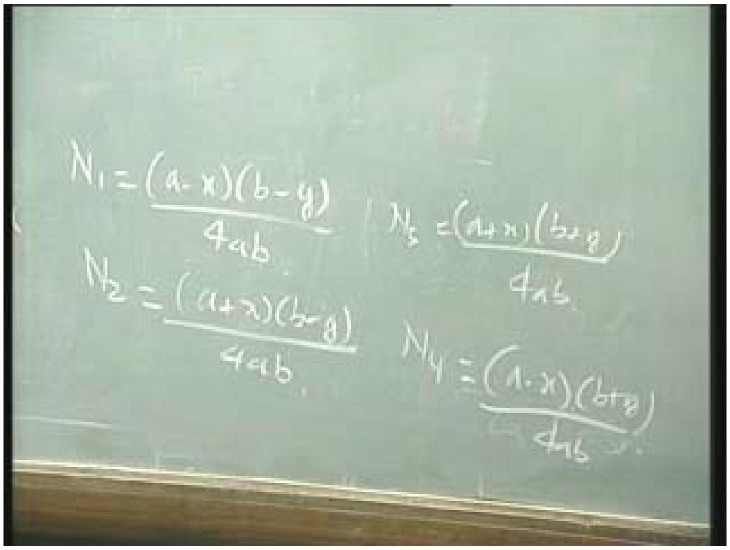

Can someone suggest a much easier method to do it? What do I mean by that? Can you use these properties of N's, can you use the properties of N's to write down the same thing by a much easier method. I will give you Do you understand? The properties of N's are well established. By inspection can I write down? What people call, finite element people call as, by inspection can I write down? One second, one second; let me give you clues. There are lot of ways, area we do not usually use for this; we use it for triangles. Let us not worry about that right now, but I will give you a clue. Do you understand? In other words, I can write down the same thing by inspection, by inspection looking at certain properties of N's. In order to do that, let me say that I can write down this line to be some equations to these lines.

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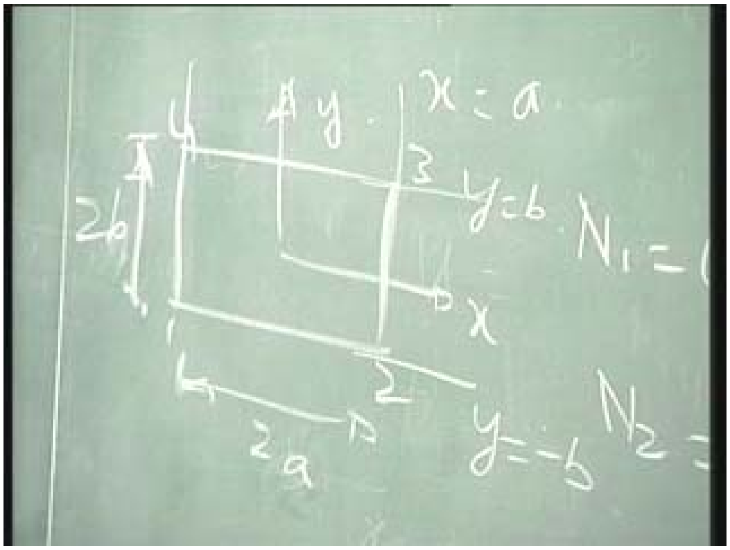

What is that for 1 2? y is equal to minus, y is equal to minus b and similarly you can write down for 2 3, x is equal to a and so on. That is your clue. I will give one minute to think about. What is that I am going to do? I want, number 1, I am going to use one property which I said that this fellow, the shape functions should be zero at all other nodes expect to the node to which it is wedded or N_i should equal to be zero, when i not equal to j, it should be equal to 1, at the node i. That is the clue. Now, think about it. In other words, let me give next clue. I can write down straight away, but I think it is nice that you think. In other words, I should have terms here in such a fashion that all the other three nodes, shape functions at those fellows should disappear. That is a next clue. Is it clear?

The third clue I am giving is that this node does not lie in these two lines. That node does not lie in these two lines and all other nodes lie in these two lines. These two lines, what is this line? y is equal to b, y is equal to b, x is equal a. If my equation involves these two lines, then obviously all other guys would go to zero, period. Is that clear? So, I can write N_1 to be, is it clear, N_1 to be some C_1 into, what is this equation? x minus a into y minus b; that is it. By inspection, I can use these properties of N to write it. Please do not forget the basis that we came from linear interpolation, that we introduced xy xy and so on so forth, all these things are there. But it is much easier to write it down by inspection, because every time I do not want to do all these things. It is easier to remember it looking at it like this.

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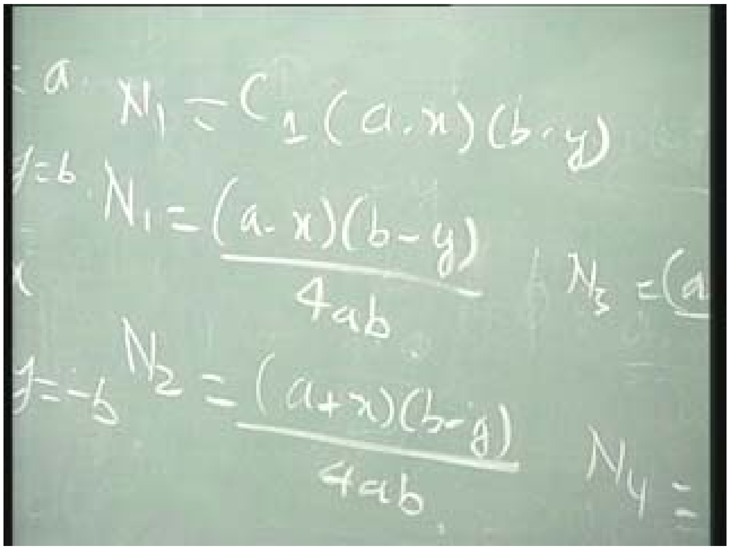

I can say that N_1 is equal to say C_1 into a minus x into b minus y is equal to N_1, the same N_1. How do I now get this fellow C_1? Period; that is it, very good. I am very happy that N_1 is equal to 1 at this x is equal to minus a and y is equal to minus b. So, you get obviously C_1 to be equal to 1 by 4ab. So, you can very easily write down this equation. Is it clear? I am going to write down in future by inspection only. I am going to just look at the properties and I am going to write down these things in future by inspection. Now, I am going to give you a small problem, a very, very simple problem, just a modification of this. Let me see whether you are able to write down the shape functions; just small modifications nothing great. In fact, you can substitute it; but I do not want you to substitute in this, but I would like you to write it by inspection. Is that clear? I want you to write down this by inspection.

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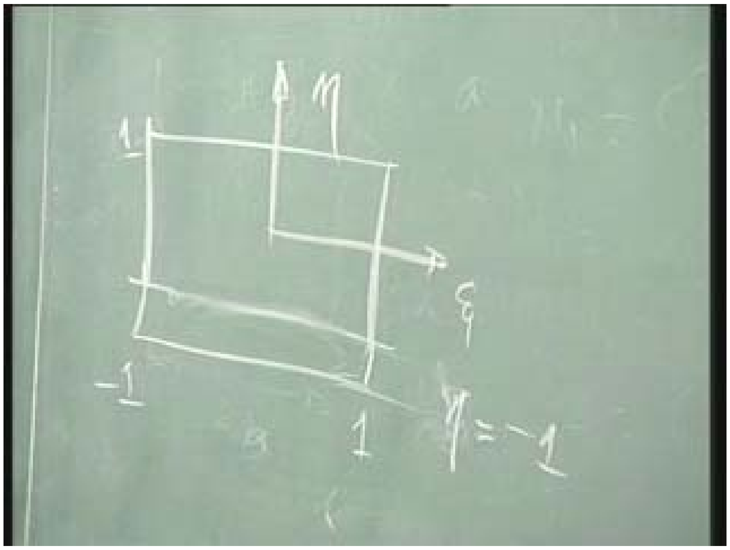

Let us take the same element psi, say, I am going to psi eta coordinates. It should be more squared and this varies from minus 1 to plus 1; minus 1 to plus 1, psi eta coordinates minus 1 to plus 1. Write down N_1 N_2 N_3 by inspection method, not by substitution; of whatever I can even substitute 2a 2b, but I would like you to write it by inspection method. What is the first thing you do? You write down the equation of all the lines in this system. What is this line? Eta is equal to minus 1 and so on.

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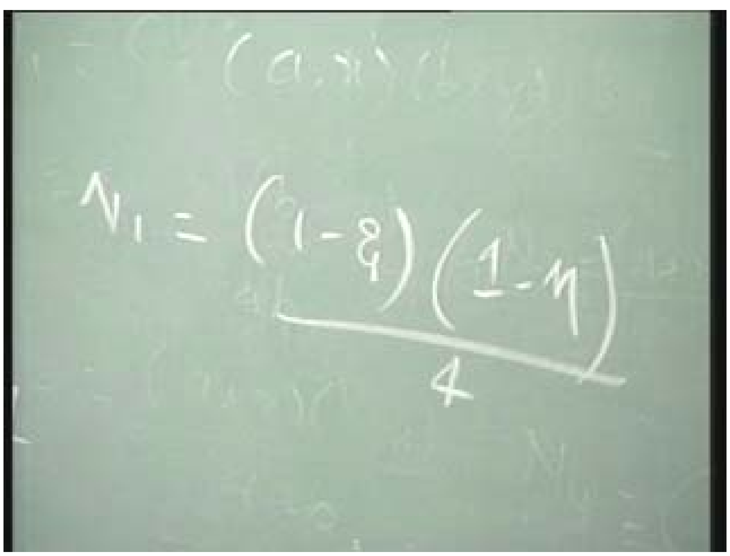

Yeah, yeah, what is that you get? N_1? What is N_1? Very simple, straight forward; but you just do it with this; 1 minus psi. Is it eta minus 1 or 1 minus eta? 1 minus eta; 1 minus eta by 4, that is it; 1 minus psi into 1 minus eta by 4. So, you can now write it down by inspection. You know, I mean I will teach you as we go for higher order elements how to do that, but right now, the concept is quite clear. I am making use of these facilities or this property of N's. Is that clear? Now, I have a problem. always we come of see How do we proceed here? We come across a problem; we try to solve it by going to next step of theory and so on. The first thing I know, in fact even before I started this class one or two people asked this, sir, if you look at the type of elements we have used yesterday also, in the last class also when we looked at the elements they were not necessarily so nice elements; they were not rectangular elements. They are quadrilateral elements and how do you now handle that kind of an element. That is number one.

In other words, all of us are worried about what we call as the integrations, integral. I know lot of people right in the first class said, Oh God we have to integrate it, triple integral and so on. I know that it is not going to be easy. It is not that you and I face; every one face this problem of integrating. It is not going to be very easy, that is number one. How do you handle shapes, which are different from simple shapes of elements like this? I cannot; obviously, when I go to very complicated geometry, I will not be able to fill up that geometry with such nice shapes. When I have automatic mesh generation, I have no control over exactly this shape of the elements.

Number 2, you have been interpolating only the field variables. You have been interpolating phi, in other words u v and so on. But, you have not interpolated, say the shape. Only if I am able to or in other words, As far as it is rectangle, it is fine. But, how are you going to introduce that shape also into the element definition? So, shape also we will interpolate. What do I mean by shape interpolation? Is it possible to say that x can be interpolated by means of the x's or x_1 x_2 x_3 x_4 of the four nodes. In other words, we now move into a realm called iso-parametric elements.

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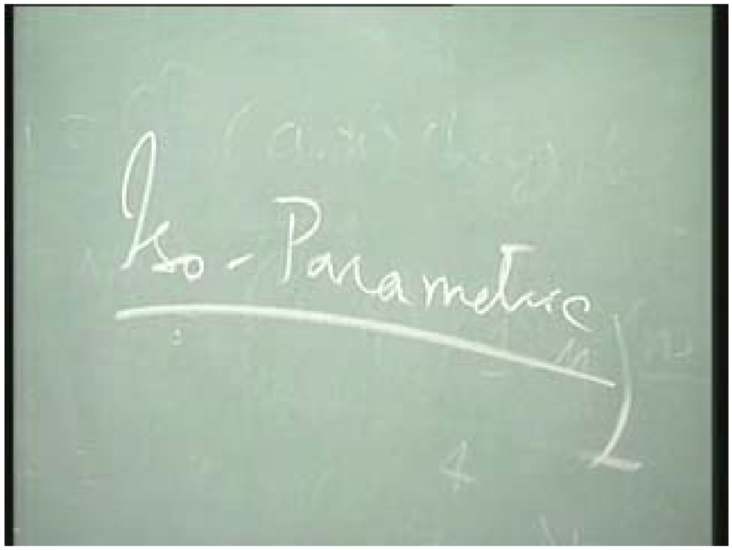

We are now into what are called as iso-parametric element. Iso-parametric element is a concept which is applicable to a number of elements. Essentially we have to understand, to appreciate iso-parametric elements, the concept of natural coordinate system. You have to appreciate the concept of natural coordinate system, but iso-parametric element is a concept which is applicable to brick element or in other words solid elements. It is applicable to plane elements, it is applicable to shell and so on; shell and may be plate. Iso-parametric element is a concept, is a generic concept. We will study iso-parametric element and why does this become important?

It is the most important element or the most important concept that has evolved in finite element analysis. The central, I would say, crux or the development of finite element analysis over the years has been possible because of this beautiful concept, called iso-parametric concept. The whole development has depended on this and the whole advantage of finite element analysis is because of the ability to handle odd shaped elements. The whole thing was possible by iso-parametric concept and it is the most important element that is used in commercial packages. Whatever element you have been handling or you have been seeing, the results of which you have seen in the past few days, have been possible because of this concept of iso-parametric elements. Hence it is important that we understand the iso-parametric elements. The concept, I know that it is a difficult concept to catch, so, I will explain that slowly in the next class, but right now, it is important to understand that the whole concept depends upon what are called as natural coordinate system, it depends upon the natural coordinate system.

So, the first thing that we are going to do is to look at natural coordinate system. What do we mean by natural coordinate system and then we will convert it, convert all these things in terms of natural coordinate system. That gives us a big leeway as far as the coding, computer coding is concerned. In other words, iso-parametric element not only makes it possible to do practical problems, but also gives a method of coding, so that your software development itself becomes very systematic. It is possible to evolve a very nice software.

We will talk about the iso-parametric concept in the next class.