Fundamentals of Operations Research

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Lecture No. # 11

Simplex Algorithm in Matrix Form

Introduction to Sensitivity Analysis

In this lecture we will study the matrix representation of the simplex algorithm. We will also derive a couple of equations that we will be using later in sensitivity analysis. We take this familiar example. First do this simplex iteration and then explaining the matrix method of solving it.

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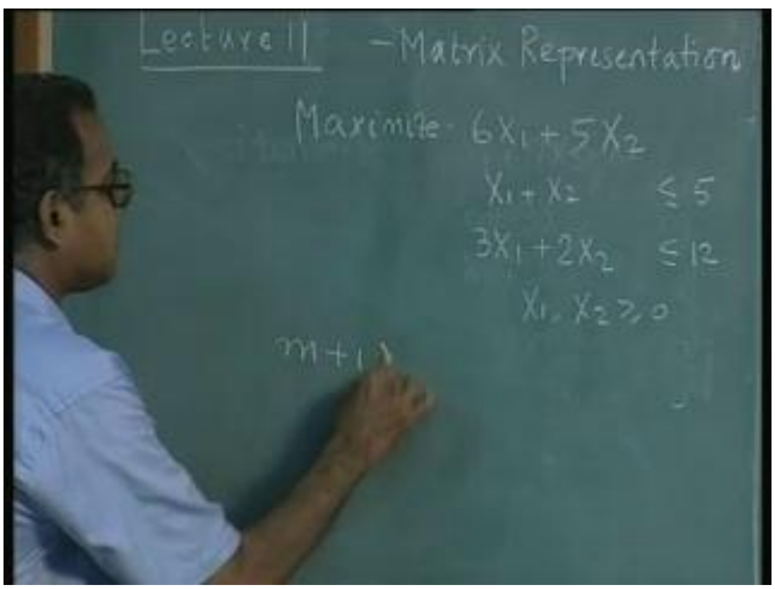

Problem is to maximize $6X_1 + 5X_2$ subject to $X_1 + X_2$ less than or equal to 5; $3X_1 + 2X_2$ less than or equal to 12; X_1, X_2 greater than or equal to 0.

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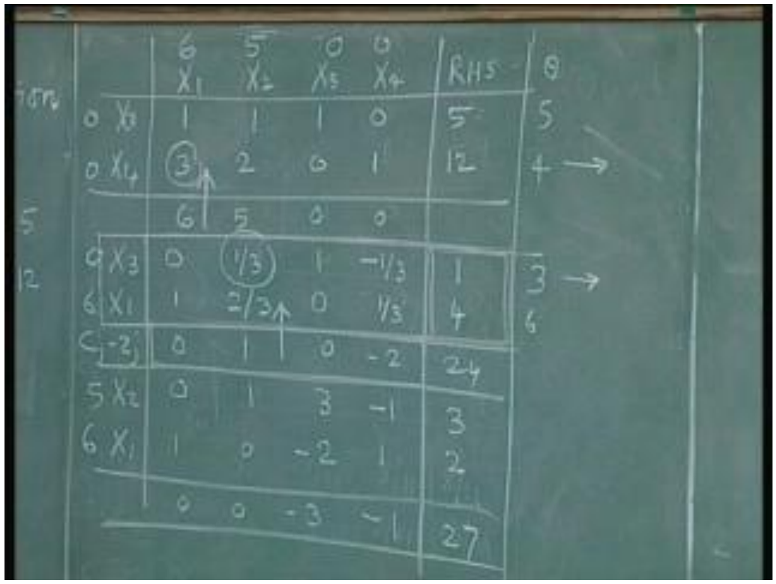

Let us quickly set up the simplex table and solve this problem. We have $6X_1 + 5X_2$. We will start with X_3, X_4. We have 1 1 1 0 5, 3 2 0 1 12, 5 0 0. Variable X_1 enters; we compute the values of theta so we have 5 and 4. Variable X_4 leaves. This is the pivot element. I have X_3 and X_1 here. 0 1/3 1 – 1/3 1 1 2/3 0 1/3 4 $C_j – Z_j$ will be 1, – 2, 24. Variable X_2 enters. Now theta is 3, 6 leaves pivot element. X_2, X_1 we have 0, 1, 3 – 1, 3, 1, 0, – 2, 1 and 2 Z = 27 we have 0, 0, – 3, – 1.

Now when we solve this problem using the simplex iteration what do we exactly do in every iteration? Suppose we take an intermediate iteration, that is shown here, the first thing we are interested in is whether this is optimal. So in an intermediate iteration the first thing we want to do is to find out the C_j's – Z_j's for the non basic variables. If all the C_j's – Z_j's are less than or equal to 0 then it is optimal and then we compute the value. It is enough if we compute the value of the objective function after we have found out that it is optimal. If it is not optimal then all that we need is what the entering variable is and what the leaving variable is. Essentially, this is the first thing that we would like to compute. The moment we know in an intermediate iteration, these are the basic variables and we will also have corresponding values here. We do not need to compute this entire matrix. We need to compute the C_j's – Z_j's only. These two numbers would be anyway be = 0 corresponding to the basic variables.

We need to compute these two C_j's – Z_j's and if one of them happens to be positive, it enters or if both are positive, the maximum enters. Then we need to find out the leaving variable for which we need this column corresponding to the entering variable and then since we have the right hand side we compute the theta or at the end of this iteration we know what the basic variables in the next iteration are. Now these are the basic variables in the next iteration and then we realize that we have to compute these $C_j – Z_j$'s. Both of them turned out to be negative. We get the optimum so we do not necessarily require computation of this matrix which we end up doing in our tabular form therefore can we work out this simplex in a different way by which we do the minimum required number of computations and not end up computing this entire matrix.

For example if after adding the slack variables in every iteration, we have m constraints and n variables.

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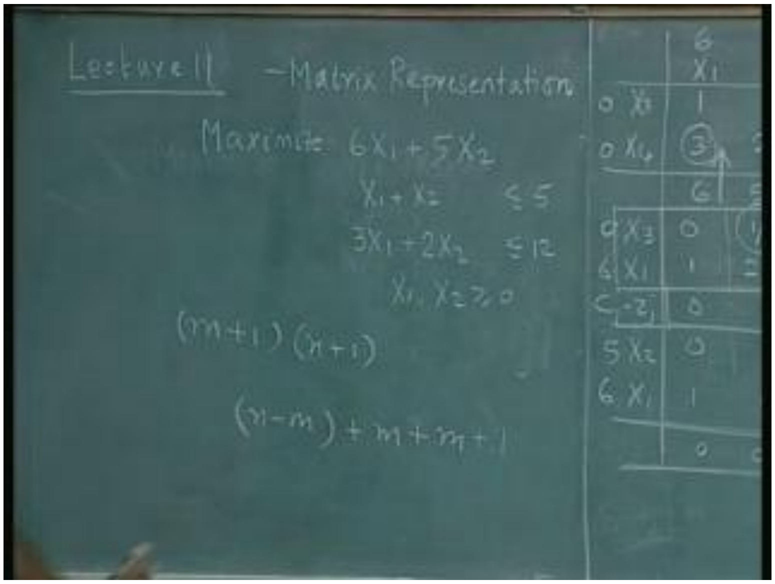

We actually compute an m + 1/n + 1 because there are n variables under right hand side; we have m constraints under $C_j – Z_j$. So in every iteration we compute an m + 1 into n + 1 where as what we need are the n – m values of the non basic $C_j – Z_j$'s plus, we need m values of the right hand side plus we need m values of the entering column plus we need one value at the optimum of this. So we need only this much. While we need m + n + 1 value, we end up computing a matrix which is m + 1 into n + 1. Can we re-do this simplex in such a way that we compute only the required number of elements of this in every iteration? In the process we can learn a couple of equations which will be used in sensitivity analysis later. Let us go back and start the simplex in what is called as a matrix representation of the simplex.

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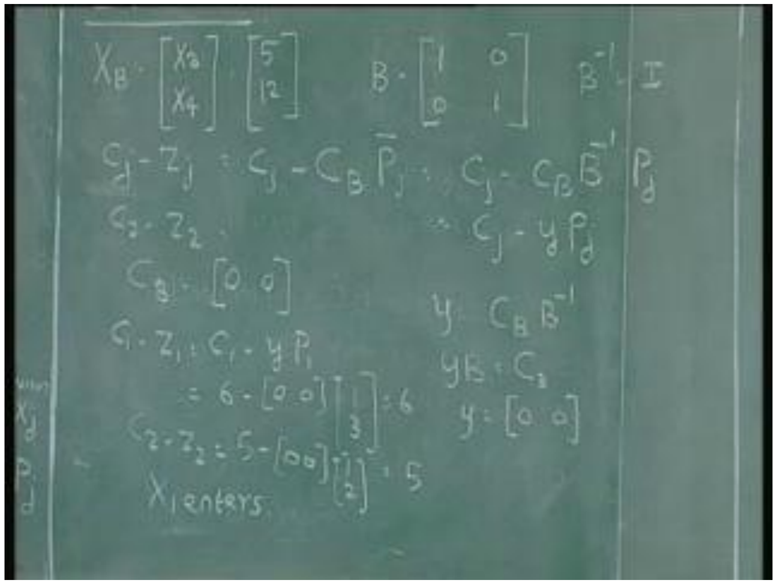

We have the first iteration. We have now defined the set of basic variables. X_3 and X_4 is now defined as set X_B, a set of basic variables. So we have X_3 and X_4 here and that is equal to the right hand side values which are 5 and 12. We have to check whether this is optimal to do that. We need to find out values of $C_j – Z_j$ for variables X_1 and X_2. So we need to compute $C_1 – Z_1$ and $C_2 – Z_2$. These are the two things that we need to compute. Now we are going to write an equation which we will derive little later and show how that equation holds but right now we will use that equation to get the values of $C_j – Z_j$. We are going to call every column of the given problem as P bar j. Now P bar j represents column corresponding to variable X_j and then the moment we define an X_B we also define something called a Basis matrix which corresponds to the P bars i.e., we define P_j. We define Pj as the column in the original matrix and then the basis matrix B is now obtained from the P_j is corresponding to the basic variables.
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In this case the basis matrix will be P_3 and P_4 which is 1 0 0 1. For variable X_3 the corresponding co-efficient is P_3 which is 1 0. For variable X_4 the corresponding column is 0 1 which is our P_4. So because X_3, X_4 are basic variables B is made out of P_3 and P_4 in the order in which they appear as basic variables. The B matrix is this. In this case the B matrix turns out to be an identity matrix so B inverse is also i in this case because B is i.
we are going to use an equation which goes like this In any intermediate iteration of the simplex for example if we are here, then corresponding to variable X_2, this column in an intermediate iteration is given by P bar j and that P bar j is going to be defined as B inverse P_j where B corresponds to the basic variables of that iteration. If we are looking at this iteration then the basic variables are X_3, X_1 the corresponding B will be 1 0 1 3. B is defined for a given set of basic variables.

Once we define a set of basic variables here any column in the intermediate simplex iteration will now be given by P bar j = B inverse P_j where P_j will be for this (Refer Slide Time: 11:41). This is for P_2 so P bar 2 = B inverse P_2. This is P bar 2. This is P_2. This is X_B for which the basis matrix will be 1 0 1 3. We can invert it and then we can actually prove that this is B inverse P_2

but right now we assume this. Any $C_j - Z_j$ for any j, for example the way we computed this $C_2 - Z_2$ is we first computed Z_2 which is the dot product of this and this 0 into 1/3 + 6 into 2/3 which is = 4 therefore the dot product of this and this. So this we call as C_B. C_B is the objective function co-efficient corresponding to the basic variables. In this case 0, 6 because of this 0 and this 6, X_3 comes first X_1 comes later so C_B will be 0, 6. Here Z_j is nothing but the dot product of C_B and P bar j, so this is C_j – dot product of C_B and P bar j. This is $C_j - C_B$, B inverse $P_{j.}$ We now write this C_B, B inverse as y and we write it as $C_j - yP_j$ where y is C_B B inverse.
Now for this problem, this is B and C_B is 0, 0 because basic variables are X_B are X_3 and X_4. Corresponding to them the values in the objective function are 0, 0 so C_B is 0, 0.
In order to find $C_j - Z_j$ or $C_1 - Z_1 = C_1 - yP_1$, I need to find out a y now. y is C_B B inverse. Pre multiplying I have $yB = C_B$. In this case B is i, so y itself is C_B = 0 0, because C_B is = 0 0; y is = 0 0. So C = 1 – $Z_{1;}$ C_1 is = 6; 6 – y P_1 1 3 which is 6 now $C_2 - Z_2$ is 5 – 1, 2 which is 5. Now the variable with the most positive $C_j - Z_j$ enters so X_1 enters.

Now to find out the leaving variable, we need to find out which is shown here. X_1 enters.

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We need to find out the column corresponding to the entering variable so we need to find out a P bar 1 which is B inverse P_1. B inverse is i, so in this case P bar 1 is 1, 3. Now to find out the leaving variable we need to find out a theta as we did in this case where we got 1, 3. We have right hand side = 5, 12. So we need to find out theta such that now there are two ways of doing it. One is we can say that we have to find out the minimum theta which is 5 divided by 1 and 12 divided by 3. The minimum one will be 4 which is what we have seen here.

The other way of doing it is to find out minimum theta such that 5 – theta 5 – 1 theta is greater than or equal to 0; 12 – 3 theta is greater than or equal to 0. The same values 5, 1, 12, 3 are being used. Either we can say theta as the minimum of 5 divided by 1, 12 divided by 3 or saying that we want to find out minimum theta such that 5 – theta is greater than or equal to 0, 12 – 3 theta is greater than or equal to 0. This would give us theta = 4. Because the second variable gives us theta = 4. Because of that X_4 leaves the basis and the new set of basic variables is X_B is equal to, X_4 leaves the basis. X_1 replaces. So you have X_3, X_1. Interestingly you can now go back and write the actual values as right hand side. By doing this the second one went out. If you substitute theta = 4 here, you get a 0. But the value here is not 0 but the value of theta itself. So you will get theta here and for the other one you can substitute 5 – theta is 1, so for the second iteration, the solution is based on a set a basic variables X_3, X_1 with solution 1, 4. So we have X_3 X_1 with 1, 4.

Now we have C_B corresponding to X_3. X_1 will be 0, 6. The basis matrix B corresponding to X_3 X_1 will be (1, 0), (1, 3). Now we need to check whether it is optimum. We need to find out the $C_j – Z_j$ corresponding to the two non basic variables X_2 and X_4. So we need to find out $C_2 – Z_2$ and $C_4 – Z_4$ as we go back, $C_j – Z_j$ is $C_j – yP_j$. We need to find out y. In order to find out y we need $y_B = C_B$ so we need to find out a y such that y_1, y_2 into (1, 0) (1, 3) is = (0, 6) from which $y_1 + 0y_2 = 0$; so y equals this gives $y_1 + 0y_2 = 0$ from which y_1 is 0; $y_1 + 3y_2 = 6$ from which $y_2 = 2$ so we have computed our y. Alternately we could have found out b inverse for this and then we could have written y = C_B B inverse and we would have got this. Let us for the sake of completion compute the B inverse and keep it now. What will be B inverse for this? Determinant B will be 3 now. Cofactor B will be 3 0, – 1, 1 adjoined B will be 3, 0, – 1, 1 and determinant and B inverse for this will be adjoined by determinant 3/3 = 1, – 1/3, 0 1/3. This will be B inverse. We could have found out y by using this equation y = C_B B inverse so, (0, 6) into this would give us 0 into 1 + (6 into 0) = 0; (0 into – 1/3) + (6 into 1/3) is = 2. We would still have got the same B.

We do not need any of these now. We have got our B inverse all here. So we have found our y so $C_2 – Z_2 = C_2$ is $5 – yP_2$ which is $5 – P_2$ is 1, 2. Co-efficient in the constraint corresponding to variable X_2 is 1, 2. So this will be 0 into 1, 2 into 2 = 4; 5 – 4 is = 1 and $C_4 – Z_4$ equals 0 – 0, 2 into 0 1 because P_4 is 0 1 from the co-efficient matrix so this will be 0 into 0, 2 into 1 is 2 – 2.
Now variable X_2 with the positive value of $C_2 – Z_2$ enters. I have variable X_2 that enters and then what is the leaving variable? We have to find out a leaving variable so we need to find out P bar 2. P bar 2 is P inverse P_2 because we have the equation P bar j = P inverse P_j. B inverse is 1, 0, – 1/3; 1/3 into P_2 1, 2. This is 1 into 1, – 2 into 1/3, 1 – 2/3 is 1/3; 0 into 1 + 2 into 1/3 = 2/3. Same 1/3 2/3 is what you see here as 1/3 2/3 B inverse P_2. Now we need to find out a theta for the leaving variable. This is the right hand side, 1 and 4. We need to find theta such that 1 – 1/3 theta is greater than or equal to 0 and 4 – 2/3 theta is greater than or equal to 0. This would give theta = 3 this would give theta = 6. The minimum value is 3 which happen for the first one, so you realize that the first variable X_3 leaves the basis. The next iteration we write it here.

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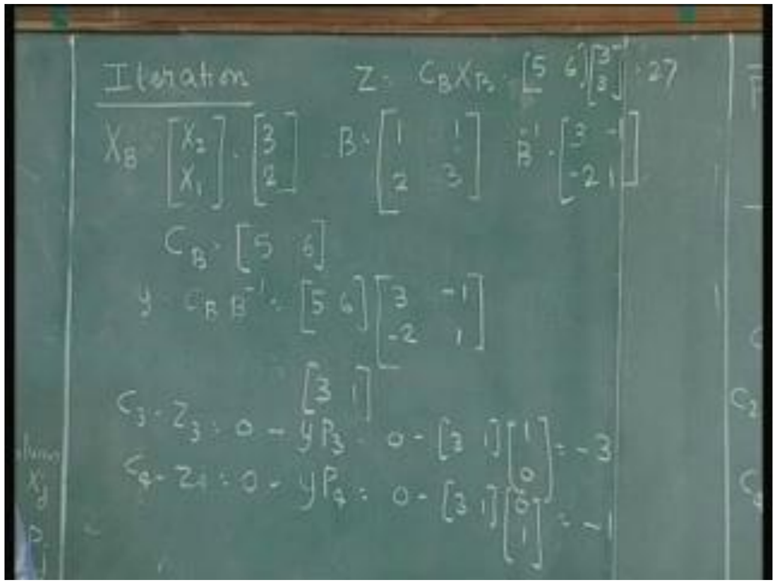

X_3 leaves the basis. X_2 enters the basis. We have X_2, X_1 and the value of the right hand sides can be got by substituting the theta. This is the equation from which we found out theta. So the value will be theta here which is 3 and here the value will be 4 – 2/3 into 3, 4 – 2 which is 2 so we have a solution $X_2 = 3$; $X_1 = 2$. We need to find out B, corresponding to X_2, X_1 as it appears. X_2 is 1, 2, X_1 is 1, 3 so you have 1 2 and 1, 3. C_B is objective function co-efficient corresponding to X_2, X_1 as they appear. 5, 6 and then we need to find out a B inverse. In order to find out y, y is C_B B inverse so we need to find out B inverse. Let us find out B inverse by the normal method. So determinant B in this case will be 3 – 2 = 1. Cofactor B will be 3 – 2 – 1 = 1. Aadjoined B is the transpose of cofactor so 3 – 2 – 1 = 1 and B inverse is adjoined by determinant which is 1 so 3 – 2 – 1 and 1 so this is the value of the B inverse. We have to find out y such that y = C_B B inverse which is 5, 6 into 3 – 2 – 1, 1 which is 5 into 3 = 15 – 12 = 3; 5 into – 1 = – 5 + 6 = 1
We have found out y. To verify whether this is optimum we need to find out $C_3 – Z_3$ and $C_4 – Z_4$. $C_3 – Z_3$ is 0 – C_3 is 0; Z_3 is yP_3 which is 0 – 3 1 1 0 P_3 is 1 0 which is here. This would give us – 3 $C_4 – Z_4$ is 0 – yP_4 which is 0 – y into 0, 1. P_4 is obtained from here so you would have 3 into 0 + 1 into 1 – 1.

Since both $C_3 – Z_3$ and $C_4 – Z_4$ are negative. The algorithm terminates with a solution $X_2 = 3X_1 = 2$. Now we compute the value of Z. Z is = C_B X_B which is which is (5, 6) into (3, 2) which is 27. The algorithm terminates. What we have actually done by calling if the matrix method is we have exactly worked out the simplex iteration. But we have understood how the $C_j – Z_j$'s are represented in terms of the P_j's and y's and so on. We have also done something more. What happens in the matrix method is if we look at the equations carefully the P_2 keeps appearing again and again. Now for example we have now shown that the values here in every intermediate iteration are actually dependent on the problem data. In the normal simplex tabular, the way we created the simplex tables, every subsequent iteration was actually dependent only on the values of the previous iteration. We did not look at these values. Now what we have done in the matrix method is we have represented every iteration in terms of only the first table of the simplex or in terms of the problem that is the change that we have made.

Secondly we have also done a lot of computations. Now we have to see whether this is better or the original simplex with n – m into is better. Now what did we do in every simplex iteration? And what did we do here?

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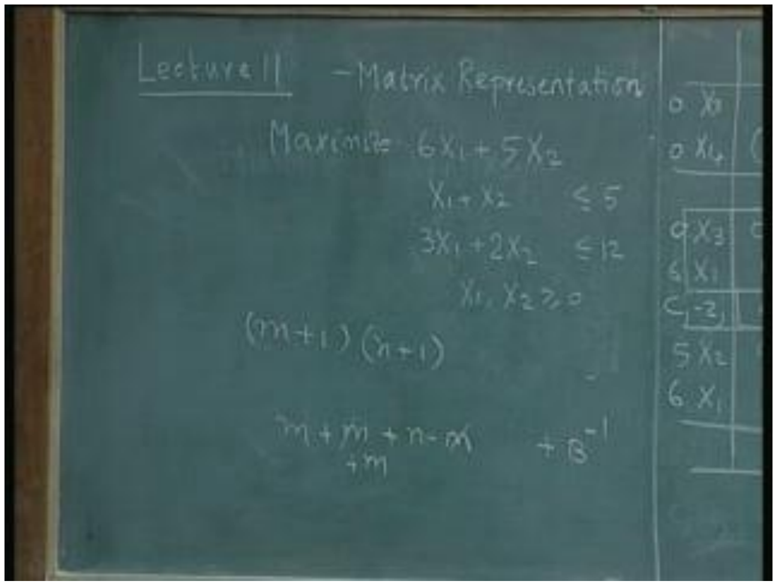

We evaluated the right hand side, so we found out m values of the right hand side to begin with and then we evaluated the y. y is a set of dual variables. So we will have as many dual variables as the number of primal constraints. So you have another m value is of y. But to calculate the y we needed the B inverse. So we said we look at + B inverse and the moment y is calculated we calculated C_j – Z_j for the non basic variables. How many non basic variables do we have? We have n variables out of which m are basic. So we calculated n – m C_j's – Z_j's. We identified one entering variable and then we calculated the numbers corresponding to the entering column which is + another m numbers. This m and this m will get cancelled, 2 m – n plus the effort on the B inverse. Now what is the effort on the B inverse? and the second is very interesting. Can we read B inverse also from the simplex table? The answer is we can read B inverse from the simplex table. Let us look at this. In the very first iteration we said B is = i, B inverse = i. The second iteration, this was the B inverse that we computed. 1, 0, – 1/3 + 1/3 and you will find that your B inverse is here 1, 0, – 1/3, + 1/3. In third iteration, your B inverse is 3, – 2, – 1 and 1. You find your B inverse here 3, – 2, – 1 and 1.

In the first iteration B inverse was identity matrix. You find the identity matrix somewhere here (Refer Slide Time: 29:52). Now B inverse is seen under the identity matrix of the original table. B inverse is always seen right in the simplex algorithm under the identity matrix corresponding to the original table. When we said that we computed B inverse, we also computed another m square term because B is a square matrix which is m/m. We computed another m square term so now here we have computed (m + 1) into (n + 1). Here we have computed 2m + n + m square and we realize that this is actually smaller than this. Therefore the matrix method is advantageous over the regular simplex method. Why do we have the B inverse under this? The answer is in every simplex table. What we actually do through our row column operations is we are actually performing one iteration at the moment. We identify that X_3, X_1 is the basic variable which means X_2, X_4 are non basic. We do certain row operations from the previous table so that we get an identity matrix corresponding to X_3 and X_1. What was 1, 0, 1 and 3 in the previous, has become 1 0 0 1 after some row operations but we continue to do the row operation along all elements. Now the moment we get this identity matrix for this, which is what we do, we can straight away read the solution from the right hand side.

So what effectively we do is we start with the 1 0 1 3 which is the basis matrix corresponding to X_3, X_1 and then do a set of row operations to get into the identity, which means if this is B corresponding to X_3, X_1, 1 0 1 3 is B then we do the row operations to make it into i. How do we do that effectively? We are pre multiplying it by the B inverse. From this when we get this part of the simplex table this is obtained by consistently pre multiplying every column by B inverse where B is 1 0 1 3. In the process this identity matrix also gets pre multiplied and that is why you see B inverse always happening under this identity matrix.

Similarly when we have this iteration we know the basis matrix is X_2, X_1. So we start with 1 2 1 3 even though we have done some row operations here effectively. We have carried out a set of row operations to get a 1 0 0 1 here, which means effectively we have pre multiplied every column of this with the B inverse corresponding to the B 1 2 1 3. So in the process, the identity matrix also gets pre multiplied by B inverse. So B inverse is always seen under this. Now not only are we pre multiplying this identity, because we are pre multiplying every column with B inverse any P bar j is = B inverse P_j. Therefore we have proved the result P bar j is B inverse P_j.

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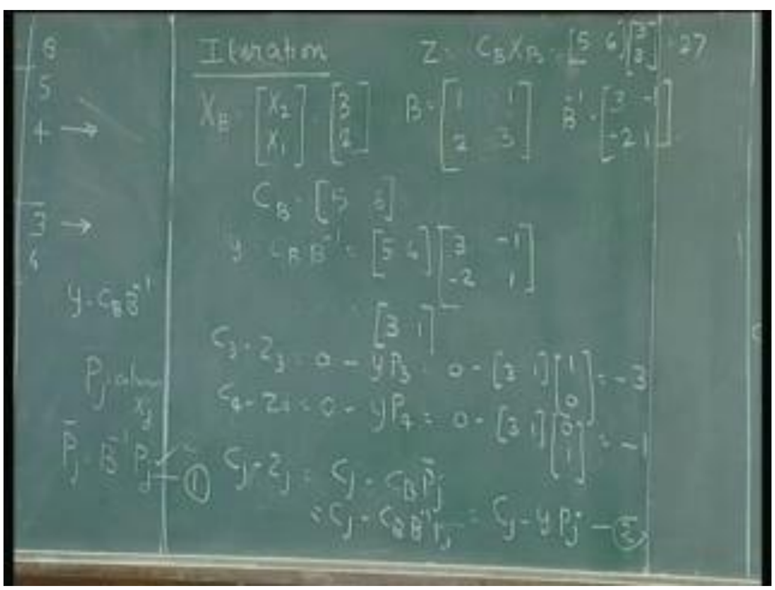

Once we know this result $C_j - Z_j = C_j - C_B$ P bar j is obvious from which $C_j - C_B$ inverse P_j comes from the dot product substituting P bar j which is $= C_j - yP_j$. So the important things we have learnt from the matrix method are these. This is the first equation and most important equation that we have learnt. This is the second equation $C_j - yP_j$ or C_B B inverse. $y = C_B$ B inverse is another equation and now in every iteration what do we do? We are actually solving in this case for B X_B = the right hand side.

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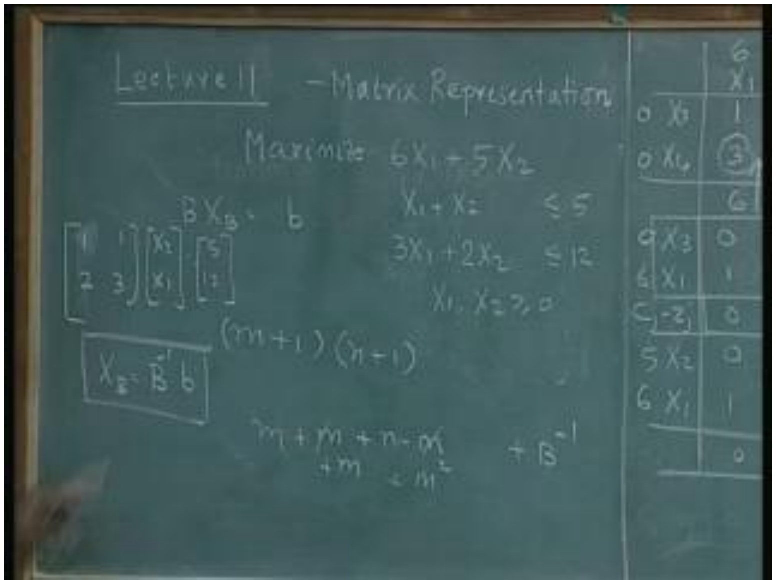

In this iteration what we are trying to solve is when we fixed X_2, X_1 as the basic variables we are going back and we are trying to solve for $X_2 + X_1 = 5$; $2X_2 + 3X_1 = 12$. So we are solving effectively for $BX_B = B$ or we are trying to solve in this case for X_2, X_1 so 2, 1 X_2 X_1 so 1 2 1 3 into X_2 X_1 = 5, 12. So you are trying to solve for B X_B = B from which X_B = B inverse b. This is another equation so will be using all these equations when we do sensitivity analysis.

Now at the end of the matrix method we have now found out a way which can be slightly faster than the tabular form of the simplex and would give exactly the same results that we have. We have also learnt that the B inverse appears in this simplex table. If you look at a simplex iteration we see a lot more in this simplex table than what we would want or we would have imagined. We see the primal solution here. We see the value of the objective function here we see the entire dual solutions here. We also see the B inverse from here so simplex table tells us almost everything about the problem that we are trying to solve. Now the next thing that we would see is called sensitivity analysis and we would be using these equations that we have seen. Now at the end of it all, let us go back and explain sensitivity analysis again through an example.

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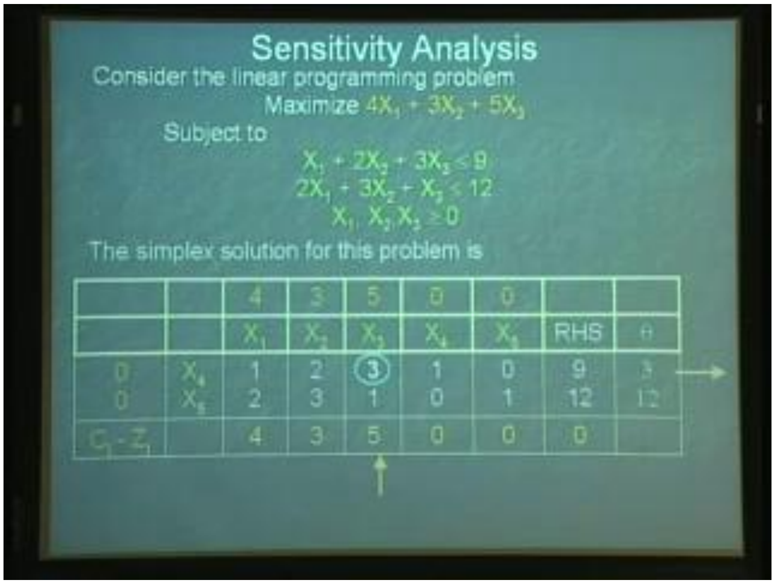

Now what do we want to see in sensitivity analysis? An ever linear program is characterized by an objective function, a right hand side value, a coefficient matrix.

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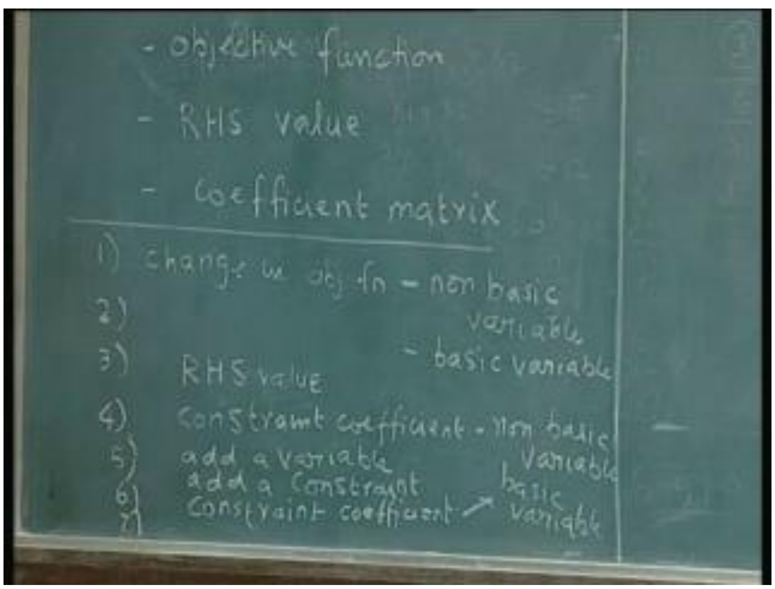

Now in the simplex table the objective function value comes in the top. The right hand side values are shown here and the coefficient matrix is shown in this way (Refer Slide Time: 37:51) Now let us assume that we have a linear programming problem that we have just now solved. After solving it we would like to see how the solution would change if there are some changes in some of these numbers or some changes in these numbers or some changes in these numbers. What will happen to the optimal solution? If there is a change in the objective function coefficient what would happen to the optimal solution if there is a change in the right hand side value? What would happen if there is a change in the coefficient matrix? We would also like to do a couple of other things. Suppose I add another variable into the problem and along, the variable comes. It is subjective function coefficient and constraints coefficient. What will happen? Does the present solution remain optimal or it would change?

Suppose I add another constraint to the problem and along with the constraint comes a right hand side value as well as the coefficients. With the addition of the new constraint, does the problem continue or does the solution continue to be optimal or is it going to be a new solution? Now all these questions we would like to address. The obvious answer to all these is, make the change and solve the problem all over again. That is a very obvious answer to the question. If after solving a problem we realize for example that a certain 5 on the right hand side becomes 6 what happens to the solution? One would always go back change the 5 to 6, solve the problem all over again. But is that the most efficient and intelligent way?

Alternately can we use the information that we have? Can we use the optimal solution that we have and then from the optimal solution can we proceed to evaluate all these changes? That is done in what is called sensitivity analysis. In sensitivity analysis we will be seeing a few things. The first thing we will see is the change in objective function coefficient for a non basic variable because every optimal solution has a set of variables which are basic variables. We would like to see what happens:

1. If there is a change in the objective function coefficient of a non basic variable
2. If there is a change in the objective function of a basic variable
3. If there is a change in the right hand side value
4. If there is a change in the coefficient matrix in the constraint coefficient corresponding to a non basic variable.
5. Adding a variable to the problem
6. Add a constraint and
7. Add a constraint coefficient for a basic variable.

Now we look at each of these one considering an example here.

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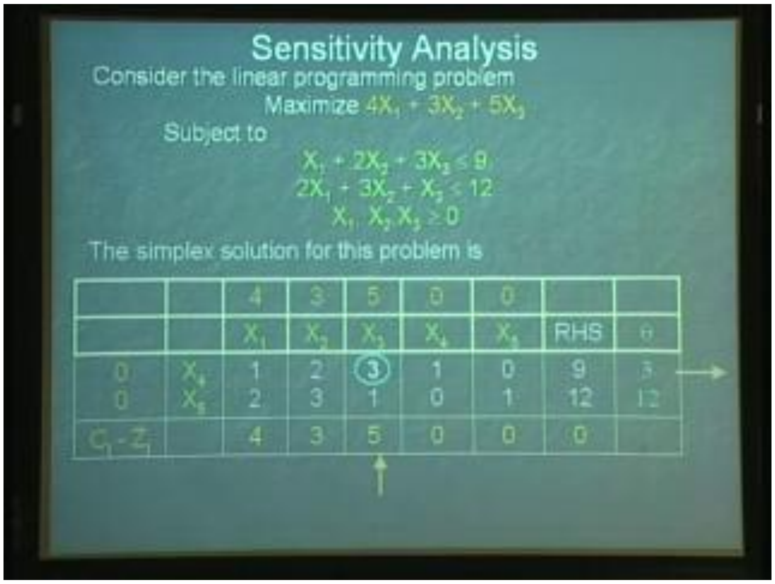

The example that we would have is this, to maximize,

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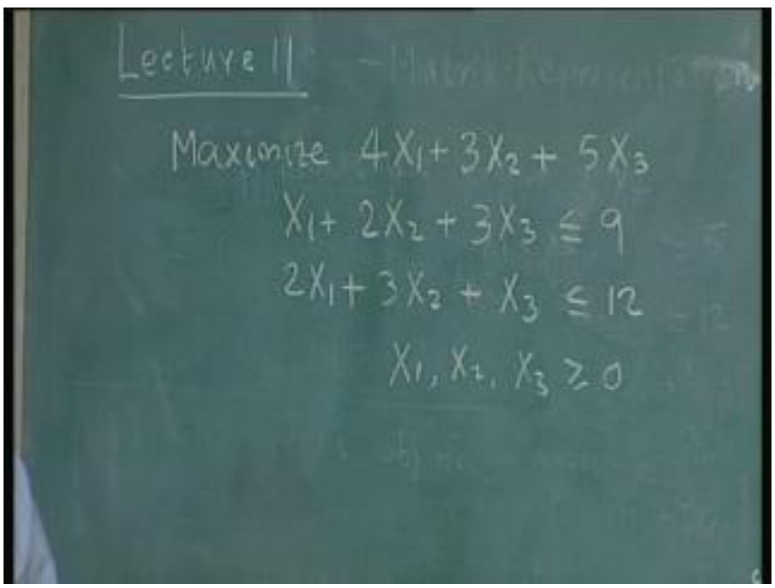

The example that we would have is this, to maximize,
$4X_1 + 3X_2 + 5X_3$ subject to the condition $X_1 + 2X_2 + 3X_3$ less than or equal to 9
$2X_1 + 3X_2 + X_3$ less than or equal to 12; X_1, X_2, X_3 greater than or equal to 0
We first assume that we have solved the problem. So we take you through the steps of the simplex algorithm to solve this problem. We quickly go through this. We start with slack variables X_4 and X_5 because they are less than or equal to constraints. The first simplex table can be seen here with the X_4 and X_5 as the basic variables with objective function coefficient 0. Right

hand side value is 9 and 12. $C_j - Z_j$ values are 4, 3 and 5. So variable X_3 with the most positive $C_j - Z_j$ enters.

Theta is computed as 9 divided by 3 is = 3; 12 divided 1 which is = 12; 3 is minimum theta. Variable X_4 leaves the basis and variable X_3 enters the basis.

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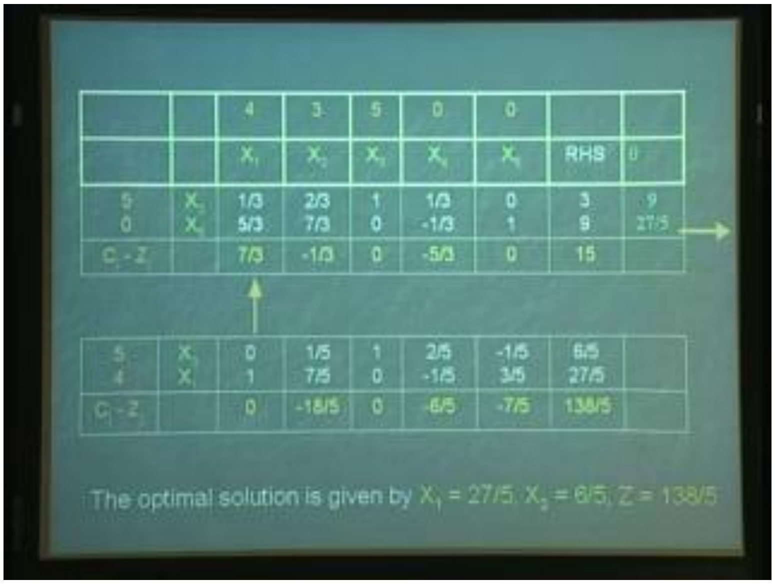

		4	3	5	0	0		
		X_1	X_2	X_3	X_4	X_5	RHS	θ
5	X_3	1/3	2/3	1	1/3	0	3	9
0	X_5	5/3	7/3	0	-1/3	1	9	27/5
$C_j - Z_j$		7/3	-1/3	0	-5/3	0	15	
5	X_3	0	1/5	1	2/5	-1/5	6/5	
4	X_1	1	7/5	0	-1/5	3/5	27/5	
$C_j - Z_j$		0	-18/5	0	-6/5	-7/5	138/5	

Next iteration would be starting with X_3 and X_5 which are here. So we have X_3 and X_5 which are here and these are the iteration with $C_j - Z_j$ values 7/3 – 1/3, 0 – 5/3, 0 and Z = 15. Now variable X_1 with the most possible $C_j - Z_j$ enters corresponding theta values are 3 divided by1/3 which is 9 and 9 divided by 5/3 which is 27/5; 27/5 is smaller than 9 so variable X_5 leaves the basis and this the pivot. So next iteration would have X_3 and X_1 we have X_3 and X_1 with 6/5, 27/5. All $C_j - Z_j$'s are negative so this is having 138/5 so this is the solution we assume now that we have this optimum simplex table along with this we just write down only the optimum simplex table here.

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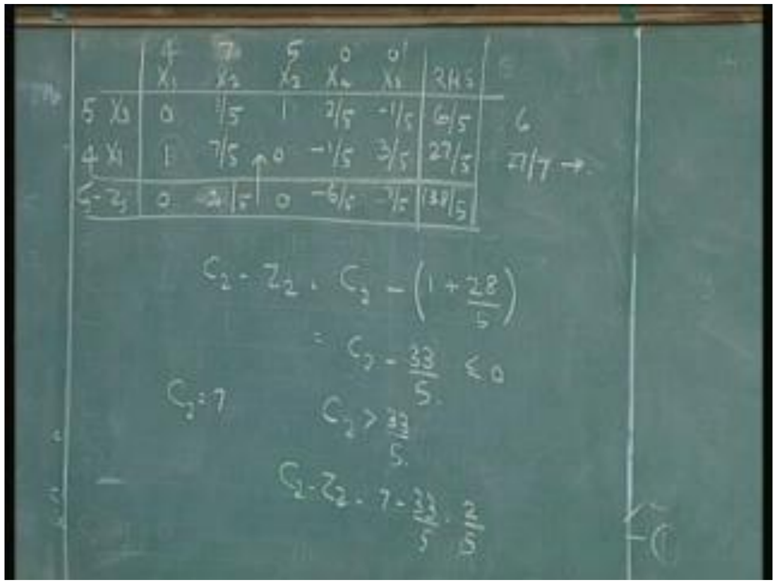

We would have X_1, X_2, X_3, X_4 and X_5 right hand side values. We just write down the optimum table with the X_3 and X_1 here with values 4, 3, 5, 0, and 0. X_3 has 5; X_1 has 4 and the simplex table would be 0, 1/5, 1, 2/5 – 1/5, 6/5, 1, 7/5, 0, – 1/5, 3/5 and 27/5 with $C_j – Z_j$ values = 0 – 18/5, 0, – 6/5, – 7/5 and 138/5.

Let us assume that we have this solution and we want to do sensitivity analysis on this problem with respect to this optimal solution. First thing we would look at is what will happen to the solution if there is a change in the objective function coefficient of a non basic variable. So we take variable X_2, which is presently non basic. There is a change in the objective function coefficient of this now which means, this 3 changes to something. We call this as C_2. Let us assume that it is C_2 now because X_2 is a non basic variable this X_2 does not appear here so C_2 will not appear here. The only place there will be a change, (the right hand sides will not change) now because let us go back to our equations. Right hand side value is B inverse B. We have not made any change in the B matrix so right hand side will not change and but $C_j – Z_j$'s are checked. We should also consider this. If this change of value from 3 to something is going to have an impact, then what will be the impact? The impact would be that a present solution is no more optimum. The present solution would no more be optimal if the right hand side becomes negative and the optimality is retained or the right hand side is still non negative but the $C_j – Z_j$'s, one of them becomes positive and it tries to enter.

The only impact that this C_2 can have is in this $C_2 – Z_2$. It does not have any impact on the other things because this is not going to affect y. It is not going to affect y because it is not going to affect C_B. C_B would remain as 5, 4. So the only change that will happen is in the objective function coefficient of a non basic variable. Then that would be reflected as a change in the $C_j – Z_j$ corresponding to that non basic variable. $C_2 – Z_2$ is the only thing that will undergo a change so $C_2 – Z_2$ will become C_2 – 5 into 1/5, 1 + 4 into 7/5, 28/5, so – 1 + 28/5 this is C_2 – 33/5. As long as C_2 – 33/5 is less than or equal to 0, the present solution will be optimal because this quantity will be negative. So the change C_2 will have an impact only if C_2 is greater than 33/5. If C_2 is greater than 33/5, $C_2 - Z_2$ will become strictly positive and variable X_2 will try to enter the basis. Now at $C_2 = 3$ we would have 3 – 33/5 which is – 18/5.

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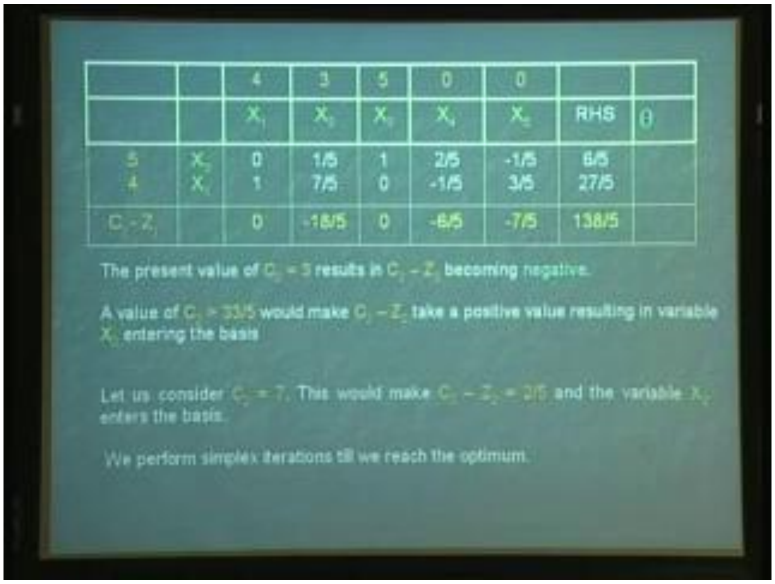

Now let us look at what happens. If present value would give negative a value of C_2 greater than 33/5, it would make $C_2 - Z_2$ take a positive value. So if you consider a value $C_2 = 7$. For example, if this 3 increases to 7 till it increases to 33/5 or 6.6 or 3 and 3 by or 6 and 3/5 there will be no impact of this change. The present solution will continue to be optimum. Only when it exceeds 30/7, something will happen. If we consider $C_2 = 7$ then $C_2 - Z_2$ will become 7 – 33/5 which is 2/5 so this will become + 2/5. If this becomes 7 this becomes + 2/5 and because of this 2/5 variable X_2 enters the basis. We need to find out a leaving variable and we need to compute theta 6/5 divided by 1/5 is 6, 27/5 divided by7/5 is 27/7 which is smaller. So variable X_2 will enter and variable X_1 will leave and have to do a simplex iteration.

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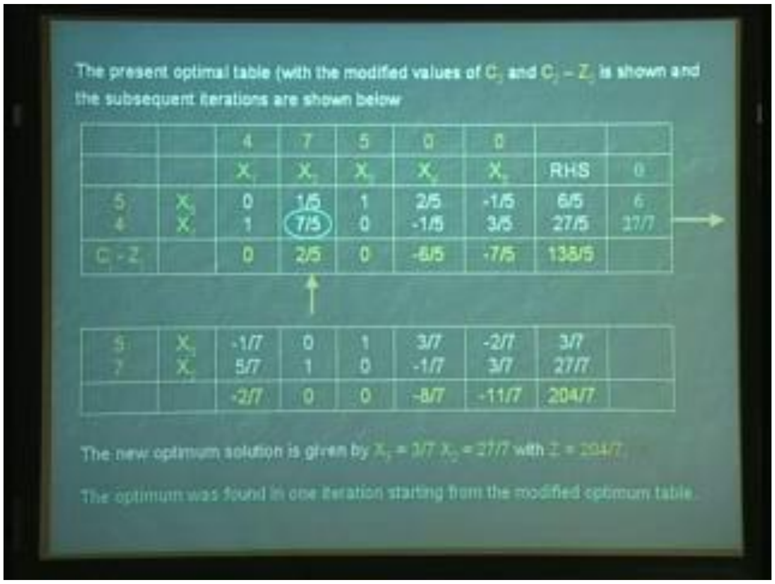

That is shown here. Variable X_2 will enter and variable X_1 will leave which is shown here. 2/5, the newly computed value is shown here. The change C_2 to 7 is shown here. So X_2 enters and the leaving variable is X_1, so we do a simplex iteration. We have identified the entering variable first and then the leaving variable. So we do a simplex iteration with X_3 and X_2 as basic variables that you can see here and after 1 iteration we realized that we have reached the optimal solution with $X_3 = 3/7$; $X_2 = 27/7$ and $Z = 204/7$.

In this example, we could reach the optimum solution in 1 iteration. Sometimes we may need more than 1 iteration to reach the optimum. But the advantage for example is we started with the previous optimum table, we made the change. And the effect of change in the same table and now we realized for this example in just 1 iteration you could get the optimum solution to this problem. So this is how we handle a case where there is a change in the objective function coefficient of non basic variables. We now look at what will happen if there is a change in the objective function coefficient of a basic variable. Let us do that once again from the optimum solution of this problem.

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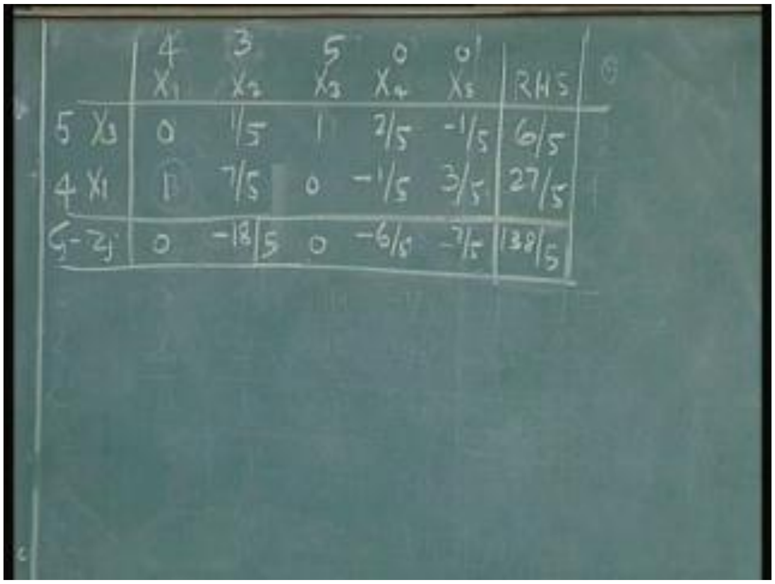

So we go back to the same table and make table appear as it did. When we started, this was 3 so you have 5 + 28 = 33, 3 – 33/5 which is – 18/5 and this is how the optimum table looks. We go to the second part of sensitivity analysis where we try to see if there is a change in the objective function coefficient of a basic variable. There are two basic variables X_3 and X_1. We could take any one of them.

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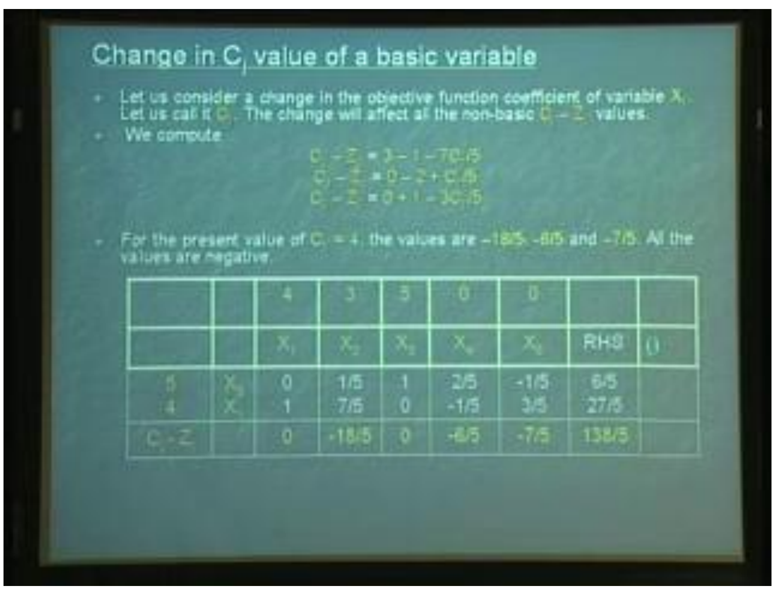

We take change in the objective function coefficient of variable X_1 to illustrate this.

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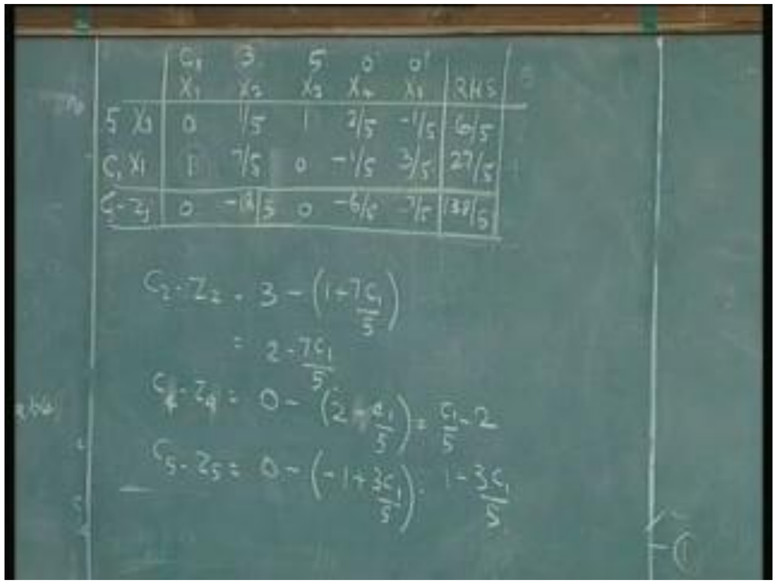

This 4 will become C_1 and this 4 (Refer Slide Time: 52:52) will also become C_1.
Once again there will be no change in the right hand side value because the basis matrix B is not affected nor the right hand side affected. So there will only be changes in the $C_j – Z_j$'s.
In spite of this change, X_1 being a basic variable will have a C_j's $–Z_j$'s = 0 for that variable but these numbers with all the three numbers can change now. They use C_B. C_B has changed therefore $C_j – Z_j$ for all the non basic variables will change. If there is a change in the objective function coefficient of a basic variable then this will be reflected as change in $C_j – Z_j$ for all the non basic variables and all of them have to be evaluated. $C_2 – Z_2$ will now become 3 – 5, 5 into 1/5 is = 1; 1 + $7C_{1/5}$ – 1 + $7C_{1/5}$ which is 2 – $7C_{1/5}$.

Now $C_4 – Z_4$ will be 0 – 5 into 2/5 is 2 + C_1 into – 1/5; so – $C_{1/5}$ which is $C_{1/5}$ – 2; $C_5 – Z_5$ will be 0 – (5 into – 1/5) = – 1 + $3C_{1/5}$ which is 1 – $3C_{1/5}$. If C_1 is such that all three of them are less than or equal to 0 then the present solution would remain optimum for example if we substitute C_1 = 4; you would realize that all of them are negative but if one them turns out to be positive then that variable would enter.

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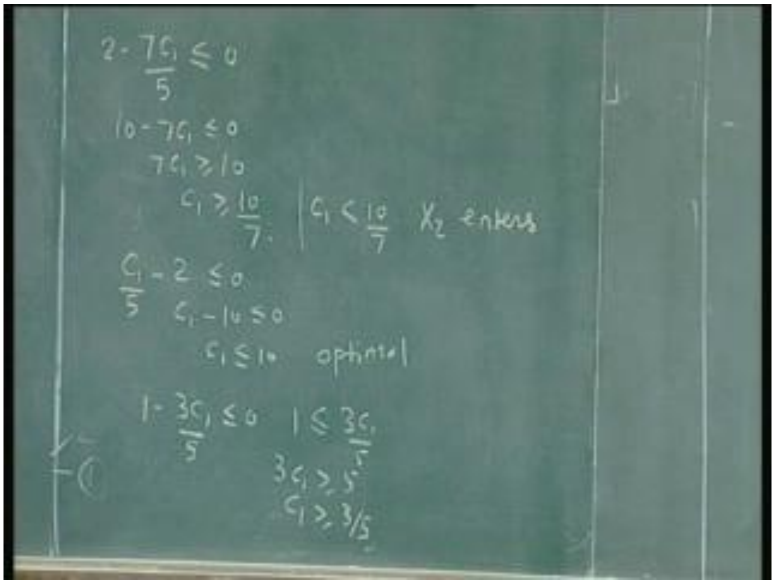

As long as $2 - 7C_{1/5}$ is less than or equal to 0 there will be not any impact so this would give us $10 - 7\ C_1$ greater than less than or equal to 0 so $7C_1$ greater than = $10C_1$ greater than or equal to 10/7. As long as C_1 is greater than or equal to 10/7, then this will continue to become negative. Now for the other one $C_{1/5}$, C_1 greater than or equal to 10/7, now $C_2 - Z_2$ is negative when C_1 is less than 10/7. Variable X_2 will enter. X_2 enters. When C_1 is less than 10/7, this would enter.
Now here $C_{1/5} - 2$ is less than or equal to 0. $C_1 - 10$ is less than or equal to 0; C_1 less than or equal to 10, it will continue to be optimum so this would be optimum. For the third one, we have $1 - 3C_{1/5}$ less than or equal to 0. 1 is less than or equal to $3C_{1/5}$. $3C_1$ greater than or equal to 5; C_1 greater than or equal to 3/5 and when C_1 is greater than or equal to 3/5, this would remain optimum. C_1 is less than or equal to 10. This would remain optimum. C_1 greater than or equal to 10/7 this would remain optimum. Now let us see this.

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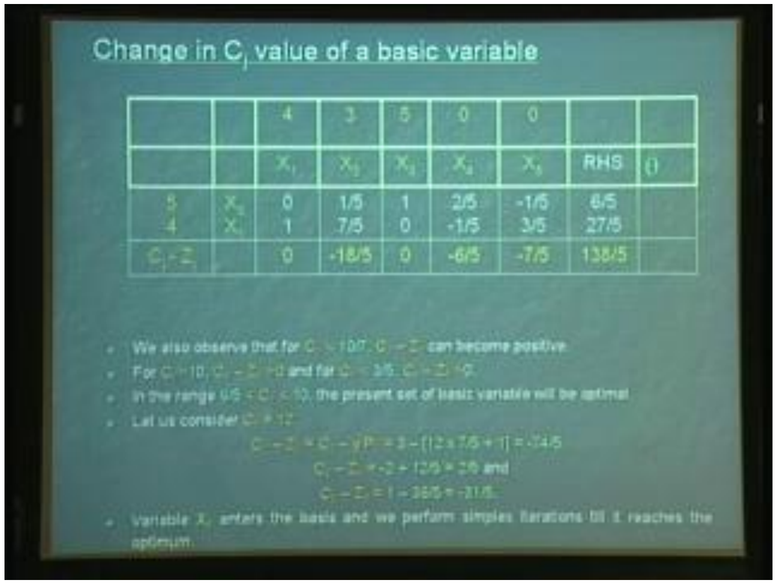

As we go here, we observed that is C_1 less than or equal to 10/7, $C_2 – Z_2$ can be can become positive for C_1 greater than 10; $C_1 – Z_1$ is greater than 0 and our C_1 less than 3/5; $C_5 – Z_5$ is greater than 0. We realize that in the range 6/5 to 10 the present set will be optimum. See what we need to look at is as long as C_1 – 7 is C_1 is greater than or equal to 10/7. C_1 less than or equal to 10; 3/5 the present set will be optimum. So from these three, we realize that we have a range. As long as it is between 5/3 and 10 it will be optimal, so when it leaves the range and then it will not be optimal and something else will enter.

We try to substitute the value C_1 = 12 which is beyond the range, so we would expect this variable to enter the basis and that is now shown in the simplex table and the next iteration.

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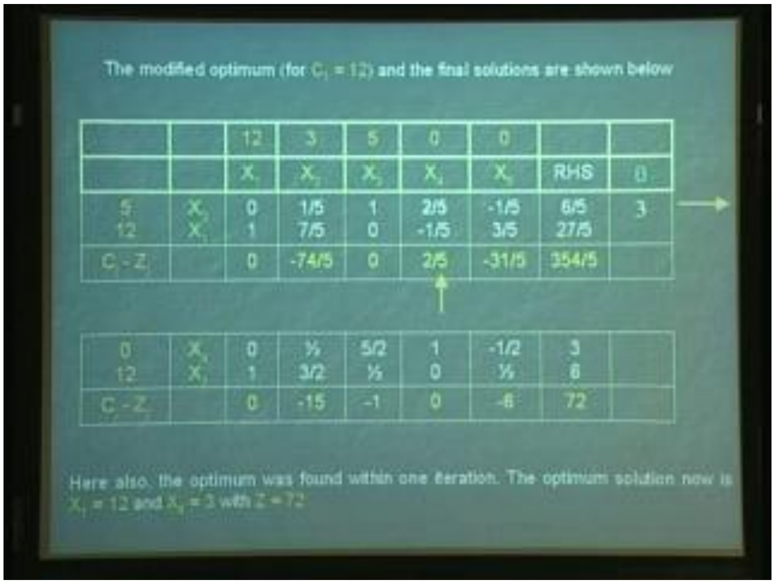

We now realize when we substitute C_1 = 12 then we would get 12/5 – 2 which is 2/5 which comes in here. So variable X_4 enters the basis and we need to find out the value of the leaving variable. There is only one theta, where theta = 3 and in the next iteration we get the optimum solution. Now the optimum solution has Z = 72. So in summary if there is a change in the objective function coefficient of a basic variable then that change gets reflected in the $C_j – Z_j$'s of all the non basic variables and then we need to find out the range of the C_1 or the coefficient which would keep it optimal. If the new value of C_1 violates the range then one of the present non basic variables will become basic and we continue doing simplex iterations till we reach the optimal.

The other aspects of sensitivity analysis will be seen in the next lecture.