Advanced Operations Research
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Lecture No. # 38

Mixed Integer Linear Programming

In today's lecture we will address the mixed integer linear programming problem called MILP. And we will see algorithms to solve the MILP problem, so we start with a numerical example.

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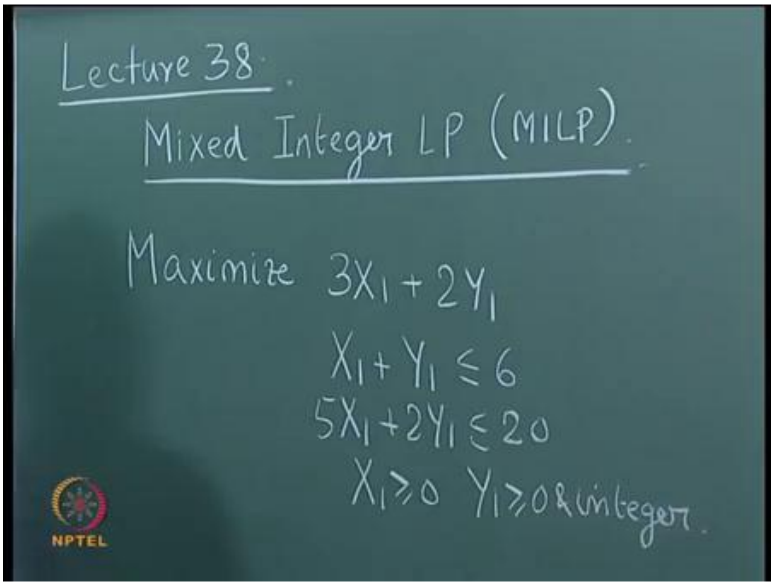

So, the example that we will use is a maximization problem. So this is the example, we use a slightly difference notation here, we have use both X and Y as variables. Because, the mixed integer linear programming problem has some variables that have to be integers and some variables that can take continuous values, it is good to use a notation by which the X variable represents the continuous variables and the Y variables represent the integer variables. So in this particular example, we have two variables one of which should take only integer value which is y 1. So, y 1 greater than or equal to 0 and integer the other variable X 1 can take continuous values, so if you had more than one X variables than we use X 1 X 2 and so on similarly, we use y 1 y 2 and so on. We will be looking at 3 different algorithms to solve this we will begin with the branch and bound algorithm then we will look at a cutting plane algorithm for the MILP and later we will look at Benders partitioning algorithm solve MILP.

So, let us begin with the branch and bound algorithm so as is customary with branch and bound for all integer programming, we first solve the LP by relaxing the integer restriction on the y variable. So we treat y also as greater than or equal to 0, to begin with and solve the linear programming problem if are some reason the optimum solution to the linear programming problem, gives y 1 an integer value then it will be optimal to the MILP. If it is not, we will then do something to try and get the integer value to y 1 and the corresponding optimal solution. So let us first solve the problem as a linear programming problem to see, what is the L P optimum solution.

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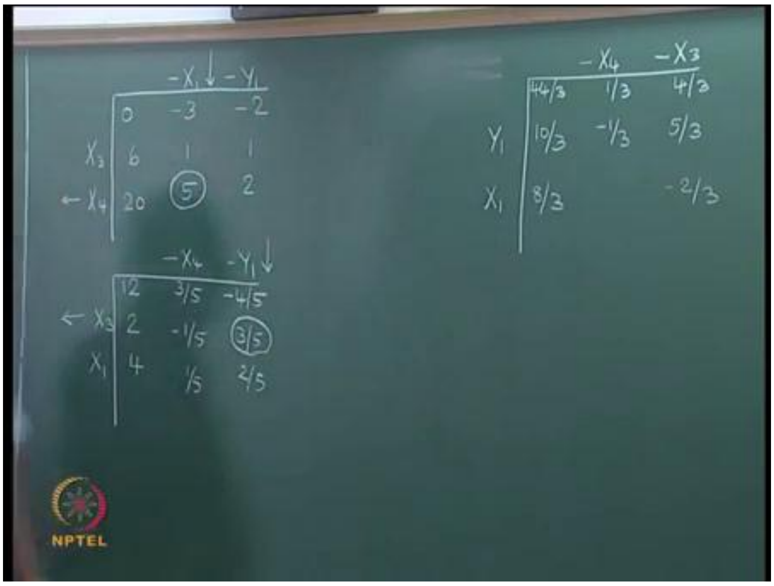

So we set up the simplex table as we have done before. This is a maximization problem so we will add 2 slack variables which are called X 3 and X 4. Now, because X 1 is continuous and only y 1 is restricted to on integer value, X 3 and X 4 will be continuous variables so they will not be integer variables they will be continuous variables; so X 3 and X 4 which are continuous variables we will start with them as the basis. So this start with X 3 and X 4 and then we have minus X 1 and minus y 1 here as currently non basic variables. So, we will have minus 3 and minus 2, here. We will have 6 1 and 1 and then we have 20 and then 5 and 2. So, this is the initial simplex table with X 3 and X 4 as basic variables and X 1 and y 1 as non basic variables. So this solution as primal feasible and dual infeasible dual has negative values. So, the most negative dual will enter which is equivalent to the largest coefficient rule where the most positive c j minus z j or the most negative z j minus c j will enter for a maximization problem. So the variable X 1 will enter the solution and then we find out the minimum ratio 6 divided by 1 is 6 20 divided by 5 is 4, so this will be the leaving variable and this will be the pivot.

So the next iteration will be like this, so minus X 4 and minus y 1, X 3 and X 1, pivot becomes 1 by pivot so 1 by 5 divide by the pivot 4 2 by 5 divide by negative of the pivot, so plus 3 by 5 and minus 1 by 5. Now, this is initially taken as 0, so 0 minus minus 3 into 4 is plus 12, 6 minus 1 into 4 is 2, this is minus 2, minus minus 3 into 2 by 5 which is minus 2 plus 6 by 5 which is minus 4 by 5, minus 2 plus 6 by 5 is minus 4 by 5, 1 minus 1 into 2 by 5 is 3 by 5. So, once again this is primal feasible, dual infeasible so the negative dual will enter, there is only one entering variable. So, variable y 1 enters the solution, now to find out the minimum ratio 2 divided by 3 by 5 is 10 by 3, 4 divided by 2 by 5 is 10, so X 3 will go y 1 will enter. So the next iteration will have, y 1 will enter so minus X 4 minus X 3, y 1 and X 1 this is the pivot, so pivot becomes 1 by pivot 5 by 3, divide by the pivot 10 by 3, minus 1 by 3, divide by negative of the pivot so minus 4 by 5 divided by minus 3 by 5 is plus 4 by 3, minus 2 by 3.

So, this will be 12 minus minus 4 by 5 into 10 by 3, so 12 plus 40 by 15 or 12 plus 8 by 3, 44 by 3, 4 minus 2 by 5 into 10 by 3 is 4 minus 4 by 3 which is 8 by 3, 3 by 5 minus minus 4 by 5 into minus 1 by 3, so 3 by 5 minus 4 by 15 is 5 by 15 which is 1 by 3, 1 by 5 minus 2 by 5 into minus 1 by 3 so 1 by 5 plus 2 by 15, 5 by 15 which is 1 by 3.So this is the LP optimum, with X 1 equal to 8 by 3, y 1 equal to 10 by 3 and objective function is 44 by 3, lets go back X 1 is 8 by 3, 24 by 3 plus 20 by 3 is 44 by 3. So this is the LP optimum for this problem, so we write know do not require this intermediate iterations, we need only the LP optimum table so we set up the first we verify whether the LP optimum satisfies the condition, that y 1 is integer. So, it does not satisfy the condition that y 1 is integer therefore, we will use the branch and bound algorithm to try and get to the optimum solution, that has y 1 as integer value write now, y 1 takes a value 10 by 3 which is a non integer value therefore, the LP optimum is not feasible to the MILP and therefore, we need to go to the branch bound algorithm.

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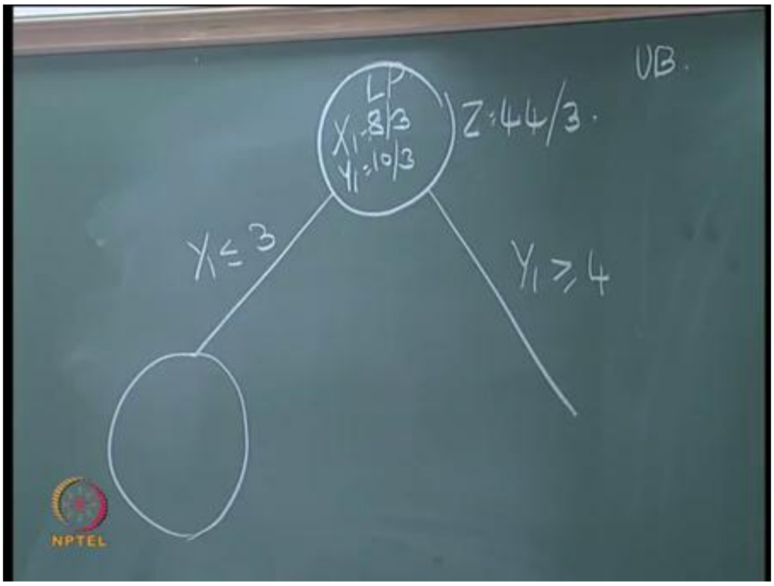

So we start with the branch and bound algorithm, with the L P optimum as the starting node which has this is L P optimum. Which has X 1 is equal to 8 by 3 y 1 equal to 10 by 3 and Z is equal to 44 by 3. Now, as we do in the all integer branch and bound algorithm, we take that variable which should be having an integer value but, right now having a non integer value which is y 1, which is the only variable in this particular example and y 1 is 10 by 3, so we take the lower integer value of 10 by 3 and we take the upper integer value of 10 by 3 to get 3 and 4. So we branch of with 2 branches which is y 1 less than or equal to 3 and then we branch with y 1 greater than or equal to 4, so we have 2 branches y 1 less than or equal to 3 and another branch with y 1 greater than or equal to 4 and because it is a maximization problem, the linear programming optimum LP optimum is an upper bound to the MILP optimum. Because, what we are essentially doing is? we are trying to create nodes by adding constraints, so you add a constraint to a maximization problem, the value of the objective function will only reduce, so the LP optimum will be an upper bound to the IP optimum incase of maximization, incase of minimization the LP optimum will be a lower bound to the IP optimum, we are now solving a maximization problem. So 44 by 3 is an upper bound to the optimum to this which means, the optimum to this will have a value less than or equal to 44 by 3 cannot exceed 44 by 3. So we now, have this constraint that is added which is y 1 less than or equal to 3, so we now solve a problem which has all this plus the condition y 1 less than or equal to 3. Now, this can be done in many ways, you can solve the linear programming problem from the beginning by adding y 1 less than or equal to 3 here, that is one way to do it.

The other way to do it is you keep the LP optimum, since we have it now add a constraint to the LP optimum which is y 1 less than equal to 3 and performs sensitivity analysis where we add a constraint and we can do that. The third of course, is to use simplex method for bounded variables, because y 1 less than equal to 3 is an upper bound on y 1. So we could use simplex method for bounded variables and use it without adding a constraint you can still use the ideas from the bounded variables to get the solution. Right now, since we are working this out an on the board its little easier for us, since we have the LP optimum solution, here which is easy for as to try and use this as an additional constraint into the problem and then solve it. So let us do that, so we are trying to introduce the constraint y 1 less than or equal to 3.

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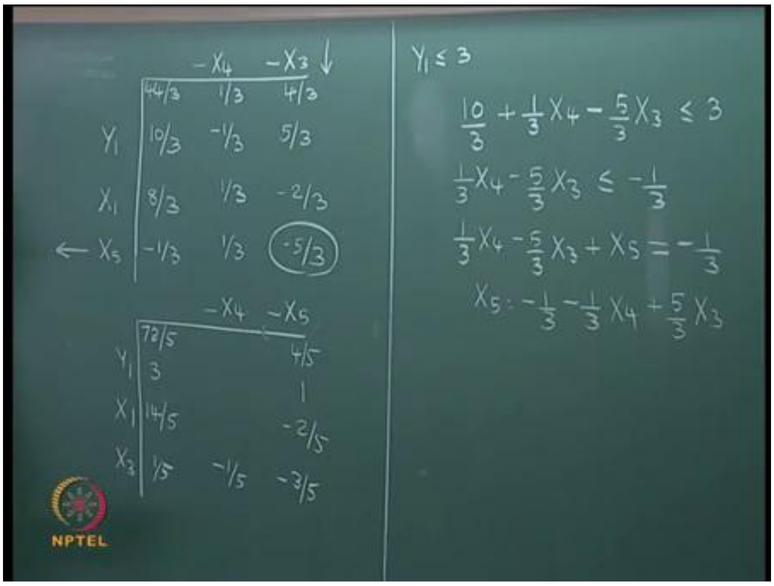

So our constraint is y 1 less than or equal to 3, from this y 1 is equal to y 1 is the basic variable, so y 1 is equal to 10 by 3 plus 1 by 3 X 4 minus 5 by 3 X 3 that is your y 1. And since we have y 1 less than or equal to 3, we are going to have this less than or equal to 3. Now, take these 2 the other side you get 1 by 3 X 4 minus 5 by 3 X 3 is less than or equal to minus 1 by 3 less than or equal to minus 3 minus 10 by 3 is minus 1 by 3. Now, add a slack variable which is X 5, so 1 by 3 X 4 minus 5 by 3 X 3 plus X 5 is less than or equal to minus 1 by 3 X 5 greater than or is equal to minus 1 by 3, now the slack variable is added to convert the inequality to an equation. Now, from this you can write X 5 is equal to minus 1 by 3, minus 1 by 3 X 4 plus 5 by 3 X 3. So writing here introduce new slack variable X 5, so X 5 is minus 1 by 3 minus 1 by 3 X 4 minus 1 by 3 X 4 and plus 5 by 3 X 3 so minus 5 by 3 here.

Now, we have added the y 1 less than or equal to 3 constraint into the simplex table to get this node. Now, if you look at the this table is dual feasible primal infeasible, so we have to do a dual simplex iteration by sending out the variable X 5 that leaves first, corresponding entering variable is chosen based on a minimum ratio but taken from the other side. So 1 by 3 divided by 1 by 3, 4 by 3 divided by minus 5 by 3 in a dual simplex iteration the pivot has to be negative so there is only 1 pivot, so 4 by 3 minus 5 by 3 this is the entering variable. In a simplex iteration the pivot has to be positive in a dual simplex iteration the pivot has to be negative, so there is only one negative pivot so this will be the one. Now, we enter this variable we leave out this variable and to the next simplex table to get the LP optimum corresponding to this node. So X 4 remains here, X 5 comes here, so we have y 1 X 1 and X 3 this is the pivot element, so pivot becomes 1 by pivot, so minus 3 by 5, divide by the pivot 1 by 5 minus 1 by 5, divide by the negative of the pivot 4 by 5, 1 2 by 5 minus 2 by 5. Now, this value will be 44 by 3 minus 4 by 3 into 1 by 5, so 44 by 3 minus 1 by 15, 44 by 3 minus 4 by 15. So 220 minus 4 - 216 by 15 which is 72 by 5, so that is the value here.

Now, this value is 10 by 3 minus 1 by 3, which is 9 by 3 and 3, 8 by 3 plus 2 by 15, 8 by 3 plus 2 by 15 is 42 by 15, which is 14 by 5. Now, this one is 1 by 3 plus 4 by 15, which is 9 by 15, which is 3 by 5. This is minus 1 by 3 plus 1 by 3, which is 0. This is 1 by 3 minus 2 by 15 is 3 by 15, which is 1 by 5. So this gives as the LP optimum with y 1 equal to 3, y 1 equal to 3, X 1 equal to 14 by 5, and z equal to 72 by 5. So the first thing we observe is the objective function value as come down, 44 by 3 is 14.666 and 72 by 5 is 14.4, so the value as come down. Now, this is feasible to the MILP, because the variable y 1 which should take on integer value is right now, taking on integer value with y 1 equal to 3. So X 1 is continuous 14 by 5, z equal to 72 by 5, so this node gives as a feasible solution to the MILP. So, we fathom, this node by feasibility, because we obtained a feasible solution to the problem.

But, we are still not sure whether it is the optimum solution. But, it is a feasible solution to a MILP, now the given problem is a maximization problem. So we have a feasible solution to a maximization problem, with objective function value 72 by 5. So a feasible solution to a maximization problem becomes a lower bound to the optimum. Now, the optimum can be greater than or equal to 72.5. From, here we also know that the optimum has to be less than or 44 by 3, so feasible solution to a maximization problem is a lower bound to the optimum. feasible solution to the minimization problem is an upper bound, so we have know obtained a node where we have a feasible solution to the MILP and this is a lower bound to the optimum. So we do not proceed here, because we have already obtained a feasible solution, so we say that fathom this node by feasibility. Now we have to look at the other one, so we will do that as well, so we will take this so we now have to add y 1 is greater than or equal to 4 to the LP optimum.

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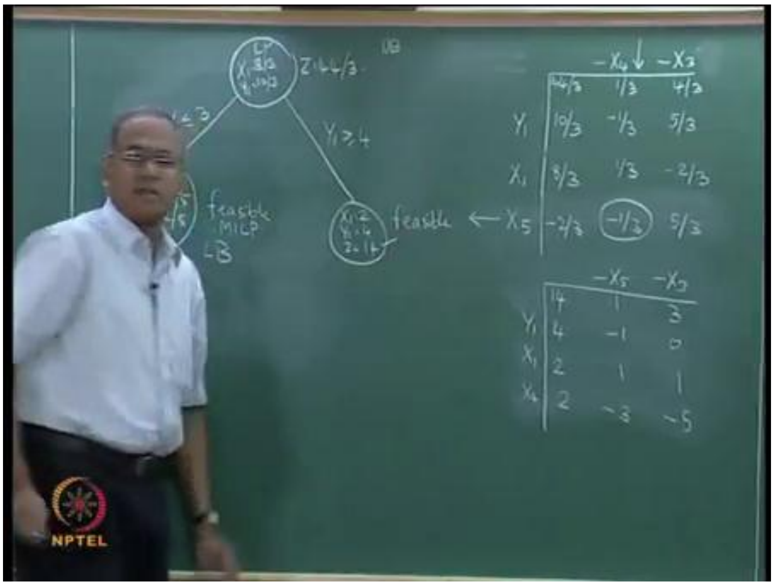

So we go back, we now have to add y 1 greater than or equal to 4, to the L P optimum. So y 1 from this table is 10 by 3 plus 1 by 3 X 4 minus 5 by 3 X 3 is greater than or equal to 4, so taking it to the other side this will be greater than or equal to 4 minus 10 by 3, which is 2 by 3 greater than or equal to 2 by 3. Now, we add X 5 which is a surplus variable so 1 by 3 X 4, minus 5 by 3 X 3, minus X 5 is equal to 2 by 3. And then you take X 5 to one side the others to the other side, so this gives as X 5 is equal to minus 2 by 3, plus 1 by 3 X 4, plus 5 by 3 X 3. So we now add this constraint into the simplex table by having on X 5 here, so X 5 is minus 2 by 3, plus 1 by 3 X 4, so minus 1 by 3 and minus 5 by 3 X 3 here, minus remains; so minus 5 by 3, so I have a plus 5 by 3.

Please note that minus 2 by 3 plus 1 by 3 X 4 minus 5 by 3 X 3, there is a minus sign here therefore, the numbers change. So once again, this is dual feasible and primal infeasible. So, we have to do a dual simplex iteration; so when you do a dual simplex iteration, this variable goes first, the negative variable goes first. Now, because it is a dual simplex iteration the pivot element has to be negative, there is only one negative element here in the pivot. So this automatically becomes the entering variable, then we carry out the next iteration, where X 4 comes here, X 5 goes outside, minus X 5 minus X 3, y 1, X 1, X 4, this is your pivot; pivot becomes 1 by pivot, so minus 3 divide by the pivot, so plus 2 and minus 5, divide by negative of the pivot, so 1 minus 1 and 1. So this will become 44 by 3 minus 1 by 3 into 2 which is 42 by 3, which is 14; 10 by 3 plus 2 by 3, which is 4; 8 by 3 minus 2 by 3, which is 6 by 3, which is 2; the other one becomes 4 by 3 minus 1 by 3 into minus 5, 4 by 3 plus 5 by 3 which is 9 by 3, which is 3, 5 by 3 minus minus 1 by 3 into minus 5; so 5 by 3 minus 5 by 3, which is 0; 2 by 3 minus 1 by 3 into minus 5, so minus 2 by 3 plus 5 by 3, which is 3 by 3, which is 1.

So, we simply getting on all integer table, but that is only incidental, there is no guarantee that will get on all integer table; it just happens because all the variables here took integer values, and the because of the nature of this problem. That it is not guaranteed that in MILP we will get, sometimes you get an all integer solution therefore, this kind of thing is chosen. So, this is the LP optimum with X 1 equal 2, y 1 is equal to 4 and z equal to 14. So this is X 1 equal to 2 y 1 equal to 4 with z equal to 14.

So, once again we observe there it is feasible to the MILP, because since we are looking at the MILP we are going to look only at the Y variables, whether they have integer values. It is only incidental that the X variable also has on integer value in this case; so it is one again feasible to the MILP with objective function value 14. And therefore, presents a lower bound, a maximization problem feasible solution is the lower bound. So, we do not proceed further we do not proceed further on this, because we have already got a feasible solution. Now, we go back and check there we had earlier, we had a feasible solution which 72 by 5, which was our best solution. Now, we have another feasible solution with 14. So, if this new solution is better than the existing best solution we have obtained.

In this case it is not because 72 by 5 is more than 14 this exactly 14; so this solution, even though it is feasible has not added much value, because we have already identified a feasible solution that better than this. So, the algorithm will terminate, because there is no more node to branch on, and this will give as the optimum solution with the current best feasible solution is the optimum solution. So, this is the optimum solution with Y 1 equal to 3, X 1 equal to 14 by 5, and Z equal to 72 by 5.

In a typical branch and bound, it is not necessary that all of them will be fathomed by feasibility, we have seen that in a branch and bound there are three ways of fathoming a node. One is by feasibility when you get a feasible solution, which is what we got in both the cases, feasible to the MILP. This one is not feasible to the MILP, but this is feasible to the LP; so when you get feasible to MILP, you fathom by feasibility. Sometimes, you will have an infeasible solution by, because you keep adding constraints at every node, there are times you may get on infeasible solution. So, you fathom by infeasibility, and the third is called fathoming by the bound.

For example, if this node had not given as a feasible solution to the MILP, but if this node had given as a solution with y taking a fractional value if - it has done so. Why taking a fractional value, and this objective function value more than 72 or objective function value same as 14; the objective function value same as 14, but if y had taken a fractional value say 4.1 or 4.2, and this value is 14.

We can still fathom it by found, because by adding constraints and by proceeding down wards, we will only get values which are 14 or less. We already have a feasible solution which 72 by 5 for a maximization problem. Therefore, this lower bound here, the feasible solution here will fathom this particular value, because proceeding from here downwards can only give a solution with 14 or less. So, there is no point in proceeding further, because we already have a solution with 72.5.

In this particular example, we are not able to show fathoming by the bound, and we are able to show only fathoming by feasibility, but in general there are three ways to do. So, this is how we apply the MILP algorithm branch, and bound to MILP; this structure is the same, but the bounds and the feasible solutions will change depending on whether it is a maximization problem or a minimization problem. We have to remember that if you are solving a maximization problem, a feasible solution to the MILP represents a lower bound, and LP optimum represents a upper bound.

If we are solving a minimization problem, a feasible solution will represent an upper bound, and LP optimum will represent a lower bound. So, we have to interpret that suitably, when we solve for this. Otherwise this is same as solving branch, and bound for all integer. In a way it is advantages, because if this problem where an all integer problem, then every time we have to take care that all the variables should take integer values.

So, the number of nodes that we generate will be more, because it is mixed integer where some of them take integer values, and some of them take fractional values. In a way branch, and bound is advantageous, because we have to consider fewer nodes for the branch. So branch and bound is an advantage, when it comes to solving on MILP, and it is very similar to the ILP. So, in some sense a same branch and bound algorithm can always be used for ILP or MILP, depending on how many variables are restricted to the integers. Whereas, the cutting plane algorithm will be different for the MILP. So, we will now see the cutting plane algorithm for the MILP, we will retain the LP optimum solution alone here.

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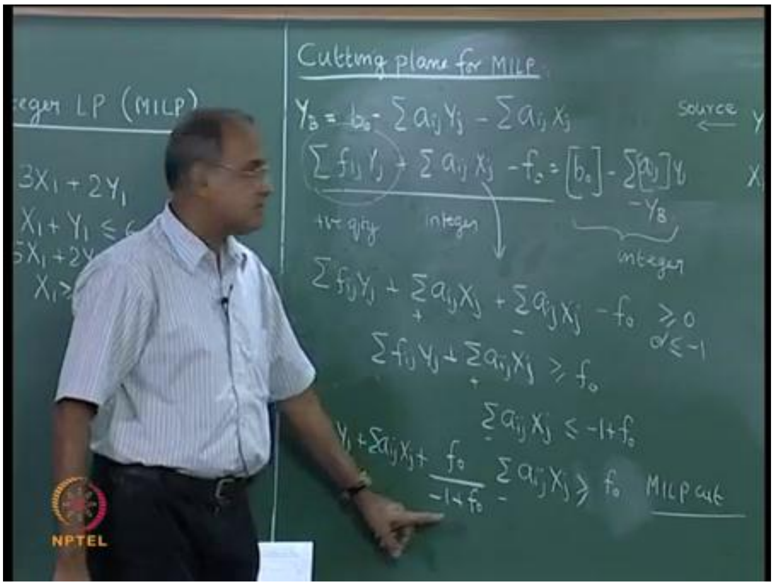

So, now let us look at cutting plane algorithm for MILP. Now, the cutting plane algorithm for the MILP is used, when the LP optimum is not feasible to the MILP, like in our case where this is the LP optimum. LP optimum by relaxing this integer restriction, we are getting y 1 equal to 10 by 3, y 1 should be an integer. So, we need to use a cutting plane algorithm, which means we have to introduce a cut. So, this row will act as source row for the cut, because that is the variable which should take on integer value, currently not taking on integer value.

So, this will act as a source row for the cut. So, let us first write a general expression for the cut, and then we will generate the cut using this row as the source row, and then performs iteration get to the optimum to the MILP. So, typically the we will use something like this, Y b is equal to b naught minus sigma a i j y j minus some sigma a i j X j. Now, this would represent the source row. For example, we will write here y 1 is equal to 10 by 3 plus 1 by 3 X 4 minus 5 by 3 X 3 is written in a general form here with b naught as the right hand side value.

And the set of non basic variables can have some variables which are X variables, and some variables which are y variables, in a general problem we may have more than 1 Y variable, more than 1 X variable. All the slacks and surplus variables will become X variables; so, we would simply have at the end, like we had here 1 X variable and 1 y variable in the basis. We could have some X variables, and some Y variables which are non basis; so we start writing the set of Y variables and the set of X variables, which are there. So, this will be the general source row equation.

Then we re-write the source row has something like this. Sigma f i j y j plus sigma a i j X j minus f naught is equal to b naught minus sigma y j minus y b. So, let me explain this, have just written this from sheet a paper. So, let me explain this to you; so what we have done here is that, we have kept the X j variables as it is, and we have brought this to the other side of the equation. So, the a i j X j terms come exactly as there, the y B goes to the other side, so there is no problem there.

Now, we are looking at these two terms. Now, this one is not integer, because this comes from the source row, you go back - this comes from the source row; source row is always a variable that is currently having a fractional value, but should be having an integer value. So, b naught is not integer; so b naught is not integer, it has the fractional component, and its greater than or equal to 0 or greater than 0. Because it has the fractional component, it is greater than 0 because it anyways it satisfies this up to this.

So for example, 10 by 3 you cannot have a minus 5 by 3 there, because it comes from a LP optimum, so b naught is now written as a lower integer component of b naught plus of fractional positive fraction component of b naught. For example, 10 by 3 will be return as 3 plus 1 by 3; 3 is the lower integer value, 1 by 3 is the difference; 3 plus 1 by 3 is 10 by 3. So, lower integer value of b naught will be 3, fractional value will be 1 by 3. So, what we have done is this b naught is written as lower integer component of b naught plus the fractional component of b naught; fractional component of b naught is called f naught. So, f naught comes to the other side, b naught remains exactly as it is.

In a similar manner for all these y j variables, right now we do not have any y j variable there, but if there a y j variable, then that y j variable is written as a lower integer component, and a fractional component is written as a lower integer component and a fractional component. So, one something is written as a lower integer component and a fractional component; the fractional component is always a positive fraction. For example, if I if you have 4 by 3 is written as 4 by 1 by 3, if you have minus 4 by 3 is written as minus 2 plus 2 by 3 because, you write a lower integer component and then a positive fraction component. Now, that positive fraction component comes to this side; so f ij y j the already a negative sign, so the fractional component comes to this side f i j y j. So, the lower integer component lies here. So, this is how this particular equation is re-written from the original equation.

Now, what is the advantage of writing it that way: The advantage of writing it that way is this portion is an integer; now this portion is an integer, because lower integer value of b naught is an integer, a ij's lower integer value is an integer, y j's have to be integers in the given problem. Y B it should also be integer, because y B is the basic variable. So, the right hand side portion is an integer, which also implies that the left hand side portion this is an equation; it also implies that the left hand side portion is also an integer. Because it is a equation, even though it has a lot of fractional components; now f naught has a fractional component, all these f i j's are fractional components and so on.

Now, then we look at - remember y j(s) are greater than or equal to 0, and these f ij(s) are fractional components, positive fractions, because these are taken from these terms, and as I said, you have a lower integer of a ij and a positive fraction. So these become positive fractions f ij(s) become positive fraction; you have to be just that be careful in the sense that there is a minus sign already here; so depending on the term, suppose it is 4 by 3, then it will become 1 plus 1 by 3, and there is a plus 1 by 3 with a minus sign, so when you bring it here it will become plus sign. If a ij itself has minus 4 by 3, then a ij will become minus 2 plus 2 by 3, that 2 by 3 is a positive fraction term with a minus sign; so when you bring it to the other side, it will become plus. So all your f ij(s) are positive fractions less than 1; and y j(s) are greater than or equal to 0; so in a way, this is a positive quantity.

Now, this one depending on the coefficient of a ij can be a positive quantity or a negative quantity. So what we do now is we re-write this as sigma f ij Y j plus sigma a ij X j with a plus sign indicating the these are positive a ij coefficients plus sigma a ij X j with a minus sign, minus f naught is an integer. And because this is an integer, it is either greater than or equal to 0 or less than or equal to minus 1. Now, this is a positive quantity, this is a positive quantity; so these two are positive quantities, so you can now write this as sigma f ij Y j plus sigma a ij X j plus quantity. Those two are positive quantities should be greater than or equal to f naught, because f naught is a positive quantity it is a positive fraction less than 1. So this quantity, this left hand side can be written as greater than or equal to f naught.

The other one can be written as sigma a ij X j with minus should be less than or equal to minus 1 plus f naught, because this is a negative quantity. Right? this is a negative quantity; so take this f naught to the other side, you can write this as less than equal to minus 1 plus f naught; f naught is always a positive fraction less than 1; so minus 1 plus f naught is a negative quantity, it is a fraction, negative fraction less than 1. So the negative part of these should be less than or equal to minus 1 plus f naught; so once we write this, multiply this equation by f naught divided by minus 1 plus f naught.

So this will become f naught divided by minus 1 plus f naught sigma a i j X j with negative is less than or equal to f naught by minus 1 plus f naught greater than or equal to. Please note that I am changing this sign of the inequality, because multiplying it with f naught by minus 1 plus f naught. Now f naught is a positive fraction less than 1; so f naught by minus 1 plus f naught is not 0, it cannot be 0, but f naught by minus 1 plus f naught is a negative quantity, because f naught is a positive fraction less than 1, so minus 1 plus f naught becomes a negative fraction; so f naught by minus 1 plus f naught is a negative term. So since I multiplying it with a negative term on both sides, which is non-zero, a sign of the inequality will change. So this is how the second one will look.

Now, I combine these two; now this quantity is greater than or equal to f naught; this has to be re-written as only f naught, because a multiplying by f naught by minus 1 plus f naught; so this has to be written as this. So, this quantity is greater than or equal to f naught, this quantity is greater than or equal to f naught; therefore, the sum has to be greater than or equal to f naught. So the general MILP cut will now become sigma f ij, so I am just adding this term here, so the final cut will become sigma f ij Y j plus sigma a ij X j plus f naught by minus 1 plus f naught is greater than or equal to f naught. So I am adding all of them, so this is called an MILP cut, this is called an MILP cut. So now what we can do is, we can apply this MILP cut on to this source row to see how we are doing.

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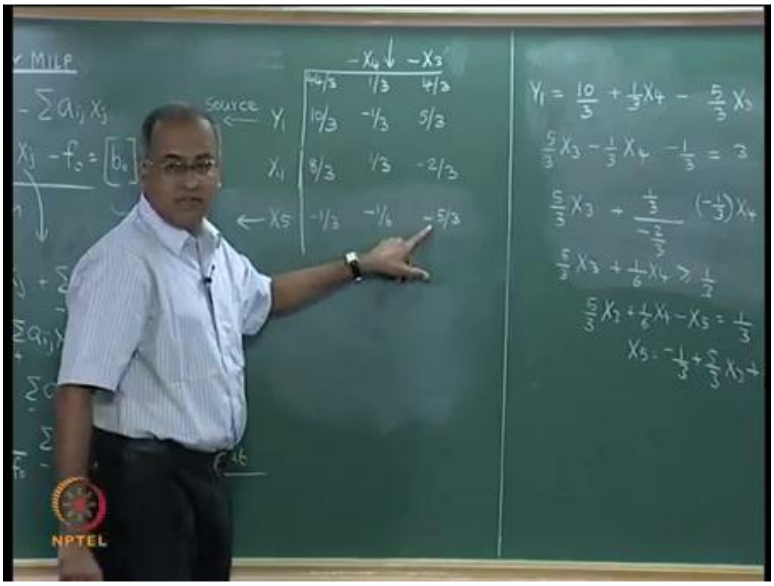

So, let us write the source row here as y 1 equal to 10 by 3 plus 1 by 3 X 4 minus 5 by 3 X 3. So you are simply bringing all the a ij X j to this side, Y b goes to the other side, so finally, what happens? There is no y term here, there is no Y variable here therefore, this term does not exist, this term does not exist. Now this is we need to bring all the X terms to this side, so you get 5 by 3 X 3 minus 1 by 3 X 4, we have got this. Now this is 3 plus 1 by 3, so minus 1 by 3 is equal to 3. Correct?

So now, you realize that this term is not there, this term is the one that has a positive value; so your MILP cut is 5 by 3 X 3, that comes from here; f naught is 1 by 3; so plus 1 by 3 divided by minus 2 by 3 minus 1 plus f naught into minus 1 by 3 X 4 is greater than or equal to 1 by 3, this is our MILP cut greater than or equal to f naught. So this MILP cut will give us 5 by 3 X 3 plus 1 by 6 X 4 is greater than or equal to 1 by 3. 1 by 3 divided by minus 2 by 3 is minus 1 by 2; minus 1 by 2 into minus 1 by 3 is plus 1 by 6 X 4 is greater than or equal to 1 by 3.

So now we re-write this we add a surplus variable X 5, so 5 by 3 X 3 plus 1 by 6 X 4 minus X 5 is equal to 1 by 3, from which X 5 is equal to take this to the other side, minus 1 by 3 plus 5 by 3 X 3 plus 1 by 6 X 4, so this will become when we add the MILP cut become X 5 as minus 1 by 3 plus 5 by 3 X 3; so you get minus 5 by 3 plus 1 by 6 X 4 so you get minus 1 by 6; so minus 1 by 3, minus 1 by 6 and minus 5 by 3.

Now we have added the MILP cut using this source row; now we realize that this is primal infeasible and dual feasible. So, this variable will go out, you have to do a dual simplex iteration, this variable will go out. The entering there are two negatives, so 1 by 3 divided by 1 by 6 is 2, 4 by 3 divided by 5 by 3 is 4 by 5, so 4 by 5 is smaller than 2; so variable X 3 will enter the solution and we need to do an iteration, a dual simplex iteration with this as the leaving variable, this as the entering variable. We will do that in the next lecture and continue with the discussion on the MILP.