Introduction to Finite Element Method

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Lecture - 13

In this class let us review certain principles of calculus of variation.

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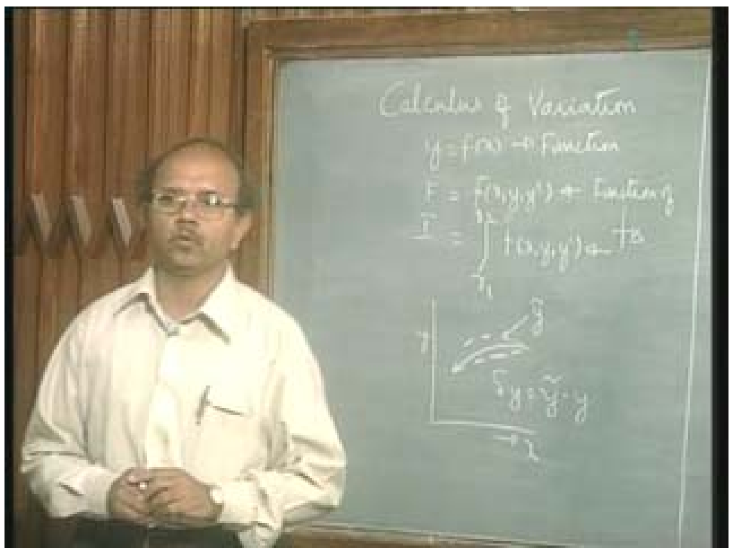

As all of you know, calculus of variation is a part of mathematics which has a lot of applications in various branches of engineering and sciences. What I have done is to get one important result in calculus of variation so that, that result can be used by us for the formulation of finite element method. Let us look at what you already know.

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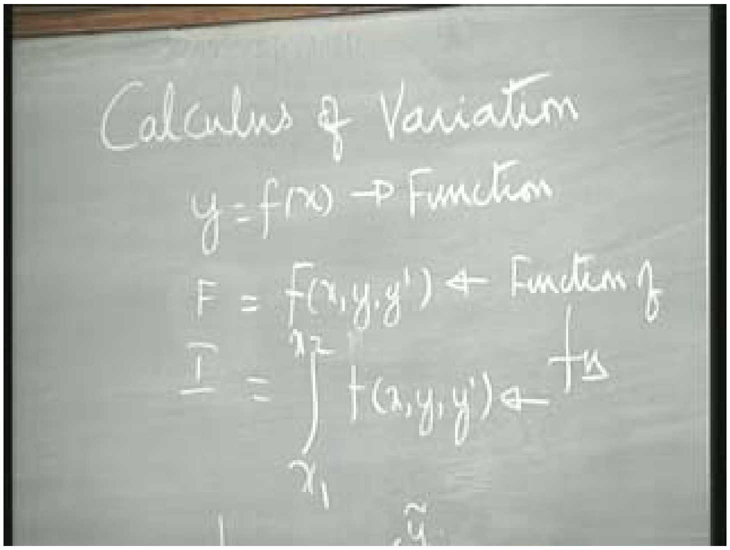

For example y is equal to function x is a function, which you would have come across in your earlier classes. Similar functions you would have come across, for example, x square plus 3x cube and so on is equal to y, is a function, which can be written something like this. You have also studied about the maximum and minimum values of a function like this; maxima, minima, even you study in high schools.

Now, let us just extend that concept; let us write some function F is equal to a function of x, y and y prime. If you look at these two functions the difference is quite clear, in the sense that in this case, you had one independent variable x and y was a function of x. In this case you have a function which is a function of functions. In fact, it can further be extended to I is equal to some integral x_1 to x_2 function of x, y and y prime. Such function of functions which is not a very correct definition for a functional is good enough for us to proceed. In other words, this is what is called as a functional; this function of functions are what are called as functional.

In the first case, what you already know, this case our whole aim was to find out that value of x at which this function becomes a minimum or a maximum, put together called extremum values. Here the situation is slightly different. In this case, our whole idea is to find out a function which would extremise or maximize or minimize a functional. What exactly do we mean by this? Suppose I have different functions of y sorry of x which is y.

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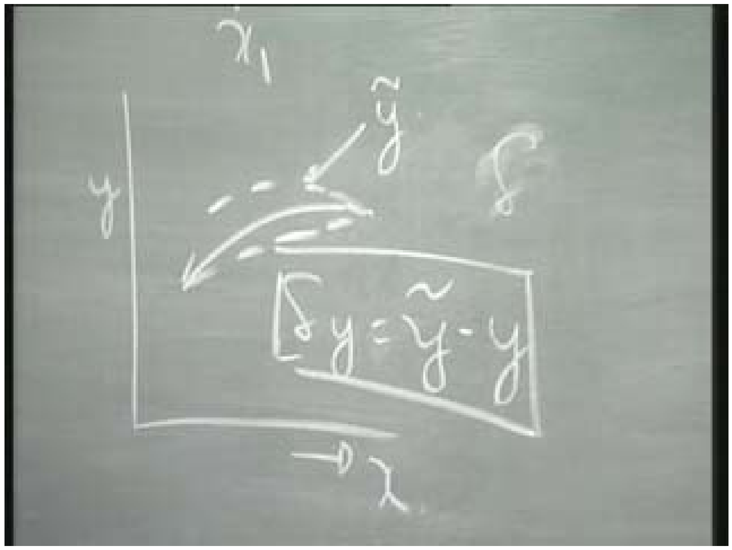

What can be that function which can give me an extremum value or in other words you can draw a number of functions like this and what is that function which will give me an extremum value of this, is our criteria. In order to find that out, like we had dy and dx, let us introduce what is called as del y. Let us introduce an operator called del y. I think I have done that already, yeah, an operator called del y. Del y is a variation in this function y. This is something like dy and corresponding dx. But please note that del y does not have a del x because we are looking at a function of x which is y.

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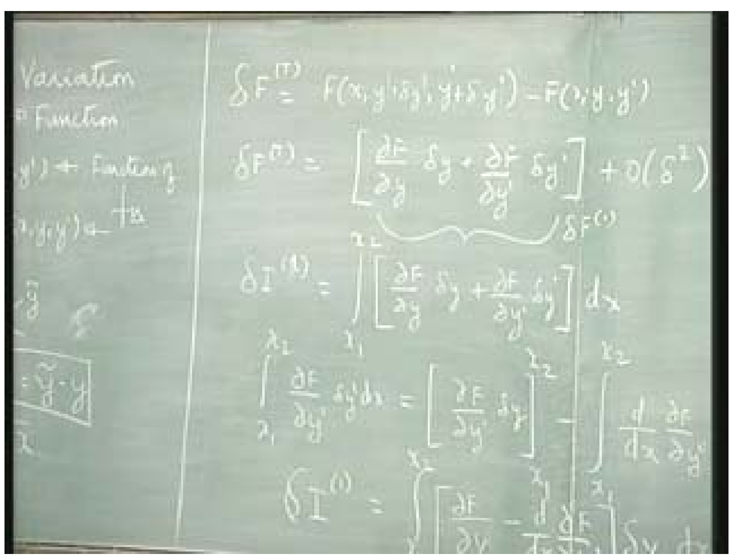

Now, let us also introduce del F; del F or delta operator as it is called, del F, what is del F? Del F is the variation of the function when my y takes the value say y plus del y or change in the function. So, del F is equal to a function of x, y plus delta y or del y and y dash or y prime plus del y prime where y prime is, you know, dy by dx minus function of x, y, y prime; that is what is del F. Let me extend del F using Taylor series approximation like what you usually do with the functions of x. The result is that you can write this as dow F by dow y into delta y plus dow F by dow y into delta y prime plus all other higher order terms.

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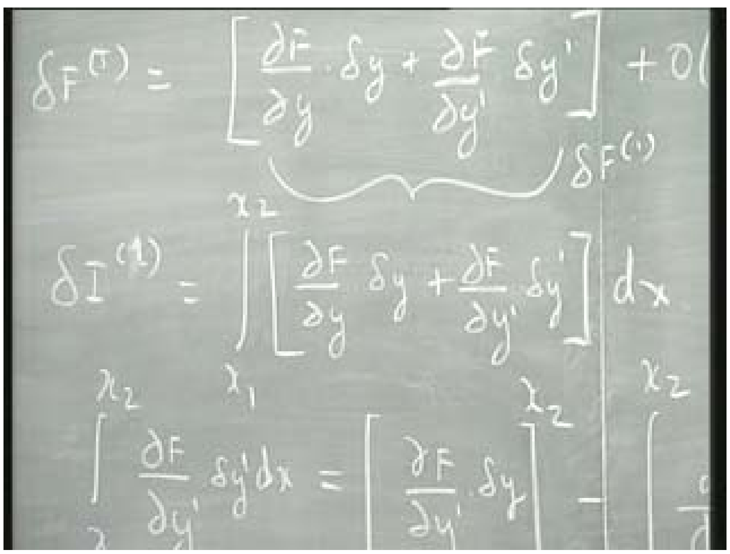

Let me define the first variation of I, delta I 1, to be the first term or involving the first term in this expansion and is written as x_1 x_2 dow F by dow y into del y plus dow F by dow y into del y prime dx. That is the first variation of I.

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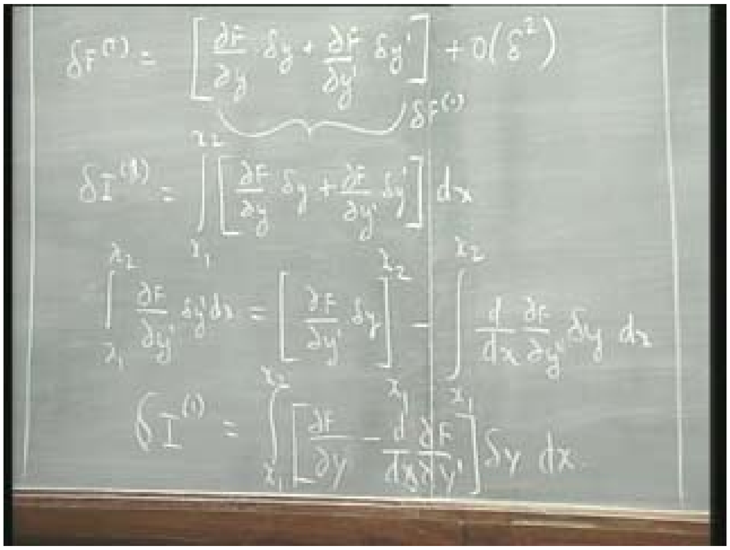

Now, let me take this second term and do our familiar operation of integration by parts. The result is that I get a function something like this, uv minus integral vdu; you will clearly know what is du or u and dv. Look at this term. That term consist of a del y. It is a variation of the function y. Let us assume that this function is such that or del y is such that it is equal to zero at x_1 and x_2. These are boundary conditions which we will come to later, when we apply this to familiar problems or in other words we assume that the del y is such that it is equal to zero at x_1 and x_2 and y takes the value, whether y or y hat or y squiggle, whatever be it, variations of y is such that it takes the values of y at x_1 and x_2. So, for the time being we can say that this goes to zero, so that del I 1 can be written in this form.

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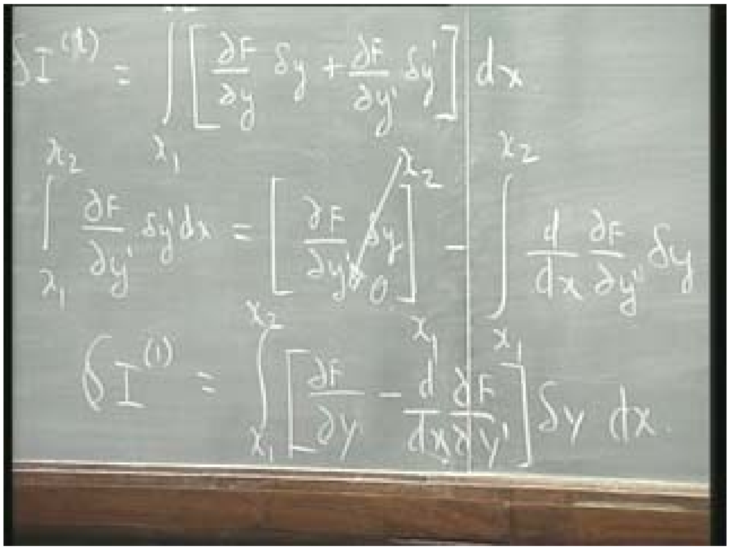

Is it clear? Del y can be, sorry, del I 1 can be written in that form. Look at this expression carefully. If I want a maximum or a minimum value of del I 1 which is my interest because del y, I want approximately equal to del I total; t means total, then the expression should be such that it should not change sign when I change del y.

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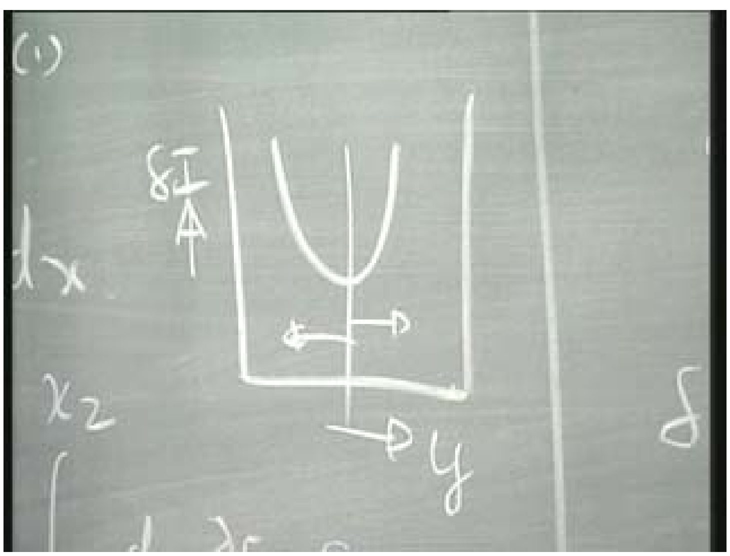

To understand this, in a very crude fashion, suppose I have a function like this and I say this is y. I cannot write it in 2D, but just to understand and I say that this is the del I function here. Suppose I am in the minimum value, whether I go this side that means del y is positive or this side when I go to negative, whatever it is, del I should be positive. That was a crude approximation just to make you understand that when I say del y takes any signs, either I increase del y or decrease del y, or in other words when I go above the line or below the line here. Let us have a look at this, this particular graph which I have drawn here then, you can see that whether you are above the line or below the line, del y, so that will show you the variation.

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Whatever be this del y, then I should be or I should have del I 1 to be to equal to zero. Then only, I would get that positive value because this is not going to be affected by changing del y, del y term is not there. So, del y is my arbitrary variation, it is in my hands. The only condition now I have put is del y is such that, because the variation of y does not happen to be present at x is equal to x_1 and x is equal to x_2. It is just a function. Is that clear? So, I am varying y, hence del I 1 has to be equal to zero, del I 1 has to be equal to zero.

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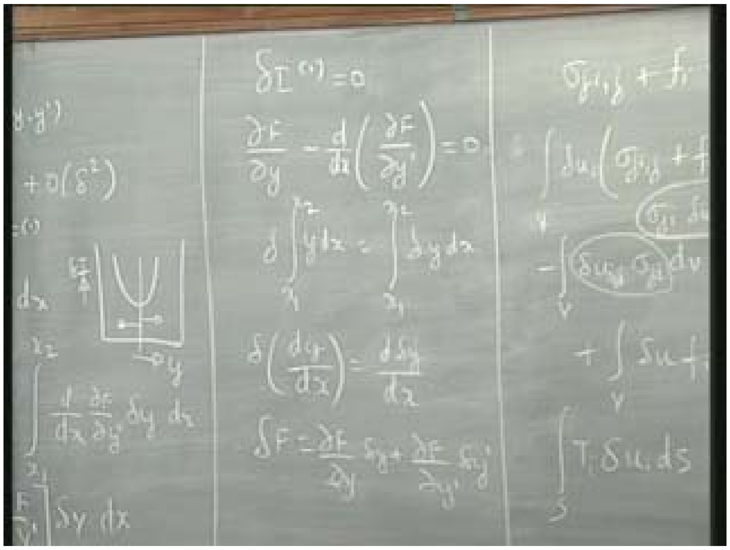

What does this mean? This means that whatever is in the bracket has to be equal to zero and hence dow F by dow y minus dow x, sorry, dx d by dx of dow F by dow y prime it should be equal to zero. Is it clear, any question? Is there any question, clear so far? This is the famous Euler-Lagrange equation. We are not going to talk much about it right now, but may be you will encounter it in other courses. That is the famous Euler-Lagrange equation. Let us summarize some of the important results that we have from calculus of variations, which will be quite useful to us. So, del of integral x_1 to, sorry, x_2 y dx is equal to x_1 to x_2 del y dx and del of dy by dx is equal to d of del y by del x dx and del F is equal to dow F by dow y into del y plus dow F by dow y prime into del y prime. They are very simple; you can easily derive them, it is not a big problem. These are important results, so, you please have a look at this carefully. These particular 5 lines summarize what all the thing that we need. Is that clear?

With this setting, let us now look at what is called as virtual work principle. I am sure all of you know what is virtual work principle? What I am going to do is Even though whatever I have done may be vague right now, just bear with me. Let me finish this calculus, sorry variational or virtual work principle, then I will apply whatever we have done now and things will become clear as to why I have written so far all this equations. But, so far whatever I have been writing till now, I hope is logical from a mathematical point of view and do not attach immediately any physical concept to whatever I have done. I will do that in a minute, but mathematically if you have any doubts on whatever I have written so far, please ask me. They are actually very simple calculus which I have just extended it a bit.

Let us look at what is called as virtual work principle.

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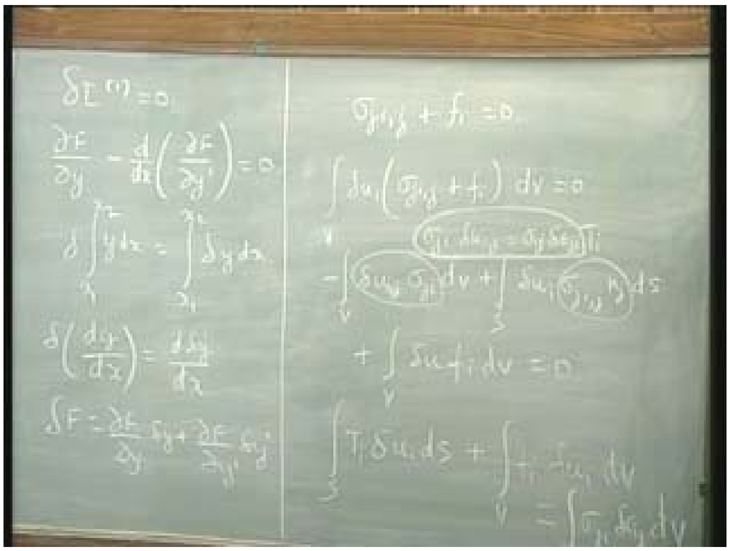

The virtual work principle starts from this first equation. What is this equation? Equilibrium equation; starts from the equilibrium equation. Since equilibrium equation says that sigma j comma ji comma j plus f_i is equal to zero, if I multiply this with any delta u what happens? That is equal to zero. So, integrating it throughout the volume will again be equal to zero. So del u is some sort of a weight function, a function. I assume that del u has only one property. It is continuous and that it satisfies the boundary condition for u, a homogenous part of the boundary condition. What is meant by homogenous part of the boundary condition?

Suppose, I put a boundary condition u is equal to some 4 or u is equal to some u_0, then del u is equal to zero at places where I have specified u. So, that is called homogenous part of the boundary condition. Since we are in the three dimensional setting that means we have del u_1 del u_2 and del u_3; note this carefully here. So, del u is a test function. Now, since this is equal to zero, the integral is also equal to zero. This is the basis for many of the approximation technique. If there is a residue here, suppose I take a u which is a displacement, which I want to find out, then it is possible that this will not be equal to zero and there will be a residue there. My whole idea can be such that I have to choose u so that integrally throughout the volume this can be equal to zero. So, that forms the basis for many the numerical terms, though very correctly, analytically, that expression has to go to zero. Is that clear?

Now, let us resort to our famous Green's theorem of application and you get a term like this.

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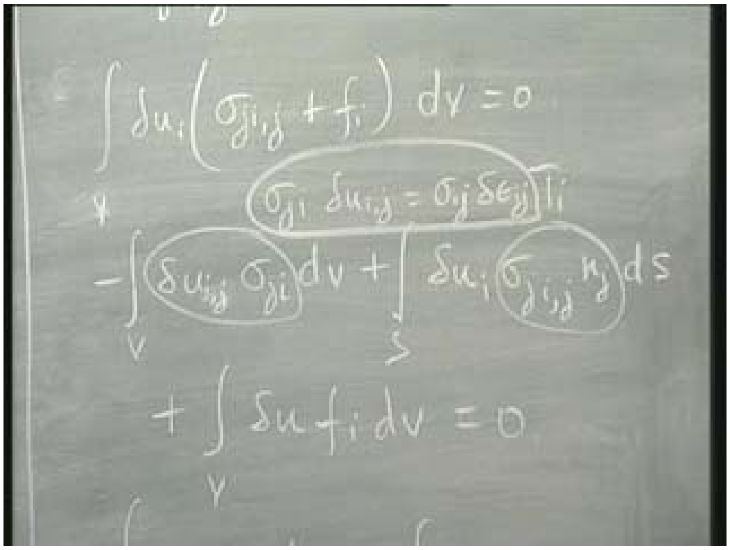

Look at this term. What has happened now? We have shifted the differential from u. Is it clear? We have shifted the differential to u rather from sigma, we have shifted it to u. So, u j comma i or j comma j, it has been shifted to del u i comma j or in other words, please note that sigma has u in it. How, because we have seen that sigma is related to strain, strain is related to displacement. Is that clear? Strain is related to displacement and hence sigma has displacement embedded in it. What is the differential or what is the order of this or of the differential that is present in sigma, first order or second order?

How is that? du by dx or dow u_i by dow x_j? First order; and then I differ in $sigma_{ij}$ or j I, I have first order. Then, again it is differentiated with respect to x. In essence this has second order d squared u by dx squared. That is what is present in that. So, what is that we have done? We have weakened, we have weakened the differential or the order in sigma by shifting it to u; by shifting it to u we have weakened. What does it mean? Suppose, I have a technique now, of searching for a solution for u. I am searching for solution for u. I need to search only those u's which need to be or which need to have the first differential. What is that we are trying to do in this whole finite element ……. There are so many other techniques. What is that we are trying to do?

We are trying to search for u in such a fashion that I can satisfy these equations; that is my whole idea. Yes; no doubt I am not going to keep on putting some u's and keep searching for it, but I am going to search it in a space. It is called search space. Let us not worry about that. There are lots of mathematics involved in this, whole of functional analysis is involved; let us not get into those details, but it should be clear that we are searching for that u which will give us a, sorry, which will give us the solution. When I now keep searching for that u, now I am quite relaxed because my condition for u has now been weakened, that I need to search u's which are such that they have or the first differential of u is only present. Is that clear?

I have weakened from the first condition to the second condition. Firstly I had to have d squared u. So, I have to search for u's which have the second differential ….. On other hand, when I come to this equation, I have weakened. The first equation is called as strong form and the second equation is called as the weak form. Further we can see that $sigma_{ji}$ or ij into del u i comma j can be written as $sigma_{ij}$ delta $epsilon_{ij}$. That is quite clear because u i comma j can be written as $epsilon_{ij}$ plus $omega_{ij}$, from which you can write that as $sigma_{ij}$ delta $epsilon_{ij}$. Is that clear? This you can do it as small exercise. Substituting it I get a final equation of this form.

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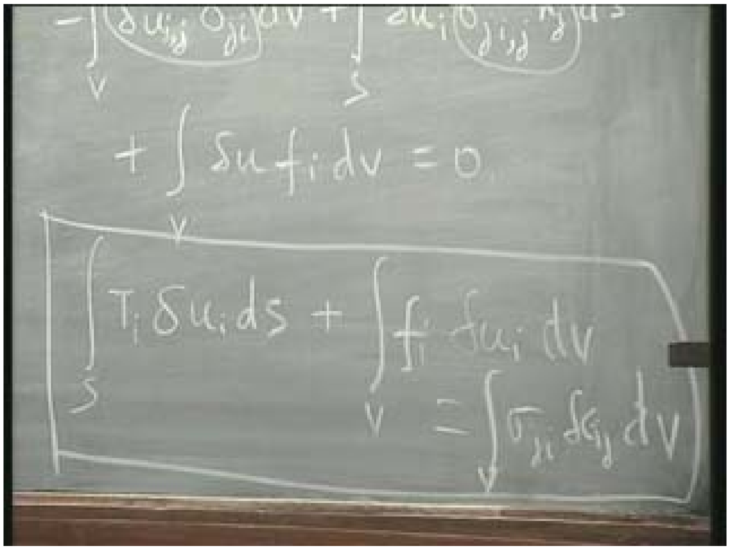

Look at this equation carefully. This is an important equation. Look at that equation carefully. I hope it is clear. the what is the now let us come to So far, it is purely mathematics search function or a function from what is called as a function space and all these thing. Let us now look at it and interpret it more physically. Let us give a name to delta u so that physically we can interpret. So far we have not given a name to delta u. It is just a function. Sometimes people call this as weight function. Hence this whole technique is also called as weighted residual method. It comes into larger class of a problem called weighted residual method.

Let us call at this point of time, more physically to understand things, let us call delta u as virtual displacement. They are not real displacements, but they are virtual displacement. You can name it anything, but right now to understand it, we are saying virtual displacement. Look at this term, first term. What is T_i into delta u_i? What is T_i? It is the stress vector acting on the surface. So, what is T_i into delta u_i? Not work, virtual work; you know, please note there is a difference. This is the virtual work; that means that the real forces act through virtual displacement. It is something like a concept which we are trying to understand the mathematics behind it. It is just a concept; conceptually we can say it is virtual work. Virtual work done by those stress vectors, which are present in the surface, wherever it has been specified. Is that clear?

What is the second term? What is the second term?

Again virtual work done by body forces. So, again it is the virtual work done by the external agencies. Is it clear? Body work done by the external body, sorry, virtual work done by the external agencies. What is this third term here is equal to this. Internal work or in other words internal virtual work done by the stresses. Stresses can be looked at as internal forces or equivalent to internal forces. Hence it is the internal virtual work. What does this whole virtual work principle states? The virtual work principle states that the internal virtual work is equal to the external virtual work. So, that is the virtual work principle. Is it clear?

There are other things we are going to talk about on boundary conditions. Right now, I am not introducing what are called as essential boundary condition and natural boundary condition. But, nevertheless I think we can have a look at this term here. Please note that the virtual work principle takes into account the stress that is specified. On the other hand it does not take into account the specification of u at certain boundary. The whole boundary can be divided into two parts. In a part of a boundary, you can give stress. In another part of the boundary you can give displacement. Is that clear? In any problem, we can give for a part of the boundary some stress or stress vector or pressure, in other words. In other parts of the boundary you can give displacement; you can specify displacement, it could be zero or some value.

Look at this here. Can someone tell me, does it ring a bell? Does it say something to you? Look at this. Now, please note that the delta u is equal to zero in those parts of the boundary condition where displacements are specified. That is correct. So, wherever displacements are specified, that is equal to zero. What does T_i delta u_i give you? It says that at this principle satisfies T because I am bringing in T here, but nowhere, I have brought in a situation where whatever is specified as u in a part of a boundary condition is satisfied; that has not been done. Is that clear?

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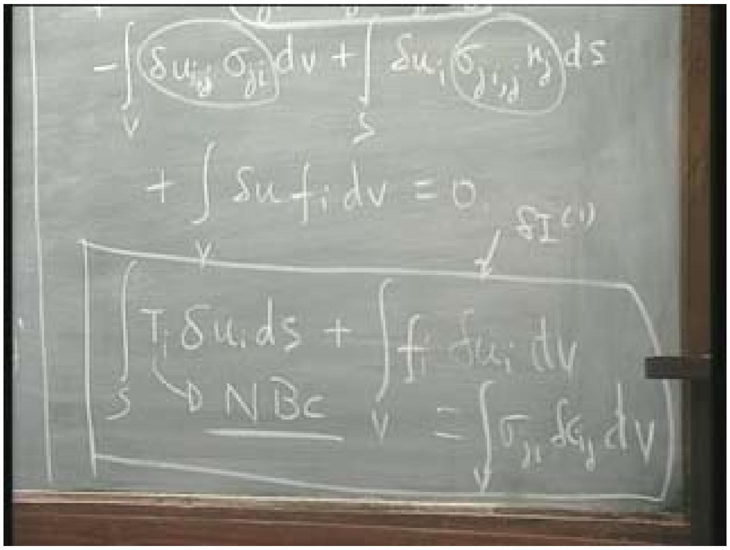

Or in other words, this formulation naturally satisfies the stress boundary condition and hence stress boundary condition or pressure boundary condition is called as natural boundary condition. This is called as natural boundary condition. Is it clear? Any question? The other one, u, which has not been satisfied in this formulation is called as the essential boundary condition. It is called as the essential boundary condition, because essentially you have to specify and the other one is called as the natural boundary condition. It is essential to solve a problem because my formulation here does not satisfy and hence that is called as the essential boundary condition.

Depending upon the way you write the equation, the equilibrium equation, the order of these natural boundary conditions would change. We will have a look at that later, in a later lecture in this course, but right now, I hope you understand what is essential and what natural boundary condition is. Is it clear? Let me call this as delta I 1. Let me call that It has delta in it. Let us say that it is delta I 1. Now, I want you to write down I 1. How do I now write this down? Is it clear? I want to write down I 1. I will do that, there itself. Is it clear? This is something like delta I 1. I want to write down u_1. What am I going to do?

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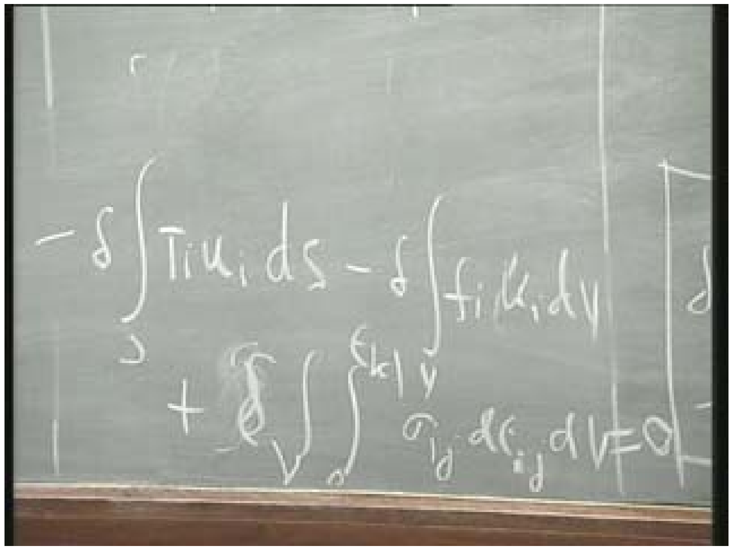

I am going to get that delta out there. I think it is better that I write it here. Delta s T_i u_i dS may be, I will put a minus there, so that the whole thing can be brought to one side. What is the next term? Minus of delta v f i sorry u_i dv plus, how do I write the last term? Delta of integral v, no, no; wait; integral zero to $epsilon_{kl}$ $sigma_{ij}$ d $epsilon_{kl}$ into dv; sorry, delta $epsilon_{ij}$ into kl, I have already put there, so ij into dv. That is actually the delta $sigma_{ij}$ delta $epsilon_{ij}$; that is equal to zero. Is that clear? What I have done is I have just taken delta out to write down the same thing using my principles, whatever I have told before; just write this down in terms of delta. What is this?

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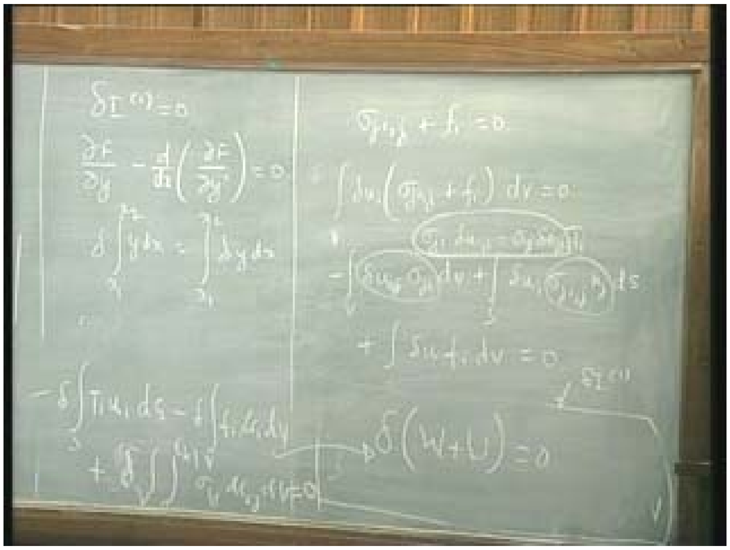

I will put it side by side so that I can say that delta W plus U is equal to zero. Before I write that a word of caution or a word to remember. What is this term, actually? What is that term? Internal energy or strain energy. More importantly, it is strain energy. For an elastic material, please note for an elastic material, this can be looked at as the strain energy. Till now I have not introduced any constitutive equation. Whatever I have done is independent of the constitutive equation. I did not say whether it is elastic or a plastic. So, virtual work principle is valid whether the material is elastic or elastoplastic. On the other hand, when I put W here, what is the assumption that I am going to make? I make the assumption that this material from now on is one for which strain energy is defined.

When I go from this statement to this statement, what is it that is involved? What is it that is involved? What is involved is assumption that we dealing with an elastic material. Then delta of W plus U is equal to zero and what is U? U is what is called as the potential lost of the external loads due to work; potential that is lost. What does it mean? It means that when the load was there without doing any work, it had potential to do work. So, as it does work, it losses the potential to do work and hence it is negative. So, one is the strain energy, the other one is the potential of the external loads to do work. Is it clear? That is why it is called delta W plus U. We will do a simple problem to understand this. Both of them put together is called as potential energy.

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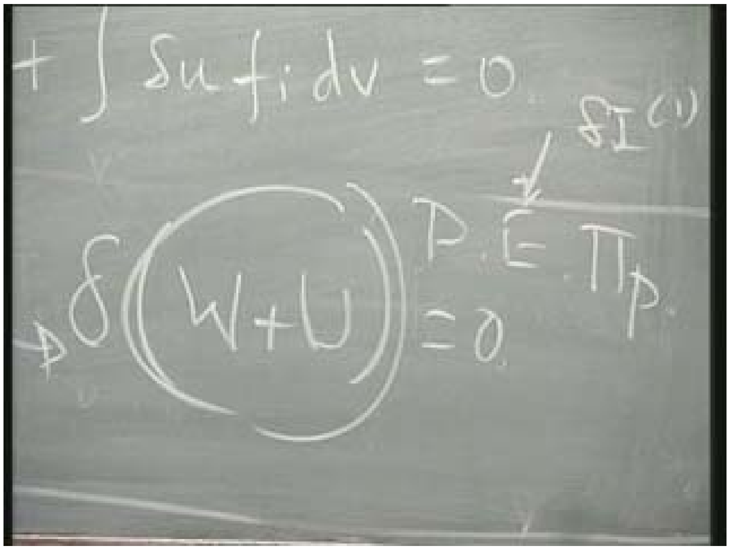

This whole thing is called as potential energy, usually put down by the symbol pi_P. Is it clear? Delta pi_P is equal to zero. That is the potential energy theorem or pi_P is equal to a constant. What does it say? It says that, potential energy theorem states that of all the admissible displacements, admissible we have already defined - continuous and so on, whatever we have given to delta u that is admissible displacement; of all the admissible displacement one which makes the potential energy take an extremum value because delta pi is equal to zero means what pi_P takes a extremum value, in this case a minimum value. But let us call this as an extremum value. Of all the configurations that which makes delta pi_P is equal to zero is the one which satisfies the equilibrium equation.

Stability concept is different. It satisfies the equilibrium equation, because we have come all the way from the equilibrium equation. Is that clear? This formulation is another way of defining an equilibrium equation. Is it clear? Yeah, any question, you can ask me. This is the potential energy theorem. One is the potential lost. What is this? We will just come in a minute. Our pi_P is equal to a constant. pi_P is equal to constant and delta pi_P is equal to zero. Now, the equilibrium equation can directly be solved by using certain numerical techniques. What is that ultimately we are studying? We are studying numerical techniques. So, the first equation, the equilibrium equation can be solved directly by certain numerical technique.

What is that numerical technique? Finite difference method; finite difference method attacks directly the first, say, you would have probably done in earlier numerical analysis courses, simple finite difference method. Finite difference method goes and attacks the first equation. Is that clear? Let us look at the second equation, the second equation, where I have shifted now the differential; order of the differential of differentiation or order of the differentiation that is attacked by, the second step is attacked by the finite element method.

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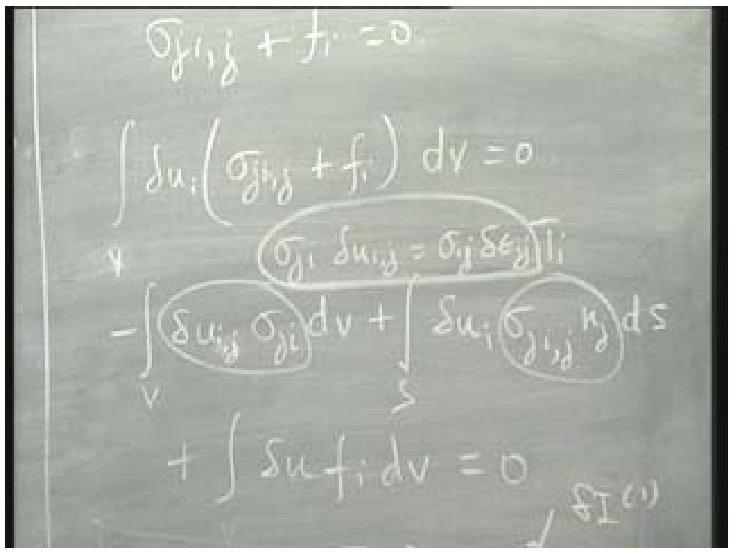

If I now go and solve this part of the equation or this restated equation of my first equation, virtual work principle at that stage then, I say that the numerical analysis falls into the category of finite element method by shifting one. I can go ahead and shift this, this differential, this differential order of differentiation again. By shifting it, by substituting for sigma and then putting del u i comma or I mean, I can go to del u squared term, a second order differential of del u, in which case I go into what is called as a boundary element method. If I again apply at this stage another Green’s theorem, then I go into boundary element method. We are not going to talk about boundary element method, but we are going to stick to, in our lectures, we are going to stick to the methods which are used to solve this problem.

So, to summarize straight again delta pi$_P$ is equal to zero. pi$_P$ is defined as the strain energy plus the potential lost due to the external of the external loads; potential lost by the external loads rather. That is the potential energy and del pi_P is equal to zero or pi_P is equal to a constant. Let us do a simple problem to illustrate this particular technique. Now, let me do a simple problem. I will just rub off all this. I will stick to this part of the board, because it is easier to look at this.

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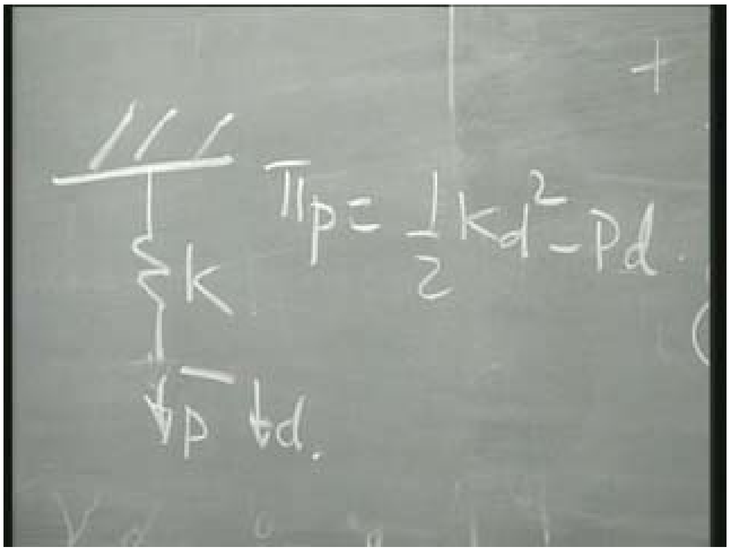

Let us say that I have a spring and I have a force P. K is the stiffness of the spring. Let us say that the spring has a displacement equal to d. My first step is to write down what pi_P is for this spring. What is pi_P? What is pi_P? pi_P consists of two terms; common, you can answer very simply. Kd squared, d is the, half Kd squared. That is the first part. That is only W. What is the second part? Is it plus? I told you, potential lost, minus Pd, because as the spring expands, the potential you can image something like this; the potential of the external loads to do work reduces. One thing; it is my personal opinion that the whole thing is actually mathematics, the whole thing we have arrived at purely from a mathematical point of view, pi_P.

The problem with engineers is that they cannot follow mathematics unless you tie it up with some physical things. In other words people do not accept complete mathematical solutions without any physical meaning. Unfortunately it is possible to do it and my whole aim ultimately is to get a u. But people do not really appreciate all the steps in mathematics. So, what they do is, common you tell me what the physical meaning is. I know, many professors would have asked you in your exams or in your viva voce or whatever it is, what is the physical meaning to it.

The problem is This is the problem with every engineer. We cannot imagine, especially mechanical engineers cannot imagine or cannot do without imagination, because we deal all the time with gears, lathes and so on and so forth; so, we have to imagine things. Actually, since I want to give physical picture to it, hence I use words like virtual work, potential energy. If I really go and ask you, do you understand what potential energy is, whatever I have given here, I do not think you would have understood. I do not think you know you can explain; yes, it is energy. Somehow it has got into us that we understand energy. But if I really ask you what is energy and if you go on, you know, philosophically arguing, then I am sure that it is not as physical as a displacement which you can see.

This potential When I say potential energy you are all happy, because I have brought in one energy term which you think you have understood very clearly. Energy, yeah, I know energy; I have been studying this from my school days, so I understand. Actually everything is mathematics. Hence when I say potential lost I ……. lost, really explain this What is potential lost? Actually what happens is I have a minus term, minus PE term, but you are asking for physics. Yes, it is also useful for you to write down simple equations. Hence I say that look, when there was no displacement, there was no displacement, the force that was there as soon as I apply, it had a potential to do work. When I get it displaced, then it loses its potential by an amount Pd. Hence I say that it is equal to minus Pd. Is it clear? It is just a convenience, so minus Pd.

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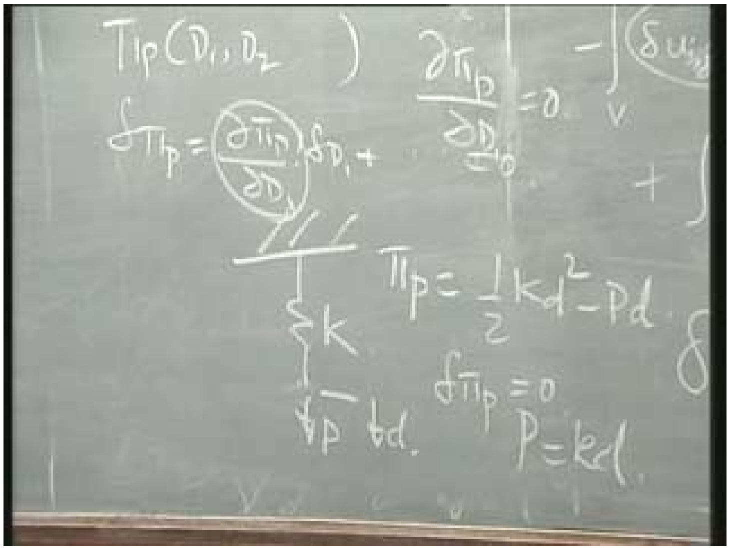

Now, I have to write down del pi_P is equal to zero. What is del pi_P is equal to zero, which means that the del pi_P can be written as, note this again, let us just look at it; say, pi_P say is a function of say D_1 D_2 and so on. It is a function of functions. What is this D_1, D_2? They can be displacements; they can be displacements or they can be something else. pi_P says is a function of D_1 D_2 D_3. So, del pi_P can be written as dow pi_P by dow D_1 into delta D_1 plus so on. This is same as del F. If you remember what I said earlier in the class, del F is equal to dow F by dow y into delta y. What is D? D is a displacement. Displacement can be a function of say x and so on. So, dow pi_P by dow D_1 into delta D_1 plus dow pi_P by dow D_2 into delta D_2 so on and so forth, so this is equal to zero. Since Del D_1, this is del D_1 or delta operator D_1, what are they? They are my perturbations to D_1. If this term has to be equal to zero, then independently these terms have to be equal to zero.

In other words dow pi_P by dow D_i should be equal to zero for every i. Then only actually del pi_P 1 or the first variation of del pi_P could be equal to zero. Is it clear? With this and this, tell me what happens? How do I write now del pi_P is equal to zero for this expression? Dow pi_P by dow D; fantastic. Dow D is equal to zero and what is that you get? P is equal to, yeah, what happens ultimately is an expression that you know quite well, P is equal to Kd. What is that? Equilibrium equation. Now, please note again you know, one more caution; please note that student makes a mistake. They always get carried away by certain energy theorems without understanding completely. What is that they say? What they feel is that there is a strain energy, there is a potential energy or potential lost due to external load, so, both of them should be equal. Whatever is gain in the strain energy should be equal to this. So, usually what they do is pi_P is equal to zero and so half Kd squared is equal to Pd. This is what they straight away usually put.

But please note that what I have written here is actually the change; change of one is the gain of the other. That is why it is del u is equal to minus del W or vice versa. Is it clear? It is like when a ball falls down, the change in potential energy is actually the change in kinetic energy, so, it is a change. Pi_P is equal to a constant. Please note that carefully or it is the total energy of the system. It is the change that matters and that is what gives raise to my equilibrium equation, P is equal to Kd. Is that clear? So, this is now going to form a basis, both the virtual work principle, what is this? This is another statement. This is called a variational or a functional statement.

I can either start from virtual work principle to arrive at equations of finite element analysis or I can start from potential energy theorem. But, when I want to define potential energy theorem, I am restating it, that it is valid only if the energy term like strain energy exists or else you cannot state this as a potential energy theorem. Because when I wrote $sigma_{ij}$ d $epsilon_{ij}$ is equal to W that is valid statement, only when there is a potential energy that is defined to it. Is that clear? But virtual work principle, I can start off from virtual work principle, what does it mean? It means that delta pi_1 is equal to zero is nothing but the virtual work principle. So, virtual work principle as well as potential energy theorem leads to the same result for a material which is elastic, for which strain energy is defined. Is that clear? Any question? If there are no questions, let us do one more problem and understand this more carefully.

The problem which I am going to do is the same as what I did in one of my earlier classes. What was the problem?

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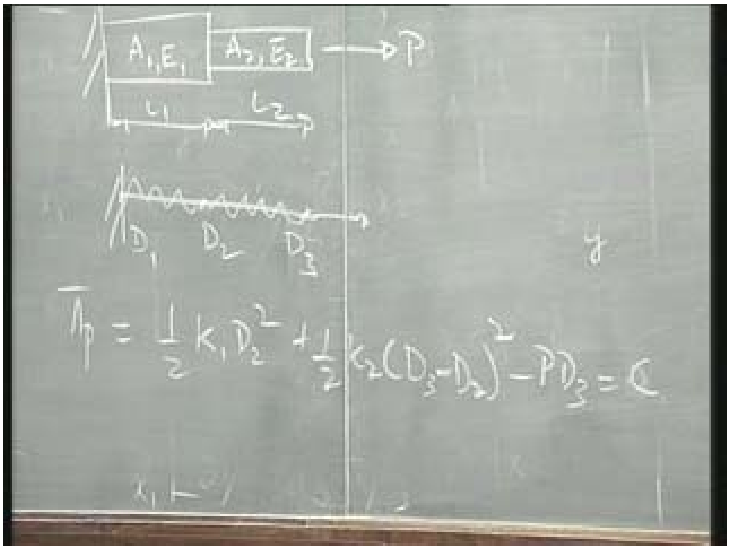

Remember this problem? L_1 L_2, if I have changed anything, tell me. I think I had put all only L_1 L_2 and A_1 E_1, A_2 E_2; K_1 and so on, we had already defined. Let us say that this thing is defined by the same fashion, by degree of freedom. We are not going to introduce elements and all that. Let us say that we are defining the movement by degree of freedom. This can be looked at like two springs. If you are more comfortable with it, say that they are defined by two springs. My whole idea right now, is to write down pi_P for this. How do I write down pi_P for this? How do I write down pi_P? What is the strain energy? Half K_1 into D_2 square; that is the displacement at 2 plus half K_2 into say D_3 minus D_2 squared minus PD_3 is equal to, sorry, is equal to constant. Is it clear? We have 2 or 3 more steps. What is the step now? Dow pi_p by dow D_2 is equal to zero, dow pi_P by dow D_3 is equal to zero. By doing these two, I will get two equations, solving which I can get D_2 and D_3.

We will continue this problem in the next class, but meanwhile if there are any questions, I can answer them. Do you understand over all concept? These things will become clear again when we do another one or two problems. I am going to do two problems in the next class to drive home this point, because this is the crux of this whole course on finite element analysis, because from here we are going to develop the stiffness matrix. We will do that in the next class.