Fundamentals of Operations Research

Prof. G. Srinivasan

Department of Management Studies

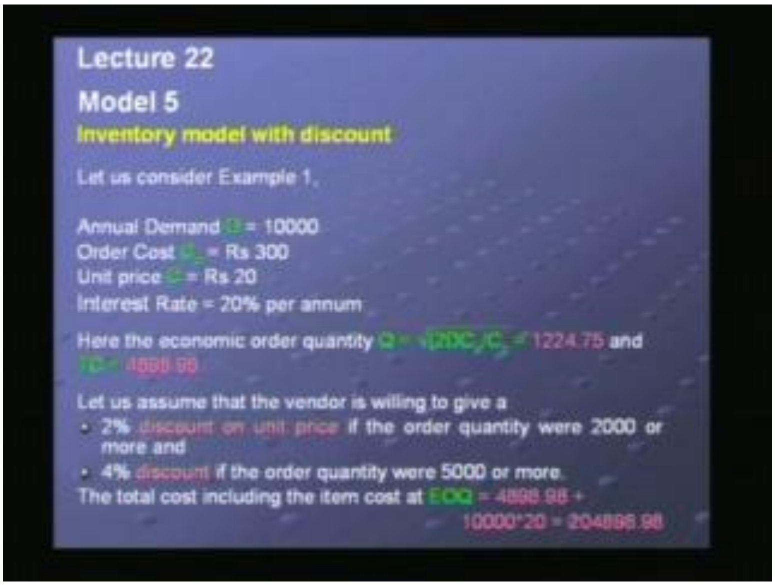
Indian Institute of Technology, Madras

Lecture No. # 22

Inventory Models - Discount Models, Constrained Inventory Problems, Lagrangean Multipliers, Conclusions

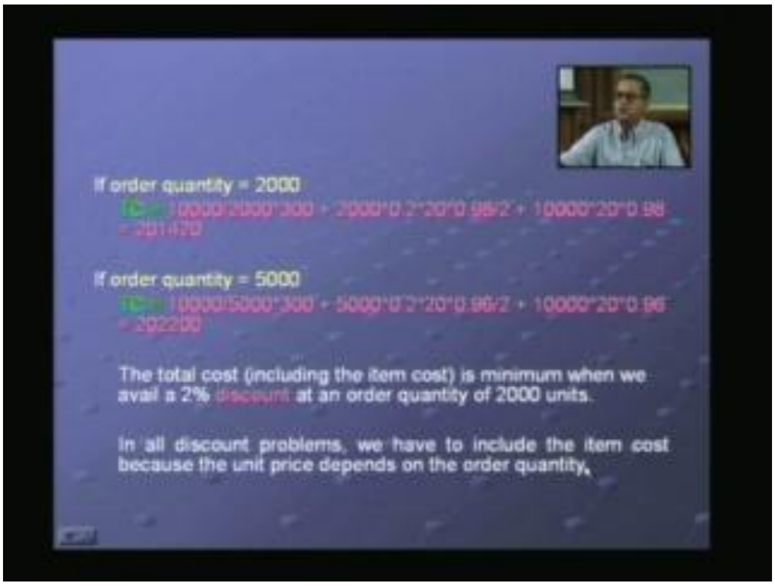
In this lecture we continue our discussion on inventory models and we begin by considering inventory models with this count.

(Refer Slide Time: 01:24)



Let us go back to the first example with a single item model with annual demand of 10,000, order cost of rupees 300 per order, unit price rupees 20, interest rate at 20 per annum. So here we will be using C_C = i into C which will work out to be the same rupees 4 per unit per year. The economic order quantity Q is given by root of 2 into D into C_0/C_C which is 1224.75. The minimum cost of ordering and carrying at the economic order quantity TC is 4898.98. We have already seen this example and these numbers. Now since we are placing an order at a time of 1224.75 and we assume that we are going to play several orders to meet this demand of 10,000 and more we could ask the supplier for a discount. So let us assume that the vendor or the supplier is willing to give a 2 percent discount on the unit price, if the order quantity were 2000 or more and give a 4 percent discount if the order quantity were 5000 or more. So let us evaluate whether these 2 price break or is favorable to us. Now without any price break at the existing price of rupees 20 at the economic order quantity, the total cost including the item cost would be 4898.98 which is obtained from this. Plus, the cost of the item is 10,000 into rupees 20 which would give us 204898.98 rupees per year.

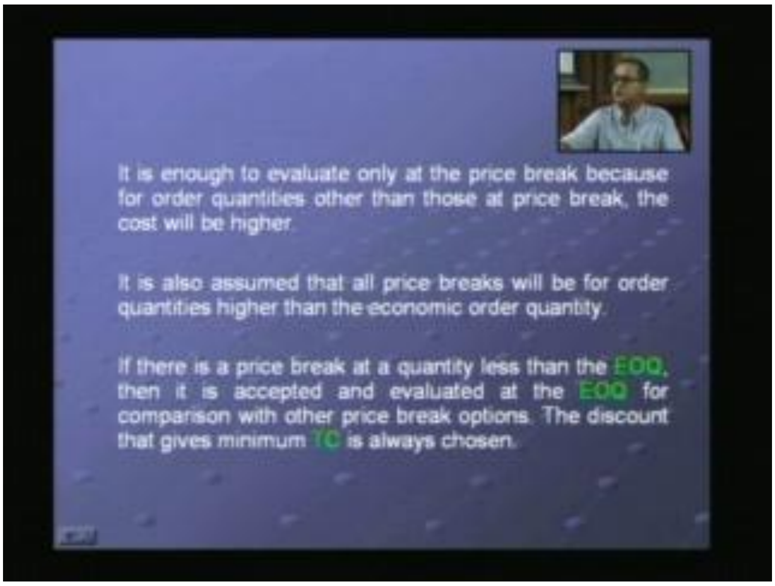
(Refer Slide Time: 03:14)



Now if we look at an order quantity of 2000 then at 2000, we avail a 2 percent discount. So the number of orders will be 10,000 divided by 2000, 5 orders in the year multiplied by ordering cost 300. Average inventory will be 2000 divided by 2 which is 0.5, so 2000 divided by 2 into the carrying cost which is 0.2 into 20 which is 4 rupees + the cost of the item which is 10,000 items into 20 per item into 0.98. There is a 2 percent discount at the price break of 2000. So what happens is this 2 percent discount gets reflected in the unit price as well as in the carrying cost. So the unit price now becomes 20 into 0.98 because of the 2 percent discount into 10,000 items. The carrying cost C_C is defined as i into C, i is 20 percent and C is 20 rupees. Now the unit price is not 20 but 20 into 0.98. So C_C is not 4 but C_C is 0.2 into 20 into 0.98. So average inventory is 2000/2000 into $C_{C,}$ which is 0.2 into 20 into 0.98. So this works out to 201420. Now when the order quantity is 5000, we get a 4 percent discount so we evaluate the total cost for an order quantity of 5000. So order quantity of 5000 means 2 orders in the year 10,000/ 5000 into 300. The carrying cost is 5000/2 which is the average inventory. C_C is now 0.2 into 20 into 0.96 Now .96 comes because of the 4 percent discount.

So the unit price is not 20 but 20 into 0.96 which is 19 rupees and 20 paisa. 20 percent of 19 rupees and 20 paisa is the carrying cost per unit per year. Unit price of the item is 10,000 items into 20 rupees into 0.960. This .96 comes because of the 4 percent discount so this total becomes 202200. Total cost including the item cost is minimum, comparing the 3 at the economic order quantity the total cost is 204898.98 at a 2 percent discount with order quantity 2000 is 201420 and a 4 percent discount at an order quantity of 5000 is 202200. So among the 3 values, the minimum value is 201420. So the total cost including the item cost is minimum when we avail a 2 percent discount at an order quantity of 2000 units. Whenever we have a price break, we need to look at the total cost at the economic order quantity and the total cost computed at the price break points. In all the discount problems we will include the item cost because the unit price now depends on the order quantity. Till now in all the models we had not included the cost of purchasing the item only in discount models we use that because the unit price depends on the order quantity. It is enough to evaluate only at the price break. For example the problem says there is a 2 percent discount, if the order quantity were 2000 or more. So should we evaluate the total cost at some other order quantity which is higher than 2000. The answer is it s not necessary. Similarly when we are looking at a 4 percent discount, we would get this 4 percent discount if the order quantity were 5000 or more. Now the question is, is it necessary to evaluate at a 4 percent discount any order quantity more than 5000? The answer is no.

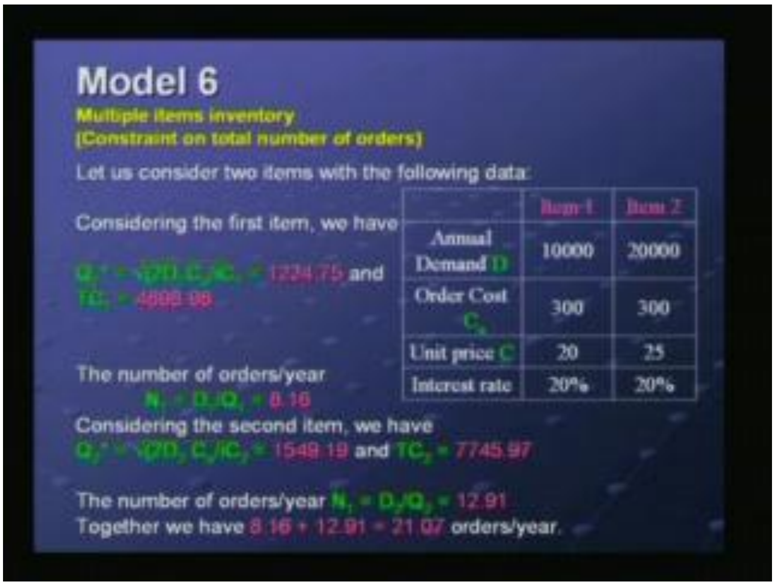
(Refer Slide Time: 07:29)



It is enough to evaluate only at the price break, the points at which we get discount because for all order quantities other than those at the price break or for any order quantity which is higher than the price break, the cost will only be higher because of the quadratic nature of the cost function. It is also assumed generally that all price breaks will be for order quantities higher than the economic order quantity. For example in this problem economic order quantity was 1224.75, we did not assume any discount at the EOQ or economic order quantity. We assumed that there is a discount at quantity 2000and 5000 which are higher than the economic order quantity. So at any point, whenever the discount is for quantities higher than the economic order quantity, it is enough to evaluate the impact of the discount only at the points at which there is a price break. If there is a price break at a quantity less than the economic order quantity then it is accepted and evaluated at the EOQ for comparison with other price break options.

Discount that gives minimum total cost is always chosen. We have not considered the case where for example there could be a 1 percent discount if the order quantity is more than 1000. So if the vendor is willing to give a 1 percent discount for order quantity greater than 1000 then we will avail the 1 percent discount not at 1000 but at the economic order quantity. So wherever there is a discount which fix into the economic order quantity, we will evaluate it at the EOQ. When there is a price break for quantities higher than the economic order quantity, we will evaluate the total cost at the points, at which there is a price break and that is what is shown here. Now if there is a price break at the quantity less than the economic order quantity then the price break is accepted not at the points at which there is a price break but at the economic order quantity then such a TC total cost is compared against other discount 0 points. Such as this and whichever is minimum is always chosen. So this is how we address problems with discounts or price break. What we have to keep in mind is whenever we are addressing inventory models with discount, it is absolutely necessary that we also include the total cost of the items, simply because the unit price now depends on the economic order quantity. It is not a constant any more. It is a variable. It depends on the quantity that is being ordered. Therefore all discount problems will include order cost, carrying cost as well as the cost of the item. This is one change from the previous models where we did not explicitly consider the total cost or price of the items.

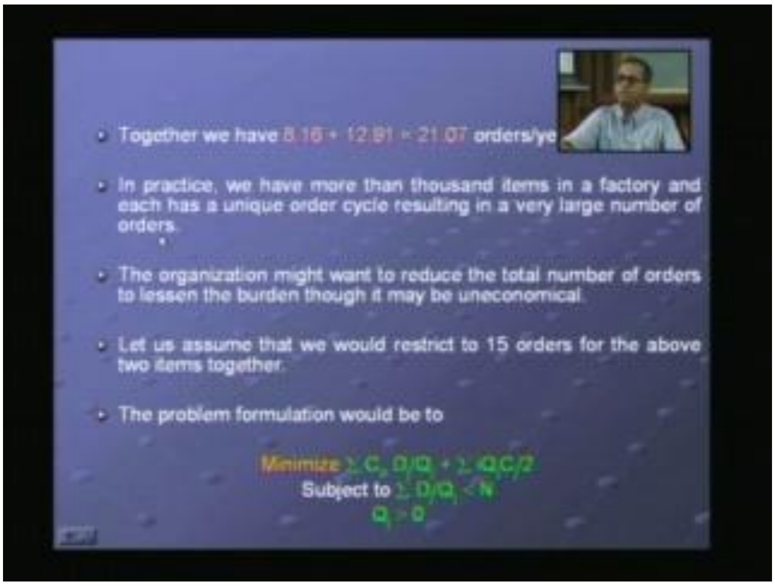
(Refer Slide Time: 10:34)



Now let us consider a few more models in inventory and we look at multiple item inventory model. So far we have seen all the 5 models and examples. We have seen a single item model. Now in the remaining couple of models that we will be discussing, we will look at multiple items. So let us consider 2 items. We call them item 1 and item 2 with the following data. The annual demand D is 10,000 for the first item and 20,000 for the second. Order cost is 300 and 300. Unit price is 20 and 25 and interest rate is 20 percent for both. Now we have kept the order cost same for both these items. We have also kept the interest rate same for both these items. Now carrying cost C_C is expressed as interest rate i into C. So carrying cost will be rupees 4 for the first item and rupees 5 for the second. But the interest rate i will be the same. Now in all multiple item problems, it is customary that we keep the order cost same as well as the interest rate same. Interest rate being same is the understandable because it is the interest that is paid on the money that is borrowed. Now no matter whichever item that money is spent on. Order cost is kept constant because sometimes one can argue that order cost could be different for different items because some items may require more intense inspection.

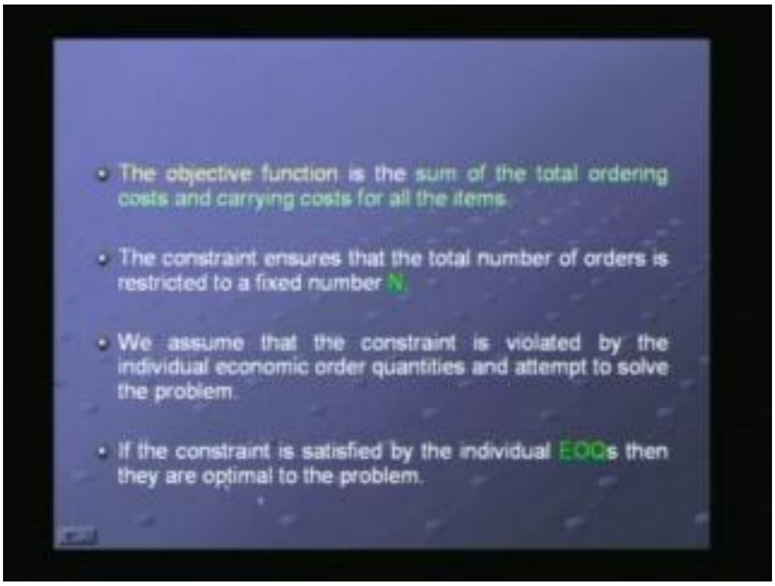
Cost of transportation could be different for different items and so on. But in all these multiple item problems, it is customary to assume that order cost is the same. The reason being, it will be very difficult for the organization to go back and compute an order cost for every item. So what the organization would do is over a period of time they will find out what is the total amount spent on ordering for all the items and then average it out with the number of items ordered and they get one average figure which is the representative order cost for all the items. Therefore we choose the same 300 as the order cost for these items. Now going back to the first model, we find out the economic order quantities for both these. We already have the result for item 1. So for item 1, economic order quantity is $2D_1$ C_0/iC_1. We use subscript 1 for item 1 and subscript 2 for item 2. So Q_1 star, economic order quantity for the first item is 1224.75 and the corresponding total cost is 4898.98. Number of orders per year would be D_1/Q_1. Total demand is 10,000. Economic order quantity is 1224.75. Number of orders is 8.16 orders per year. Considering the second item, we compute Q_2 star economic order quantity for item number 2, as root of 2 D_2C_0/i C_2 which works out to 1549.19 and the total cost TC_2 is 7745.97. Number of orders for the second item is given/D_2/Q_2 which is 12.91. If we consider these 2 items now, we observe that together we place 21.07 orders per year for these 2 items put together because the number of orders are different, the points in time, where these orders are made are also different. So we make 21.07 orders per year.

(Refer Slide Time: 14:26)



In practice when we look at more than thousand items in a in a factory or in an organization each has a unique order cycle resulting in a very large number of orders. Now the organization might want to reduce the total number of orders to lessen the burden though it might be uneconomical. It would be uneconomical because we have already seen that any quantity other than the economical order quantity would increase the total cost of ordering and carrying. So it could be uneconomical but the economic order quantity is now posing a large burden on the people who are doing the purchasing therefore the organization might want to reduce the total number. Let us assume that we would restrict to 15 orders for the above 2 items. Together that was 21.07 orders per year. We would assume that we will not reduce the burden therefore we are going to restrict it to 15 orders and number 15 is obviously smaller than 21.07 so that the burden is reduced. Now the problem would be for these 2 items, put together sigma C_0 is constraint D_j/Q_j + sigma i into $Q_jC_j/2$. Now i C_j is the carrying cost. C_0 is the order cost. D_j is a demand for the item and so on. So for j for many numbers of items, we could submit up. For 2 items, we could write it as C_0 into D_1/Q_1 + D_2/Q_2 and so on. The constraint now will be sigma D_j/Q_j is less than or equal to m. Now this capital N is 15 .The restriction that we have D_j/Q_j will be the number of orders per year for item j, so sigma D_j/Q_j is the total number of orders for these 2 items put to together, should be less than or equal to 15. Q_j greater than or equal to 0.

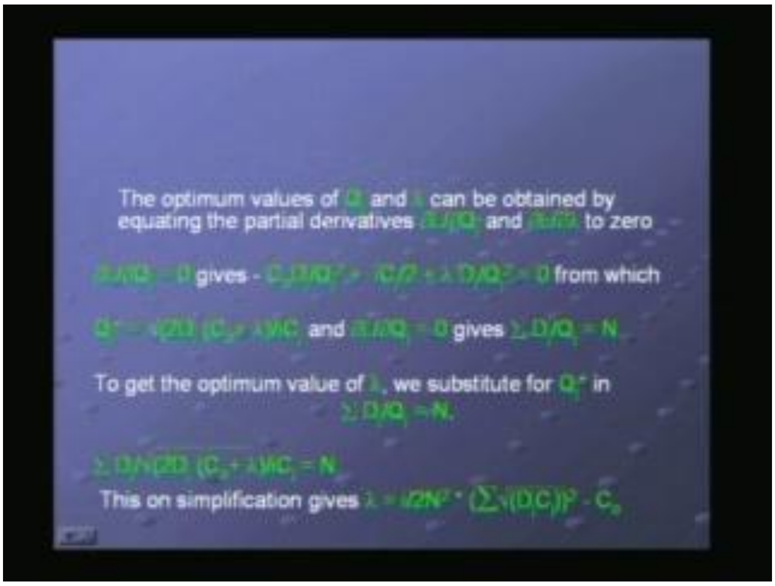
(Refer Slide Time: 16:22)



Now the objective function is now the sum of the total ordering costs and carrying cost for all the items. The constraint will ensure the total number orders are restricted to a fixed number N. We assume in this case which is also true. Let the constraint be violated by the individual economical order quantities. The constraint said N is = 15, together these 2 items have 21.07. So the constrain is violated when we substitute Q_1 star and Q_2 star. We want to solve this now. The constraint is satisfied by the individual EOQs. Then the individual EOQs are optimum to the problem. For example if we had said that this N is 25 if we had said we would like to restrict it to 25 orders per year. Then we are happy doing 21.07 at the economic order quantity. Only when the constraint is violated, the problem becomes significant. Now when the constraint is violated by the individual EOQs, we can easily show that the constraint will be satisfied as an equation not as an inequality when we actually solve the constraint version of the problem even though this constraint is an inequality. Now if the present solution is violating this constraint and when we solve the problem with the constraint now we can easily show that it is okay if we if we substitute this with an equal to sign and not solve it as an inequality.

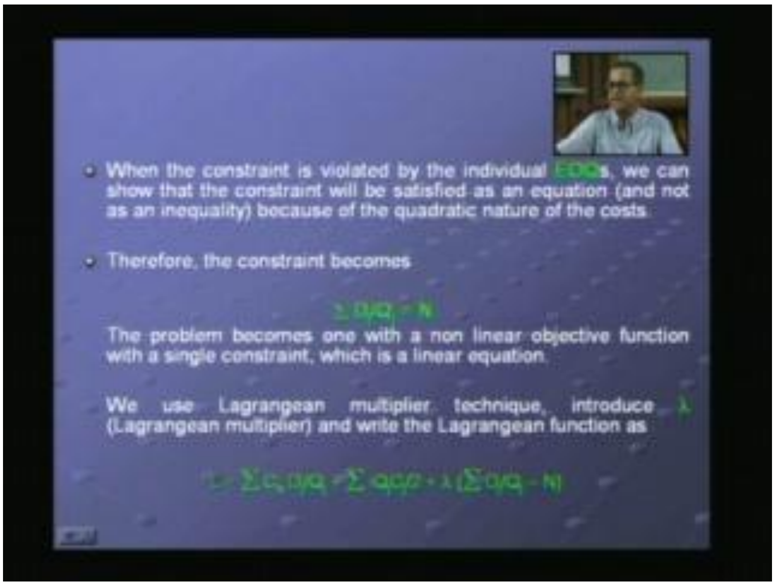
So we can convert this into an equation and sole we do this because it is easier to solve nonlinear problems when the constraints are equations nonlinearity comes in the objective function because of Q_j being in the denominator as well as in the constraint because Q_j in the denominator. Constrained nonlinear problems are easier to solve with equations than with inequalities. In this case because we know that if the constraint is binding it will be satisfied as an equation. We convert the inequality to an equation and then we solve. So it is becomes $D_j/Q_j = N$. Now the problem becomes a nonlinear objective function with a single constraint which is the equation in this case. Now we use the Lagrangean multiplier technique. We introduce a lambda which is a Lagrangean multiplier and write the Lagrangean function. We take the constraint into the objective function and write the Lagrangean, so the objective function remains as it is. Lagrangean has the lambda which comes in which is a multiplier into sigma $D_j/Q_j - N$.

(Refer slide time: 18:54)



Now once the Lagrangean is written, the optimum values of Q_j and lambda can be obtained by equating the partial derivatives tau L/tau Q_j and tau L/tau lambda to 0. We are not going to explicitly evaluate the second derivatives and show there is a minimum for all inventory problems which is assumed that second derivative indicates a minimum. So tau L/tau Q_j will give – C C_0 D_j/Q_j square + iC/j_2 + lambda D_j/Q_j square = 0

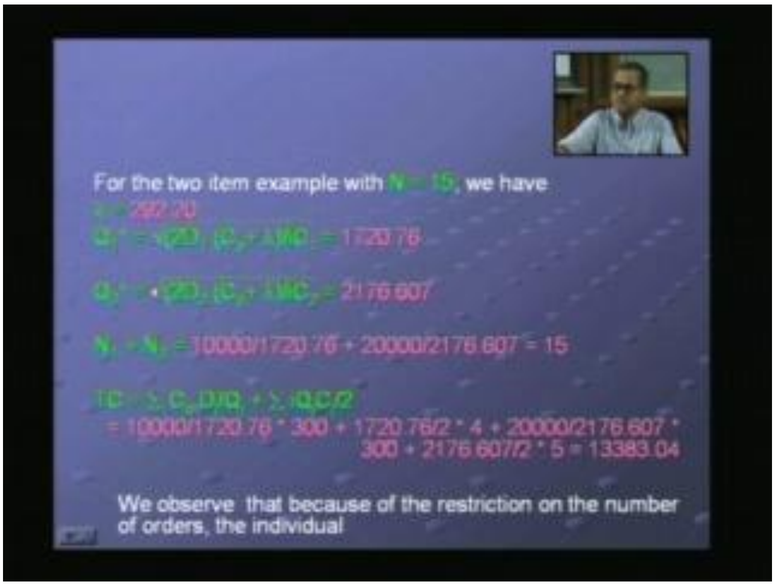
(Refer slide time: 19:25)



Now this comes because there are 3 terms. This would give – C_0 D_j/Q_j square. This would give $iC_j/2$. This would give – lambda D_j/Q_j square. So we have 3 terms = 0 from which Q_j star is root of 2 D_j into C_0 + lambda/iC_j. The square root is for the entire term tau L/tau Q_j or tau L/tau lambda is = 0 would give sigma D_j/Q_j = N. Now we use this formula Q_j star = root of 2 D_j into C_0 + lambda/iC_j. We realize that we do not know the value of lambda for a particular item j. The value of D_j is known, C_0 is known, i is known, C_j is known, but lambda is not known. Unless lambda is known, we cannot compute the economic order quantity Q_j

star. So in order to find out this lambda, we go back and partially differentiate this with respect of lambda which would give us sigma D_j/Q_j = N. So that is written here. Sigma D_j/Q_j = N. Now we go back and substitute here sigma D_j/Q_j = N. Q_j, we go back and substitute Q_j star = root of 2 D_j into C_0 + lambda/iC_j. So we substitute for this Q_j from this and we get an expression like this, sigma D_j/root of 2 D_j into C_0 + lambda/iC_j = N. Now we can simplify this. We can pull out the CC_0 + lambda outside. D_j and C_j are only dependent on the item j. So we can pull out C_0 + lambda and 2 outside of this, then we have sigma root of D_j C_j because there is a D_j term here. There is a root of D_j term here and there is a root of C_j term here. So it will simplify itself to give us root of D_jC_j. So finally on simplification we would get lambda = i/2N square into sigma root of D_j C_j the whole square – C_0. So we have a nice solution here with lambda given by this quantity. We know all the terms here. We know N, we know D_j, we know C_j, we know C_0 and i. We can calculate lambda and once we calculate lambda, we can go back and calculate the economic order quantity Q_j star.

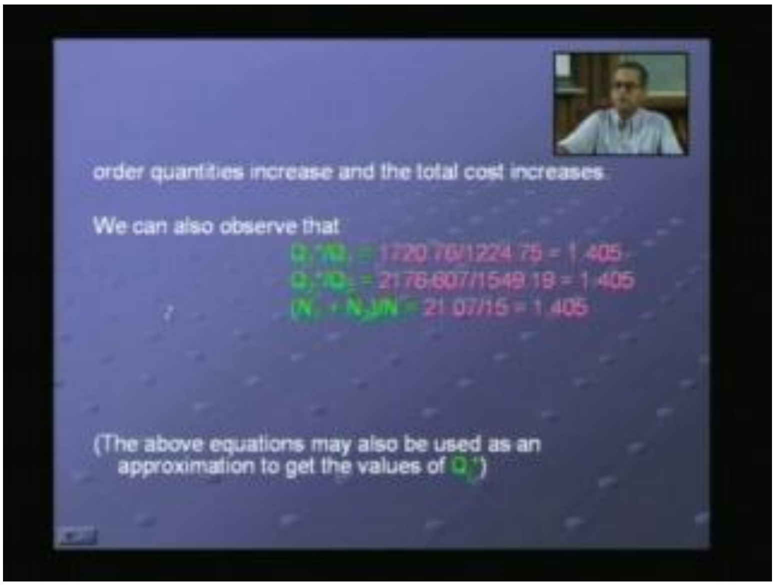
(Refer slide time: 22:02)



Now for the same example we can show that when we substitute N is = 15. Rest of the things is known. i is 0.2, 20 percent. For the first item, D_j is 10,000, C_j is 20, and C_0 is 300. So we substitute and we get sigma D_j C_j is here. So D_1 is 10,000; D_2 is 20,000; C_1 is 20; C_2 is 25 so we substitute here and we get lambda is 292.20. Now once we find out lambda, we can go back and get the economic order quantities Q_1 star and Q_2 star. $2D_1$ into C_0 + lambda/iC_1, Q_2 to star is root of 2 D_2C_0 lambda /IC. So they become 172.76 and 2176.607 respectively. Now to verify, we can go back and find out $N_1 + N_2$, Number of orders we get exactly 15. We have solved with 15, we have solved for N is = 15. So the total number of orders becomes 15. Now the order quantities increase. Previously 1224.75 and 1549 have now gone up to 1720 and 2176. This is because the restriction on the number of orders will try to increase the order quantity. Now total cost also goes up to 13383.04.

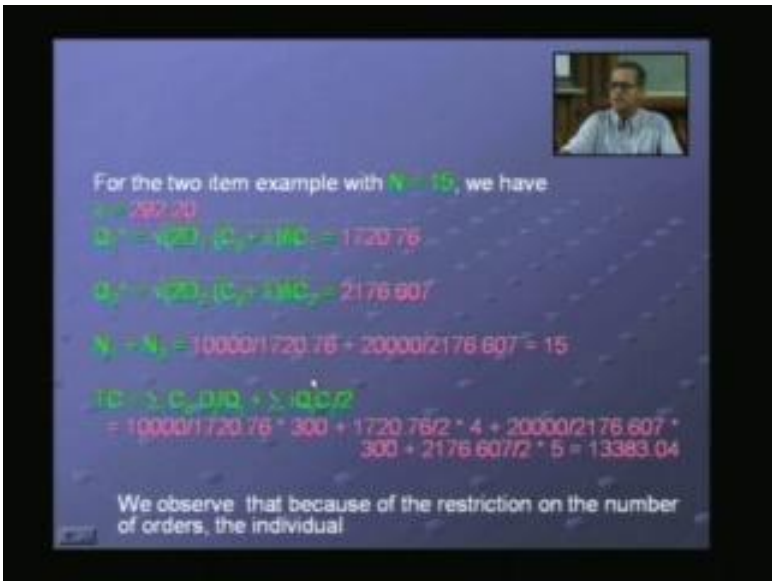
Individually C's are also higher than 42 hours. Any constrained problem minimization would always give a higher value to the objective function than the unconstrained problem. So the total cost increases. Order quantity has increased, but the total number of the orders now becomes exactly 15. Now we observe that because of the restriction on the number of orders the individual order quantities have increased.

(Refer slide time: 24:15)



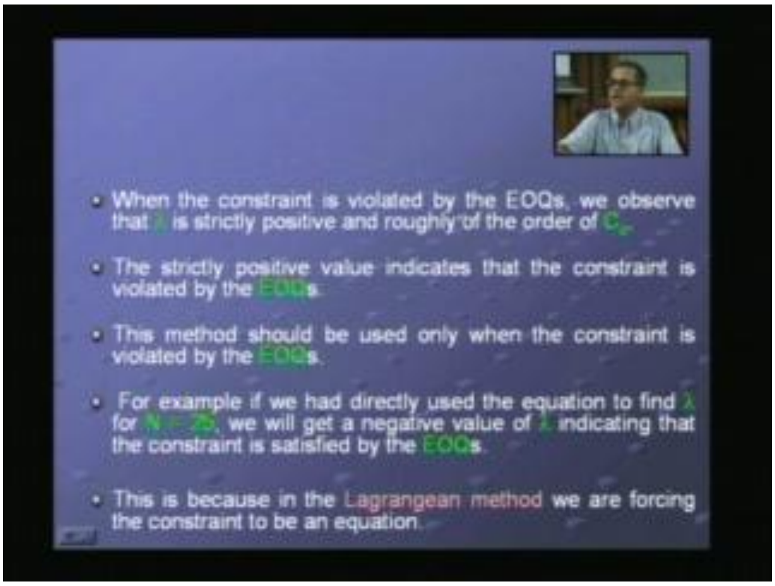
We can also observe this the new Q_1 star/Q_1 is = 1720/1224.75 which is 1.405. The new Q_2 star/the old Q_2 economic order quantity Q_2 is 2176.607/1549.19 which is 1.405 which is the same as N_1 + N_2/N, originally without the constraint 21.07 orders with the constraint 15 orders same 1.405.

(Refer slide time: 24:47)



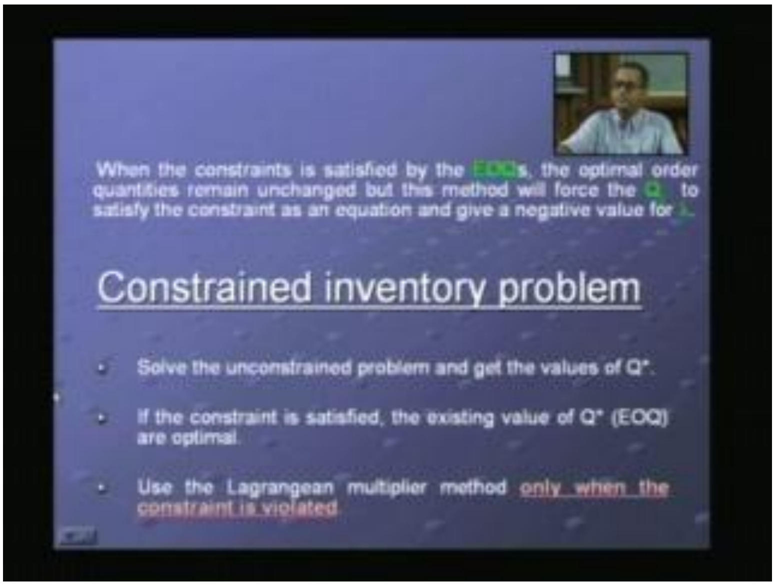
We can solve the problem optimally by using the Lagrangean multipliers, get the values for lambda, go back and substitute or we can simplify this, use this as an approximation to quickly get the values of Q_1 star and Q_2 stars because we know $N_1 + N_2 = 21.07$. We want to bring it down to 15, so the factor is 1.405 and is multiplied with the factor of 1.405 to get 1720.76 and 2176.607. So this could be used as an approximation as well.

(Refer slide time: 25:23)



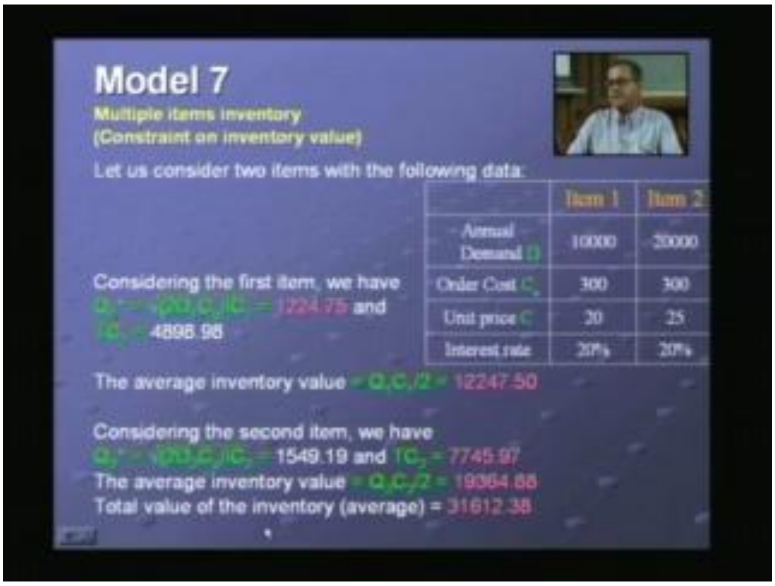
Now when the constraint is violated by the EOQ, we observe that lambda is strictly positive and roughly of the order C_0. The lambda was 292.20 which are roughly of the order of 300. For example you will not get lambdas like 0.1 and so on. So we will have high values for lambda which is of the order of C_0. Strictly positive value indicates that the constraint is violated by economic order quantities. This Lagrangean method should be used only when the constraint is violated by the EOQs, we have already seen that. For example if we had directly used equation to find lambda with N = 25, we would get a negative value of lambda. Now we had already seen that it is a less than or equal to condition. In case instead of 21.07, we said 25, we will be happy to order the EOQs and the constraint is satisfied. So we should use this model only when this constraint is violated by the economic order quantity. If we blindly use the lambda equation which we have here and try to solve for N = 25, then we will get a negative value for lambda. This negative value for lambda implies that the constraint is actually satisfied by the EOQ. There is absolutely no need to have used the value for lambda. Negative comes because we had forced an inequality to an equation. So we will try to solve it as an equation and give a negative value because in Lagrangean method we are forcing the constraint to be equation. So the Lagrangean method should be used only when the constraint is violated.

(Refer Slide Time: 27:05)



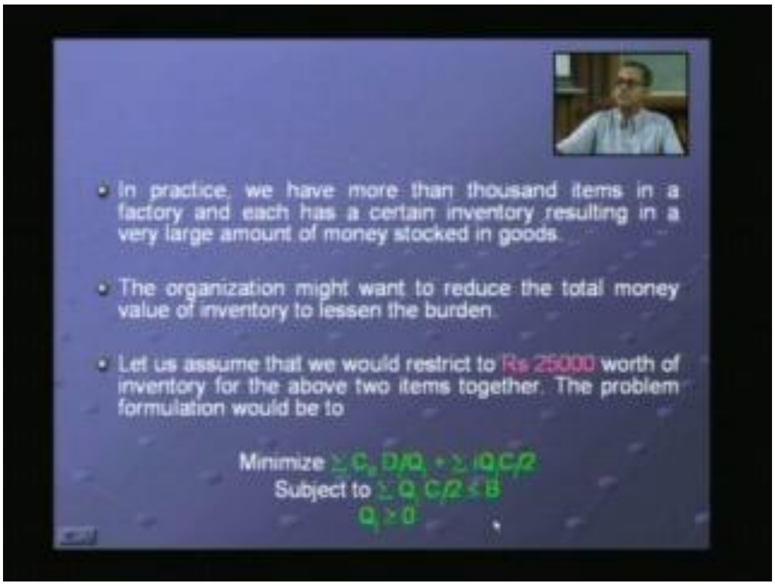
And the constraint is satisfied by the EOQs, the optimal order quantities remain unchanged but this method will force the Q_j to satisfy the constraint as an equation and give a negative value for lambda. Now what are the steps? Solve the unconstrained problem and get the values of Q star like what we did? We solve the unconstrained problems first and we got the values of the economic order quantities which are total cost and the number of orders. Then go back and check whether the constraint is satisfied or violated now we found 21.07 orders which violate the 15, then use the Lagrangean to solve. So the model constraint is satisfied. Existing Q star EOQs are optimal. Use the Lagrangean multiplier method only when the constraint is violated.

(Refer Slide Time: 27:58)



Now let us look at a different kind of a multiple item model. Now we look at the same problem here with the same economic order quantities 1224.75, 4898.98. What happens is the maximum inventory in the system is 1224.75. The average inventory is Q/2. Now that much of average inventory is held right through which means the certain amount of money is being locked up in the inventory. So money value or the average inventory value is Q_j $C_j/2$ or $Q_1/2$ into C_1 which is 12247.5 in this example and for Q_2 star for the second item the Q_2 star is 1549.19 into 25/2 would give 7745.97. So average inventory value $Q_2C_2/2$ is 19364.88 for 1549.19. And 1549.19 divided by 2 into rupees 25 would give us 19364.88. So both the items put together have a total money value or average inventory of both these items is 3162.38.

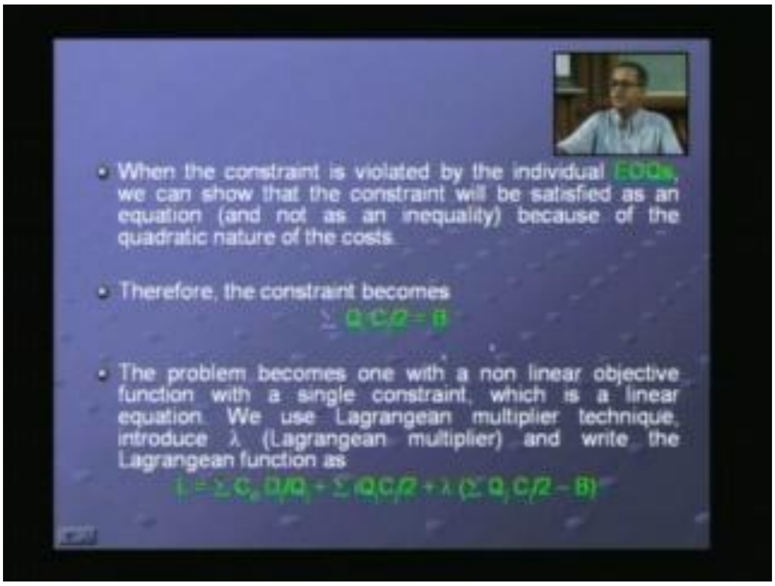
(Refer Slide Time: 29:26)



Now as we saw in the previous example, we have many items. More than thousands items in a factory. Each has a certain inventory resulting in a very large amount of money locked up in goods or stocked in goods. Organization might want to reduce the total money value of the inventory to lessen the burden and let us assume they want to restrict it to 25,000. Now the money value for these two is 31,612. We want to bring it down to 25,000. So we have the same objective function, to minimize the total of the ordering cost and caring cost subject to the condition Q_j $C_j/2$ less than or equal to B. This is the money value of the items. This is the restriction. This is the 25,000 that we have. This is the money value of the items at an average locked up in inventory Q_j greater than or equal to 0.

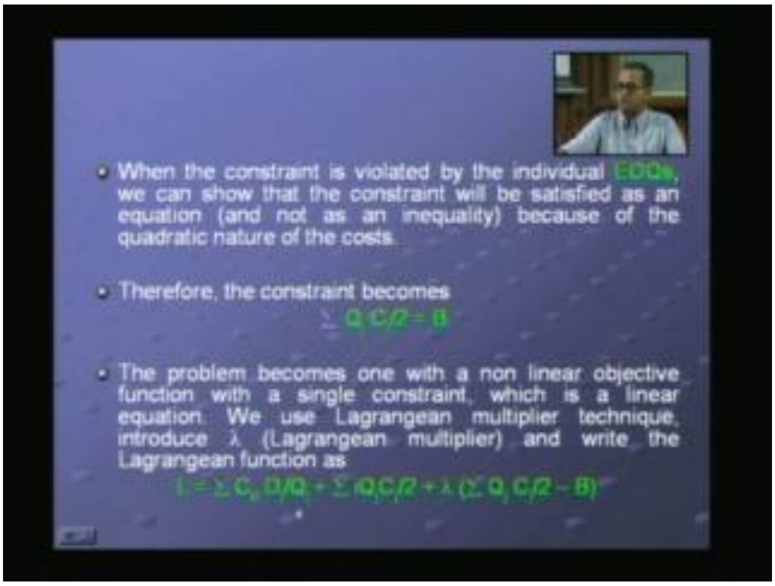
Once again we realize that this constraint is violated and when this constraint is violated. It can be solved as an equation. Objective function is a sum of the total order cost. Caring cost constraint ensures that the fixed amount B is met. We assume that individual economic order quantities violate this constraint, convert it to an equation and then solve it as we did in the previous case.

(Refer Slide Time: 30:43)



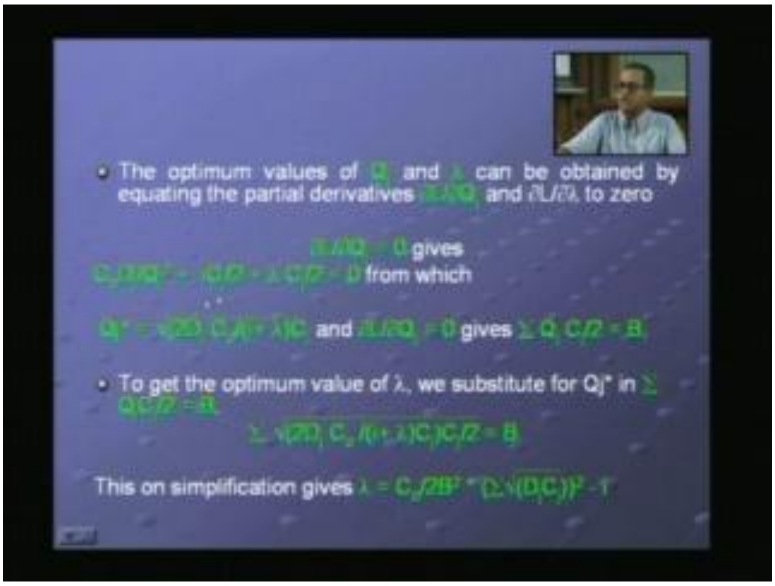
So when the constraint is violated, we could convert it to an equation sigma CQ_j $C_j/2$ is = B. Now we have a nonlinear objective function with a linear constraint is only on the money value of inventory so Q_j appears in the numerator. So we have a linear constraint objective function which is nonlinear because order cost component has Q_j in the denominator. So the Lagrangean function is given by sigma C_0 into D_j/Q_j. D_j/Q_j is the number of orders C_0 is individual order cost, so D_j/Q_j into C_0 is the money value of the order cost per year. Q_j $iC_j/2$ is inventory caring cost, iC_j is inventory cost and lambda is Lagrangean multiplier. The constraint Q_j $C_j/2$ = B is taken into the objective function. Lagrangean function is created by introducing a multiplier lambda which is multiplied by sigma Q_j $C_j/2$ – B. Now once again we have to find the optimum value of Q_j and lambda and these are obtained by equating the partial derivatives tau L/tau Q_j and tau L/tau lambda to 0. tau L/tau Q_j = 0 gives – C_0 D_j/Q_j square + $iC_j/2$ + lambda $C_j/2$ = 0.

(Refer Slide Time: 32:17)



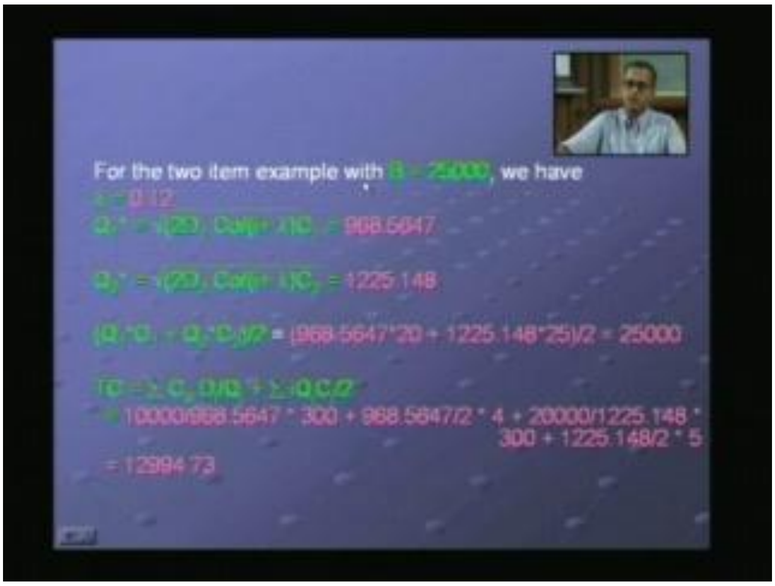
now this would give a – C_0 D_j/Q_j square because Q_j is in the denominator here Q_j is in the numerator so you get $iC_j/2$ here again Q_j is in the numerator you get lambda $C_j/2 = 0$ from which Q_j star is root of 2 D_j C_0 into i + lambda into C_j.

(Refer Slide Time: 32:39)



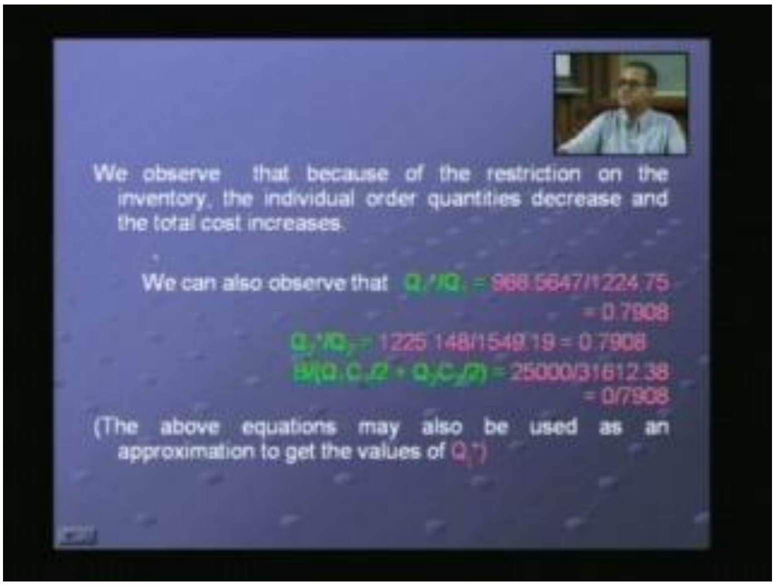
Now we can calculate this Q_j star. Only when the value of lambda is known right we do not the value of the lambda now to find out the value of lambda we partially differentiate with respect to lambda and get sigma Q_j $C_j/2 = B$. So we substitute for this Q_j from here root of 2 D_j C_0 into i + lambda C_j. Now to get the optimum value of lambda we substitute for Q_j star. In this now, this sigma root of 2 D_j C_0 into i + lambda C_j into $C_j/2 = B$. This on simplification would give lambda = $C_0/2$ B square, sigma root of D_j C_j the whole square – i.

(Refer Slide Time: 33:36)



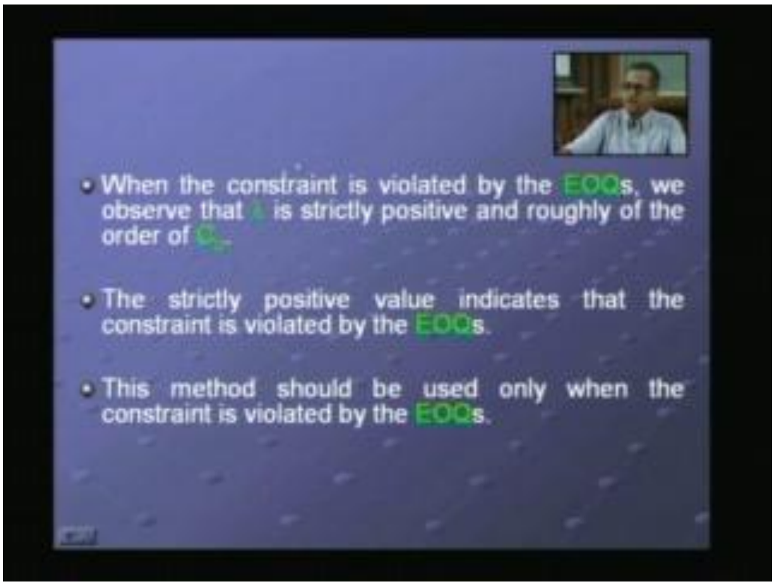
One needs to simplify this expression to get the value of lambda. Now for the 2 item example when we substitute B = 25,000 in this and all other values known C_0 is 300sigma D_j C_j root of D_j C_j can be calculated D_1 is 10,000, C_1 is 20, D_2 is 20,000, C_2 is 25, I is 0. 2. So if we substitute all these values, we would get lambda = 0.12. Now here we realize that lambda is roughly of the order of i. i is 0.2, so lambda is roughly of the order of i. Now Q_1 star is root of $2D_1C_0$ into i + lambda C_1 968.5647. Q_2 star is root of $2D_2$ C_0 I + lambda C_2 which is 1225.148. verifying Q_1 $C_1/2$ + Q_2 $C_2/2$ would give us exactly 25,000. We have solved this problem for B = 25,000. Sigma Q_j $C_j/2$ is 25,000. Now total cost for this will increase. 12994.73 is higher than the economic costs 4898.98 + 7745.97. So with the new values, the total cost would be higher. 12994 is higher. The individual order quantities have come down. They have come down because of the restriction on B equal to 25,000 and because they have come down, the total cost goes up.

(Refer Slide Time: 35:19)



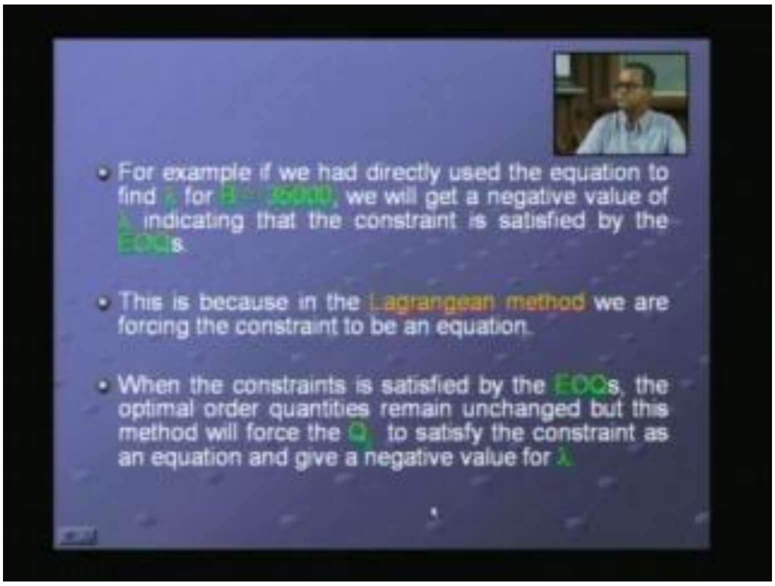
We observe that because of restriction, the individual order quantities have decreased and the total cost has increased. We also observe something which we did. Very similar to what we did in the in the previous model. With the new one divided by the old one is 968.5647 divided by 1224.75 which is roughly 0.7908. Similarly Q_2 star/Q_2 the new 1225.148 divided by the economic order quantity 1549.19 with also 0.7908. Now the budget is restricted to 25,000. Previously it was 31612.38 dividing one by the other, we would get 0.7908. Now once again these approximations can also be used to get the new values very quickly. Now this 25,000 is known, 31612 is known, from the economic order quantity. Dividing one by the other 0.7908 is known. So go back to the old economic order quantity, multiply by 0.7908, you will get the new value Q_1 star. So we either we can go back and use the Lagrangean and solve it, or we could approximate it by this simple term.

(Refer Slide Time: 36:33)



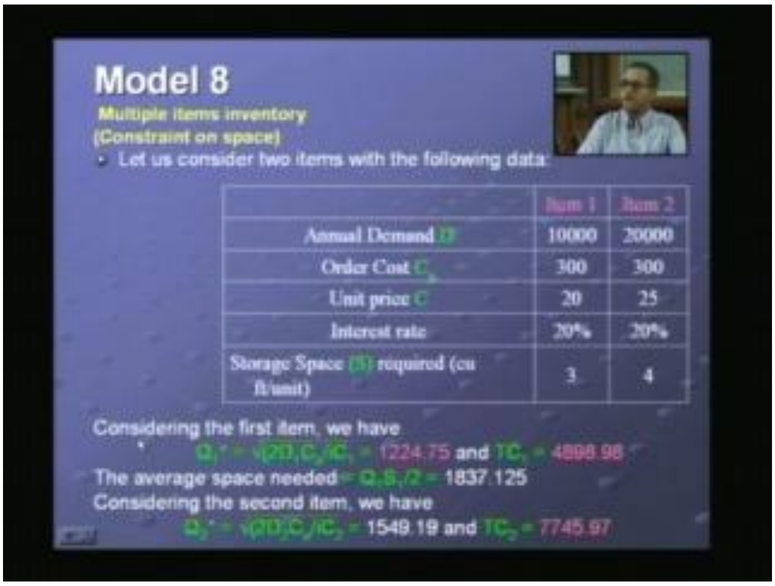
Once again we need to use or we can use the Lagrangean model only when the constraint is violated and lambda will be strictly positive roughly of the order of i when the constraint is violated. Strictly positive value of lambda indicates that the constraint is actually violated. Once again the method should be used only when the constraint is violated by the EOQs. If we had used it for B = 35000, now let us go back here. Now we had 31612. Now we place restricted B to 25,000. If we had put B = 35000 then the economic order quantity satisfies the budget restriction therefore the economic order quantities themselves will be optimum. However if we make the mistake of evaluating lambda for B = 35000 and use this expression we would get a negative value of lambda.

(Refer Slide Time: 37:33)



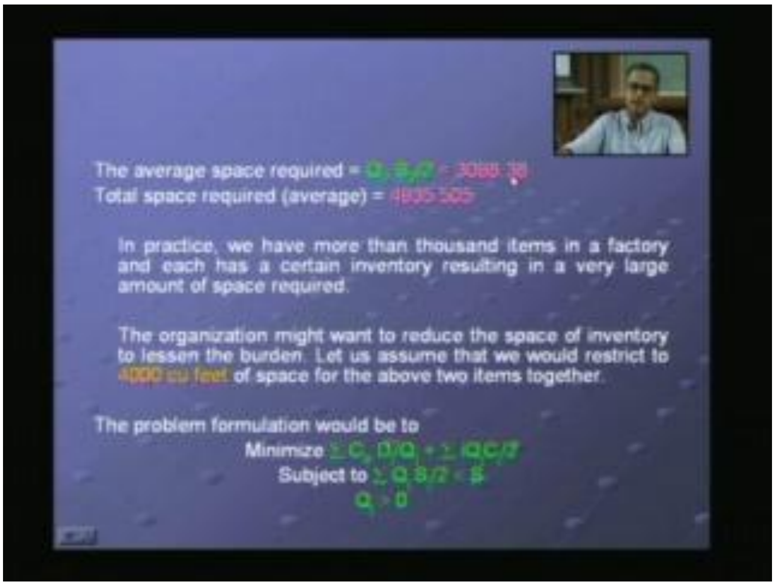
A negative value of lambda indicates that the constraint is actually satisfied by the EOQs. Once again this is because in the Lagrangean method, we are forcing the constraint to be an equation and we are solving it. So this is how we solve the budget constraint inventory problem.

(Refer Slide Time: 37:55)



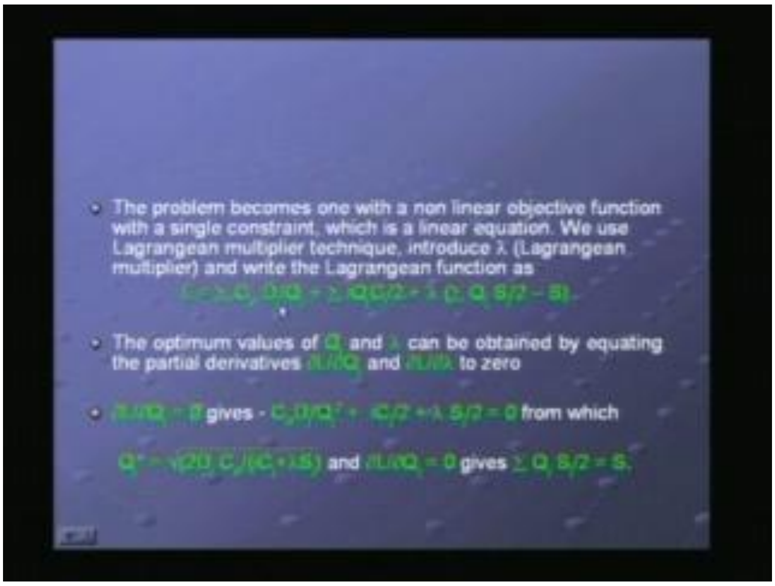
Now let us look at one more problem which is a space restriction so we again look at the same 2 items with the additional condition that there is a storage space that is required for each of these items and say we represent it as some kind of cubic feet per unit. We assume that item 1 requires 3 units of space and item 2 requires 4 units of space. So at the economic order quantity, we have Q_1 star as 1224.75 and the space requirement will be $Q_1S_{1/2}$ which is 1837.125 on an average. $Q_{1/2}$ is the average inventory. So we calculate the space requirements only for the average inventory not for the maximum inventory. This is because at any point in time not all items are going to be at their maximum levels, they are going to be at different levels of inventory and an average is a very reasonable approximation for the present state of the inventory. So we compute the space requirement for the average not for the maximum.

(Refer Slide Time: 39:00)



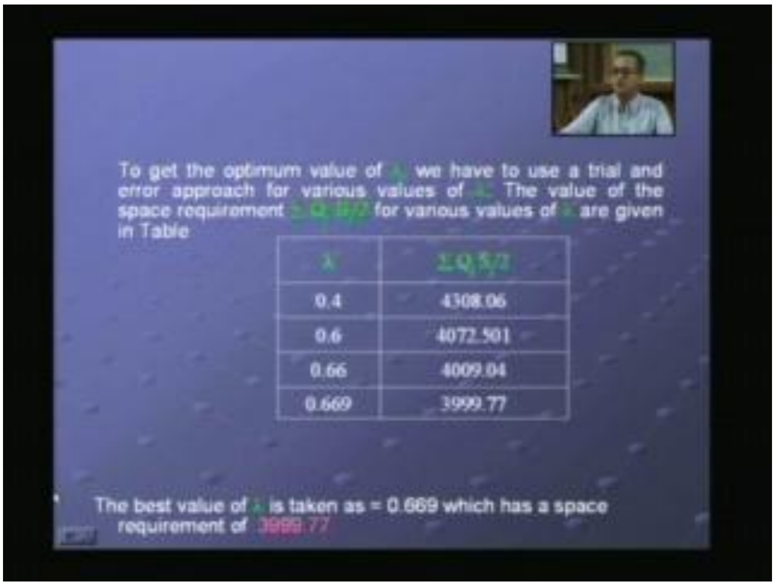
Similarly for the second item, the economic order quantity is 1549.19 TC is 7745 and the space requirement is 3098.38. Together the 2 items require 4935.505. Once again we argue that we will have many items in the factory, each requiring a certain amount of space. So we would like to minimize the quantity so that the available space can only be used. So we assume that we want to bring down the 4935.505 to 4000 cubic feet. Once again we have the same objective function with the constraint alone changing to Q_j $S_j/2$ is less than or equal to S. S is the restricted storage space of 4000. Q_j greater than or equal to 0, once again we can show that we can convert this into an equation and solve it and using the method of Lagrangean multipliers, the economic order quantity violates this 4000. In our example the economic order quantities require 4935.5. The restriction of 4000 is violated by them so the constraint is not satisfied by the economic order quantity. They now need to solve the constrained problems and get the new values of Q_j star and we can solve it as an equation because the constraint is binding. So we convert it to an equation as we have been doing right through and then the constrained becomes sigma Q_j $S_j/2$ is = S.

(Refer Slide Time: 40:37)



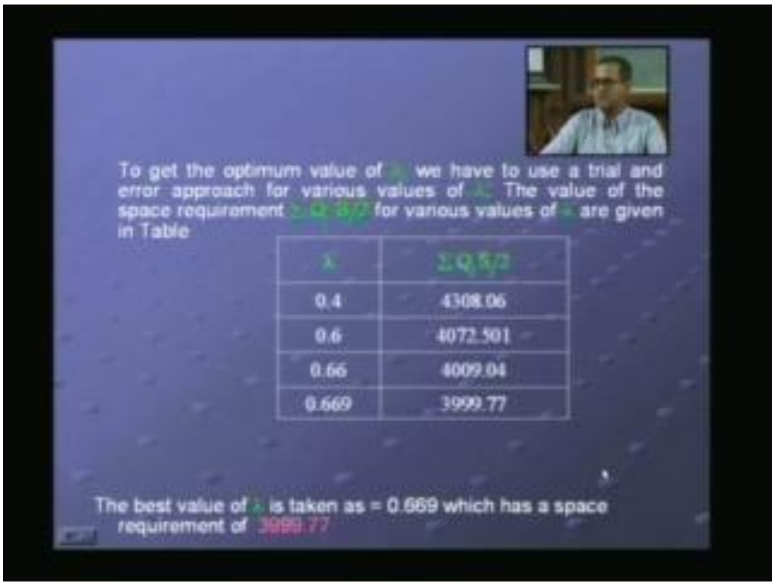
Now we set up the Lagrangean function same objective function sigma C_0 D_j/Q_j which is the total ordering cost iC_j $Q_j/2$. Total caring cost of inventory lambda is Lagrangean multiplier Q_j $S_j/2$ – S. This represents the constraint. So constraint is brought into the objective function. Now once again the optimum values of Q_j and lambda can be obtained by setting the partial derivatives to 0, you get – C_0 D_j/Q_j square + $iC_j/2$ + lambda $S_j/2$ = 0. This Q_j is in the denominator. So you get – C_0 D_j/Q_j square. Q_j in the numerator $iC_j/2$ Q_j again in the numerator lambda Sj/2 lambda Sj/2 = 0 from which Q_j star is root of 2 D_j C_0 into iC_j + lambda $S_{j.}$ Once again we can compute that 2j star only if we know the value of lambda and in order to find out the value of lambda we go back and substitute sigma Q_j Sj/2 = S

(Refer Slide Time: 41:45)



Now when we go back and substitute, we cannot get an expression for lambda as we did in the previous case or we do not get nice compact expressions as we did in the previous 2 cases. In this case we have to substitute for various values of lambda and then find out whichever works for this constraint. That value of lambda brings a left hand side close to right hand side. This is the optimum value unlike in the 2 previous cases.

(Refer Slide Time: 42:10)



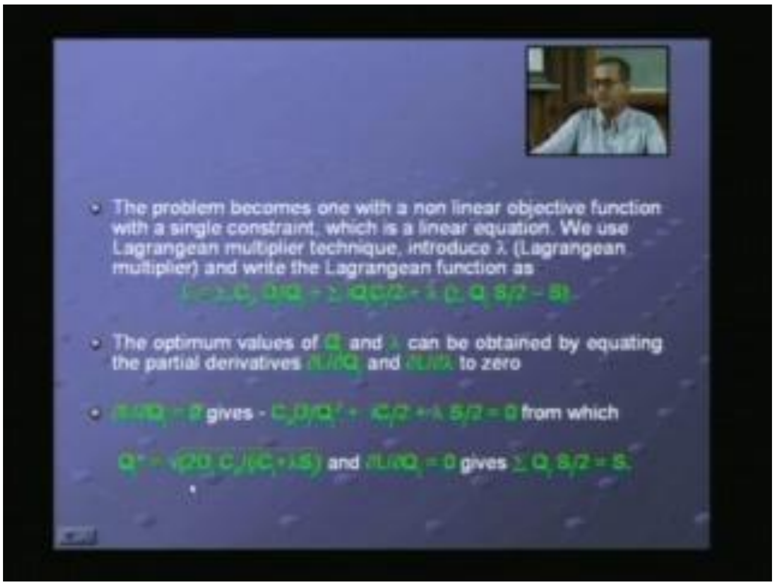
So we compute Q_j Sj/2 for various values of lambda and we get for lambda = 0.669, we get 3999.77, which we think is reasonably close to 4000. So we stop lambda is = 0.669 which has space requirement of 3999.77.

(Refer Slide Time: 42:35)

Now the new values of Q_1 star and Q_2 star will be this. We have to substitute iC_1 + lambda S_1 iC_2 + lambda S_2 to get 9990.41 and 125.326. Because of the space restriction, the order quantities come down. The total space is 3999.77 and total cost would increase to 12925.15. Now we also observe that because of the restriction in space individual order quantities decrease and the total cost increases. Now you can also have an approximation once again as we had previously. So new value by old value of the order quantity gives us 0.816 Q_2 star/Q_2

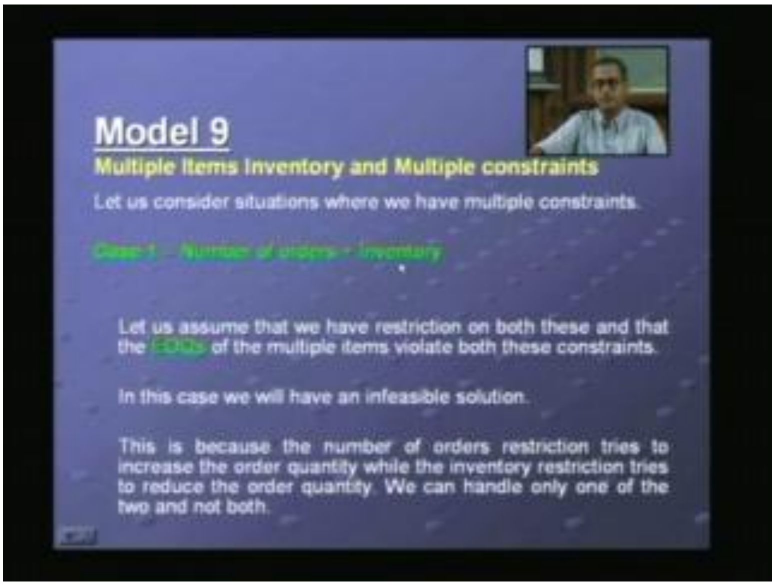
gives us roughly 0.807 and the 4000 by the actual space taken by the economic order quantities also give us a 0.81. So we may assume 0.81 to be a reasonable approximation for this and we can go back and get approximate values of Q_1 and Q_2 square instead of taking the trouble of setting up the Lagrangean, getting this expression and substituting to get the best values of lambda.

(Refer Slide Time: 43:49)



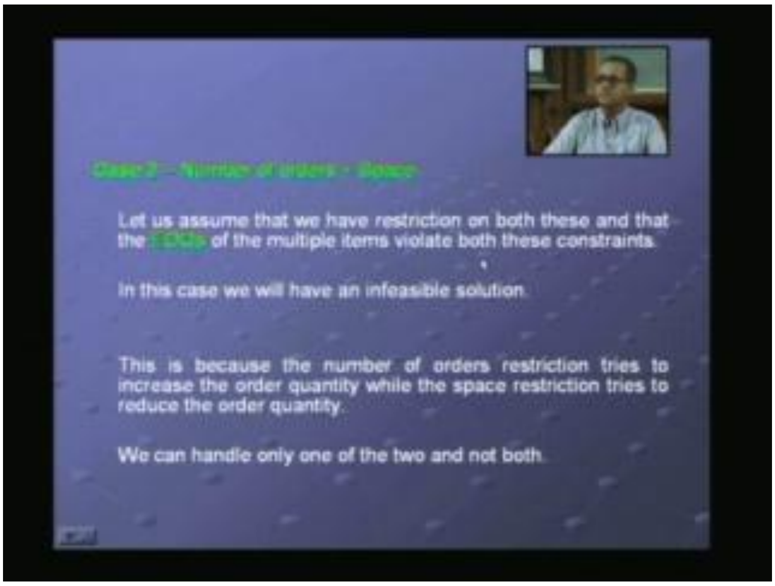
So if we want the optimal, we get this Q_Q star expression but to get the value of lambda, one need to go back and look at substitution or one can alternatively use this approximation. We know this 44935. So we know this fraction 0.81. Go back and multiply the economic order quantities with that fraction to get the approximate value of Q_1 star and Q_2 star. Now when the constraint is violated by the economic order quantities we observe that lambda is strictly positive and roughly of the order of and lambda in this case is strictly positive. Now the strictly positive value indicates that the constraint is violated/the EOQs and Lagrangean multiplier should be used only when the constraint is violated by the EOQs. Otherwise it would end up giving us a negative lambda and so on. So we would get a negative value and so on.

(Refer Slide Time: 44:42)



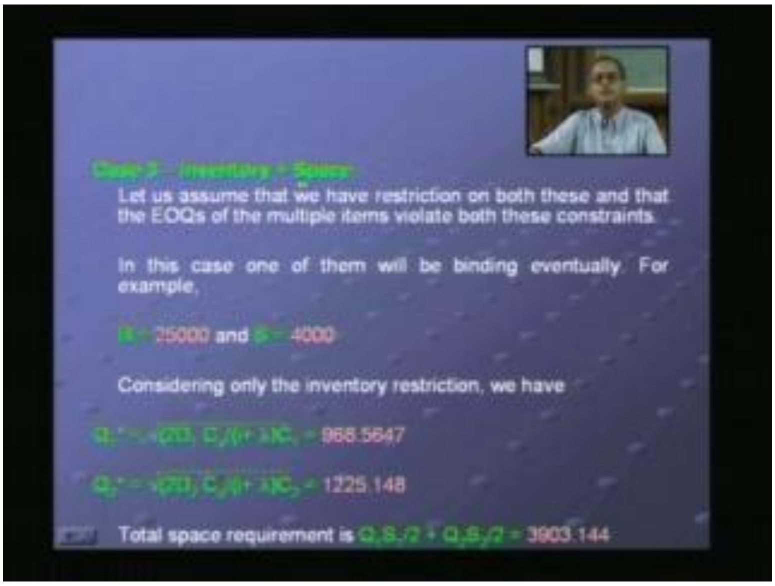
Now let us go back to the last model. We look at multiple items and multiple constraints. Now we could have a situation where we would like to have restrictions on the number of orders as well as restriction on the inventory. For the same problem let us assume that we have restriction on both these. We want to bring down the 21.07 to 15 as well as bring down the inventory. Now let us assume that we have a restriction on both. The EOQs violate both these restrictions. When both of these are violated, we will have an infeasible solution. This is because the number of order restriction tries to increase the order quantity. The inventory restriction tries to reduce the order quantity at any 0. In time we can handle only one of them and the economic order quantity is violating both so we cannot have a feasible situation in case 1. When the EOQ violates both and there is a restriction on both. We will be able to satisfy only one of them.

(Refer Slide Time: 45:39)



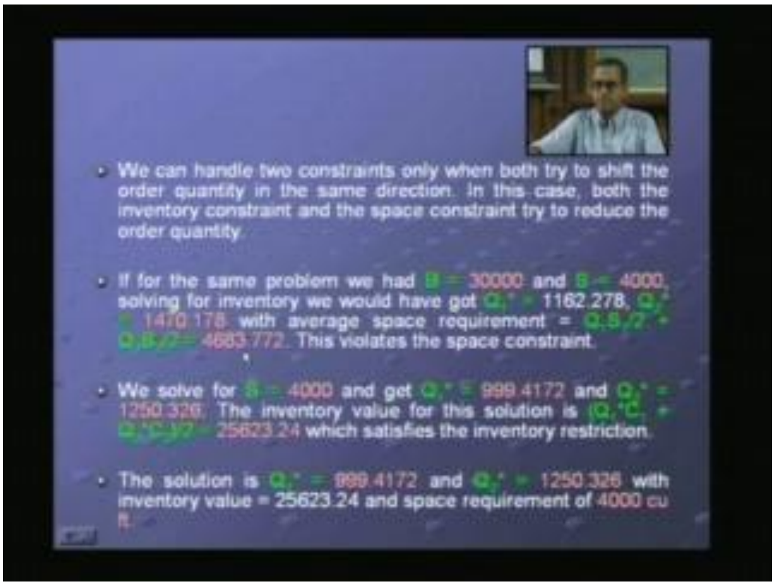
When we have a number order restriction and space restriction once again, we have a restriction on both EOQs. Let us assume it violates both. As in the examples that we have seen Once again we will have an infeasible solution because the order restriction tries to increase the economic order quantity space restrictions tries to reduce the order quantity they conflict each other the EOQ is violating both so we cannot have a situation that satisfies both these we can handle only one of them and not both.

(Refer Slide Time: 46:10)



In our case 3, where we have restriction on inventory space, this is possible. We assume that we have restriction on inventory and space and the EOQ violates both considering our own example we looked at 25,000 and 4000. And we consider only the inventory. We got these values 968.56 and 1225.148 using Lagrangean multipliers. Now the space requirements for this is 3903 which is okay, which satisfies this 4000. Therefore we assume that this is optimal. We cannot actually satisfy both. We will only be able to satisfy one and the other will be got via solving for one and substituting in the other. We find out which one works. In this case we have solved for inventory and when we substitute, the space works therefore this is the optimum solution.

(Refer Slide Time: 47:08)

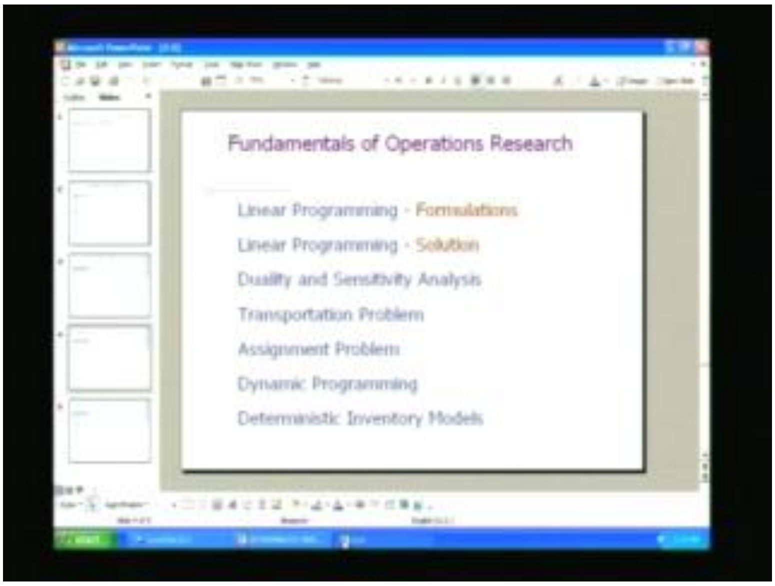


We can handle 2 constraints only when both try to shift the order quantity in the same direction, both the space constraint and the inventory constraint try to shift the order quantity and reduce the order quantity. In this case both reduce therefore we can consider. If for the same problem we had a budget restriction of 30,000 and space restriction of 4000 first, we would solve for inventory. We would have got Q_1 as 1162 Q_2 is 1470 with the space requirement of 4683. So when we had solved for budget of 30,000 we find the space restriction is violated. So we go back and solve for the space 4000. We already know the answers 999.150. Now we compute the inventory values of this which is 25623 which satisfies. When we have more, we first solve for one and substitute in the other. When the solution for 1 satisfies the other, we get the optimum.

Normally the practice is for this problem. The solution is 999.4172 or 15.326, inventory value of 25623 which is less than the 30,000 and is okay. Space 4000 is exactly satisfied. Now it is customary that we start when we have these 2 restrictions between the inventory as well as the space. It is easier to solve for the inventory because we know the expressions for lambda. So when we have a problem like this, we first solve for the inventory and if the inventory solution satisfies the space then they are optimal. But if the inventory violates, as in this example, then we have to go back and solve for the space and then go back and substitute the inventory which will eventually be satisfied.

Only one of them will be the optimal solution. So with this we can end the discussion on deterministic inventory models. We sum it up by saying that we have looked at several aspects of deterministic inventory models. We saw the 4 basic models to find out the economic order quantity as well as the economic batch quantity for production. We then looked at the inventory models with discount and then we looked at the inventory models for multiple items with 3 types of constraints on the number of orders, constraints on the money value of inventory and constraints on the space. We solved all 3 of them individually and then we also looked at combination and said that we can solve only the combination of inventory and space and we worked it out through an example.

(Refer Slide Time: 49:57)



Now at the end of this inventory we come to the end of the first course on fundamentals of operations research and before we wind up let us go back and try to give a summary of what we have seen in this lecture series on the course fundamentals of operations research. We have 22 lectures as part of this first course or the introductory course on operations research. We looked at 7 modules in this course the course can broadly be divided into 7 modules. The 7 modules are linear programming problem - Formulations, linear programming - solutions, duality and sensitivity analysis, transportation problem, assignment problem, dynamic programming and deterministic inventory models. So these were the 7 modules that we have addressed. If we go back and try to recapitulate what we saw in each one of these linear programming formulations introduced, we saw multi various formulations.

We saw 4 distinct examples of formulations. We saw how to formulate a real life situation into a linear programming problem. We saw maximization and minimization objectives. We defined the terminologies, objective function, constraints, and decision variable. We saw how to write objective functions. There are 2 types of objective functions, types of constraints, such as inequalities, equations, inequalities with greater than or equal to less than or equal to equations. Types of variables such as greater than or equal to, less than or equal to and unrestricted. We saw enough examples to illustrate the various types of objective functions, types of constraints and the types of variables. In this course when we moved to linear programming solutions, we saw 3 methods to solve linear programming problems.

The graphical method which was applicable for a 2/2 problem or a 2 variable problem could be represented in a graph irrespective of the number of constraints. Then we looked at the algebraic method which is an improvement because it could handle multiple variables and then we saw the limitations of the algebraic method and then we introduced the simplex algorithm both in algebraic form as well as in tabular form as a method which is superior to the algebraic method. We also saw the various aspects of simplex. We saw the various aspects such as initialization, iteration and termination. How to construct the initialized simplex stable, what happens during iteration and what are termination conditions etc. We solved minimization problems, particularly, problems with greater than or equal to constraints. We saw aspects such as degeneracy, alternate optimum and roundedness and infeasibility and defined the termination conditions for linear programming problems. Then we looked at the duality and sensitivity analysis. We introduced the dual. We saw mathematical expression to the dual. We also saw the physical meaning or economic meaning of the dual and economic interpretation of the dual.

We also looked at some duality theorems and we also showed that the simplex algorithm satisfies or solves both the primal and the dual and it is an example of primal logarithm where it always keeps the primal feasible and when the dual becomes feasible, becomes optimum. We also looked at the economic interpretation. We also saw the dual simplex method which is another variation of the simplex algorithm which is used to solve particularly minimization problems with greater than or equal to restrictions. Under the sensitivity analysis, we looked at what happens to the problem when there are changes in the coefficients. We looked at 7 aspects in the sensitivity analysis, changes in the objective function coefficient of a non basic variable, changes in objective function coefficient of a basic variable, changes in the right hand side values, changes in the constraint coefficient of a non basic variable adding a new product or adding a new variable or adding a new column.

They all mean the same. Adding a new constraint, adding a new row also means the same. So we looked at these 7 aspects and finally we said that if there is a change in the constraint coefficient of the basic variable then we do not use sensitivity analysis and solve the problem all over again because the matrix B would get affected by such a change. If the change is such that the B inverse is not affected then it is it is good to do sensitivity analysis and sensitivity analyses is useful under those circumstances. Now these 3 modules is the entire linear program in portions of the course. Modules 4 and 5, we looked at transportation problem and assignment problem which could be formulated and solved as linear programming problems. They are formulated as linear programming problems but both are not solved using the simplex algorithm directly but they are solved using algorithms which have all the properties and characteristics of the simplex algorithm. So we formulated the transportation problem.

We showed that it is a linear problem and then we said we could solve the problem faster and better compared to the simplex and we said there are 2 aspects of solving the transportation. One is to get a good basic feasible solution and then to try and improve this basic feasible solution to get to the optimal solution. We looked at 3 methods rule, minimum cost method and Vogel's approximation method to get the initial basic feasible solution. We also looked at the 2 face. We looked at the stepping stone method and the UV method to solve the transportation problem optimally. We also solved some degenerated cases of transportation problem to show the possibilities that can happen when one tries to solve transportation problems. We then moved to the assignment problem which is the special case of transportation problem.

We explained Hungarian algorithm that is used to solve the assignment problem. We also showed Hungarian algorithm is optimal and why we need a special algorithm to solve the assignment problem even though it is a special case of a transportation problem. We then moved to a different aspect called dynamic programming. Dynamic programming problems were different from the problems that we encountered in the first 5 modules. Dynamic programming is a very special part of operation research where we solve the problem by defining the stage, state decision variable and criterion effectiveness. We solve the problem at various stages, one variable at a time with a dependency being modeled through the use of state variables. We saw different aspects. We saw problems where the decision variables took discrete integer values where they took continuous values. We saw situations where we maximized, minimized. We used calculus to find out the first derivative. There were situations where we used linear functions and evaluated the function at the extreme points. We also through 2 examples attempted to show how integer programming problems and linear programming problems can be solved using dynamic programming. Lastly, we addressed deterministic inventory models and we saw 9 different models to cover the various aspects of inventory including economic order quantity, economic batch quantity, discount and multiple item models. So these 7 modules and the topics that we covered would kind of take us through the first course in operation research. There are other topics which are covered under the fundamentals of OR course but we are going to restrict ourselves only to these 7 modules. There are many more topics that are there in the field of operation research. It is a very vast field with lot many tools and techniques available.

There are other topics that have to be studied. Topics such as advanced topics and linear programming methods like how to handle bounded variables, decomposition algorithm, column generation methods, goal programming that looks at multiple objectives and so on. How to efficiently invert a matrix and how to make the simplex algorithm better and faster, what are the various other aspects of linear programming in terms of its complexity and so all these have to be studied.

We will have to cover additional topics such as integer programming, optimization, network models, nonlinear programming, game theory, and queuing theory and so on. All these need to be covered as topics of operation research. All these topics have not been covered in this first course that we have seen in 22 lectures. Now we need to look at all these aspects as well topics such as integer programming, network flows, traveling salesman problem, queuing theory, quadratic programming, and nonlinear programming are all usually covered in the advanced operation research course and they are not being covered and have been left behind in this course.

So with this we essentially come to the end of our treatment of the first course in operations research course with these 7 modules and we look forward to another course advanced operations research which is handled separately. We hope you benefit by viewing this and listening to whatever we have covered in 22 lecture hours. You are also free to give your feedback to the people who have made this possible.

Thank you!