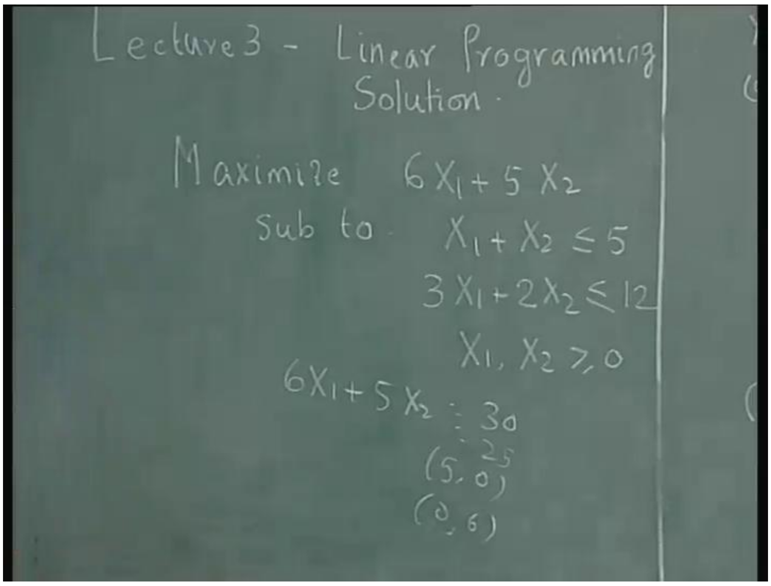
Fundamentals of Operations Research
Prof.G. Srinivasan
Department of Management Studies
Lecture No. # 03
Indian Institute of Technology, Madras

Linear Programming solutions - Graphical and Algebraic Methods

Today we shall look at Solving Linear Programming problems.

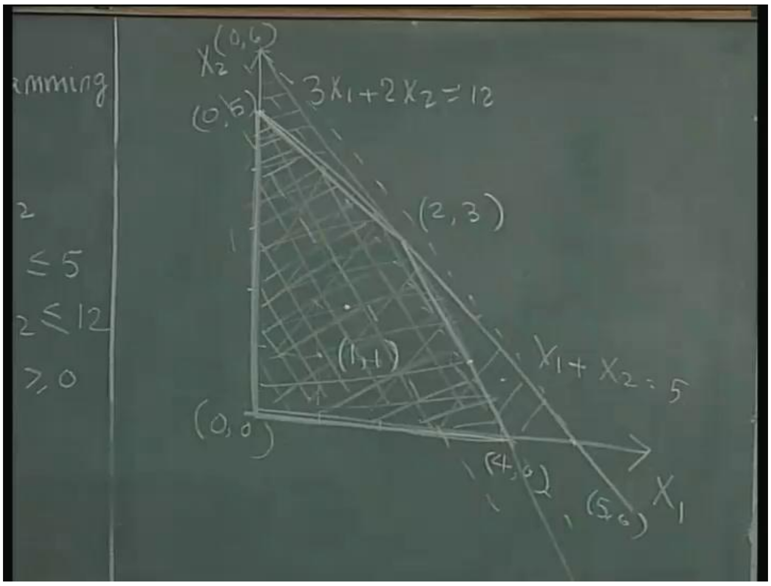
(Refer Slide Time: 01:23)



We shall start by considering a simple example and then see the first graphical method to solve linear programming problems. The problem that we will take up will include maximizing, (i) $6X_1 + 5X_2$ subject to (ii) $X_1 + X_2$ less than or equal to 5 (iii) $3X_1 + 2X_2$ less than or equal to 12 and (iv) X_1 and X_2 greater than or equal to 0.

You may recall that this is the problem that we formulated in our first example. We will first look into the graphical method to solve this. I am sure you would have learnt how to draw graphs corresponding to inequalities earlier.

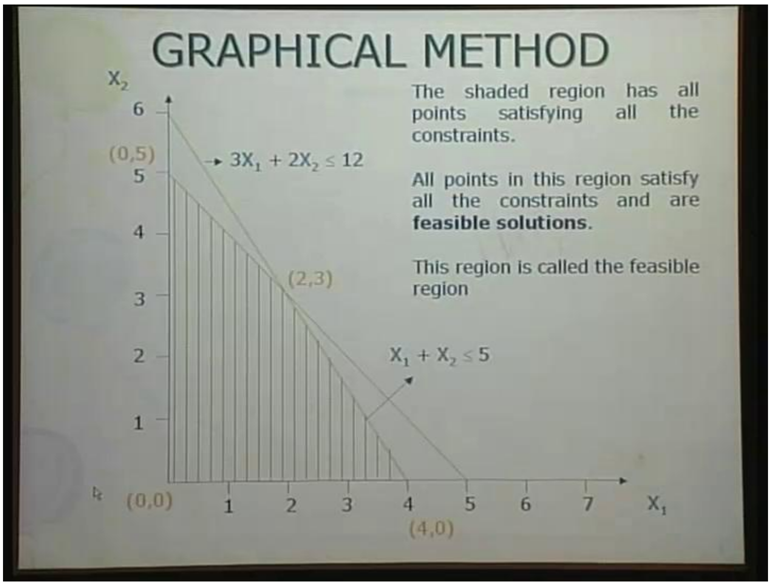
(Refer Slide Time: 02:20)



Let us first try to represent both these constraints in the form of a graph. We call this $X_1$and X_2. We are only looking at the first quadrant because we have both X_1 and X_2 greater than or equal to 0. So we will not look at the remaining three quadrants. We will first try to represent $X_1 + X_2$ less than or equal to 5 in the form of a graph and we first draw $X_1 + X_2 = 5$. We now try to identify two points that are on the line $X_1 + X_2 = 5$. So the two easy points would be (5, 0) and (0, 5). So we draw the line this is $X_1 + X_2 = 5$. Now this line divides the quadrant into two regions. One, which will be less than or equal to 5 and the other greater than or equal to 5. The origin (0, 0) (0 being less than 5) would mean that this is the area which is $X_1 + X_2$ less than or equal to 5. So we just mark the region $X_1 + X_2$ less than or equal to 5. Now we try to plot the other one, $3X_1 + 2X_2$ less than or equal to 12. So we first plot $3X_1 + 2X_2 =12$. So two corner points that we can think of are (4, 0) and here this is (5, 0) and this is (0, 5).

The other one is (0, 6) which lies on the line $3X_1 + 2X_2 = 12$. So we draw the line joining this. So this is the line $3X_1 + 2X_2 = 12$.This (Refer Slide Time 04:23) is $X_1 + X_2 = 5$. Again this line divides the first quadrant into two regions. One which is less than or equal to it and the other greater than or equal to it. So once again we look and evaluate $3X_1 + 2X_2$ at (0, 0) and realize that this is the portion that is towards the left of it and this is less than or equal to 12. So this is the region which is $3X_1 + 2X_2$ less than or equal to 12.

(Refer Slide Time: 04:53)



You can see this on the graph sheet as well here. You will find that the first one is between (5, 0) and (0, 5). The other is between (4, 0) and (0, 6) which is shown here. The shaded region shown in this figure is actually the region that is common to both these constraints. This (Refer Slide Time: 05:32) constraint as well as the other is shown here (Refer Slide Time: 5:35) in the shaded form. So in this graph you will see that this represents the region which is common to these two. So these two constraints are now represented by this region which is shown in this graph. This is the point of intersection which is (2, 3). Now let us look at this region. Now every point inside this region satisfies both these constraints. Now for example if we take a point, for example (1, 1) which is inside this region, then Now (1, 1) satisfies both these. (1, 1) has an objective function value of 6 + 5 = 11. Now we are interested in trying to maximize the value of the objective function.

So what we can easily do is if we are considering a point (1, 1) here, you could always move either in the right or in the upward direction or left or downward direction and start moving. You can try to see if there are better points than the point that we have chosen. So if we take (1, 1) because both these coefficients are positive, then it is possible either to move in the right hand side direction or in the upward direction to get points which are better than (1, 1) with respect to the objective function $6X_1 + 5X_2$. Every point inside this region satisfies all this. We also know that every point outside this region violates at least one constraint. For example a point here (Refer Slide Time: 07:18) would satisfy $X_1 + X_2$ less than or equal to 5 but would violate $3X_1 + 2X_2$ less than or equal to 12.

So our region of concern and interest is only this shaded region. So if we now try to get better points by moving either to the right or to the above then we realize that the farthest we can go is actually up to the boundary of this region. To put it simple terms, for any linear programming problem, for every point that is inside this region, there is always a point on the boundary which will dominate that point with respect to the objective function. If there is a negative coefficient then you would move either to the left or below depending on whether it is for X_1 or X_2. If there is a positive coefficient then it is always advantageous to move to the right or move to the top to find a point which is on the boundary.

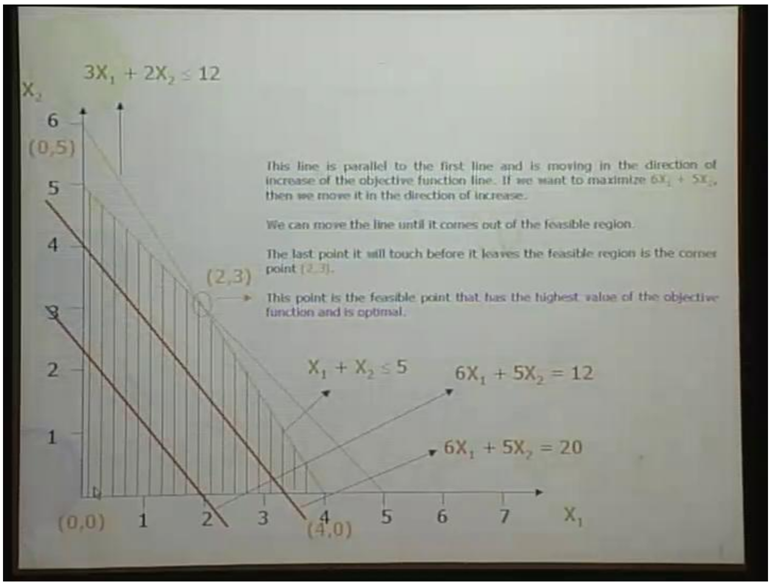
Therefore we can now say that even though every point in this region satisfies these two constraints (because we have an objective function which is linear), you will have a situation where we are interested only in the boundary points. For every point inside this region there is always a point on the boundary which will be better with respect to the objective function even though all these satisfy the constraints. So our focus immediately shifts now to only looking at all the points which are on this boundary rather than points which are inside this visible region. Now let us look at a point on the boundary for example we consider this (Refer Slide Time: 08:53) point Now with respect to this point. If we draw this objective function line, slope of the objective function line being different from the slope of this line, we can always show that a point which is above or below will Now be superior with respect to this objective function. Therefore what will happen is, for example is if we try to draw the objective function line $6X_1 + 5X_2$ say equal to some number like 30, (a convenient number), then we end up getting two points (5, 0) and (0, 6). This is (5, 0) and this is (0, 6) (Refer Slide Time 09:26). So this is the objective function line. If we draw the objective function for a value like say 25 then you will find another line which is parallel to this which will have a value of 21.

Now it is always possible to show this band since these slopes are different for every point on the boundary, there will always be points which are above or below lying on the line which will be better with respect to the objective function. So this brings us to the point that we are now not interested in any point that is on the boundary but we are only interested in the corner points. Now the ones in this example has four corner points therefore the corner points are the points that will dominate every point.

Alternatively, for every point that is inside this region, it is always possible to identify one corner point which will be better with respect to the objective function. So the four corner point that are of concern to us are (0 0), (4 0), (2 3) and (0 5). Now the moment we identify the corner points of this region, all we need to do is to find out the value of the objective function for these four corner points. Now for the point (0, 0) the value of the objective function is 0. For the point (4, 0) the objective function value is (4 into 6) + (5 into 0) which is 24. For the corner point (0, 5) the objective function value is 25. And for the corner point (2, 3) the objective function value is (2 into 6 = 12) + (3 into 5 = 15) which is 27. So among the four corner points that we have, this (Refer Slide Time 11:17) corner point has the maximum value with respect to the objective function and since we are trying to maximize, we realize that this gives the best solution for this example. We can also get to the solution in an indirect way for example, if we draw the objective function line (for instance, we have already drawn it for 30 and 25) for say 15 then we will have the objective function line once again parallel and l it would be something like this (Refer Slide Time :11:49). Now we are interested in maximizing our objective function. So we want our $6X_1 + 5X_2$ to try and take as big a value as it can.

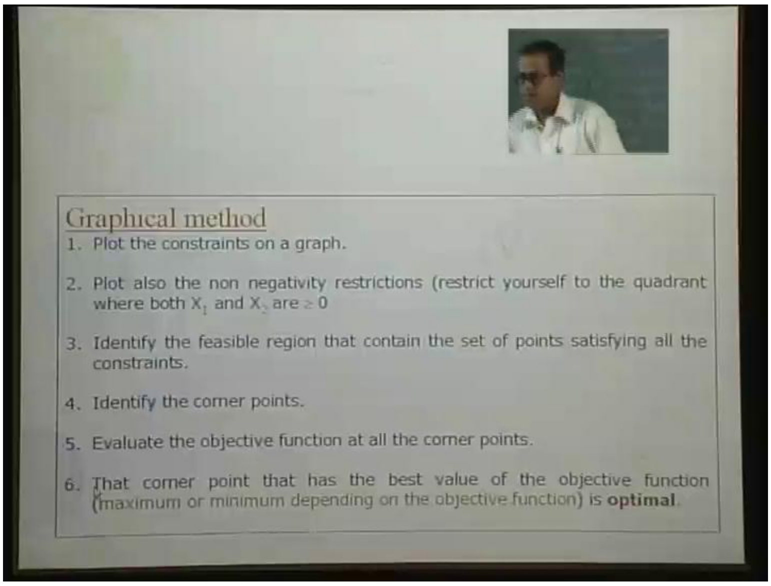
So if we try to increase the value of the objective function from say 15 and above you will see that this line is actually moving parallel to itself but in this (Refer Slide Time :12:13) direction. Now as the line moves in this direction, it will at some point leave this region and go away. When it when it actually leaves this region, it will touch a corner point and will go away. The corner point which it touches last before it leaves is the region is the best value. Hence once again you can show that the same point (2, 3) is the point. As this line moves parallel to itself the last point of the region it will touch is (2, 3) and this (2, 3) is the best solution for this problem. So this is how the graphical method works. But let us also try to introduce some terminologies using the graphical method.

(Refer Slide Time: 12:45)



So if we go back here, you will see the same thing that is happening here. You will see that the objective function is plotted for values say 12 and 20. You can see that the 12 line Refer Slide Time: 13:06) is here and the 21 that parallel is here. As it moves you will see this is the feasible point that has the highest value of the objective function and is optimal. We will try and define the terms, Feasible, optimal and so on.

(Refer Slide time: 13:11)

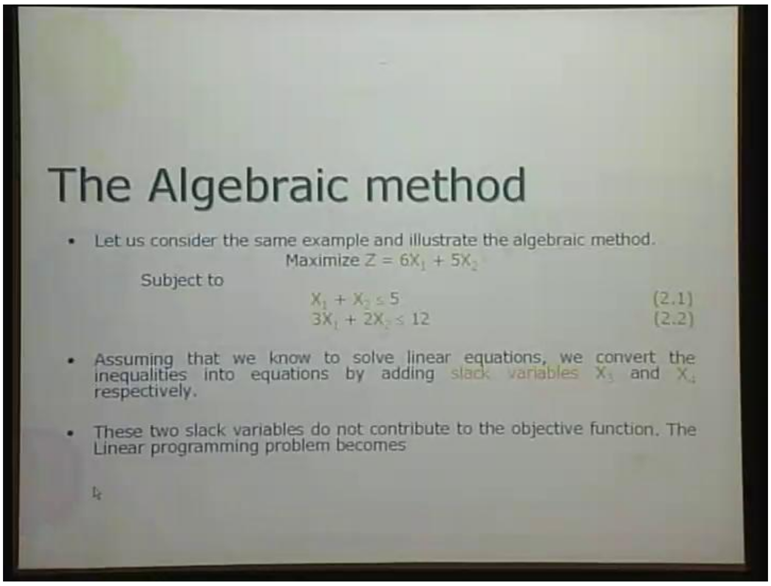


Before we do that let us also define some terminologies. As I had been saying the first important thing is this region (Refer Slide Time: 13:25). This is the common region which satisfies all these constraints. This is called a feasible region. Every point inside the feasible region is called a feasible solution. And the best solution that we have identified at the end is called the optimal solution or optimum. In fact both means the same and both are used interchangeably.

So what we need to do is first identify the feasible region by plotting the constraints and then try to find out the corner point solutions which are feasible solutions. They are also called Basic feasible solutions. We will see that later.

Right now we identify the corner point feasible solutions and then the best among the corner points with respect to the objective function is the optimum. So this is the summary of the graphical method. In the next slide, the graphical method works like this. Plot the constraints on a graph. Also plot the non negativity constraints i.e., restrict yourself to the quadrant where both X_1 and X_2 are greater than or equal to 0 which is the first quadrant. So you do not have to plot anything in the second third and fourth quadrants. Now identify the feasible region that contains a set of points satisfying all the constraints. Identify the corner points. Evaluate the objective function at all the corner points and that corner point which has the best value of the objective function maximum or minimum depending on the objective is optimal.
We can use the term optimal or optimum respectively. The graphical method works very well. It is a very easy method to solve linear programming problems. It has one very serious limitation. The limitation is that you cannot solve for more than two variables using the graphical method. So if you have a linear programming problem with only two variables irrespective of the number of constraints, you can still solve it using the graphical method.

(Refer Slide Time: 15:50)

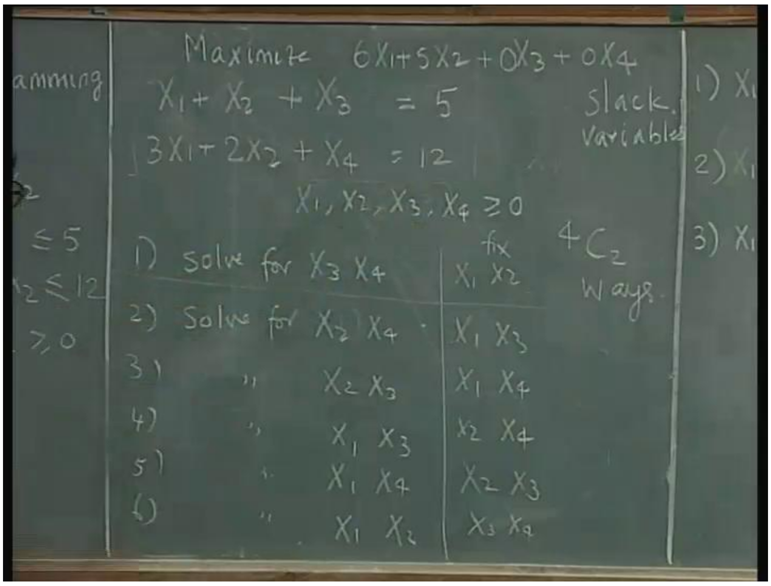


So this slide would tell us that we need to look at a method which is better than the graphical method, that which can handle more than two variables. The graphical method cannot be used if we have more than two, three or more variables. So we need a method which can handle multiple variables.

Now what we are going to do is we are going to show the algebraic method, which is the next method. Here we are going to illustrate it using the same example, so that we can understand how the algebraic method works with respect to the same problem and what are the relationships or what is similar, dissimilar or how we see all these solutions coming into the algebraic method. So we will also show the linkage between the graphical and the algebraic method by considering the same example.

Now let us go back to the algebraic method. We now look at the same linear programming problem. Right Now what we have to do is we need to solve these inequalities because the solution to this problem is actually a value of X_1 and X_2 which satisfies both these, which means we need to solve these two inequalities. Right Now we assume that we do not know how to solve inequalities. We also assume we know how to solve equations.

(Refer Slide Time: 17:19)



So what we try to do is to convert this inequality into an equation. So this is first written as $X_1 + X_2 + X_3 = 5$. Now this $X_1 + X_2$ should be less than or equal to 5.We therefore make it equal to 5 we can add an X_3 and because $X_1 + X_2$ is less than or equal to 5, this X_3 is also greater than or equal to 0 (because if X_3 is negative then it would mean $X_1 + X_2$ is more than 5 which we do not want). So the inequality $X_1 + X_2$ less than or equal to 5 is now rewritten as an equation, $X_1 + X_2 + X_3 = 5$ and the condition that X_3 greater than or equal to 0. Similarly the inequality $3X_1 + 2X_2$ less than or equal to 12 is now written as $3X_1 + 2X_2 + X_4 = 12$ and X_4 greater than or equal to 0. Therefore the problem now becomes maximize $6X_1 + 5X_2$ subject to these equations under the condition X_1, X_2, X_3, X_4 greater than or equal to 0. So this representation is useful to us because the new variables that we have introduced are also greater than or equal to 0 and are consistent with this requirement. Now how do we define these variables in the first place? Secondly do they have any effect on the objective function? Now these variables that are used or introduced to convert the inequality into an equation are called Slack variables. Now we have two slack variables X_3 and X_4, which have been used to convert these inequalities into equations. Do these (Refer Slide Time: 19:27) slack variables contribute to the objective function?

What do they represent? Do they have any physical meaning like the decision variable? Now in this problem X_1 and X_2 would represent the number of tables and chairs that the carpenter or the producer makes. Now what does X_3 indicate? What does X_4 indicate? For example if the person decides to make four tables and no chairs (4, 0) then this resource (where five of this resource is available), four of this resource is going to be utilized. Now X_3 takes value 1 because $X_1 = 4$ and $X_4 = 0$, X_3 takes value 1 and this 1 is nothing but, any resource that is not fully utilized and left available at the end of the problem and is represented by the slack.

Now we are going to make an assumption that such a resource is not going to add to the profit of the carpenter or the producer. So the slack variable contributes a 0 to the objective function. Even though it is available as an unspent resource, this is not going to add to what the person is going to make. Therefore slack variables do not contribute to the objective function. So you have a $0X_3 + 0X_4$. So now this problem is complete. We have two equations and we have four variables. We also have the additional condition that all four of them should be greater than or equal to 0 and we have an objective function which has been written in terms of all these four variables. Now we should solve this problem and we also realize that solving this (Refer Slide Time: 21:12) problem is the same as solving this problem.
The only thing is here we assume that we do not know how to solve inequalities. We have now converted it to equations. We at least know to solve this part. We will see how we solve the whole thing.

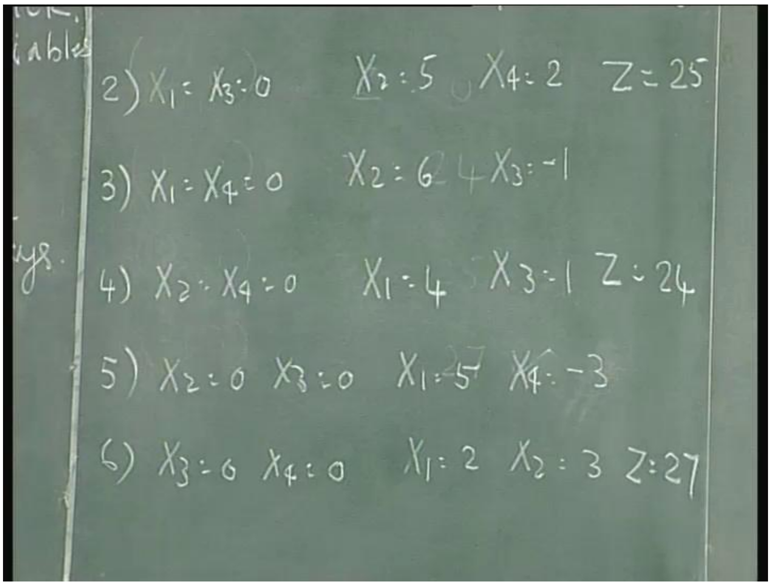
Now let us look at this problem. Now this problem has four variables and two equations. Let us assume that we know how to solve, for example, we have two equations. So we can solve two variables using two equations.
Right now we assume we do not know to solve four variables in two equations. So what we have to do now is if we want to solve this, we want to solve two equations at a time.
Therefore in order to do, that we need to solve for two variables at a time using the two equations. So we need to fix two out of these four available variables to a certain value. So we rewrite it in terms of the two variables that we want to solve. For example if we want to solve for X_1 and X_2 we can fix X_3 and X_4 to some arbitrary value and then rewrite it as two equations involving only X_1 and X_2 which we know how to solve. Now to do that there are four variables and two equations which means we can solve only for two at a time, so we can do this in $4C_2$ ways. For example the first out of these $4C_2$ is to solve for X_3 and X_4 by fixing X_1 and X_2 to some arbitrary value.

The second one would be to solve for X_2 and X_4 by fixing X_1 and X_3 to orbits. For example I am going to write another thing called fix. So here I would say,

i. Fix X_1 and X_2 to some value so that you can solve for X_3 and X_4. Here you fix X_1 and X_3 to some values. Solve for X_2 and X4.
ii. Similarly solve for fix X_1 and X_4 to solve for X_2 and X_3
iii. The fourth problem would be to fix X_2 and X_4 to solve for X_1 and X_3
iv. The fifth one would be to fix X_2 and X_3 to solve for X_1 and X_4 and
v. The last one would be to fix X_3 X_4 to solve for X_1 and X_2.

Now these are the six problems that I have mentioned here by saying that you can solve it in $4C_2$. Now the next question if we take the first one i.e., if we decide to fix X_1 and X_2 to some arbitrary value and then solve for X_3 and X_4 even this can be done in many ways because these X_1 and X_2 can be fixed to any arbitrary value.
So the easiest thing that we can do is to fix them to 0. In fact when we fix X_1 and X_2, the only thing we need to make sure is we do not give them negative values because you do not want to violate this. We want our final solution to be feasible, so we do not want to give negative values here. Therefore the only condition is when we fix X_1 and X_2 to some arbitrary value you can give any non negative value. Therefore this problem alone can be solved in infinite ways by fixing X_3 and X_4 to any arbitrary non negative value. The easiest thing that we can do is to try and fix them to 0. If we now force the condition that whenever we are fixing something, we fix them only to 0, then we get exactly six problems and six solutions, one corresponding to each problem. So let us see what happens when we fix all of them in each problem to 0 and see what happens to the solution.

(Refer Slide Time: 25:08)

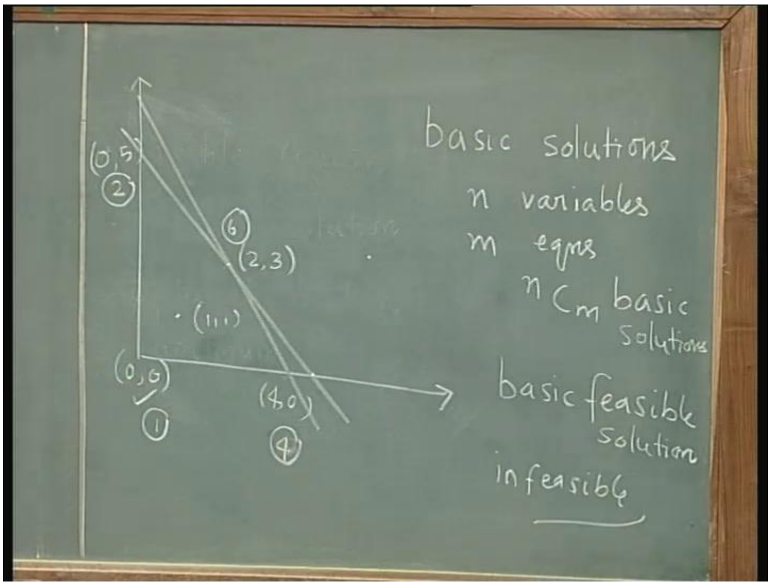


Now the first thing that happens is if we fix X_1 + X_2 to 0 which is the first problem, we straight away get X_3 = 5; X_4 = 12. So the first problem is $X_1 = X_2 = 0$. We get a solution X_3 = 5, X_4 = 12 and Z the value of the objective function is 0. Now let us look at the second problem. The second problem is that we fix X_1 and X_3 to 0, so $X_1 = X_3 = 0$.
So X $_1$ X_3 = 0 gives X_2 = 5. Now go back and substitute $2X_2$ is 10. This is 0. So this is $-2X_4$ = 2 to give you Z = 25. The third one where we fix $X_1 = X_4 = 0$ so X $_1$ = 0 X_4 = 0 would give X_2 = 6. Substituting here, X_2 = 6, X $_1$ = 0 gives X_3 = – 1.I do not evaluate the value of Z here because I do not have a feasible solution here.

It does not make sense for me to spend the extra effort to evaluate the objective function for this solution because I am interested only in feasible solutions. This is not feasible. The value that a variable X_3 takes – 1 violates this condition. So this solution is not feasible. So I look at the fourth one. The fourth one has $X_2 = X_4 = 0$. When X_2 = 0, X_4 = 0 here gives me X_1 = 4. Now X_1= 4, X_2 = 0 gives me X_3 = 1and Z = 24. Now the fifth problem that I solve Now has X_2 = 0, X_3 = 0. So X_2 = 0, X_3 = 0 gives me X $_1$ = 5. Now X_1 = 5 makes this 15.So this would make X_4 – 3. So once again I get into a situation where I violate this condition therefore the problem is not feasible or this solution is not feasible. Therefore I do not evaluate the value of the objective function.

The last one is X_3 = 0, X_4 = 0 and for this example Now, which means I have to solve X $_1$ + X_2 = 5; 3X $_1$ + $2X_2$ = 12 which is slightly more involved than any of the five cases. You finally get the solution here which gives me X_1 = 2; X_2 = 3. So 6 into 12 + 15 Z = 27. What we have done is we have first converted the inequalities to equations and then we realized that we have now more variables than equations. So we said we can (since we have only two equations) solve only for two variables which means we have to fix the remaining two to some arbitrary value and then we realized that each of this can be done in many ways. We then reduced our search to the only condition that if we fix some variables to a known value, then that value is 0 which means we end up getting only six problems. Now we have solved all the six problems and we have got this. We find something very interesting between these six solutions and the solutions that we obtain using the graphical method.

(Refer Slide Time: 29:56)



Now we quickly go back to the graphical method and try to draw the graph again. We had have had something like this. We had one graph like this. We had the other coming something like this (Refer Slide Time: 29:51). So let us assume this is the (2, 3). This (Refer Slide Time 31: 05) is (0, 5), this is (4, 0) this is (0, 0). If we go back we can see the relation. The first one being $X_1 = 0$ and the next $X_2 = 0$ is shown here. The second one is $X_1 = 0$; $X_2 = 5$. This (Refer Slide Time: 30:34) is what I call as 1, this I call it as 2. Right now we do not consider this. What we call as three is $X_1 = 4$; $X_2 = 0$. This is point number 4 and 3 and 5 are not represented here because 3 and 5 do not give us feasible solutions. So there is definitely a one to one relationship between the algebraic method that we have seen under the condition that we fix them at 0 and the graphical method. Now the four corner points that were of importance to us are seen right here when we fix them to 0. So on one hand it validates our assumption that it makes sense to fix at 0 but on the other hand, we need to explain why fixing to 0 has the same one to one relationship between the graphical methods. Now the relationship is also reasonably obvious.

Any point that is inside the region will be dominated by a corner point. So any point inside the region implies that (this does not consider your X_3 and X_4) you are actually fixing X_3 and X_4 to an arbitrary non negative value and then you are solving for it. So you are actually inside for any arbitrary non negative value that we fix here. We actually end up evaluating a point which is inside the feasible region in the corresponding graphical method. So every corner point is evaluated by fixing these things at 0 so that we are able to solve the corresponding equations. After all every corner point is obtained by solving the corresponding equations.

This corner point is obtained by these two equations $X_1 + X_2 = 5$ and $3X_1 + 2X_2 = 12$which means you are fixing X_3 and X_4 to 0. For example this corner point is obtained by fixing $3X_1 + 2X_2 = 12$ and the X_1 axis which is fixed, $X_2 = 0$. This implies that X_4 is fixed at 0. So we can now show that every corner point is actually got by fixing these values at 0 and not at any other non negative value. To put it differently it makes sense only to fix these at 0 and not to fix these at any other value other than 0 or any positive value other than 0 because by fixing them to any positive value other than 0 means that we would end up evaluating a point which is inside the region.

Also we now know that any point inside the region is not important to us because every point inside the region is going to be dominated by a corner point and since we are going to look only at corner points we do not have to fix it at any other value other than 0. So in the algebraic method when we fix some variables to a value and then solve for the remaining, we fix them only at 0. Now these six solutions that are obtained by fixing them at 0 are called Basic solutions. So in the algebraic method after we convert the inequalities to equations and solve for a certain number of variables by fixing the remaining ones at 0, we get basic solutions. So in this example there are $4C_2$ or six basic solutions. In general, after converting if we have n variables and m equations then we have nC_m basic solutions. So in this example there are $4C_2$, six basic solutions.

Now any other solution that is evaluated by fixing them to any value other than 0 (any positive value other than 0) it is called a Non basic solution and we are not interested in non basic solutions at all. Now there are six basic solutions that we have looked at. But of these six we realize that four of them are feasible and these two are infeasible. What is a feasible solution? Any solution that satisfies all the constraints including the non negativity is a feasible solution. So out of these six we find that we have four solutions that are feasible. So those basic solutions that are feasible are called Basic feasible solutions. For example if we look at a point here (for instance 1, 1), it is a feasible solution. But it is a non basic feasible solution and we are interested only in basic feasible solutions.

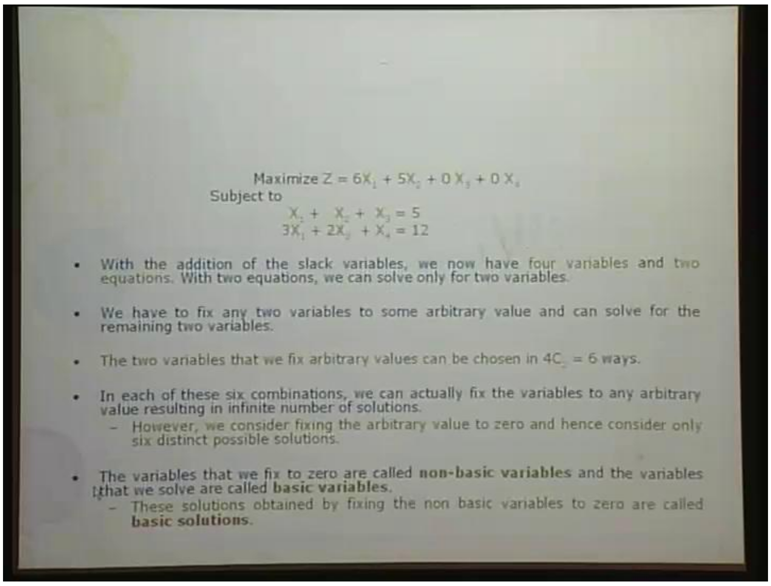
The remaining two are not feasible because these values violate the non negativity restriction. Now these have to be called as Basic infeasible solution. They are not usually called as Basic infeasible they are just called Infeasible because when a solution is infeasible it does not matter whether it is basic infeasible or non basic infeasible. I mean typically you can show that some of these points are actually this (Refer Slide Time: 37:04) point and this point which are not of concern to us does not matter. There is no need to categorize them once again saying this is non basic and infeasible, this is basic and infeasible and So on.
We are interested in basic feasible solutions any basic solution which is not feasible automatically becomes infeasible. So we have six basic solutions in this example out of which four of them are feasible and two of them are infeasible. We can also show that all the basic feasible solutions correspond to the four corner points that we have. Now these four basic solutions that are feasible are 1, 2, 4 and 6. They (Refer Slide Time 37:43) are the four corner points, 1, 2, 4 and 6. We are explaining this assuming that there is a solution to the problem. Later we will look at situations where the linear programming problem may not have a solution at all. Now those aspects will be taken up during that time. Now it is under the assumption that a solution finally exists, for example we are not getting into situations about singularity of the matrix and etc. You can always go back and say that when I am Solving for this it means I am Solving for $X_1 + X_2 = 5$; $3X_1 + 2X_2 = 12$.

Now the implicit assumption is that the rank of this matrix is not 1 and I do have a solution with respect to this. So that is an assumption here. So right now when we illustrate the algebraic method through this example we have taken a convenient example where we have a solution to the problem.

Much later when we work out the simplex method we will show you examples where the problems may not have a solution at all. We will do that subsequently. Right now we are only trying to bring the one to one relationship between the algebraic method and the graphical method and that is the purpose of this discussion. We now have six basic variables or basic solutions out of which four are basic feasible and two are infeasible. We need to introduce one more terminology. In this example of these four variables that we have, we have fixed these two to 0 and we have solved for these. Those variables which we fixed to 0 are called Non basic variables and those we had solved are called Basic variables. So every basic solution has a set of basic variables and a set of non basic variables.

If we have two equations then we have two basic variables the remaining becomes non basic. So in this case there are four variables two equations two basic variables two non basic variables. If we had 5 variables and two equations then you will still have two basic variables. The remaining three will be non basic and they will be fixed at 0. Now the corner points are basic feasible solutions and we have already seen through the graphical method that the corner point solutions are important and we have now shown that every basic solution corresponds to the corner point. Now in the graphical method we said, having identified the corner points all we need to do is to evaluate the objective function at these corner points and find out that corner point which has the best value of the objective function. So we do the same thing in the algebraic method also now we have already evaluated the objective function values for these four corner points. The objective function values are 0, 25, 24 and 27. We are interested in maximizing our objective function therefore we choose this solution as the best solution which has the maximum value of the objective function. In this example it turns out to be $X_1 = 2X_2 = 3Z = 27$ which is the same solution that we obtained using the graphical method.

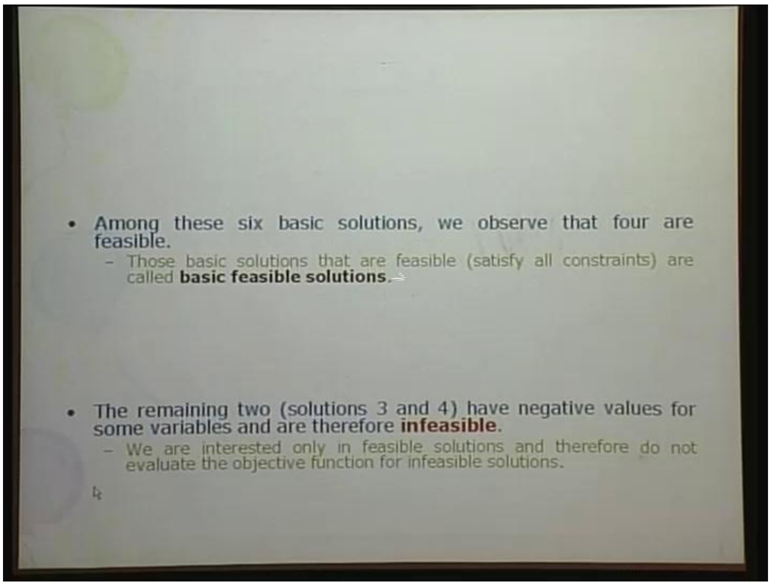
(Refer slide Time: 42:05)



Let us look at these. First what we do in summary for the algebraic method is we add the slack variables. So we now have four variables and two equations and with two equations you can solve only for two variables.

So we have to fix any two variables to some arbitrary value and solve for the remaining two. The two variables that we fix can be chosen in $4C_2$ ways, each of these six combinations can actually fix the variables to any arbitrary resulting in infinite number of solutions that we saw but we are going to consider fixing this value only to 0 and then we are going to consider only six distinct possible solutions which we have listed here and will be shown to you later.
Now the variables that are fixed to 0 are called non basic as I had indicated here and those variables that we solve are called basic. Solutions obtained by fixing the non basic variables to 0 are called basic solutions. We have defined the basic solutions here by saying that we fix the non basic variables to 0. Now these are the six solutions that we have seen.

(Refer Slide Time: 43:13)



The same six that are shown here and among these six we find four are feasible and they are called basic feasible. The other two are infeasible. They are called infeasible. They are not specifically called basic infeasible. The values of the objective functions are now shown here (Refer Slide Time: 43:20). Every non basic corresponds to interior point in the feasible region that we saw and in the algebraic method it is enough only to evaluate the basic solutions because all the basic solutions are corner point solutions and the best value is optimum.

Next in summary in the algebraic method, we convert the inequalities into equations by adding the slack variables and then we evaluate all the basic solutions for all the combinations. We verify the optimum solution as the one with the maximum or minimum value of the objective functions. We also said that the graphical method is very good when we solve for two variables. We also realize that the algebraic method can be used to solve for any number of variables and any number of constraints, provided we are able to solve the linear equations. Now are there other distinct advantages or disadvantages of the algebraic method. There are some distinct disadvantages of the algebraic method. Now we end up evaluating some $4C_2$ or in general we end up evaluating nC_m basic solution if after writing the slack we have n variables and m equations. This is done in nC_m ways. Now nC_m is a large number. nC_m can become exponential when n is large and m tends to n/2.
So we do end up evaluating a very large number of solutions before we reach the optimum. This is the first disadvantage of the algebraic method. The second disadvantage, a very distinct one is that we evaluated six solutions here and we found that two out of these six were really useless and unnecessary. We did not look at these two at all the moment the solution becomes infeasible. It does not add any value to the solution. So if the algebraic method is good enough to eliminate the infeasible ones it will be much better than what it is in its present stage.

Therefore we need to look at a method which does not evaluate infeasible solutions at all and has an internal mechanism not to evaluate any infeasible solution. In the algebraic method only after finding out a solution we realize that it could be infeasible and we are not interested in this. The other thing that happens is that there are four solutions here. If we look at these four solutions in the same order that we have seen, we first evaluated this (Refer

Slide Time: 46:05) and the moment we have a feasible solution we will evaluate the objective function value because that is what we are interested in. We are interested in the basic feasible solution which has the best value or in this case the largest value of the objective function.

So first feasible solution that we looked at is Z = 0. The second one that we looked at or we obtained is this that gives me Z = 25. Now this solution to me is much more valuable than this because this (Refer Slide Time: 46:22) has a higher value of the objective function. Now when I come back to the fourth, (I have already seen the third, I realized, it does not add value) I realize I have got a solution which is basic feasible but with Z = 24. Now this also does not add any value to me because I already have something that is better than this. So among my basic feasible solutions I would ideally like my solutions to become better and better as I evaluate it. I would rather not like to evaluate a solution with a lesser value of the objective function than something that I know. This is the most desirable one because this has a value better than this 27 and we stop. So what are the disadvantages?

1. It evaluates a large number of solutions.
2. Out of this large number some of them could be infeasible so we do not want to evaluate the infeasible ones in the algebraic method.
3. Among the feasible solutions, (there are four in this case) we would ideally want the solutions to become better and better as we move because we spend certain amount of effort in evaluating this solution and you realize it is basic feasible but then when you come to this 24, you realize it is not that important because you already have something with 25. So ideally you would want a method which would progressively evaluate better and better solutions.

The last one is this. In this algebraic method, there are four solutions. It turned out in this case that the sixth one, the last one that we evaluated was the best. Now what we would want is, just in case this had become solution number four, (which means the third basic feasible solution that we have evaluated), I have obtained this 27. Now if this 27 is the best one, that I can get then I need a mechanism which can tell me right here that you have got your optimum.

You don't have to look at the remaining two. So we would want the algebraic method to be smart enough to be able to identify the optimum as soon as the optimum is found, rather than make you work the rest of the things i.e., complete the entire Cn_m then at the end of it all make a comparison and then say this solution is better. So basically we want a method which is better than the algebraic method that is shown here (Refer Slide Time: 48:45) which is better on three counts.

1. It should not evaluate any infeasible solution
2. It should be capable of giving us progressively better solutions. We do not want, for example, to evaluate this 24 after we have evaluated this 25.
3. The moment it has found the optimum, it should be able to terminate. It should not put you in a situation where even you do not know that the optimum has been reached. You evaluate the rest of them and end of it all you realize somewhere here I might have found out the optimum.

So if there is a method which can do all these, then that method would add more value to the algebraic method that we have seen. And obviously that method might require some more computation or extra efforts.

So now there is a trade-off between getting these three advantages, if we can and then spending some more time and effort to do that. Now if we can get a method which can give us all these three advantages that we seek from the algebraic method and if we are guaranteed of such an advantage then we do not mind actually spending a little more time and effort towards that method.

Now the next thing that we are going to see is that method. It is called simplex method. This is essentially an extension of the algebraic method but the simplex method exactly addresses the three concerns that we spoke about. The simplex method will not evaluate any infeasible solution. The simplex method will give progressively better and better solutions. The method also has an inbuilt mechanism to show that you have reached the optimum. So before you enumerate or evaluate all possibilities, simplex would tell you that the optimum is reached.

So the simplex method is the most important tool we have come across and we will see that in the next lecture.