Advanced Operations Research

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Lecture - 3
Simplex Method for Bounded Variables

We discuss the simplex algorithm for bounded variables. If you look at this example, maximize $8X_1$ plus $3X_2$, subject to three constraints, also subject to two additional restrictions, that X_1 is less than or equal to 2 and X_2 is less than or equal to 6. There are actually five constraints, per say, but two of them are called bound, because they simply place a restriction on a single value that a single variable can take, in order to simplify this problem. Otherwise we will be solving a problem with five constraints. So, we take out the bounds and we treat this as a three constraint problem and then suitably incorporate the effect of these bounds into the constraints and we solve it. Let us see how we go about doing that.

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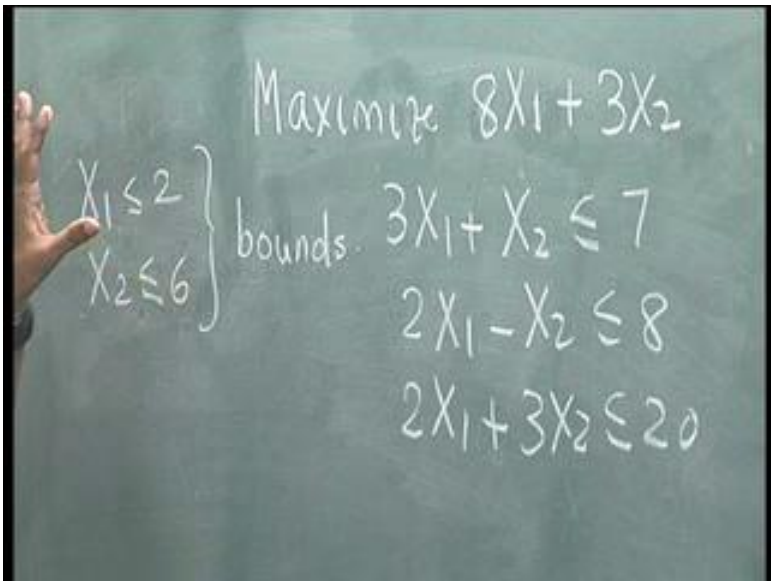

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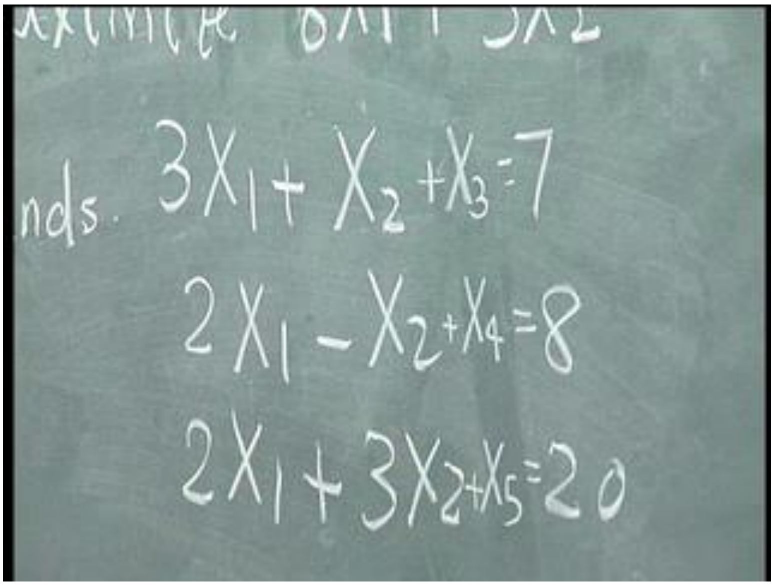

The first thing that we do is to convert these inequalities to equations, by adding appropriate slack variables. So, this will become, plus X_3 equal to 7; this will become, plus X_4 equal to 8 and this will become, plus X_5 equal to 20. We already know that we have less than or equal to constraints, they have been converted to equations by the addition of slack variables. These slack variables automatically qualify to be basic variables, so we can start the algebraic form of the simplex by treating X_3, X_4 and X_5 as basic variables.

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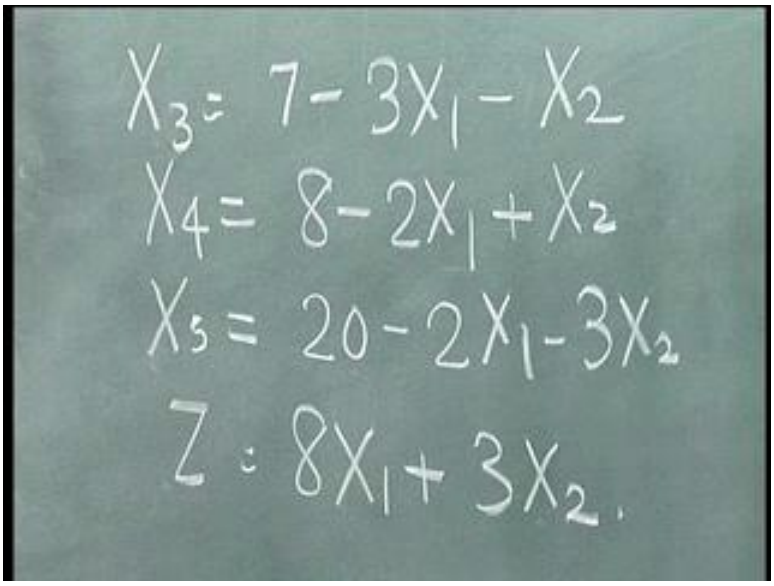

We write X_3 is equal to 7 minus $3X_1$ minus X_2; X_4 is equal to 8 minus $2X_1$ plus X_2; and X_5 is equal to 20 minus $2X_1$ minus $3X_2$; Z, which is the objective function, is $8X_1$ plus $3X_2$. We have now written all the basic variables in terms of the nonbasic variables and the objective function in terms of nonbasic variables, so that the solution can be read: X_3 equal to 7, X_4 equal to 8, X_5 equal to 20, and Z is equal to $8X_1$ plus $3X_2$.

Now we want to increase Z. Right now Z is at 0, because X_1 and X_2 being nonbasic variables are at 0. So Z is equal to 0; we want to increase Z, which can be done by either increasing X_1 or increasing X_2, both of them are at 0. Based on the largest coefficient rule, we enter that variable, which has the largest C_j minus Z_j or the largest coefficient here, so variable X_1 will enter.

Now we have to find out the leaving variable corresponding to variable $X_{1.}$ If variable X_1 enters and starts taking a positive value, X_3 is going to come down as X_1 increases and when X_1 takes value 7 by 3, which is the limiting value, X_3 will become 0.

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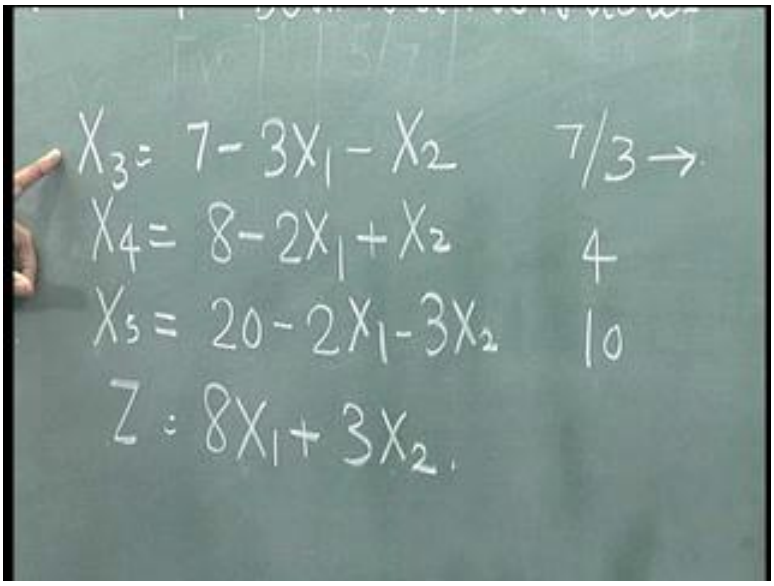

So the limiting value for X_1 based on this equation is 7 by 3; the limiting value for X_1 based on this is 4, beyond which X_4 will become negative and the limiting value for this, based on this equation is 10. The limiting value is the minimum of these three values, which happens to be 7 by 3 in this case. In a normal simplex iteration, we would enter X_1, we would also say that X_3 is the leaving variable and X_1 will take a value, 7 by 3.

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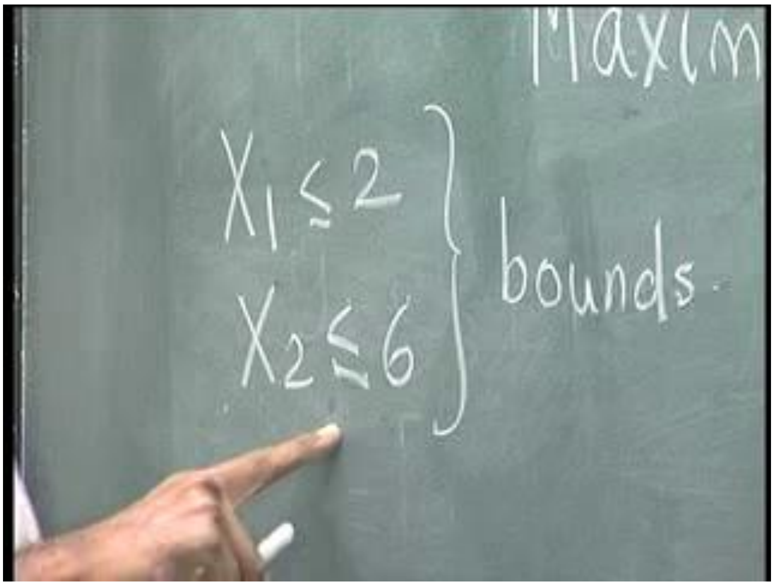

But we also have the additional restriction that X_1 is less than or equal to 2 and X_2 is less than or equal to 6, which is very much part of the problem, which we have actually separately kept. We also have to check that this 7 by 3, which is the limiting value also fulfills or satisfies this constraint, that X_1 is less than or equal to 2. Now 7 by 3 is bigger than 2 and therefore the limiting value for X_1 is not 7 by 3, but it is 2. X_1, will now increase from its lower value of 0 to the upper value of 2.

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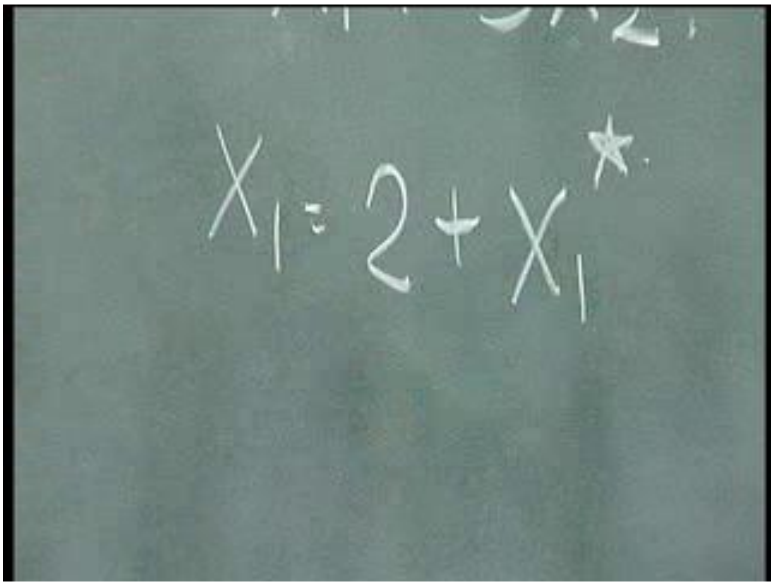

Now we start indicating this as X_1 is equal to 2 plus X_1 star. So we replace X_1 as 2 plus X_1 star. The X_1 star is used only when the variable has reached its upper limit or its upper bound.

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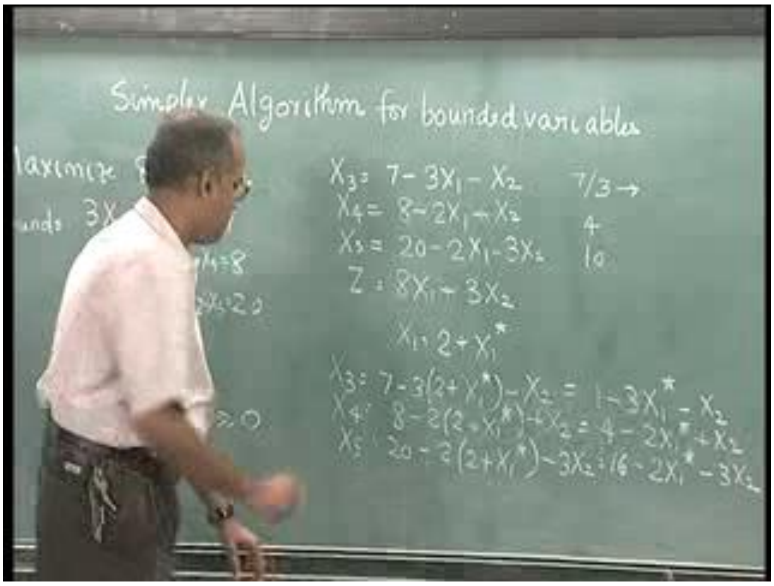

We now substitute X_1 is equal to 2 plus X_1 star in these three equations, to get X_3 is equal to 7 minus 3 times 2 plus X_1 star minus X_2, which will be 7 minus 6 is 1, 1 minus $3X_1$ star minus X_2. Now X_4 is written as 8 minus 2 times 2 plus X_1 star plus X_2, which will become 8 minus 4 is 4, 4 minus $2X_1$ star plus X_2 and X_5 will become X_5 is equal to 20 minus 2 times 2 plus X_1 star minus $3X_2$, which is 20 minus 4 is 16; 16 minus $2X_1$ star minus $3X_2$. X_3 is equal to 1 minus $3X_1$ star minus X_2. X_4 is equal is to 4 minus 2 X_1 star plus X_2. X_5 is 16 minus 2 X_1 star minus $3X_2$.

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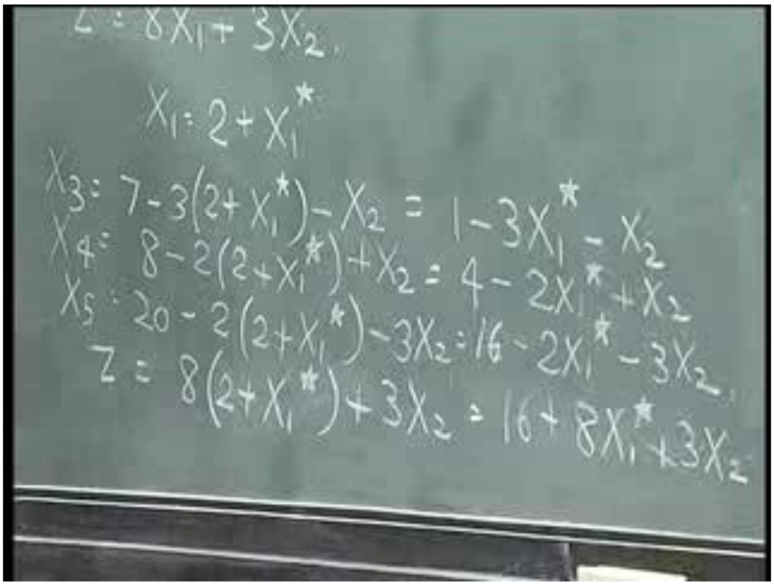

Z, which is the value of the objective function for this solution is $8X_1$ plus $3X_2$; so 8 into X_1, X_1 is 2 plus X_1 star, plus $3X_2$, which is 8 into 2 is 16, plus $8X_1$ star plus $3X_2$. Now we want to increase the objective function value Z further. From this expression, we know that, Z can be increased if X_1 star can be increased or if X_2 can be increased. Before that, we also realize that the actual solution here is X_3 equal to 1; X_4 equal to 4; X_5 equal to 16. X_1 is at its upper limit of 2 with Z equal to 16. So, X_1 is at the upper limit of 2; 8 into 2 is 16, you see in the objective function. Now $3X_1$ plus X_2 plus X_3 is 7, so 3 into 2 is 6 plus X_3 equal to 1 is 7; 2 into 2 is 4 plus X_4 equal to 4, so 4 plus 4 is 8; $2X_1$, X_1 is at 2, so it is 4 plus X_5 is 16, which is equal to 20. If Z has to be increased further, based on this expression, we can either increase X_1 star or increase X_2. Now increasing X_1 star is not possible, because if we increase X_1 star from this, X_1 will exceed its upper limit of 2. So X_1 star can only be decreased, so we cannot increase X_1 star, but increasing X_2 is possible, because currently X_2 is nonbasic at 0. We try and enter X_2 into the basis and see the limit to which X_2 can be increased.

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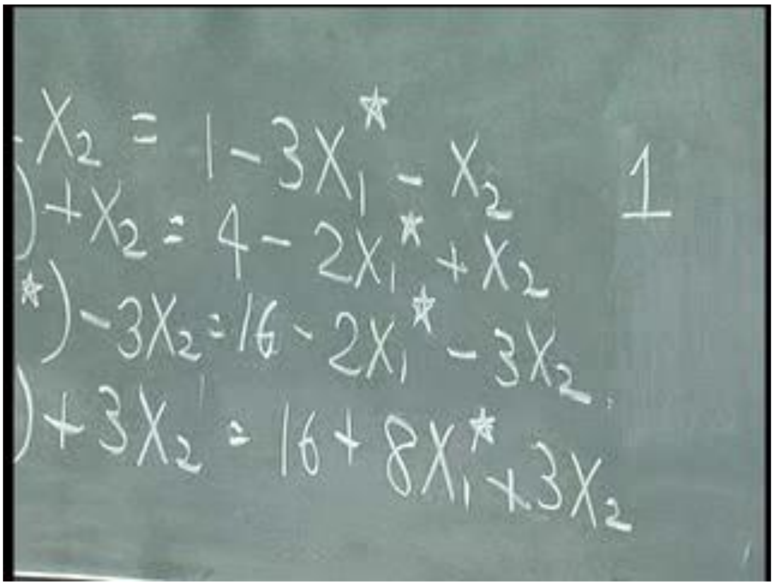

From this equation, as X_2 increases, this value of X_3 is going to come down and as it will allow a limiting value of X_2 equal to 1 at which X_3 will become 0, increasing X_2 beyond 1 will make this negative and bring infeasibility; so the limiting value is 1, based on this.

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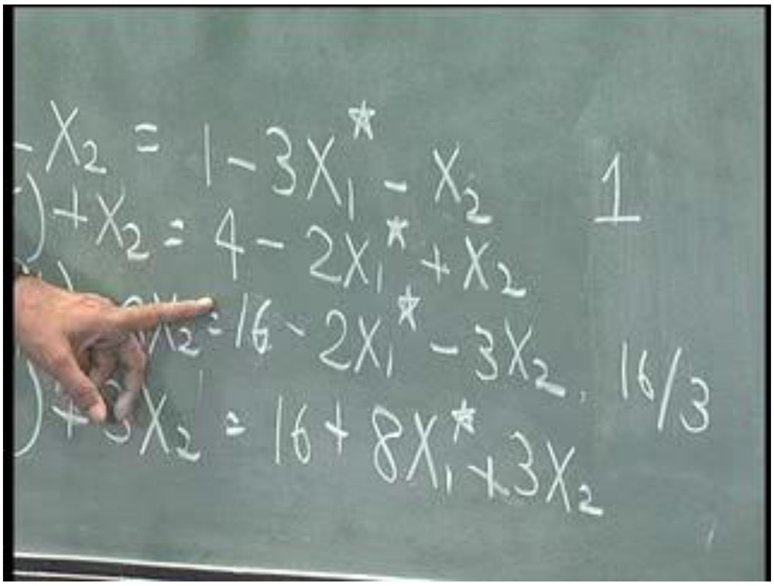

Based on this equation, increasing X_2, is not going to bring down X_4, it is only going to increase X_4. So, we do not have an issue with respect to this constraint. With respect to this constraint, increasing X_2 is going to bring down X_5 and the limiting value that X_2 can take is, 16 by 3, at which, this will become 0, so 16 by 3 is the limiting value. So, minimum of the limiting values is X_2 equal to 1. Before we fix the limiting value to 1, we also need to check whether it is within the bound value of 6. It is within the bound value of 6, therefore we would do a normal simplex iteration by entering X_2 and by replacing X_3 with X_2. So we rewrite these.

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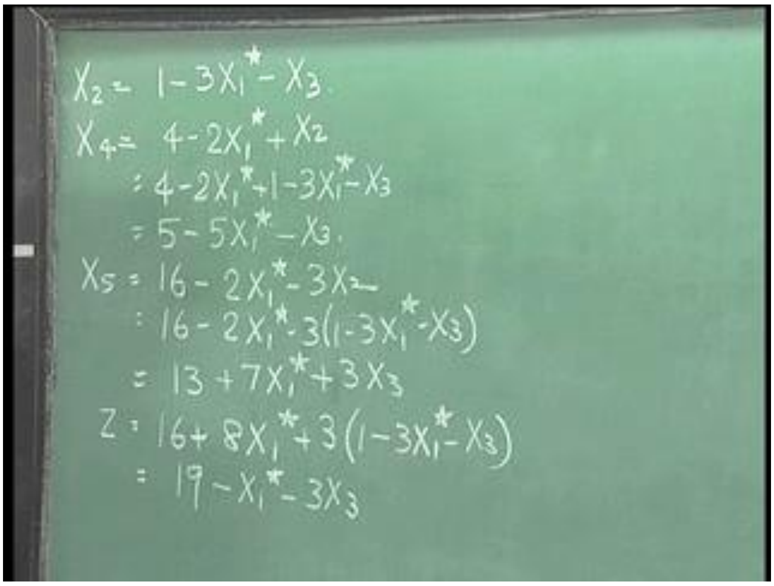

From the first equation, we write this as X_2 is equal to 1 minus $3X_1$ star minus X_3, which we get from this equation. From this equation, X_4 is equal to 4 minus $2X_1$ star plus X_2, which is 4 minus $2X_1$ star plus 1 minus $3X_1$ star minus X_3, which is 5 minus $5X_1$ star minus X_3. X_5 is equal to 16 minus $2X_1$ star minus $3X_2$, so 16 minus $2X_1$ star minus 3 into 1 minus $3X_1$ star minus X_3. This will become 16 minus 3 is 13, 13 minus $2X_1$ star plus $9X_1$ star is plus $7X_1$ star plus $3X_3$ from here. Now the Z value is 16 plus $8X_1$ star plus $3X_2$, so 16 plus $8X_1$ star plus $3X_2$, X_2 can be got from here, 1 minus $3X_1$ star minus X_3, which will give us, 16 plus 3 is 19 plus $8X_1$ star minus $9X_1$ star is, minus X_1 star minus $3X_3$. Now let us quickly verify the solution. Now the solution is X_2 equal to 1, X_1 equal to 2, so that gives us 8 into 2 is 16 plus 3 is 19.

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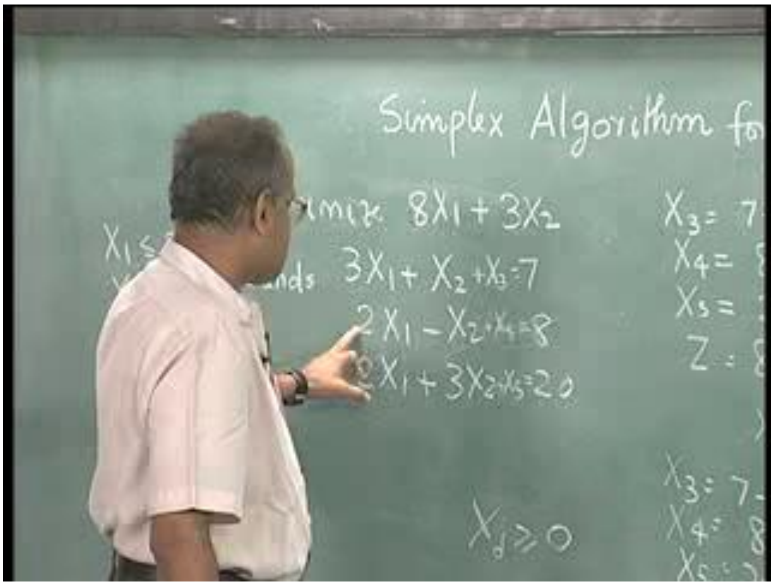

We have X_1 equal to 2, so 6 plus X_2 equal to 1, so 7, X_3 is non basic at 0. X_1 equal to 2 gives us 4, 4 minus 1 is 3, X_4 is at 5, so 3 plus 5 is 8. X_1 equal to 2, so 4 plus 3 is 7, so X_5 is at 13, which gives us 20. So from this, we get a solution, X_1 is already at 2, because X_1 star is here, which means X_1 is at 2, X_2 is at 1, X_4 is 5, X_5 is 13 and Z is this. We want to increase Z further. Increasing Z further can be done by either decreasing X_1 star or by decreasing X_3. Now decreasing X_3 is not possible, because X_3 is non basic at 0. So if we decrease, it will become negative and the feasibility will be affected, therefore we can decrease X_1 star. Decreasing X_1 star is possible, because X_1 star is defined as 2 plus X_1 star, so decreasing X_1 star is allowed, because it will only minimise or reduce the value from 2 to some other value.

So, we look at the possibility of decreasing X_1 star. Now what happens when we decrease here? When we decrease X_1 star, X_2 is going to increase. X_2 is at 1 already, so decreasing this is going to increase X_2 and we can decrease X_1 star, till X_2 reaches its upper limit of 6, because X_2 is restricted up to 6. So this would allow us to decrease X_1 star by an amount 5 by 3, so that 5 by 3 into 3 is 5, 5 plus 1 is 6. Decreasing beyond 5 by 3, will increase this beyond 6 which is not allowed. As far as this is concerned, decreasing X_1 star is only going to increase X_4 further and therefore this is not going to affect us at all; X_4 is at 5, so it is not going to affect us.

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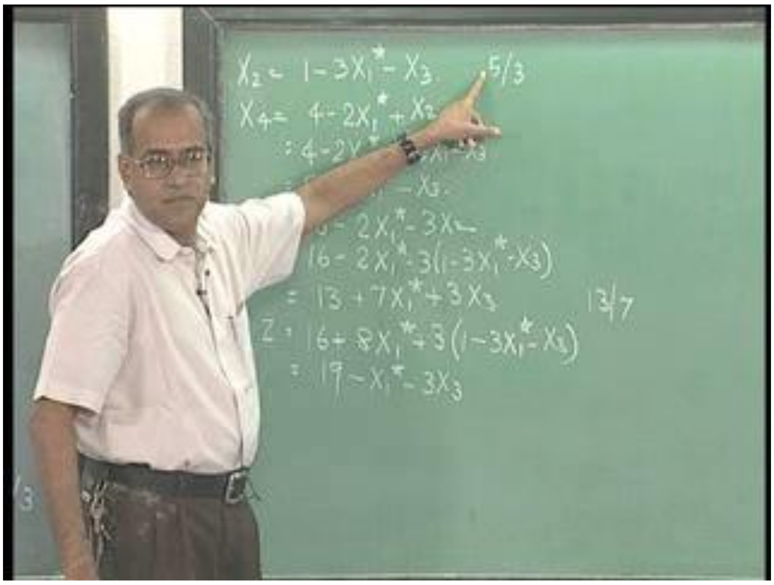

Now as far as this is concerned, decreasing X_1 star is going to decrease X_5, so this would allow X_1 star to be decreased up to 13 by 7, so that, this value will become 0. Between these two limiting values of 5 by 3 and 13 by 7, we also know that 5 by 3 will be the limiting value because decreasing this by 13 by 7 would make this go beyond the upper limit. So the limiting value is given by 5 by 3. We should also check that, with this limiting value of 5 by 3, X_1 which is 2 plus X_1 star does not go below 0. So 5 by 3, make sure that this does not go below 0, which also means that, the limiting value that we define here, which is the minimum of these two values, should also be less than or equal to 2, which is satisfied.

Now, we choose this as the limiting value and now we start rewriting this particular equation. So we write this particular equation using this. What happens when we decrease X_1 star by 5 by 3? In order to do that, X_1 star has to be written in terms of X_1, then this makes X_2 take its upper limit of 6; so now X_2 has to be written as 6 plus X_2 star. So, what we first do is this, now we write this as follows.

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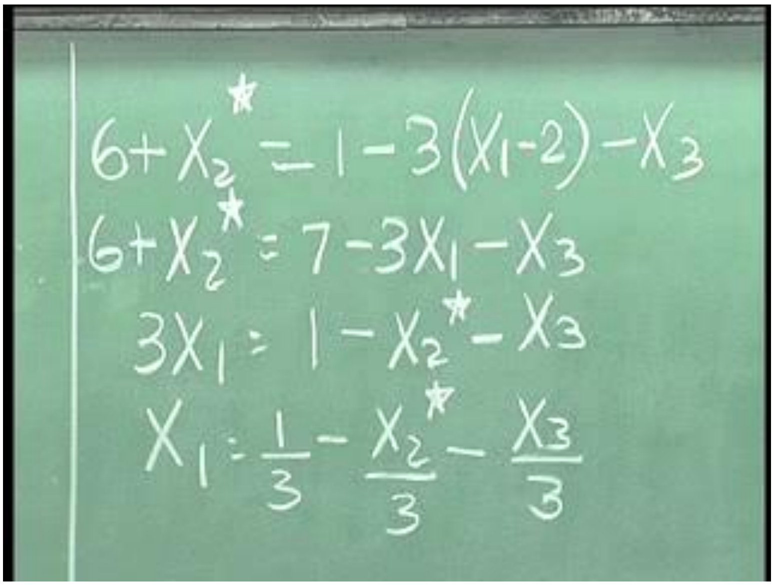

6 plus X_2 star is equal to 1 minus 3 times X_1 star is X_1 minus 2, so 3 times X_1 minus 2 minus X_3, from which, 6 plus X_2 star is equal to, this is another 6, 7 minus $3X_1$, minus X_3. Now bring this (Refer slide time:17:25) to this side; $3X_1$ is equal to 7 minus 6 is 1, so1 minus X_2 star minus X_3, from which, X_1 is equal to 1 by 3 minus X_2 star by 3 minus X_3 by 3. We also verify that, we decreased X_1 star by 5 by 3, so X_1 will become 2 minus 5 by 3, which is 1 by 3, which is what we get here. Now the rest of the things we need to write now.

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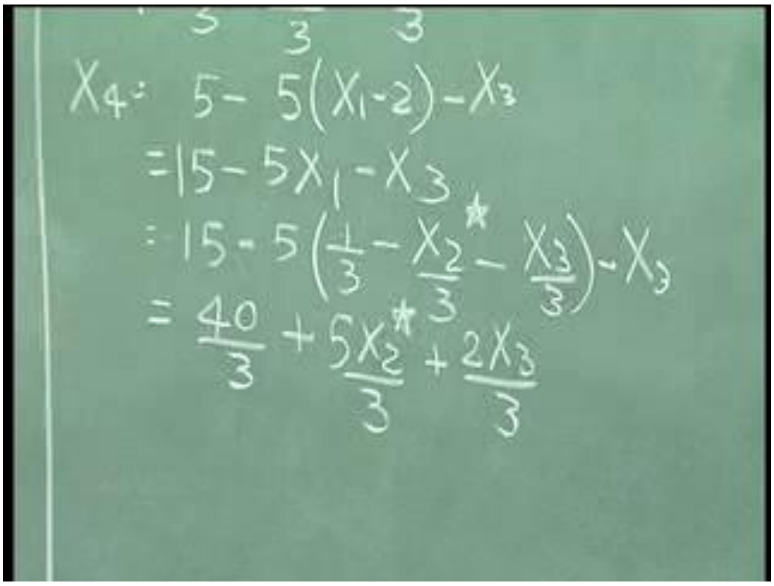

X_4 is 5 minus $5X_1$ star which is 5 into X_1 minus 2 minus X_3; so this is: 5 minus $5X_1$; this is 10 plus 5 is 15, so 15 minus $5X_1$ minus X_3. So this is 15 minus 5 into 1 by 3 minus X_2 star by 3 minus X_3 by 3 minus X_3. This becomes 15 minus 5 by 3 is 40 by 3 plus $5X_2$ star by 3 plus $5X_3$ by 3 minus X_3 will give us plus $2X_3$ by 3.

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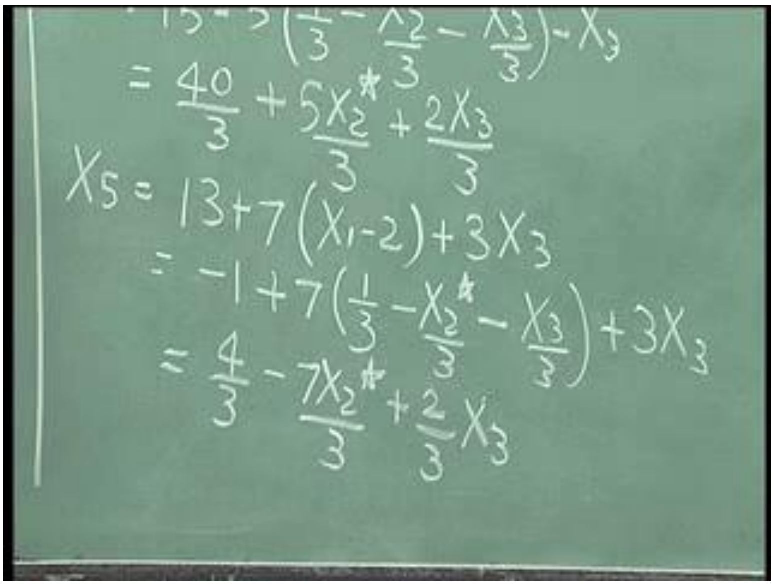

X_5 is equal to 13 plus $7X_1$ star, so 13 plus 7 into X_1 minus 2 plus $3X_3$. Now this is 13 minus 14, so minus 1 plus $7X_1$, X_1 is here, so 1 by 3 minus X_2 star by 3 minus X_3 by 3 plus $3X_3$. 7 by 3 minus 1 is 4 by 3 plus 7 minus $7X_2$ star by 3, from this; this is, minus 7 by $3X_3$ plus $3X_3$ is plus 2 by $3X_3$.

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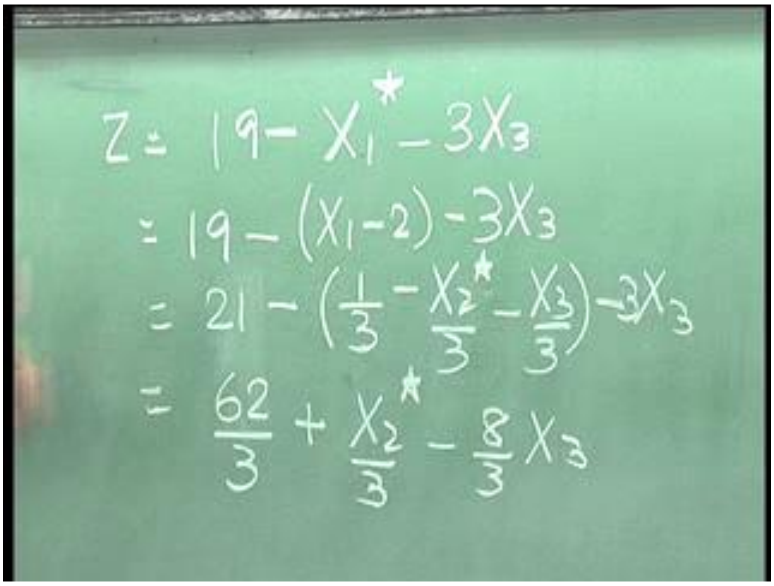

Now Z is equal to 19 minus X_1 star minus $3X_3$, 19 minus X_1 star is X_1 minus 2 minus $3X_3$. So this is, 19 plus 2, so 21 minus X_1 is 1 by 3 minus X_2 star by 3 minus X_3 by 3 minus $3X_3$. So, 21 minus X_1 is 1 by 3 minus X_2 star by 3 minus X_3 by 3 minus $3X_3$. So 21 minus 1 by 3 is 62 by 3, plus X_2 star by 3 plus X_3 by 3 minus 3X3 is minus 8 by $3X_3$; this is plus 1 by 3 minus 3 is minus 8 by $3X_3$. The present solution is right now X_1 equal to 1 by 3; X_2 is equal to 6 because you find X_2 star in this equation; X_2 is at 6; X_4 is 40 by 3; X_5 is 4 by 3. So let us quickly check that. X_1 is 1 by 3, this is 6, 6 into 3 is 18, 18 plus 8 by 3 is 62 by 3, which you find here for Z: 62 by 3. Now let us check the rest of the values.

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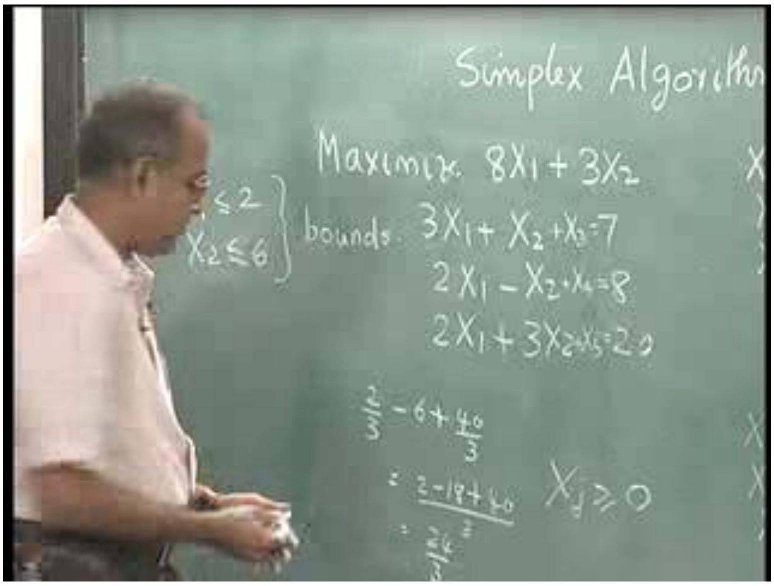

$3X_1$ plus X_2 plus X_3 equal to 7, $3X_1$ is 1; X_1 is 1 by 3, so 3 X_1 is 1, 1 plus 6 is 7; X_3 is non basic at 0. 2 by 3 minus 6, so 2 by 3 minus 6 plus X_4 is equal to 8; so 2 by 3 minus 6 is minus 16 by 6, so minus 16 by 6 plus 40 by 3, which is 80 by 6 is 64 by, $2X_1$ is 2 by 3, minus X_2 is minus 6 plus X_4 is 40 by 3. This will give you 2 minus 18 plus 40 by 3, which is 24 by 3, which is 8.

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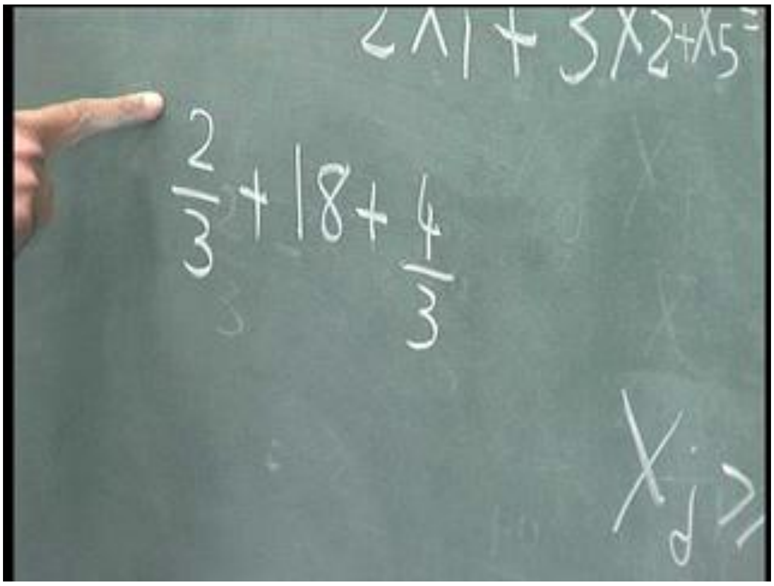

The third one is: $2X_1$ plus $3X_2$, $2X_1$ is 2 by 3, $3X_2$ is 3 into 6 is 18 plus X_5, X_5 is 4 by 3; so this is 54 plus 4, 58 plus 2, 60 by 3, is 20. So it satisfies all these and Z is equal to 62 by 3. If we want to increase Z further, it can be done by, decreasing X_3 or increasing X_2 star. Now decreasing X_3 is not possible because X_3 is non basic at 0, so it cannot be decreased. Increasing X_2 star is also not possible because X_2 star is now defined, we have already defined this X_2 star, X_2 is equal to 6 plus X_2 star. So increasing X_2 star will make X_2 go beyond 6, which is not allowed. Both these things are not possible.

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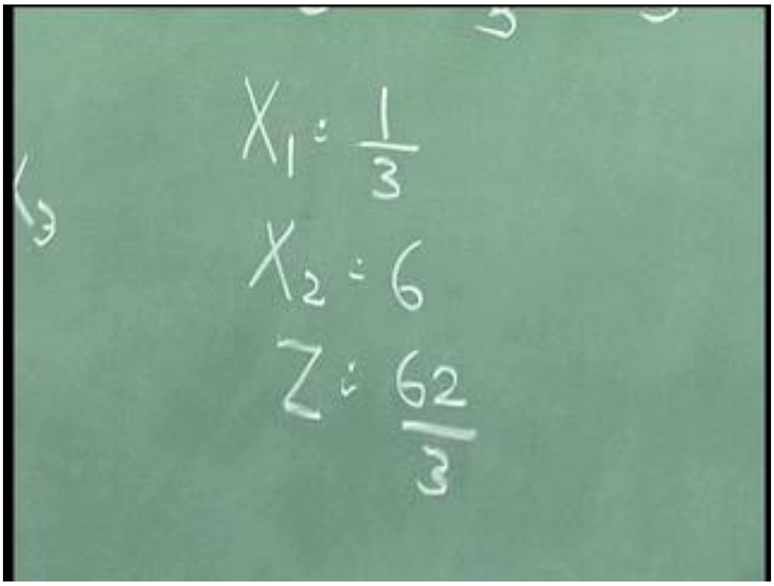

Therefore the algorithm terminates with the optimum solution: X_1 is equal to 1 by 3, X_2 is equal to 6 and Z is equal to 62 by 3. This is the algebraic version of the simplex method for bounded variables. The advantage is that, as we already mentioned, we did not include these two as explicit constraints, we included these two as bounds. So we took away the bounds from the original problem, reduced the number of constraints and every time we were entering a variable or leaving a variable, we were indirectly considering the effect of the bound. The presence of the star variable indicates that the corresponding variable is at its upper limit or upper bound. There are times you will have to see the effect to find out, whether a currently basic variable because of the increase or decrease can either get a lower value or a higher value. For example, here we realised that we are going to decrease X_1 star. As a result of decrease of X_1 star, X_2 went up to its upper bound, whereas X_5 went towards its lower bound. All these will have to be taken into consideration when we use the algebraic method. This can also be written in a tabular form; more for the sake of convenience and ease of understanding, I have explained it in the algebraic method.

One can write the simplex method for bounded variables in the tabular form as well. But more importantly, what we have learnt is that, if there are explicit bounds in the problem. It is possible to separate these bounds, treat the problem and make the problem smaller by looking only at the valid constraints. Each constraint should have more than one variable and then use the algebraic method or the tabular method and suitably incorporate the effect of the bound, both in the entering variable, as well as in the leaving variable and then proceed to get to the optimal solution. This way, by which we take away or separate the bounds and solve a smaller sized problem, in terms of the number of constraints, makes the simplex method for bounded variables a very efficient and superior way to solve linear programming problems, when we have bounds on certain set of variables.

What we do next, is to understand something called column generation, which we will explain through a very well-known problem called the cutting stock problem. Through the cutting stock problem, we will see one application of linear programming and also understand the concept of column generation. Now, as we go through this example, I will also try and explain, what column generation is. Let us take an example to learn the cutting stock problem.

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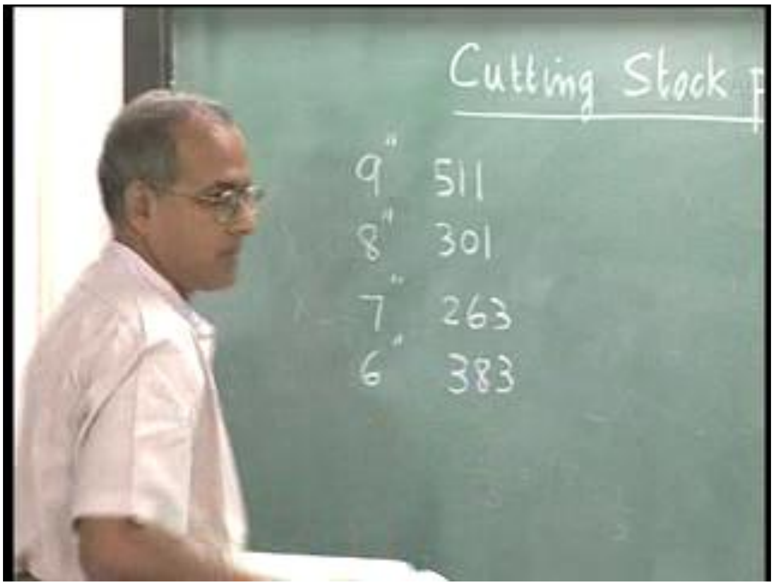

We address the one-dimensional cutting stock problem here. The one dimensional cutting stock problem is as follows: we will assume that we have, say sheets with 20-inch width that are available; we will also assume that a large number of these kinds of sheets are available. From these 20-inch sheets, we need to cut sheets with width 9-inch, 8-inch, 7-inch and 6-inch, in only one direction, which means, cutting in this direction is only allowed, we do not cut in the other direction.

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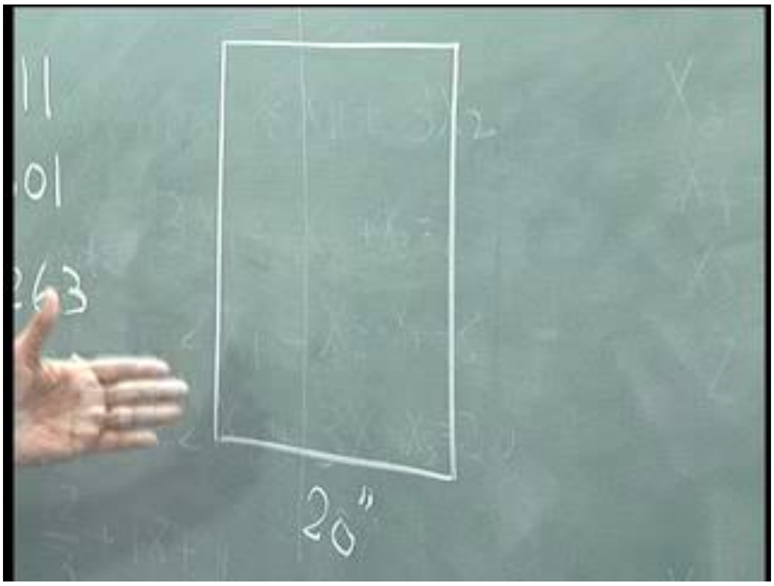

So we cut only in the 20-inch direction that is why it is called one-dimensional cutting stock problem. The demand for 9-inch is 511; demand for 8-inch is 301; demand for 7-inch is 263 and demand for 6-inch is 383. What we want to do is, to try and use minimum number of these sheets. To begin with, we would even say that we want to use these sheets, such that, we minimize the wastage, as well as we wish to use minimum number of sheets out of this. We have already seen some aspects of this one-dimensional cutting stock problem in the earlier lecture series on fundamentals of operations research, where we looked at the formulation of the one-dimensional cutting stock problem.

Let us spend a few minutes on the formulation and then we go back into, how we solve the one-dimensional cutting stock problem. Initially, the best thing to do is to try and create patterns that can be cut.

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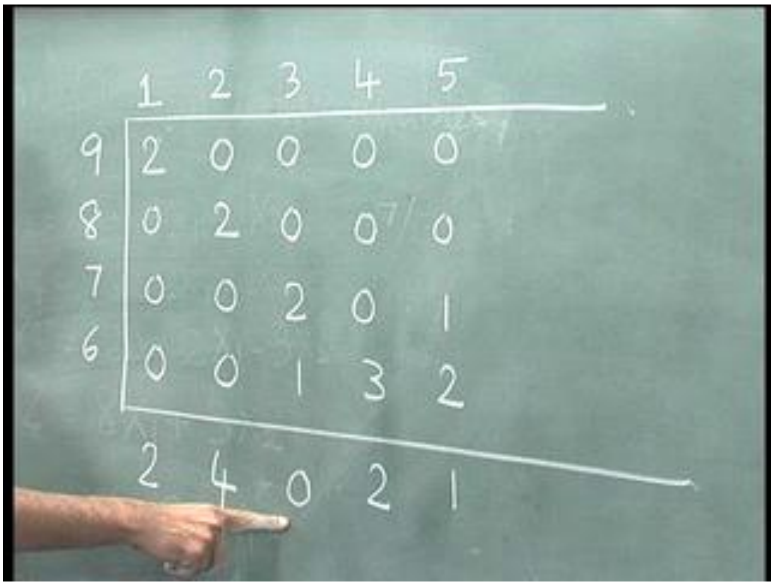

For example, we may say that since we want 9-inch, 8-inch, 7-inch and 6-inch, we can start generating patterns. We might say, pattern number 1 is to cut 2 into 9, which is 18 and a waste of 2, which will make it 20. Another pattern would be to do, 2 of 8 and to have wastage of 4. Third pattern is to have, we cannot have 3 of 7, because it exceeds 20, so we could have 2 of 7 and then we have a remaining 6, so you could have 1 of 6 and define this as a waste of 0. Another pattern could be to have 3 of 6 and have wastage of 2.

Like this we can create patterns, for example, we could have a fifth pattern, which is 0 0 1 and 2; so 2 into 6 is 12 plus 7 is 19, with wastage of 1 and so on. So, we can do this kind of a thing; we can create as many patterns as we can. In this particular example, there are 10 possible patterns, provided we define the wastage in a certain way. Right now we have defined wastage as, the wastage has to be less than or equal to this 6. For example, we did not consider till now a pattern which is, 0 0 2 0 with wastage equal to 6 because we knew that with a wastage equal to 6, we can generate 1 of 6, so it became 0 0 2 1, with waste equal to 0. The way we have generated patterns till now, please note that this is only an indicative set of patterns, not exhaustive; there are 10 possible patterns, we have written only 5 of them, but we have made sure that the wastage we have here, is less than this 6.

What will happen is, if we formulate this problem as, so there will be actually 10 patterns; so X_1 is the number of sheets with which we cut pattern 1; X_2 is the number of sheets with which we cut pattern 2 and so on; X_5 is the number of sheets with which we cut pattern 5 and X_{10} is the number of sheets with which we cut pattern number 10.

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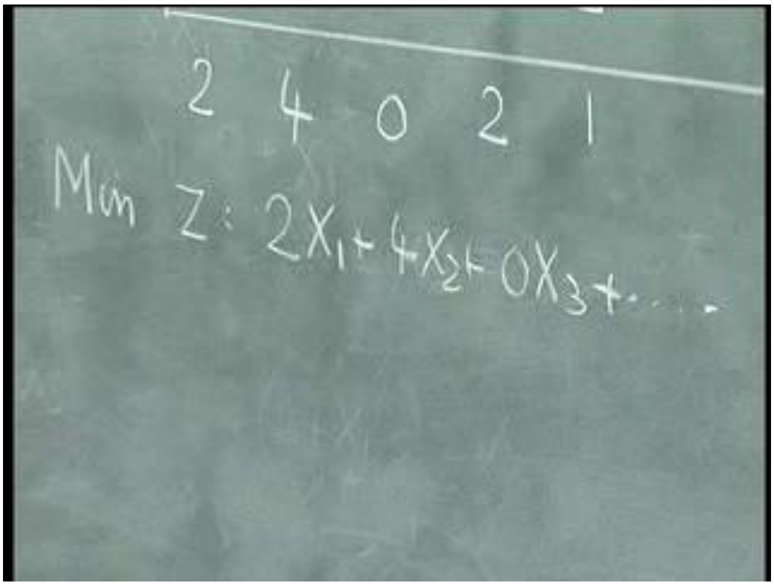

We want to minimize the wastage. So the wastage which is the objective function, will become minimize Z is equal to $2X_1$ plus $4X_2$ plus $0X_3$ and so on subject to, if we have these ten patterns, let me write down all these ten patterns, so that we also understand what they are.

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I have written the patterns 2 0 0 0; 0 2 0 0; 0 0 2 1; 0 0 0 3; 1 1 0 0, with waste is equal to 3; 1 0 1 0, 9 plus 7 is 16, waste is equal to 4; you could have 1 0 0 1, with waste is equal to 5; 0 1 0 2, with waste is equal to 0; 0 1 1 0, with waste is equal to 5. So these are the 10 possible patterns in this case, so we call these as X_6, X_7, X_8, X_9, X_{10}; so X_{10} would represent this variable.

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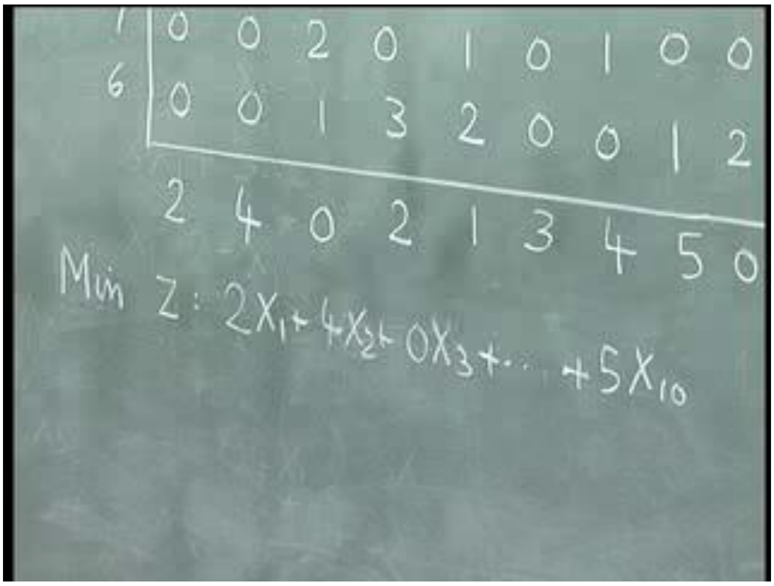

The objective function will become: $2X_1$ plus $4X_2$ plus $0X_3$ etc., plus $5X_{10}$, subject to the condition. Now, how many 9 inches we will get? We will get $2X_1$ plus X_6 plus X_7 plus X_8 is greater than or equal to 511. This is for the 9-inch.

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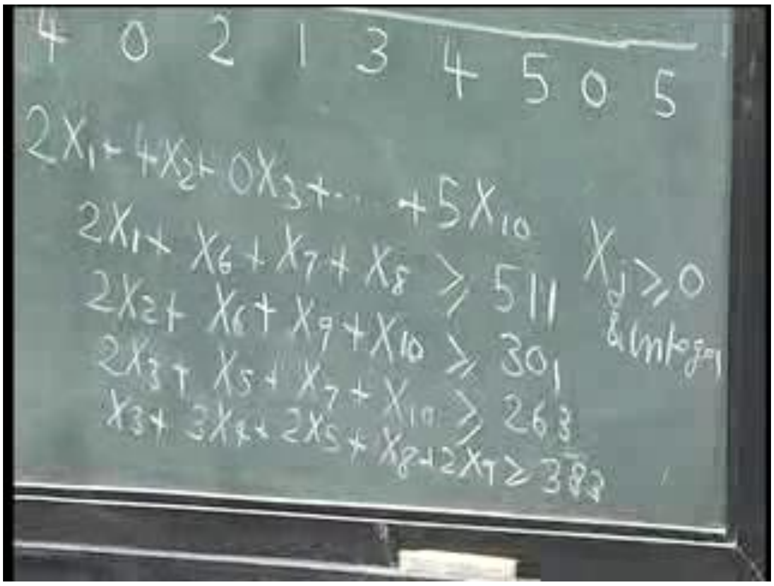

Similarly, we will have one constraint for the 8-inch which is, $2X_2$ plus X_6 plus X_9 plus X_{10}, is greater than or equal to 301 and so on. So we will have three more constraints here, those are: $2X_2$ plus X_6 plus X_9 plus X_{10}, is greater than or equal to 301. $2X_3$ plus X_5 plus X_7 plus X_{10}, is greater than or equal to 263. X_3 plus $3X_4$ plus $2X_5$ plus X_8 plus $2X_9$, is greater than or equal to 383 and X_j greater than or equal to 0 and integer. The integer becomes important, because X_1 to X_{10} represent the number of sheets that we are going to cut. So, these have to be integers.

In principle, this problem is not a linear programming problem; it is an integer programming problem. But so far we have not learnt about integer programming, we will learn about integer programming a little later in this lecture series. For the present, we will relax this integer restriction and treat this problem as a linear programming problem, by not bothering so much about the integer values that the variables can take. So it becomes a linear programming problem. Now one of the reasons I have put an inequality here, and not an equation, is this. These are the possible patterns and since we have, depending on the pattern, different numbers, that are being generated, 2, for example 3 here and so on, it may be possible that when X_j

takes certain values, we may end up having more than 511, particularly when you restrict X_2 integer, you may have the situation where the left hand side can become slightly more than the right hand side. We may end up having something slightly more than the right hand side. For example, because X_j is integer, we may end up getting 384 sheets of 6, instead of 383 sheets of 6; the left hand side can become more. In such a case, the extra sheet can be treated notionally as a waste. It is not a waste, but from the point of view of this problem, it may be treated as unwanted. If we really wish to minimise the waste, the waste is not only this, plus the additional sheets.

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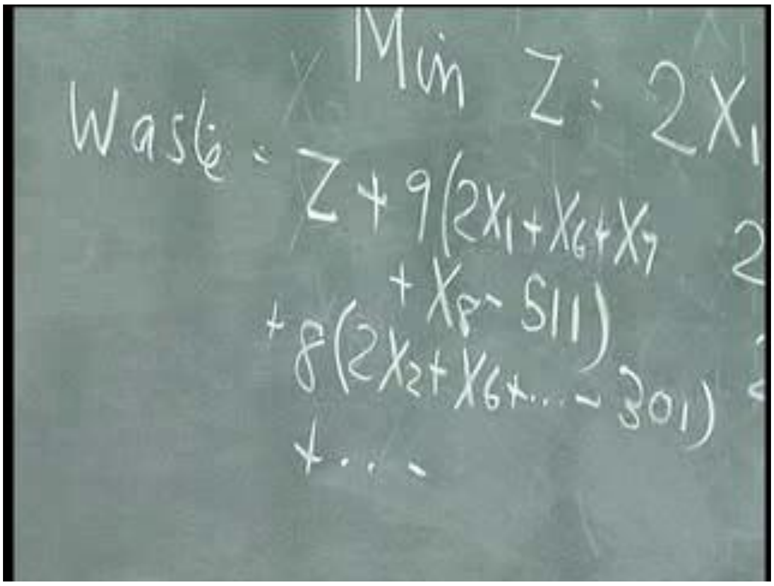

So the waste will be, waste is equal to Z, which is this expression plus extra sheets over and above 511, so this will be: 9 times $2X_1$ plus X_6 plus X_7 plus X_8 minus 511 plus 8 times extra 8-inch sheets that we have got, so this will become $2X_2$ plus X_6 plus etc., minus 301 and so on. We have already seen in the earlier course, that when we simplify this expression, we will simply get: minimize Z is equal to, this Z is different from this, you may call this as Z_1, waste is called as Z; so Z will become 20 times X_1 plus X_2 plus X_{10} plus sum constant K.

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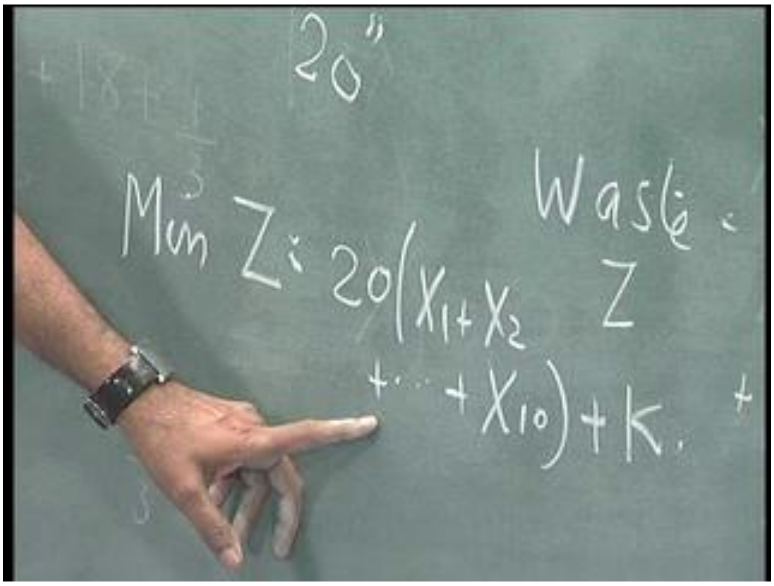

We already know from linear programming, that this K can be ignored and since 20 is a common product of all of them, this can be ignored. So the problem actually becomes, minimize summed over j X_j because, this objective function reduces to minimizing X_1 plus X_2 up to X_{10}.

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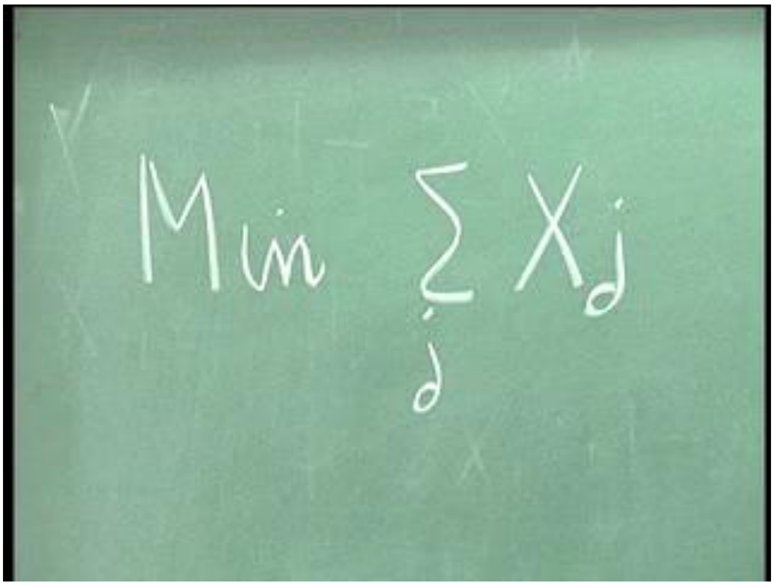

The problem of cutting it such that, we minimise the waste, essentially reduces to the problem of minimising the number of sheets that we cut. Subject to the condition This can be written in general as follows:

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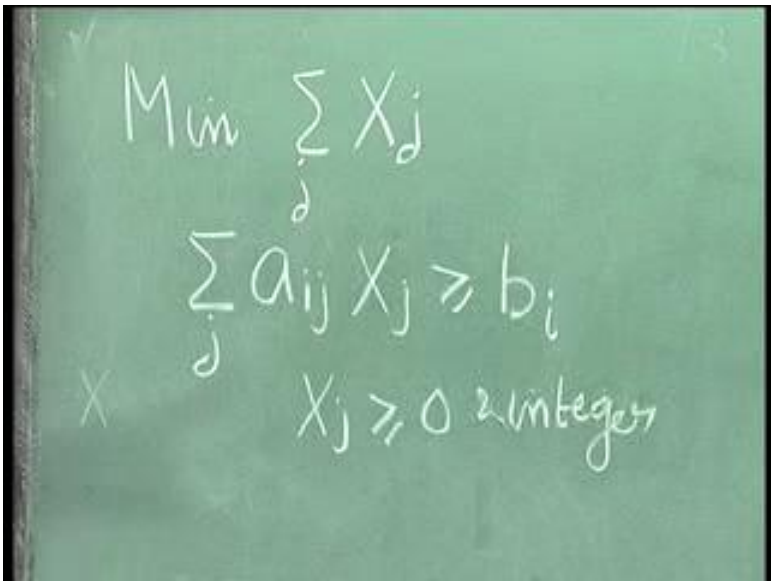

$a_{ij}X_j$ is greater than or equal to b_i summed over i, where j is 1 2 3 4 in our case. Now, X_j summed over j, is greater than or equal to b_i, i equal to 1 2 3 4 represent, the four types of sheets. X_j, j equal to 1 to 10 represent the ten patterns and X_j greater than or equal to 0 and integer. Now this greater than or equal to comes, as already explained, because when we treat X to be integer, there is a possibility that we may end up generating more sheets than what is required. Another way of looking at this problem is, so far we did not consider any pattern where the waste was 6 or more. For example, it is possible to consider a pattern which can look like this; pattern number 10 was 0 1 1 0 with 5; we may consider for example, eleventh pattern as, 1 0 0 0 with waste equal to 11, because we just cut 1 of 9 and say, that the waste is 11. When we make this assumption and generate patterns, the number of patterns becomes very large. For example, nothing prevents us from even considering a pattern 0 0 0 0 with waste is equal to 20; so the number of patterns becomes very large. Even though the disadvantage that the number of patterns have become very large, the advantage is the fact that, this inequality will now become an equation.

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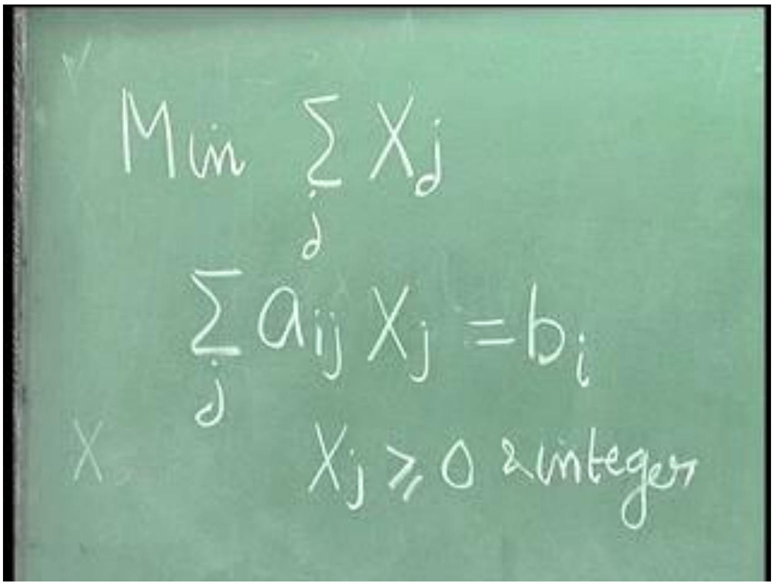

When we generate all these patterns and then use these patterns suitably, we will not encounter a situation where at the end of the solution, this one will be slightly more than the right hand side; it will become exactly equal. The problem becomes, from $a_{ij}X_j$ greater than or equal to b_i to $a_{ij}X_j$ equal to b_i, if we consider all patterns which can be possible and some of these patterns may even have a waste or wastage more than the minimum of this.

There are certain advantages of using this equation here because already from the duality theory, that we saw in the earlier lecture series on fundamentals of operations research, we know that if this problem is treated as primal, now the primal has all constraints which are equations and its dual therefore will have all variables that are unrestricted. There are certain advantages in having primal constraints as equations, particularly when we are not going to solve the primal directly. If we are going to have an algorithm which uses the relationship between the primal and the dual, an equation in the primal would give us unrestricted variables in the dual, which is a great advantage. Therefore, between choosing one possibility, the first possibility was to leave out patterns like this, to look at patterns which have waste less than 6, have a smaller number of patterns and solve the primal where the primal is $a_{ij}X_j$ greater than or equal to b_i, is one way of looking at the problem.

The other way of looking at the problem is to consider more patterns, introduce more variables and consider patterns which seem to be unacceptable, in the sense that the waste is more than even 9. But trying to exploit the advantage mathematically, the advantage that the primal now has equations. Then derive an algorithm which is based on the primal and the dual and exploit the fact that, the dual has unrestricted variables. Now between these two options, we are now going to see the second option and how nicely an algorithm can be created or can be made to solve this problem. We will look at this problem, where $a_{ij}X_j$ is equal to b_i, X_j greater than or equal to 0 and integer. This problem is an integer programming problem. It still holds, minimizing wastage is equal to minimizing the total number of sheets that are being cut. Once again we have not learnt integer programming and the basics of it. We will do that later in this lecture series.

What we will do now, is to treat this problem as a linear programming problem and solve this. As I said, we are not going to solve the primal; instead we would exploit this condition, so that the dual has an unrestricted variable. Because of this equation, we have also realized that the number of patterns becomes very large. First of all we do not know what that number is; the number can become very large. The second point is that, are we going to store all these, somewhere when we have this algorithm, because if you are solving a linear programming problem, you have to somehow store the coefficient matrix. Remember that, these numbers are reflected here, as coefficients. So the question is, when these numbers become large, are we going to store all these numbers? The answer is, we are not going to store these numbers. Then how are we going to solve it? We use this method called column generation to take care of that.

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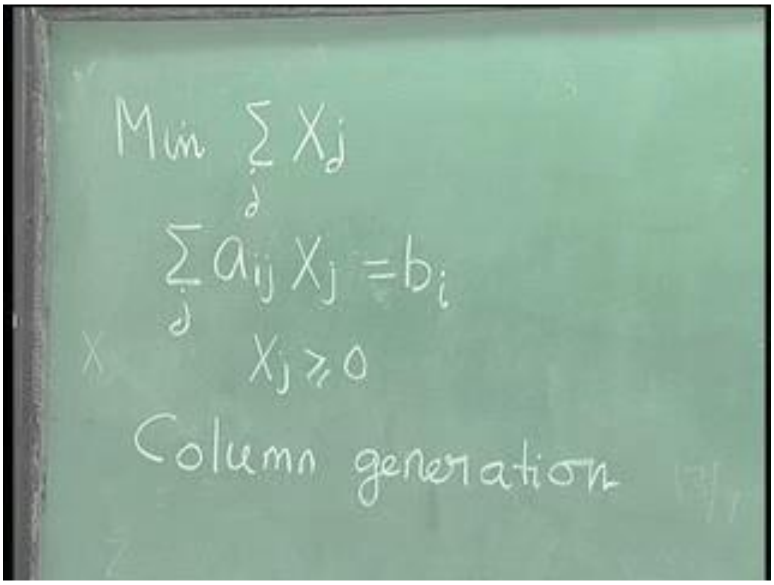

We will not store all the columns; instead we will generate the column as we move along. We will start only with, if there are four types of sheets required, four different types: 9-inch, 8-inch, 7-inch and 6-inch, we will now generate to begin with, only four patterns. In fact, one good look at this will also tell us that, there are four constraints. Therefore, if it is a linear programming problem, there will be only four basic variables. Only four patterns will effectively be chosen, if it is a linear programming problem. What we will do is we will not generate all the exhaustive set of patterns; we would simply start with four patterns at a time. The four most meaningful patterns are this, this, this and even 0 0 2 0 instead of 0 0 2 1, are the four most meaningful patterns, because of the fact that these patterns are cut only with 9-inch, 8-inch, 7-inch and 6-inch.

For example, 0 0 2 0 would mean, you are cutting only 7-inch here. We take these four patterns and then in every iteration, we will find out a new pattern that can enter. That pattern is generated using, this column generation technique. How we solve this linear programming problem, using the column generation technique, we will see in the next lecture.