Advanced Operations Research

Prof. G. Srinivasan

Department of Management Studies

Indian Institute of Technology, Madras

Lecture - 9
Goal Programming - Formulations

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Today, we will consider a new topic called Goal Programming. Here, we look at multiple objectives or goals. So far, in all our study in linear programming and related topics, we have looked at OR problems which have a single objective, which either minimizes or maximizes a linear function. Now, do we or can we have situations where there could be multiple objectives? If so, what do we do? Do we solve as many linear programming problems as the number of objectives or what do we do there? First of all, how do we model such a situation?

These things are handled in what is called Multiple Objective Programming and goal programming is one such method to handle multiple objectives. There are other methods, but goal programming is the most commonly used approach to address situations which have multiple objectives. As usual, we will take a small example and then try to illustrate how these multiple objectives or goals are met.

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Let us look at a problem, say for example, you make two products A and B, and say they are made out of three chemicals I, II and III. A requires 3 of I, 5 of II and 1 of III; B requires 4 of I and 6 of II. Profits are 70 and 80 respectively. Availabilities are 50 and 60 respectively.

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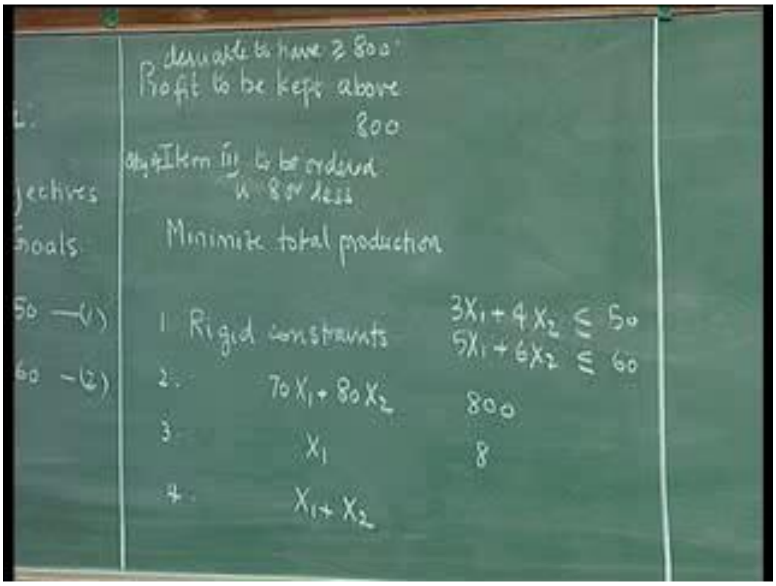

Two additional sentences are given, which are like: profit is to be kept above 800; quantity of item III to be ordered is 8 or less. Normally, if we were to model this as a linear programming problem, what we will try to do is we would first define the decision variables - say X_1 and X_2 be the quantities of A and B produced. It is a very standard problem that you can directly formulate. You will have something like $3X_1$ plus $4X_2$ is less than or equal to 50. $5X_1$ plus

$6X_2$ is less than or equal to 60. X_1 is less than or equal to 8, because, you do not want to order more than 8. Then you will have an objective function. You will not use this 800; you would simply say, maximise $70X_1$ plus $80X_2$ and solve the problem. Then, if you come up with an optimal solution whose objective function value is more than 800, you say that the 800 target can be achieved; otherwise, you would say the 800 target cannot be achieved and only this much can be achieved. That is what we would do normally in a linear programming formulation of this problem.

In a goal programming formulation, what we do is, we first take these sentences as they appear in the first place (Refer Slide Time: 04:32 min). Then, there is another sentence which would say - the effort is to minimize total production, so that transportation is easy. So, you can assume that another sentence is also given which says: minimize total production such that transportation is easy. Now this adds to the conflict, because, if you leave out this sentence you have a clear single objective which would be to maximize $70X_1$ plus $80X_2$. If you want to include this also, what you can do conveniently is change the objective function to minimize X_1 plus X_2 and try to put this as a constraint, which is like $70X_1$ plus $80X_2$ to be greater than or equal to 800. That way, you use all the numbers that are given to you.

When we want to do this as a goal programming problem, we need to do this: one, identify what are called rigid constraints and then look at all these. For example, the availability of this 50 and 60 are very rigid as a constraint, whereas, if you had written a constraint which is like $70X_1$ plus $80X_2$ greater than or equal to 800 here, this will not qualify to be a very rigid constraint because it depends again on how you interpret it. For example, if this sentence is written as - it is desirable to maintain a profit, say, it is desirable to have profit greater than or equal to 800. It is desirable to order a maximum of 8 for item III. Suppose, we start using sentences like this which happen in practice, when you produce these are the only rigid things. For example, these have to be bought from vendors or suppliers. You know that these vendors cannot give or in the present system, you cannot have more than 50 or 60 per day. So, they become rigid constraints; whereas, something like profit to be kept above 800 or quantity of item III to be ordered being 8 or less, these things become what are called goals; they are not rigid constraints, but they become goals. So you have to categorize them as - what are rigid constraints? And what are targets or goals? This 800 is like a target or a goal that you want to achieve.

You want your profit to be as much as possible, but you want it to be around 800. You do not want to put a rigid constraint saying I want a profit of 800 or more. If you start putting those rigid constraints, you may get into an infeasible situation. You do not want to get into that kind of a situation, because, you still want to produce something and make some money. The rigid constraints have to be separated; the goals have to be separated. As far as this problem is concerned, there are two rigid constraints. These two are your rigid constraints (Refer Slide Time: 07:51 min). For example, we define the same X_1 and X_2 as the quantities of A and B produced. So your rigid constraints will become $3X_1$ plus $4X_2$ less than or equal to 50 and $5X_1$ plus $6X_2$ less than or equal to 60. So, this is your rigid constraints.

Now, the rest of them are taken as they appear; unless otherwise stated they are taken as they appear. These targets or goals are now assumed to have different weightages. These weightages depend on the order in which they appear unless otherwise stated. The rest of the three things, for example, the first one that I want to satisfy is this 800. The second thing I want to satisfy is this 8 or less. The third thing I want to satisfy is to minimize the total quantities to be produced unless otherwise stated. So I take them in this order. These are written as 2, 3 and 4 respectively. All the rigid constraints are bunched together; the rest of the targets are taken as they appear. It is also assumed unless otherwise stated, that it is most important for me among the three, to consider this first (Refer Slide Time: 09:16 min) and after I have set or achieved this target, I will try to achieve this target and then I will try to achieve this target.

Right now, we have not given a target for this; we will come to that as we proceed. These two have targets (Refer Slide Time: 09:28 min). So, the third one is $70X_1$ plus $80X_2$. The moment I put it as greater than or equal to 800, it becomes a rigid constraint and then there is an 800 here. Then as far as the third part is concerned, this one is item III. So X_1, then, there is an 8 associated here and here there is an X_1 plus X_2 which is to be minimized; we do not know the target. In all goal programming formulations, you will have only equations and you will not have inequalities.

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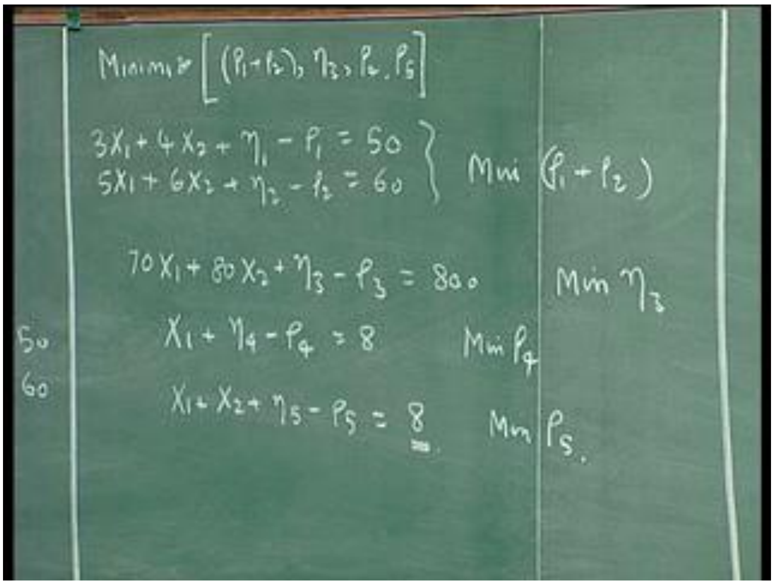

So, the rigid constraints will now be written as follows: $3X_1$ plus $4X_2$ plus eta_1 minus rho_1 is equal to 50. The reason why you do not want to solve this straightforward LPs is, you could get into infeasibility. If you look at very practical problems, if you are trying to formulate a practical situation into an optimization problem, what you will know among the data are the resources and what you will not know are the costs and the profits. An organization would say clearly that this 50 and 60, I know very well; I want to make a profit of 800, but I am not going to put it as rigid constraint. At the same time, it is not my objective either. I have different things that I want to do. I am worried about a profit of 800. I am worried about this resource because I have a weak supplier who is not able to give me what I want. This guy says he can give me only 8 or less and I am also worried about this, because, I want to minimize the transportation and may not have trucks to dispatch things. So, it is not that I have a single objective. I have multiple things which are there with me, which make the modeling as well as the real life problem little more difficult.

Yes, I would prefer an 800 plus, but if it is a 750, I will accept it. I want your problem to tell me - what exactly is the maximum achievement? Similarly, this 8 here. There are several ways of solving. We still have not come to that. One, you can do is just keep only the rigid constraints, take all the targets into the objective function. So if you have three objectives, you could put a W_1O_1 plus W_2O_2 plus W_3O_3 where O_1, O_2, O_3, are the objectives; you can do that. That is called a weighted approach and that will reduce to a single linear programming problem. It will finally reduce your single objective now. This is called lexicographic minimization. You take these as they appear and first satisfy the first and having satisfied the first go back and satisfy the second; having satisfied one and two, look at three and so on. There are different ways of doing this. I am going to touch upon only the second way, that is, the lexicographic way.

In a typically lexicographic approach, if you have taken the first objective and if you are not able to satisfy even the first, you will not look at the second and so on. We will see as we solve these problems. We are just now trying to formulate them. We will take two formulations and two solutions. You will see all those aspects as we solve this.

We write this in a form of an equation; because, it is a rigid constraint, you know that the rho does not make any sense, because it is a rigid constraint. It is like your eta_1 takes on the role of a slack variable; rho_1 basically takes on the role of a negative slack and since it is a rigid constraint, the rho_1 does not make any sense to you. However, for reasons of consistency, every constraint or equation that you write will have a corresponding eta and rho, even if the rho is irrelevant. You will see that the rho is relevant in a different way. So, this becomes $5X_1$ plus $6X_2$ plus eta_2 minus rho_2 is equal to 60. These are the only two rigid constraints that you have. If you want an X_1, X_2 which is a solution to this problem, that X_1, X_2 must satisfy all the rigid constraints. It has to satisfy all the rigid constraints, which means that I should not have rho_1 and rho_2 in the basis of a simplex table. The moment I have a rho_1 and a rho_2, it implies that on the left hand, this $3X_1$ plus $4X_2$, is greater than the right hand side; only then I have this rho coming in. Remember, I have an eta minus rho, which is like an unrestricted variable modeling. Only eta or rho will remain in the basis, both will not remain in the basis.

If I have a rho positive, it implies that the rigid constraint is violated. So, any X_1, X_2 that has rho_1 plus rho_2 equal to 0, because rho_1, rho_2 are greater than or equal to 0, if I have an X_1, X_2 such that rho_1 plus rho_2 is equal to 0, then it is feasible to the rigid constraint. So, what I want to do is I want to minimize, here I want to minimize rho_1 plus rho_2 (Refer Slide Time: 15:09 min). If I get a 0 there, it means any X that gives me rho_1 plus rho_2 equal to 0, is feasible to the rigid constraint. These are like artificial variables; yes, in a sense artificial variables, because, when you solve them finally you are only going to do this - minimize rho_1 plus rho_2, which means they will have a coefficient of 1, which is like a two-phase method of linear programming, which we did not cover in detail. For example, if you take a linear programming problem that has artificial variables, this is for the general class, we looked at the big M method to solve such problems. We gave a big M into the objective function, so that it does not appear in the optimum basis. There is something called two-phase method, where you give a +1 to the artificial variables, give a 0 to all other variables – decision and slack, and then minimize; for example, if you have two artificial variables, you will just have minimize a_1 plus a_2 subject to the whole thing. Now, if there is a basic feasible solution to the original problem, the artificial variable should not lie in the basis. So, they will go out of the basis and you will get Z is equal to 0. Then, leave out the artificial variables and start from this basic feasible solution and proceed. That is called two - phase method, which many books will cover, which we did not do in detail in the earlier course. What we will do while solving is something very similar. We will give a +1 to these eta(s) and rho(s) always unless there is a different weightage, but the eta(s) and rho(s) will always have a +1.

This is my third thing. This is $70X_1$ plus $80X_2$ plus eta_3 minus rho_3 is equal to 800. Remember, I would like to have a profit of 800 or more. What does that mean? It means I want to minimize the eta, because the moment I have an eta coming in, it means I am going to have a profit which is less than 800. Remember again, between these two, only one of them is going to be in the basis. So, if this eta is in the basis with a positive value, it means that I am having a profit of less than 800. In order for me to have a profit of 800 or more, which is desirable, I would like to minimize eta and keep eta at 0. It also tells me something, if I am not able to get eta_3 equal to 0, I get eta_3 equal to a positive value, it means I cannot get this 800. Here, the objective is to minimize eta_3.

There is a simple way to look at it. See any less than or equal to constraint will push rho into the objective function. Any greater than or equal to type constraint will push eta into the objective function. Even though this interpretation is not very technically correct, it is a very nice thumb rule to understand. You want to keep something more than 800 or above, which means it is like having a greater than or equal to constraint. This will automatically push eta into the objective function. As far as the fourth one is concerned, it is less than or equal to. So, X_1 plus eta_4 minus rho_4 is equal to 8 and I want to minimize rho_4. Now, the last one is X_1 plus X_2 plus eta_5 minus rho_5 is equal to what? There is no target that is given in this problem. You can actually assume a target; so let us assume a target of 8.

At this point, you may be tempted to say that depending on what target I assume my solution may change; because, it is like explicitly changing a right-hand side in a linear programming problem. Actually speaking, it would not because it is the last of the conditions that you are trying to modify. When you solve it, you will see this. So right now, I am going to ask you to just assume that some number here; you can put 0; you can put 100; you can put anything. In fact, the easiest thing to do is to put a 0 and proceed. I mean, it might give you infeasible, but still it does not; it is all right; you can put a 0 and proceed. When we solve it, you will see why the whole thing is. We will try to solve it today or in the next class.

What will be the objective? This is like minimizing; this is minimizing rho_5 because it is like saying I want my X_1 plus X_2 to be less than or equal to 8. This is like minimizing my rho_5. What will you do? All these will remain as they are (Refer Slide Time: 20:18 min). Your objective function will look like this: minimize, I am going to take all this into the objective function. I will just say rho_1 plus rho_2, eta_3, rho_4, rho_5. So I will take this out, subject to X, eta and rho greater than or equal to 0.

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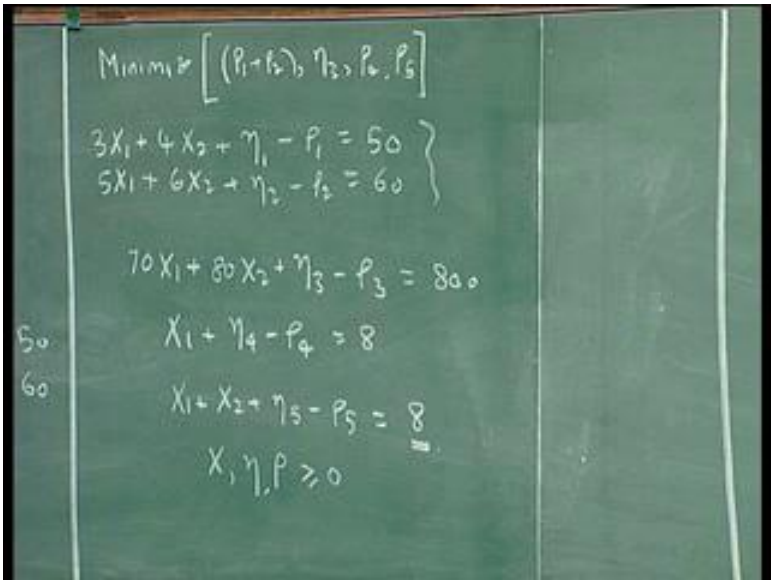

So this is your goal programming formulations for the first problem. We will see later how we solve it. This particular example we will solve using the graphical method. We will do another formulation, which we will try to solve using simplex, so that you get a feel of how to solve it using graphical method and how to solve it using simplex. One can go on explaining goal programming for hours, but in this course, we will not spend more than 2 or 3 classes on how to do this goal programming.

This is the formulation. So what have we learnt? One, all goal programming formulations will have equations. Every constraint is written as an equation ultimately with an eta and a rho. Depending on the nature of the constraint, whether you want to minimize something or maximize something or whether you want to keep something below a certain level or above a certain level, you will push the corresponding eta or rho into the objective function. All rigid constraints are bunched together. They are bunched together and then their corresponding things are written here. For example, if you had a greater than or equal to rigid constraint, a third rigid constraint, then it would have become rho_1 plus rho_2 plus eta_k. So it can be a mixture of eta(s) and rho(s) that is also allowed; it is not that it should have only this. You will have as many terms here as number of rigid constraints and depending on the nature of the rigid constraints, you will have this. There is actually nothing wrong in doing it. One of the reasons you bunch them together is you want to give equal importance to all of them. The moment you say rho_1, rho_2 it means you are giving more importance to this constraint than this constraint. So, you do not normally do that. This is the first example. We will take another example and try to understand a couple of things and then proceed to solving this problem.

Always, the objective is a minimization. You will not have maximization problems here. All these will be greater than or equal to 0. Eta(s) and rho(s), sometimes books will give d plus plus d minus, so it is like d_j, which is called a positive deviation and a negative deviation from the target or the goal; we use eta and rho. It is just matter of terminology.

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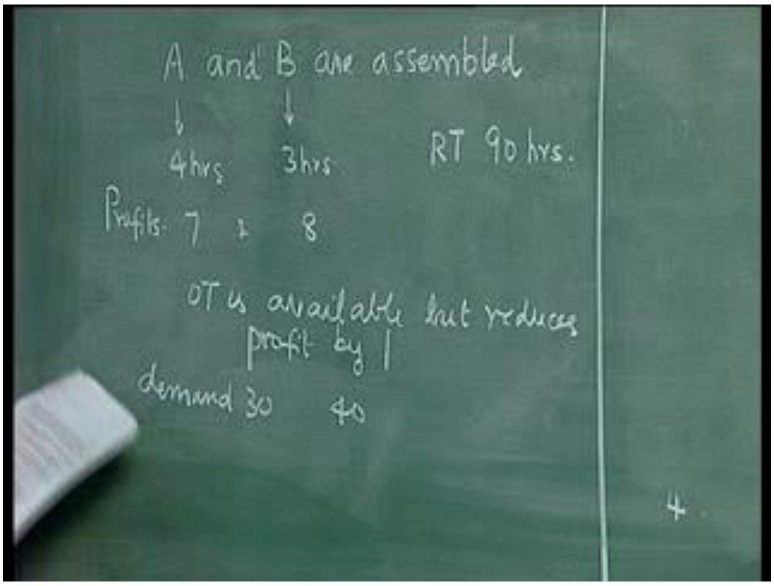

Now, let us look at a second example. It says that two products A and B are assembled. A requires 4 hours in assembly and B requires 3 hours of assembly. Regular time available in assembly is 90 hours. Profits are 7 and 8. Overtime is available, but reduces the profit by 1. Demand is 30 and 40 respectively.

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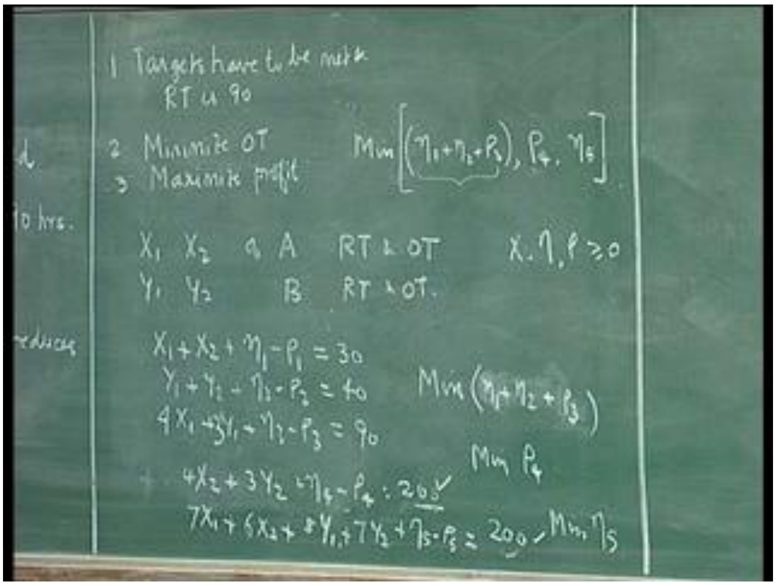

Goals are: One, targets have to be met and regular time is 90; two, minimize over time and three, maximize profit. Let us start the formulation as we try to formulate a linear programming problem and then proceed. Let X_1 and X_2 be the quantities of A produced in regular time and over time. Let Y_1 and Y_2 be the quantities of B produced in regular time and over time. There are basically two things in the rigid constraints, which are: target has to be met and the regular time availability is 90. The target that has to be met is given by this (Refer Slide Time: 26:29 min): X_1 plus X_2 plus eta_1 minus rho_1 is equal to 30; Y_1 plus Y_2 plus eta_2 minus rho_2 equal to 40; $4X_1$ plus $3Y_1$ plus eta_3 minus rho_3 equal to 90. 90 is the regular time availability and these are your rigid constraints. Regular time 90 is a rigid constraint. When you are given this information in this form, what ever is given first in a bunch is the rigid constraint. That is also an assumption, unless and other wise stated. One of the difficult things in goal programming is to first understand and interpret it suitably. So, I am trying to tell you what you should do unless and other wise something is stated. So, this 90 is not a goal. I mean, you should also look at it very practically.

Every organization will be able to define its regular time capacity very accurately and nicely. For example, if an organization runs 5 days a week and 2 shifts a day, it is 8 into 2 is 16 hours per day into 5 days a week, 80 hours is what you have as regular time capacity. The over time is a very tricky thing. You may work on the 6th day and get another 16 hours, you may work third shift and get some more hours; they will only say these are the possible things that are available. So you can say that the regular time is a rigid constraint. Now both these are you targets or goals (Refer Slide Time: 28:13 min), without the actual target being specified. For example, both are kept free as open objectives. It does not say, for example, keep overtime restricted to 50 hours; it does not say keep my profit close to 400 or what ever. So first, let us do this. The regular time is: $4X_1$ plus $3Y_1$ plus eta_3 minus rho_3 is equal to 90.

Yes. It depends again. Right now, we will not question this. Questioning this is something else; it is one level above. You should look at this whole thing not from solving an optimization problem or an OR problem, but you should try and understand the issues that are there in modeling real life situations or complex situations; that is it. At this point, I would not ask you to worry about why this is a higher priority thing than this (Refer Slide Time: 29:25 min). I would possibly ask you to take it as it is, but if you want to question it, that questioning will come later; after you solve it and say this is a solution. Then, if I change this what happens? If it is beneficial then I do that. This is like minimize rho_1 plus rho_2 plus rho_3. That is what you have - rho_1 plus rho_2 plus rho_3.

Target has to be met, so you will have eta_1 plus eta_2 plus rho_3; because, you want to meet the demand. So, you will have eta_1 plus eta_2 plus rho_3 coming here. Now these two things we do not know, so this will have left-hand side of $4X_2$ plus $3Y_2$ and we will right now keep the target as 20. So, plus eta_4 minus rho_4 is equal to 20; we will just keep this. The other one would be $7X_1$ plus $6X_2$ plus $8Y_1$ plus $7Y_2$ plus eta_5 minus rho_5 will be equal to - keep it at 200 as the profit. Both these are now defined by us; they are not defined in the problem. I want to minimize the overtime, so I will keep overtime less than something, so minimize rho_4, this will be minimize eta_5. I will have minimize eta_1 plus eta_2 plus rho_3, rho_4 and eta_5. This is the second formulation. Any questions? Again, X, eta and rho greater than or equal to 0.

Any questions? We have made two important assumptions here. We have made a 20 here and we have put a 200 here. We are not sure whether these are right or you know these are poor estimates or these are good estimates, we do not know. The only guideline that we may have in the absence of these numbers not being given to us is something like this. Look at this example.

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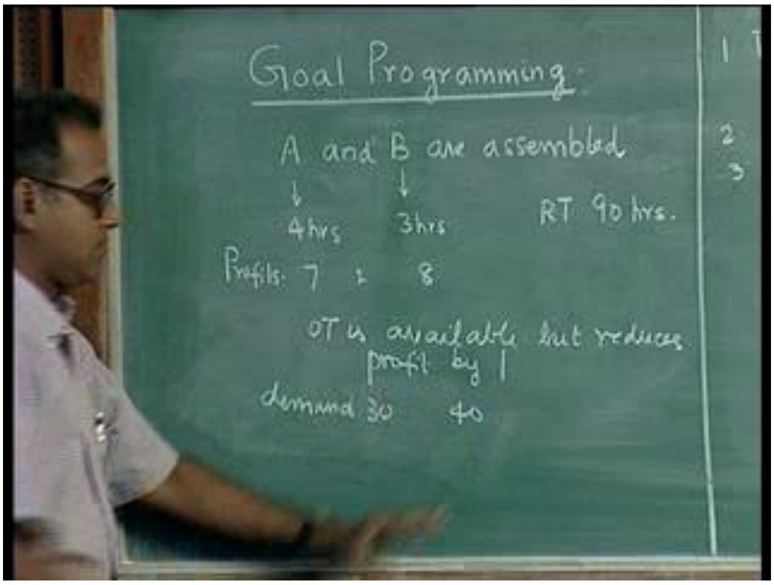

I have a demand of 30 and 40 of these (Refer Slide Time: 32:44 min). I may need, for example, so many hours, 30 into 4 is 120; plus 240 hours is what I may need. 40 into 3 hours, 30 into 4 hours; so 240 hours is what I need. I have 90 hours of regular time. If I want to meet all these things, I may need another 150 hours. So may be my estimate on this is poor (Refer Slide Time: 33:18 min). I may put a reasonably higher estimate and work. Now, what is the risk in doing this? This is the question that was asked earlier. The one thing that I want is when I solve this - what I am going to do is I am going to take the first objective, I am going to solve with all these constraints. I will get an answer. Now what I will do is, I will retain that answer and then try to solve the second. Whatever I have as a solution to the second should not deviate from the first, which is what was asked.

For example, if I think that a minimum over time that is needed is some 100 hours or what ever when I solve this, when I solve the third, I should not end up creating more overtime for the sake of the more profit. Now how does this contribute (Refer Slide Time: 34:18 min)? All the rigid constraints are modeled first and any X, Y, in this case that satisfies all the rigid constraints will and should have an objective function value of 0. If either eta_1 or eta_2 or rho_3 is positive, which means the sum is positive, it means that the corresponding rigid constraint is being violated. They are also very loosely defined. It is not that there is only point which satisfies eta_1 plus eta_2 plus rho_3 equal to 0; there are plenty of points, if you look at it carefully. The moment we draw the graph you will realize.

So, the first part of this formulation should always have an objective function of 0. If you have an objective function value of 0, it means these rigid constraints have a feasible solution set. Then you will move to the next one. Now, I will minimize this rho_4, subject to the condition that this is satisfied, which means subject to me being inside the feasible region, I am trying to satisfy this. Suppose, I put a poor target like 20 here and if it turns out that in order to meet the rigid constraint, which is to meet this 30 and 40, I may need 140 hours extra, which means this will become 120, if I have put a 20 here (Refer Slide Time: 35:50 min).

If I put a 100 here, this would become 40, but if I had put a 200 here, then this will become a 0. It will not become minus 60, it will still become a 0. If I put something sufficiently large here, then it means I am playing very safe, but then I am also making a mistake of a different kind because my organization may not like to start with very large acceptable value. Why are we discussing this so much? The moment I put a smaller value here (Refer Slide Time: 36:27 min), like 20, and I realize that I get a positive value here, it means that my target is to be revised and I may not be able to achieve this 20. Suppose my total needed is 140, I have put a 20, which means I will get 120 here. The moment I get a positive value here, I will not proceed further. I will stop the algorithm and I will evaluate the eta_5 for the solution that I already have. Now, that is the advantage or disadvantage of this lexicographic approach. The moment I am unable to proceed and the fact that I am unable to proceed will be indicated by a positive value coming here in any one of these. The moment I get that positive value, I will not optimize further, but I will evaluate the rest of the objectives further.

So, it becomes very important to define this. The last one is fine. For example, if the organization has agreed for 200 overtime, say organization is okay, I go back, discuss with them what is a realistic overtime you want and they say 200, whereas I need a 140. So, rho will not be negative, but rho will be 0, eta will take a value. Now I can go further and optimize this. So, if this being the last one, whether I have put 200 or 2000 it does not matter; because, if this estimate is very high, then this also going to be 0. If this estimate is poor, this is going to give me a positive value and I will stop.

So when it is last of the targets, that for which you do not have the value, you can have any arbitrary value and proceed. But the moment you do not have a value assigned to more than one, then what you assigned to the last but one, is going to decide whether you can optimize further or not. You need to be careful about that; that is what this example explains to you.

There can be a situations where this may be tight, this may be loose (Refer Slide Time: 38:42 min), but the order in which it has come would ensure that the tight one would give a positive and I do not optimize for that. Whereas, if I had interchanged these, the loose constraints would have come first and then it would not have affected. So the order in which they appear are also important in all these kinds of problems.

Now with this we will go back to the first one and we will try to solve the first one and then we will try to solve the second one. The first one we will solve by the graphical method and the second one we will solve by the simplex method, so that you understand how both these methods are used to solve goal programming problems.

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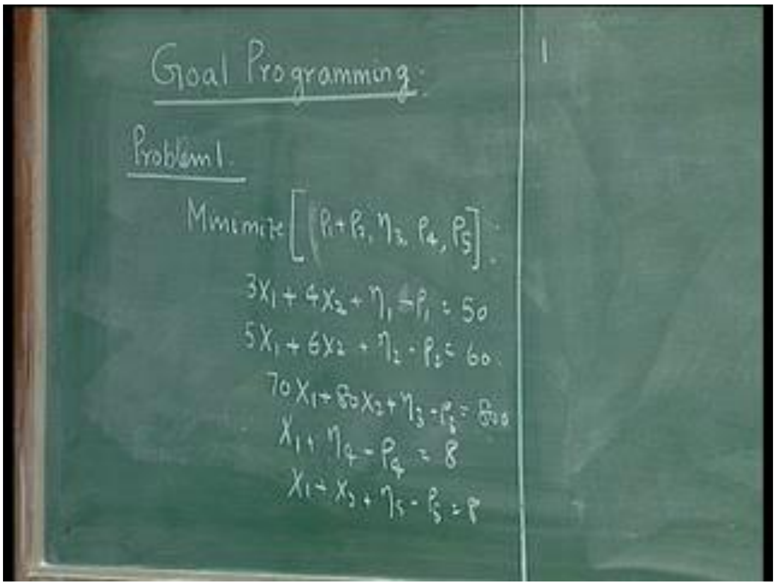

We go to problem one again and minimize rho$_1$ plus rho$_2$, eta$_3$, rho$_4$, rho$_5$ subject to $3X_1$ plus $4X_2$ plus eta$_1$ plus rho$_1$ equal to 50. $5X_1$ plus $6X_2$ plus eta$_2$ minus rho$_2$ equal to 60. $70X_1$ plus $80X_2$ plus eta$_3$ minus rho$_3$ is equal to 800; this is minus rho$_1$ (Refer Slide Time: 40:36 min). X_1 plus eta$_4$ minus rho$_4$ is equal to 8; X_1 plus X_2 plus eta$_5$ minus rho$_5$ is also equal to 8. Now, you can leave out these eta(s) and rho(s) because they are like slack variables. You go back to your linear programming, you will always leave out the slack variables. With the slack variables, a two variable problem will become four variables, but this is still a two variable problem, in spite of the additional five eta(s) and the additional five rho(s); so, it is a two variable problem; we can solve it by the graphical method. Let us go back and draw the graphs for this.

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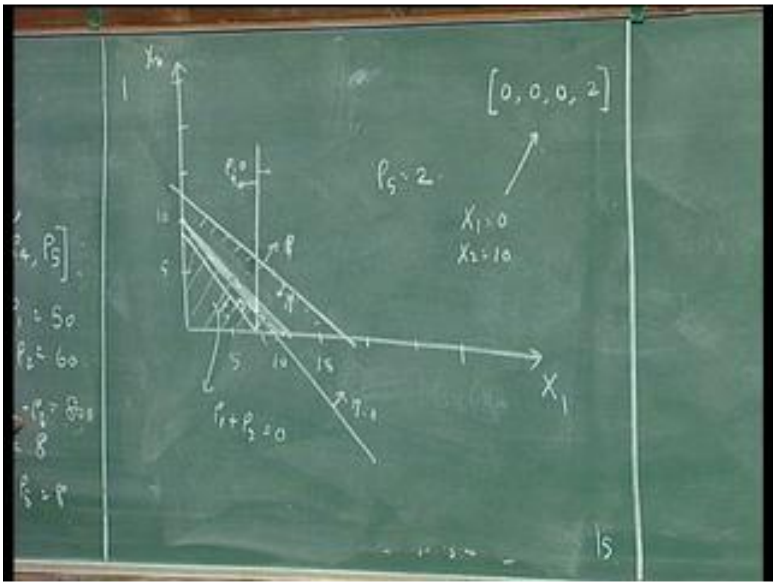

You will have X_1 and X_2 here. $3X_1$ plus $4X_2$ equal to 50. So let us put some numbers, say 5, 10, 15, 20, 25, 30; 5, 10, 15, 20, 25, 30. $3X_1$ plus $4X_2$ equal to 50 will give me X_1 equal to 6, implies X_2 equal to 5; so I have one point which is like this. X_1 equal to 10, X_2 equal to 5, it will come here (Refer Slide Time: 42:15 min) X_2 equal to 10, would give X_1 equal to some 3 point something. I will have something like this: (0,12.5), this will be (12,0) and (0,10). That is what I have. First one is 16 point something; both are less than or equal to type. Now what happens? This is the first constraint; if I put this as say 60, then this will come here, parallel to itself and it will move above. So, 60 would imply eta_1 is equal to 10. This is the direction of increase of rho and this is the direction of increase of eta. I will explain it again. So if I put this equal to 60, that is, if I put $3X_1$ plus $4X_2$ is equal to 60, then it means that if I shift this, a point here would mean that $3X_1$ plus $4X_2$ is greater than 50 and therefore this has a positive rho.

A point inside (Refer Slide Time: 44:18 min) will have $3X_1$ plus $4X_2$ less than 50 and will have rho equal to 0 and a positive eta. If I want to minimize rho, then this is less than or equal to this thing. This rho equal to 0 and any point under this will have rho equal to 0. Now, similarly this, any point here will have rho_2 equal to 0, it turns out that this constraint dominates the other. So, any point inside this now has rho_1 plus rho_2 equal to 0. Your first rigid constraint is actually represented by this particular triangle. Now you write this one - $70X_1$ plus $80X_2$ is equal to 800. This is given by the point (0,10) which is here and the other one is 8 by 7, is roughly how much? 1.4; this is 80 by 7, 11.4, say somewhere here is less than this. This is my point and this is the other one. So, this is increasing rho. This is eta equal to 0. So my feasible region reduces to something like this. This is my feasible region here. So, any point inside this region now satisfies, this is equal to 0, as well as this is equal to 0.

Now I have another one: X_1 less than or equal to 8, which is somewhere here (Refer Slide Time: 46:38 min). This will be my X_1 less than or equal to 8. This is the direction of increasing rho. So, this is rho equal to 0 or rho_4 equal to 0. Now your feasible region reduces to this alone. I am just shading out the entire thing; so your feasible region reduces to this. This is not included, so this part alone is your feasible region. You still have a feasible region; so any point inside this region will give you all this equal to 0, including rho_4 equal to 0.

Every point inside this region is capable of satisfying whatever conditions that you have. You can choose any point that way. The last one is X_1 plus X_2 is less than or equal to 8. So I have to go back here. So, (8,0) and (0,8). Let us assume (8,0) is here and my (0,8) is here. So I get something like this (Refer Slide Time: 47:53 min). (8,0) is that vertical line; (0,8) is somewhere here, so I get this (Refer Slide Time: 48:04 min). I get this as the last one. This is my last one. This is the target that I have given.

Now, I want to minimize rho_5. This is the direction of increasing of rho; this is rho equal to 0. So, every point here has rho equal to 0, but I am not getting any feasible point. I now have to increase this rho in order to get a feasible point. This will force me to get a feasible point by increasing this rho_5, which means you move this line parallel to itself upwards; upwards till you touch this 10. You are at 8 here; so you need to move this a little upward, till you touch this 10. You will finally get rho_5 is equal to 2 and the point that you will touch will be X_1 equal to 0, X_2 equal to 10. That is the point that you will touch.

The solution to your goal programming problem will be 0, 0, 0, 2 given by this. These two are satisfied (Refer Slide Time : 49:29 min). This will also be satisfied, because you are still at 0, which means you have got that 800 satisfied, this is the only one that will be violated. So, instead of setting a minimum target of 8 for these, you will end up setting up a minimum target of 10. This is the goal programming solution to this problem. We will look at the goal programming solution to the next example in the next class.