Advanced Finite Element Analysis

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Lecture – 7

In the last class we were talking about the loading and unloading criteria.

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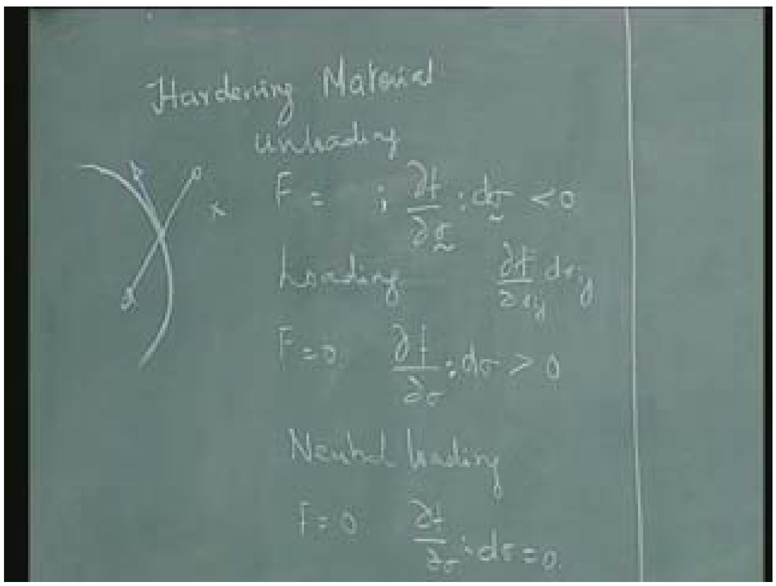

We just saw how it is for hardening material. We said that there is unloading when dow f by dow sigma double dot d sigma, of course all of them are tensors, is less than zero. This double dot indicates that it is a double contraction or in other words, this is nothing but dow f by dow sigma$_{ij}$ d sigma ij and that is the quantity which we are looking at. Loading, when this quantity is greater than zero, neutral loading, which means that we are moving along the yield surface. So that is what we call as the neutral loading.

Towards the end of the class, there was a very good question as to whether a point can lie outside. In fact, I made a statement subtly that you are looking at rate independent materials and so, does it mean that there is a rate dependent material?

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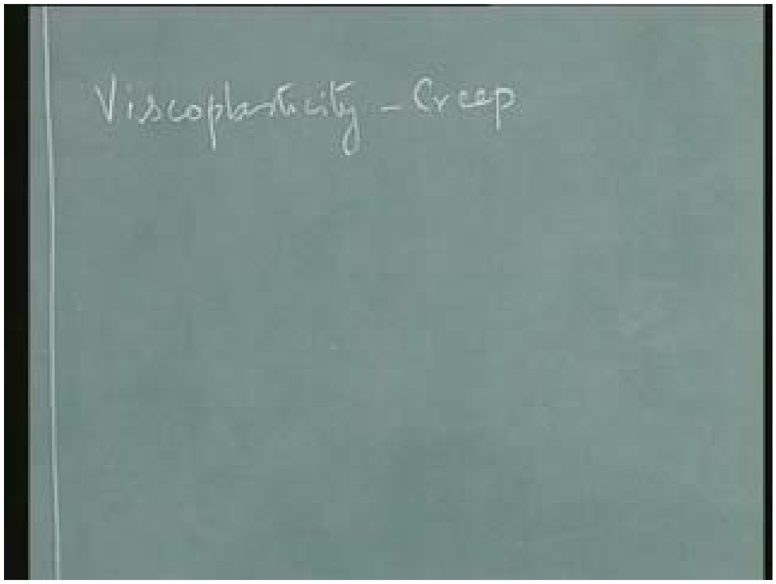

Yes, when materials are subjected to very high temperatures, they will have another effect which is added on to this which is called as viscoplastic effect or viscoplasticity is the governing phenomena for bodies which, say for example, is subjected to creep and when you say creep, you immediately think of so many other issues like relaxation, stress relaxation and so on.

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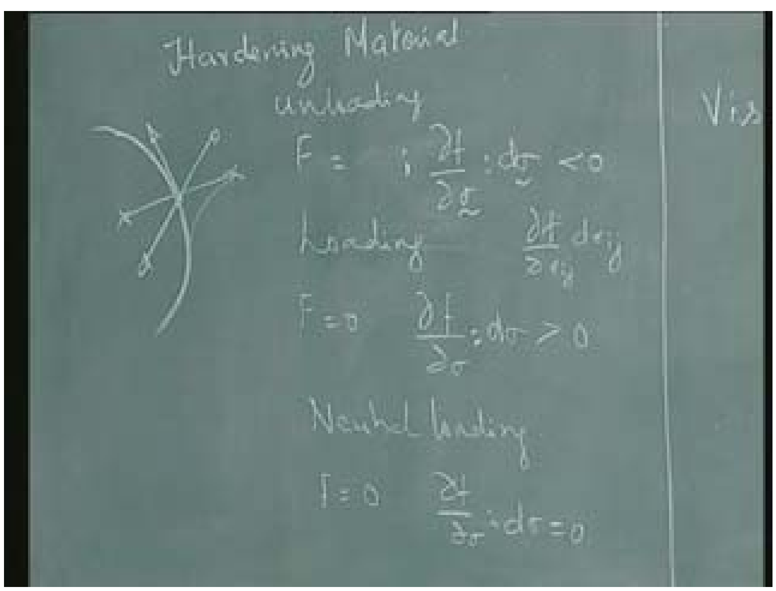

In the rate independent case though the point may lie in the yield surface, for a rate dependent case points can lie outside the yield surface and then with time they will relax back and fall at the yield surface. Is that clear? That it is possible for a point to lie outside the yield surface, for a viscoplastic case. In other words, consistency condition is not satisfied, but with time this point would fall back into the yield surface. The behavior which takes the point from inside the yield surface to this, outside the yield surface is an elastic behavior and when it falls back, then it would be a viscoplastic behavior or in other words as t tends to infinity, when times are very large, the solutions of viscoplastic problems would be the same as that of the plastic problems. So, it is possible that you can do a finite element analysis for a material with, of a material with assumption of viscoplasticity, look at the solution as t is very large and say that that is also a solution for a plasticity problem. Is that clear?

Student: consistency condition is nothing but a kind of equilibrium equation. The loading, one side we have the loading and other side is the material response sigma_y into epsilon.

Absolutely

Student: even if it is a viscoplastic material, the equilibrium conditions must be satisfied; only thing it will be dynamic equilibrium.

Yeah, equilibrium in this case is between two things. One is the material response to it. Please note this carefully that I have two terms here in f.

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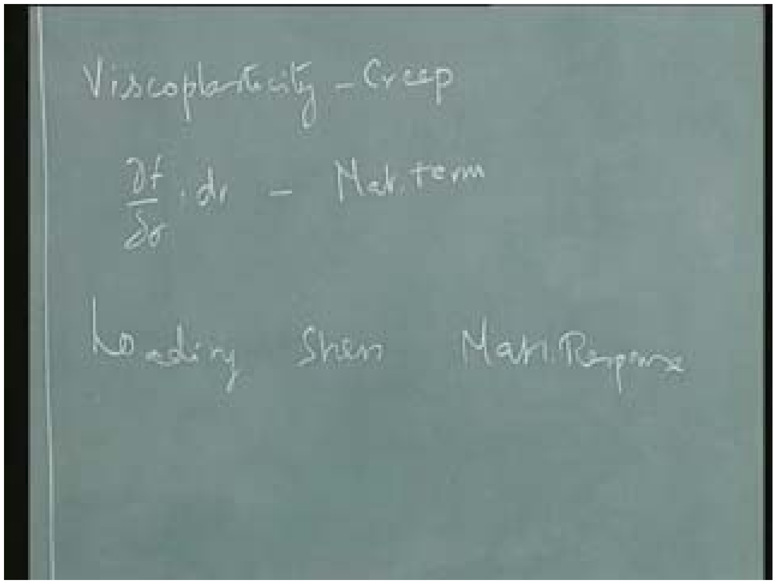

That is dow f by dow sigma d sigma term and a material term. Let me call this as material term; there are lot more to it, material term. Now, the equilibrium in a conventional sense is between the loading, so I have three things actually, between the loading and the stress developed; loading and the stress developed. Here it is between the stress developed and the material response, material response. So, these are the three things that are there - external fellow which is loading, the internal chap which is the stress and then the response of the material in terms of its dislocation motion and so on, that is the third thing that we are looking at. It is the material response which decides whether we are looking at a rate independent solution or a rate dependent solution. Because material responds in a particular way at a higher temperature, we are looking at the material to behave in a viscoplastic fashion. So, plasticity and viscoplasticity are mathematical models for the phenomenological issues that take place. Is that clear? So, this is the equilibrium equation. In other words, note this carefully that in equilibrium equation, we will not talk about the constitutive equations. Is that clear?

In one sense what we say is right. It is the equilibrium between, “equilibrium” within quotes, equilibrium between the stress, internal stress and the corresponding performance or the response of the material. Is that, is that, I hope it clarifies the issues. So, viscoplasticity has its own laws and in fact own algorithms. We have to modify these algorithms whatever we are doing now, let us not look at that right now. So, having studied this issue of what we called as loading and unloading criteria, we will quickly look at what is called as the hardening and see what happens under certain very interesting situations.

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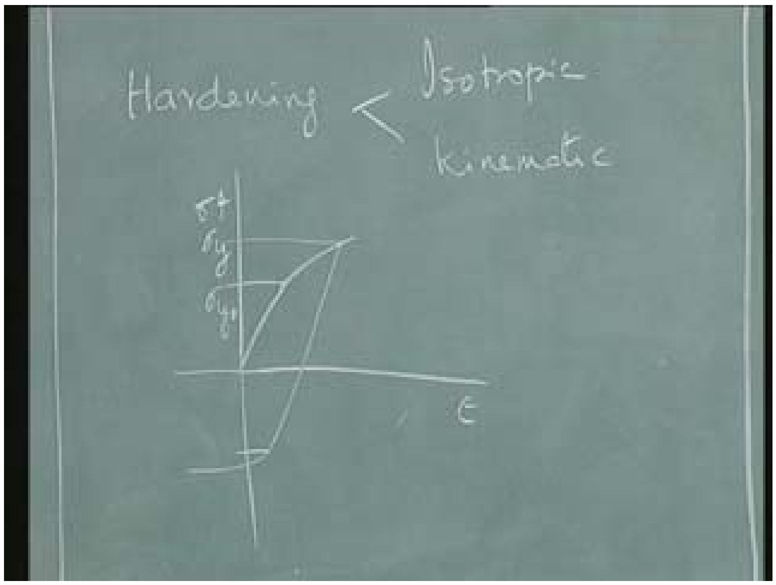

Though we are not going to do the mathematics completely now, may be we will do it later, let us now understand the different types of hardening. Hardening can broadly be classified into an isotropic hardening and kinematic hardening. Have you heard about this term kinematic hardening? No? Have you heard about the term Bauschinger effect? Let us now look at that from one dimensional perspective and understand what this kinematic hardening is and what is the phenomenon that is behind this kinematic hardening and then how to extend it to the multi action case?

When I have say, the stress strain curve, as long as I load it, it is fine. But, what happens when I unload it and reload it in the compressive regime? So, I unload it, reload it in the compressive regime; of course, it has to yield in the compressive regime. Now, what is that yield point with respect to this or with respect to that or in other words if I call this as sigma$_{y0}$, the initial yield and this to be sigma$_y$, then does this point correspond to sigma$_{y0}$, does it correspond to sigma$_y$ or it corresponds to neither of them? Depends on isotropy, very good.

What is this isotropy, what happens when there is isotropy?

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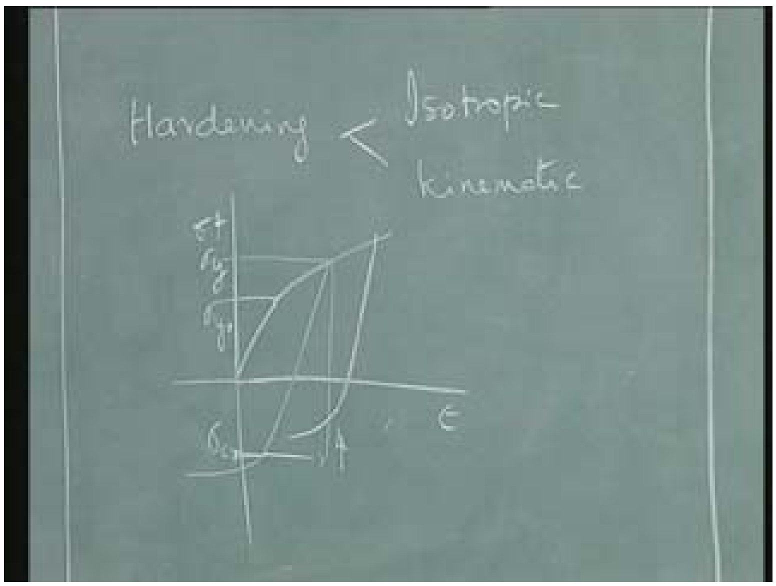

That means that this sigma$_y$, let me call this sigma$_{cy}$, where sigma$_{cy}$ is compression yield, when it is equal to sigma$_y$, the behavior is you can call this as isotropic. But, unfortunately many of the materials do not behave in an isotropic fashion, when they, when you load it and unload it. It would so happen that the yield strength of the material keeps shifting upwards, it goes up and in such a fashion that this total length between this and this happens to be 2 times sigma$_{y0}$, many of the times. It is not that every time, every time it does not happen and many of the times it so happens that the height may be 2 sigma$_{y0}$ or in other words, as I keep on going up the curve and start unloading it, my yielding or my yield point would start shifting upwards; my yield point would start shifting upwards. This effect is called as the Bauschinger effect or in other words, it is called as the kinematic hardening.

Why does this happen? We can, we can put a crude model to it. Say for example, this would, why does this happen? We can put a very crude model to it that we had already understood that in plasticity we are looking at a dislocation motion and that there is not going to be one dislocation. There is going to be a number of dislocations, which are going to move in a three-dimensional fashion. There is going to be so many other phenomena like climb and so on and so forth that are going to happen. In other words, the dislocation may get entangled with loading. This is like if you have a ball of thread and start pulling it, the thread may become entangled. So, like that these dislocations may get entangled, one of the reasons why this guy starts, you know, hardening. Yeah, I am, that is what I am doing. I am repeating what is, what is this effect or the reason?

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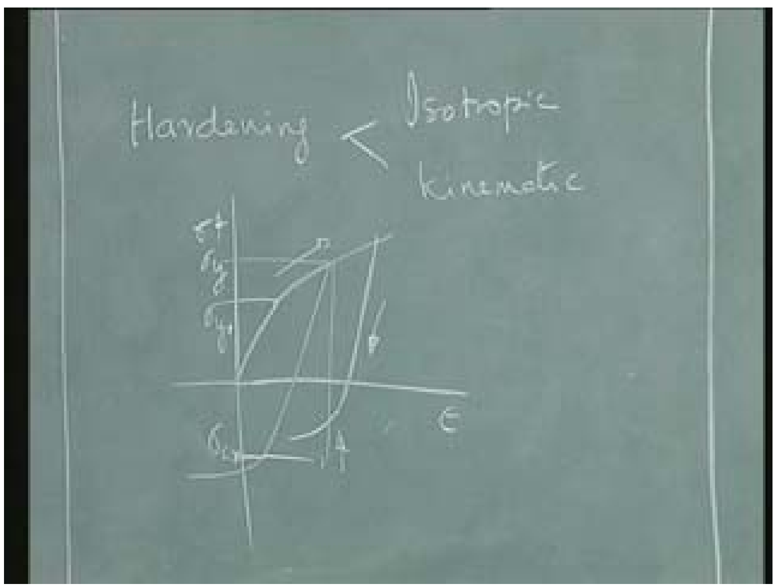

So, once I start up this curve, climb up the curve, the hardening effects are there and so, more and more stress is required. There are two reasons to it. One, of course you know that the second face particles may act as hindrance to the dislocation motion. The other is that these dislocations themselves may start interacting with each other and that would prevent or that would be an obstacle for further motion. When there are obstacles for further motions of dislocations, obviously the stress or the load that is required to move them is going to be higher. Clear? So, that is what is called as the hardening effect.

We called two types of hardening, strain hardening and work hardening; we will not worry about that right now, but that we are, as we move up this is what happens. Now, what happens when I reverse the load? When I reverse the load that means I am coming like this. Though I first elastically unload and keep moving in the elastic fashion recovering all my distortions, the lattice distortions, at one stage I will operate on the dislocations. So, now because there is a resistance in this direction for it to move, when I start operating in the opposite direction, the dislocations may not face and it does not face the impediments as it had faced for its forward motion. So, less force is required in the opposite direction to move it now.

You moved a guy like this. He is now facing resistance and I want to move him a bit in the other direction, it may be easier to move. So, from a very crude sense that this kind of entangling effect will have an opposite effect or opposite I would say phenomena for the load which now acts in the opposite direction. Hence there is a climb of the yield point and it goes up. Is that clear? So, this is what is called as Bauschinger effect, where there is a shift of yield point. Now, how does this get translated in the multiaxial case?

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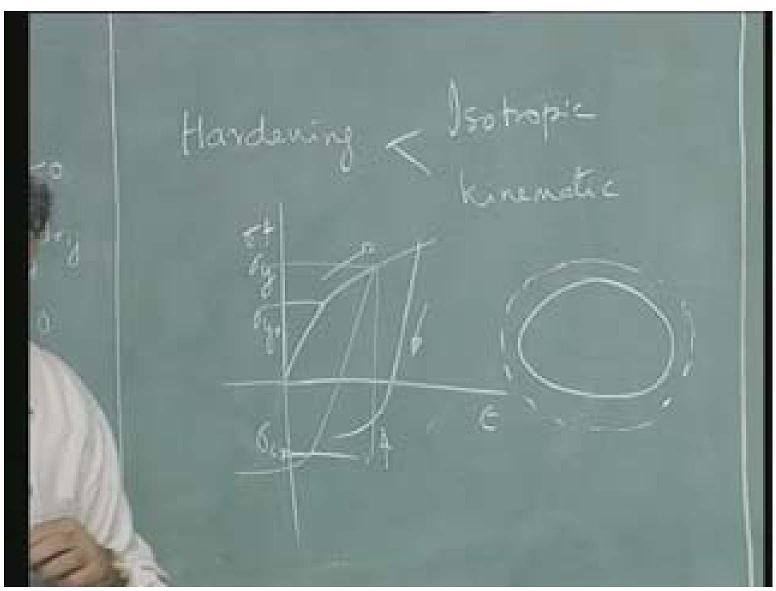

So we have an yield surface. What is yield surface, what do we mean by yield surface?

It is nothing but, you know this is a very sketchy schematic picture of my yield function. That is all, nothing more than that. So, there are two possibilities. One, we already saw yesterday that this yield surface may keep dilating like a balloon, blowing a balloon. It keeps dilating. So, if you are sitting here you will move as if you are sitting on the balloon, you will keep dilating. This is isotropic hardening. Now, forget for a moment, forget for a moment that there is isotropic hardening.

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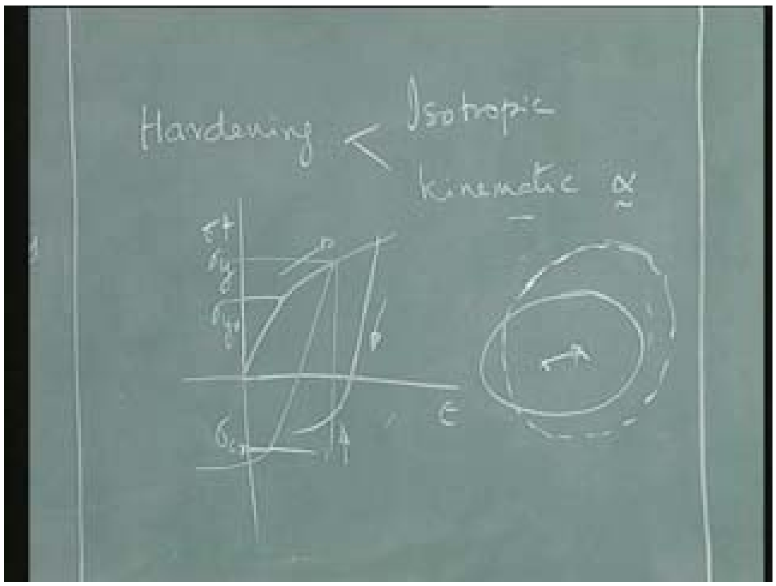

So, remove this. Of course, materials have both kinematic and isotropic hardening acting together; most of the times it happens like that. Now, what happens when there is kinematic hardening? When the kinematic hardening is there, the yield surface starts shifting; shifting. So, this shift here is manifested as a shift in the yield surface. So, when there is both a dilation as well as motion or in other words when it is, when it has both kinematic as well as isotropic hardening, the new yield surface would now be something like that. So, it would not only expand, but it would move like that. Is that clear? So, this kind of motion for the yield surface is brought about by certain tensors, which is called as back stress. So, kinematic hardening has other tensor variables apart from stress and these are called as back stress, which pulls now the point. Let us say that within quotes again the center has now shifted. This shift is represented by means of this kind of back stress which is called as alpha. Let us not worry about the kinematic hardening right now, because we, as it is we have to develop some algorithms and it is going to be quite difficult, even if you do it for isotropic hardening, we will do kinematic hardening later.

So, what is that other thing? That is isotropic hardening. So, we will concentrate on isotropic hardening. That is the third thing and lastly we have to put down what is called as a flow rule or sometimes called as normality flow rule.

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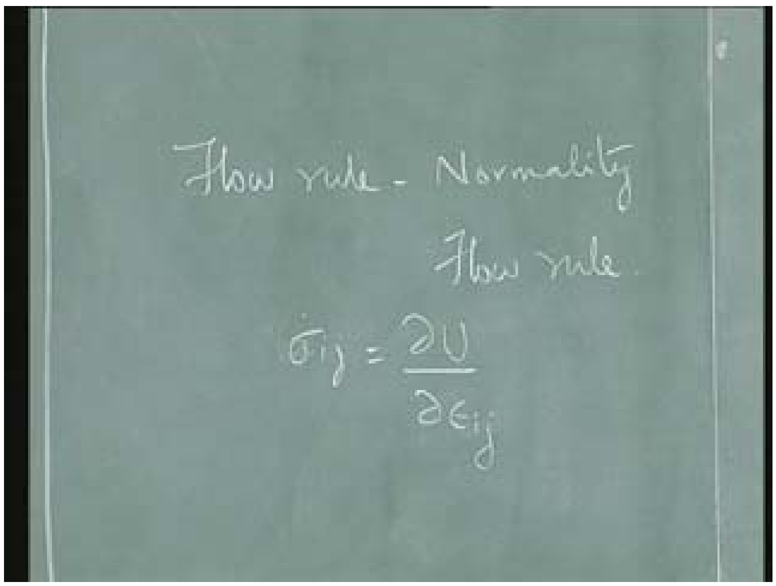

What is that we require now? We have, we have seen many of the things, but what we require is actually some sort of a relationship between stress and strain in a multiaxial case. So, I have to calculate what is say, delta epsilon ij P. There has been lot of efforts in the mid 40's to 50's as to how this can be expressed. So, how to express this relationship and one of the, I would say inspirations for putting down this normality flow rule is the developments in elasticity, where elastic strains for hyper elastic materials were expressed in terms of a function U; dow U by dow epsilon_{ij}; sorry, sigma_{ij}, sorry sigma_{ij} is written by dow U by dow epsilon_{ij}, where U can be treated as a potential function of strains and so on. So, this was an inspiration for further work in plasticity, though the mathematical rigor of plasticity is still questioned by many of the purists.

In other words, there are still some loop holes in mathematical theory of plasticity, but it is now good enough for us to apply to many practical problems. For example, if used for hardening this epsilon P, there are purists who question how far are you catching all the effects that are happening inside by epsilon P? But, does not matter; for all practical purposes the current theory of plasticity explains and is valid.

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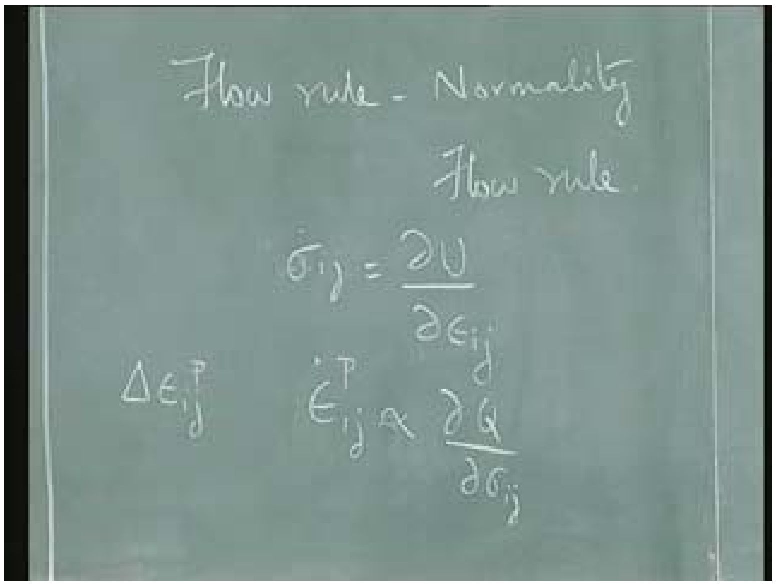

So, inspired by this kind of relationships, Mises was one of the first persons to write the rate epsilon dot ij P. Note that I am going to now use epsilon dot and delta epsilon in a, in a very, very interchangeable fashion. He said that, I will show that in a minute what I mean. Epsilon dot ij P is proportional to some potential dow Q by dow sigma$_{ij}$. P is the plastic strains, because we are looking at the increments in plastic strain. See that there is a difference if sigma is shifted to strains, then. So, that is what I said we operate at the stress space, stress space or the strain space.

Now, this is a very popular way of writing normality flow rule and that epsilon dot ij P can also be looked at as delta epsilon ij P as well, only thing is delta t is divided on either side. We already saw that time is only a pseudo term and that time has no effect, we are looking at rate independent effects. So, people write this also as delta epsilon ij P, instead of epsilon dot ij P and the proportionality constant if I introduce, then this can be written as lambda dot dow Q by dow $sigma_{ij}$.

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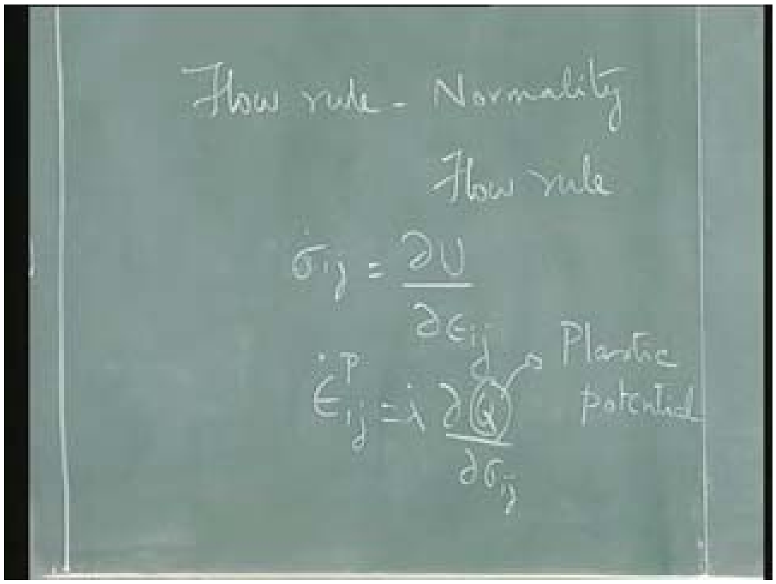

This Q, the potential function is called as plastic potential. Is it clear? It is called as plastic potential. Now, what is this plastic potential, how do I get this? This is, this has been a very interesting aspect of research in plasticity and it has been found that when Q is replaced by my yield function f, yield function f, then it is possible to model most of the metallic materials like steel, metals like steel and so on. So, this has been rigorously, experimentally done and theory followed from Drucker and that is what is called as a Drucker's postulate.

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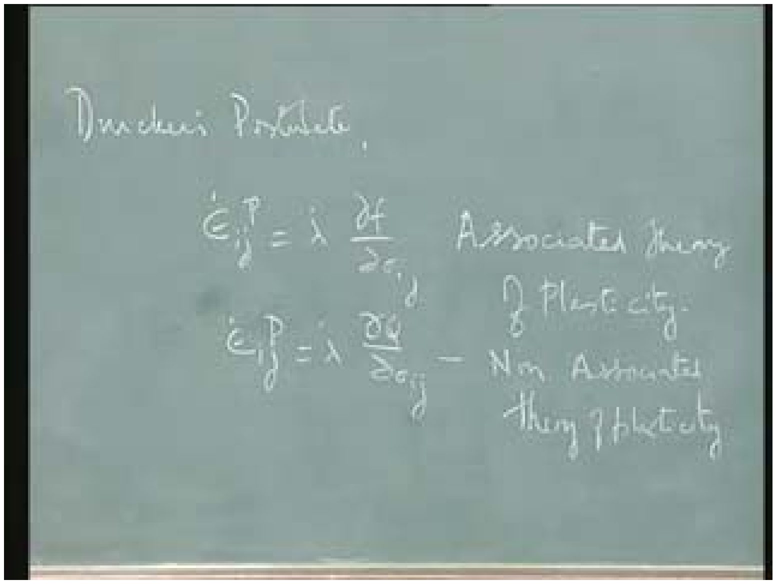

I am not going to, I am not going to the theory of this, because we will be going into plasticity, my aim is not teach plasticity though these are some of the things that are important for us to derive or use in our algorithms. So, one of the things that was done from an experimental perspective was to replace Q by f and say that epsilon dot ij P is equal to lambda dot dow f by dow sigma$_{ij}$. I mean of course capital f has only small f in that as a function of sigma, so, you can write it as lambda dot dow f by dow sigma$_{ij}$. Now, such a situation where the plastic potential is replaced by the corresponding yield function is what is called as associated theory of plasticity.

Once I say that it is an associated theory of plasticity, then there should be a non-associated theory of plasticity; there should be a non-associated theory of plasticity. What is non-associated theory of plasticity? Logic tells us that this potential is cannot be f; so, Q is different from f. There are two theories, associated theory of plasticity, where Q and f are the same or non-associated theory of plasticity. Is that clear? Now, you see a proportionality constant lambda dot. We are going to have very interesting things from lambda dot later. But, where do we apply non-associated theory of plasticity? No; we are talking about plastic materials. So, soils, rocks and concrete and other, many of the civil engineering materials undergoes what is called as non-associated theory of plasticity, where one of the things that we have to find out is what is the plastic potential Q? What is its expression and that should be written in such a fashion that experimental verifications are possible or in other words, you find out that epsilon dot ij P from experiments and it would coincide with what you get from the theory.

There are now new theories to express even for metals, behavior of metals in terms of non-associated theory of plasticity, very latest things that have happened, in order to predict some of the failures that take place in sheet metal forming, which is a very difficult area to look at. Probably all of you know forming limit ….. and so on and probably you would have looked at how experiments are done to determine what is called as forming limit diagrams, but when you want to model the plastic behavior and determine numerically the forming limit diagrams, there are still lot of issues which have to be sorted out and some of these issues, I mean still it is in the research stage, some of these issues can be looked at from a non-associated theory of plasticity as well. In other words, though I make a general statement that metals follow associated theory of plasticity, there are situations where the same metals may be modeled also as non-associated theory of plasticity. So, having come to that third thing, obviously what is this dow f by dow sigma_{ij}? What is dow f by dow sigma_{ij} in the yield surface?

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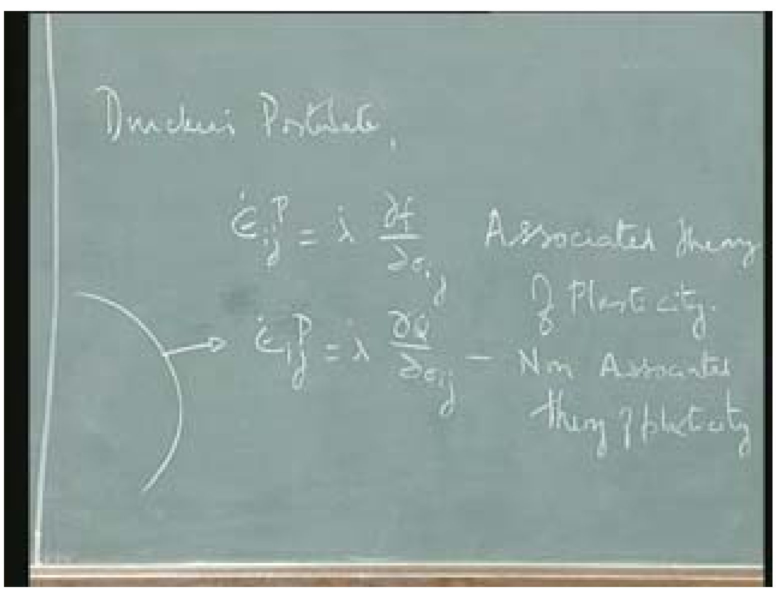

What is this? No; dow f by dow sigma$_{ij}$, yeah, it is the normal of the yield surface in the stress space. In other words, epsilon dot ij P or delta epsilon ij P, increment in plastic strain is in the same direction as that of the normal of the yield surface. Now, let us determine what is this dow f by dow sigma$_{ij}$, say for example, for Mises yield criteria? Let us see how we determine dow f by dow sigma$_{ij}$? But before that, we have to sort out a small confusion which may arise when I link up this epsilon dot ij P or delta epsilon ij P and the plastic strain which I have to use to model isotropic hardening. Is that clear? There is a, what is the confusion?

The confusion is very similar to that you would have had in the case of yielding where I had one value for the yield strength of the material and I have six values of stress in comparing these two become a problem, state of stress. So, that is why we developed the equivalent stress or the Mises stress in order that we have a one parameter characterization of the whole of the stress state and that can directly be compared with the yield strength of the material. Like that, now how do I now express these combinations of epsilons which come about, which are very neatly written here, so that I can tie up this multiaxial case with a uniaxial case.

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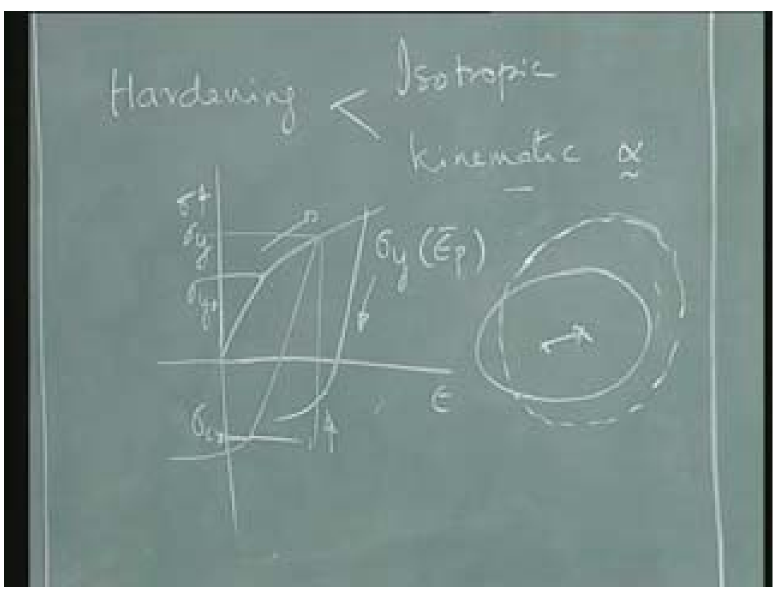

Remember that we said sigma$_y$ can be written in terms of epsilon P and very slowly I had put a bar there. What is that bar? If you look at uniaxial case, in fact I can remove that bar and write that sigma$_y$ is a function of epsilon P. Of course, it is not right to write sigma$_y$ as just a function of epsilon P all the time, all the time. It can be, it can be viewed as an internal variable or in other words, whatever is happening internally is now depicted in terms of epsilon P; may or may not be the case and it is a common practice to write down internal variables as some, some thing like as q_i and say sigma$_y$ is a function of q_i, but in many practical situations again sigma$_y$ is expressed only in terms of epsilon P by various equation, by various equations.

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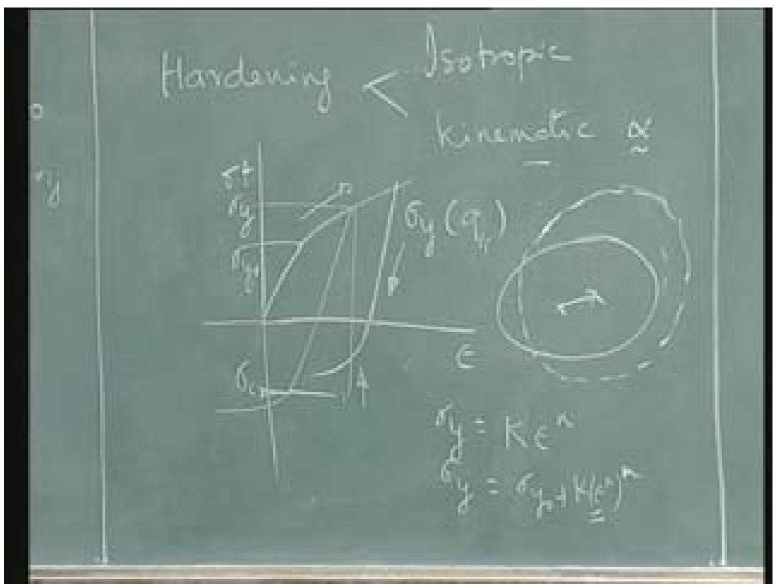

So, sigma$_y$ can be written as sigma$_{y0}$ or K into epsilon power n, where epsilon is epsilon plus epsilon P power n or sigma$_y$ is equal to sigma$_{y0}$ plus K epsilon P power n and so on. We have various ways of writing this. An exponential term may, may enter here. This, this depends upon where the stress strain curve is. Let us not worry about that. That is more say, material science, but just understand that they can be written like that. Now, the question is what is this epsilon P when in a multiaxial case? We now define what is called as an equivalent plastic strain in order that we can understand the uniaxial case. So, what is an equivalent plastic strain?

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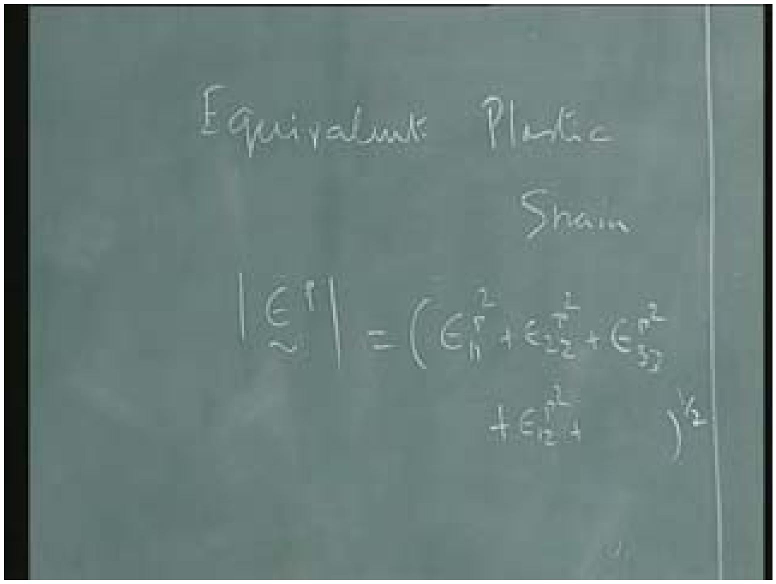

Now, what should be the condition for an equivalent plastic strain? Can you come out with a condition for equivalent plastic strain from what we have as an equivalent stress? So in a uniaxial case, my stress terms would reduce to single value which is equal to the yield function or yield strength of the material. So, in the same fashion in this case as well, I should have combinations of epsilons in such a fashion that it would reduce to that single value which I will observe in a uniaxial case. How do I write this down? Of course, this, this would depend upon the norm or length of this vector. That is very obvious, so, you can write down that as epsilon P, rather of this vector, which would be in terms of epsilon 11 P squared plus epsilon 22 P squared plus epsilon 33 P squared plus 12 P squared plus and so on, whole power half.

Now this epsilon, this norm of this epsilon or that should be equal to, in a uniaxial case should be equal to, just epsilon P. But will it be so? How do I find that out? Let us say we are subjecting this, a body to a uniaxial load. Now, what would happen to all these terms? Yes, of course; very good.

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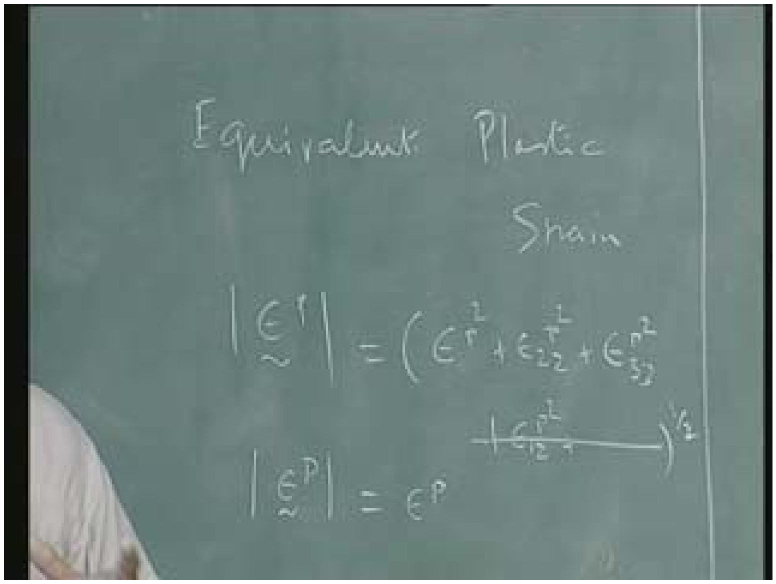

So, 12 13 will go to zero. I will not have these terms; I will have the first three terms. Let us say that my axis of deformation is 11. So, I will have epsilon 11 P. Let me call this as epsilon P, just epsilon P; so, I will have it as epsilon P squared and what would happen to the second term here? It will be zero. Poisson's ratio effect will be there, exactly; so, I should have a Poisson's ratio effect, but what would be the Poisson's ratio in a plastic regime or a plastic strain? 0.5, yeah; why is it 0.5? Incompressibility condition, yes; so, incompressibility condition has to be satisfied in the plastic strains, so the Poisson's ratio will be 0.5. So, when I, when I look at elastoplastic situation, we are not in incompressibility regime, we are nearly incompressibility regime. So it is possible to find out, see, there are some issues in incompressibility, which is different from compressibility.

When it is totally incompressible, the algorithms which I have to use later will be slightly different from nearly incompressible materials. So, just a word of warning that the material which we are looking at, if we look at it as elastoplastic or materials which are nearly incompressible. Why it is nearly incompressible, because in the elastic regime I have or elastic strains, still I have my Poisson's ratio to be 0.3. I mean, typically; not that every, every material has 0.3, but typically say 0.3 for steel. We are talking about steel, so, it is 0.3. It can be 0.2 or so on. Fine, so, what would I do?

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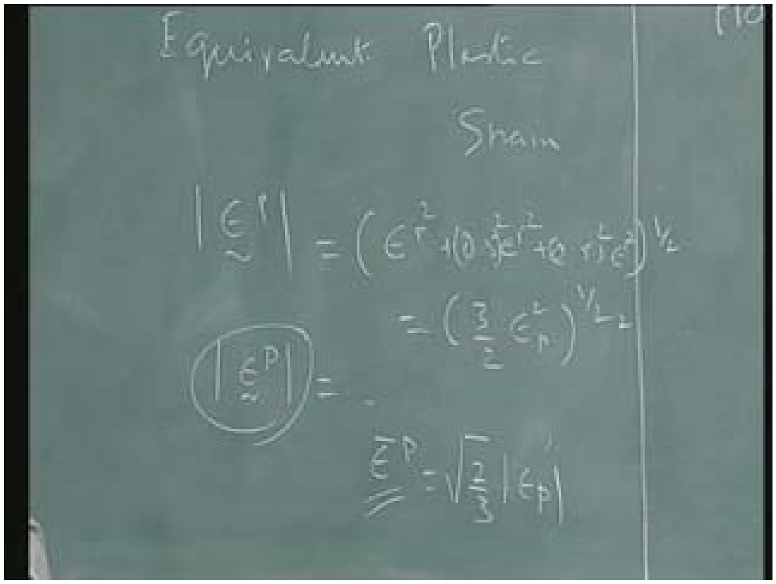

This, I would replace the second two things by 0.5 epsilon P squared plus point, sorry 0.5 squared, 0.5 squared epsilon P squared whole power half. So, this will be 3 by 2; so, this whole thing will be 3 by 2 epsilon P squared whole power half. But, what do I want? I want this to be epsilon P, an equivalent plastic strain to be epsilon P. So, if I now write this with root of 2 by 3, so, if now write epsilon bar P, that bar is for equivalent plastic strain, to be root of 2 by 3 of this norm, then what is that I get? I get that in the, in the uniaxial case this equivalent plastic strain would coincide with my uniaxial case.

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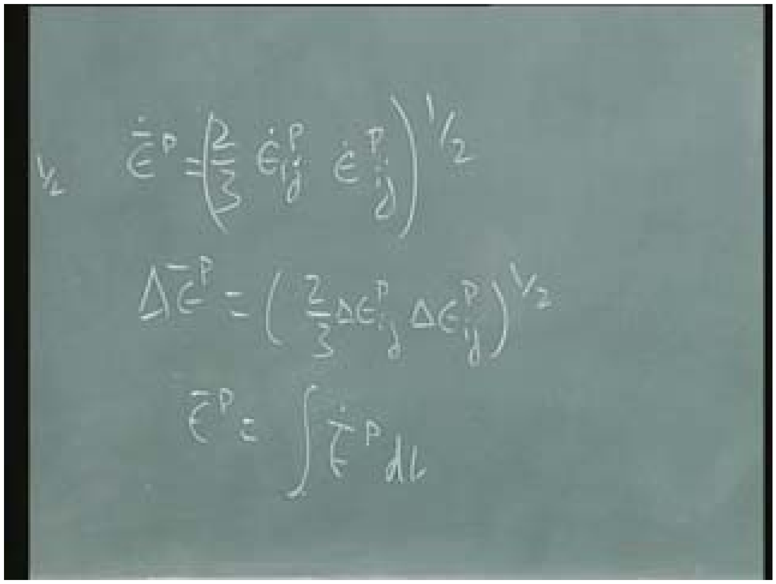

So, in other words, this particular combination, which in the indicial notation can be written as 2 by 3 epsilon dot, epsilon say dot ij P; in fact, I should put as an equivalent, now with a dot, because we are looking at the whatever we have been looking at is with dot, so epsilon dot ij P, I will, I will just modify it again in a minute, epsilon dot ij P whole power half or delta epsilon bar P can be written as 2 by 3 delta epsilon ij P delta epsilon ij P whole power half. Strictly speaking, epsilon bar P should be written as integral of epsilon bar dot P dt with time or sigma of delta epsilon bar P.

Note the difference between this and the Mises stress. What is Mises stress? Mises stress had in this case 3 by 2, 3 by 2 s_{ij} s_{ij}. Now, we have 2 by 3 delta epsilon ij P, delta epsilon. Now, one of the important quantities that come out during plasticity is what is called as the plastic work. What is plastic work? Plastic work is a double contraction on sigma and epsilon P; we had seen that as an example in our previous course.

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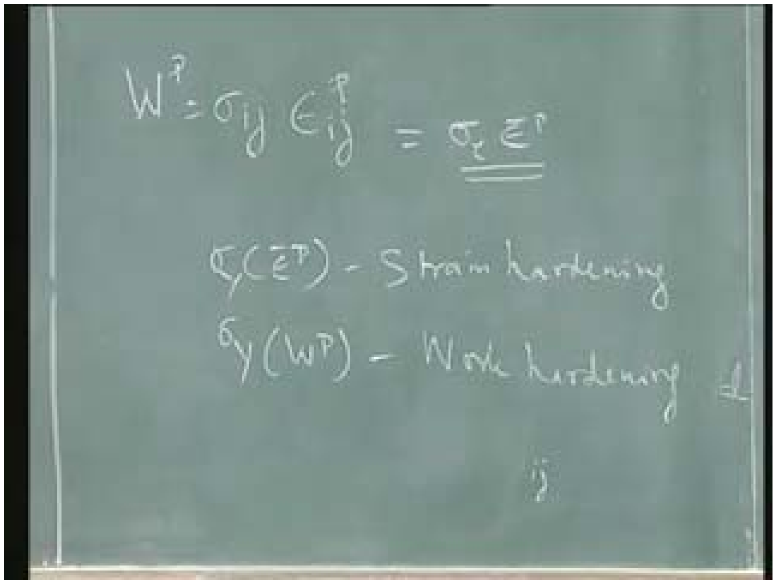

So, W is equal to $sigma_{ij}$ epsilon ij P. This is the work. Why is plastic work important? It is important in many, many applications, basically because it can also be used as damage criteria for the material in order to predict failures or more importantly this can be used as an input for the calculation of heat or temperature, because it has been found experimentally that 90% of the plastic work that goes in, people write usually with the superscript P, so, the 90% of the plastic work goes as heat. So, plastic work calculation is very important.

Interestingly, if you now substitute the corresponding terms, you would see that this happens to be sigma bar epsilon bar P or sigma equivalent epsilon bar P. So, sigma equivalent into epsilon bar P also gives me the plastic work. It is possible to calculate that, you can do that as an exercise and many of the metallurgists also base this hardening, hardening expressed by $sigma_y$ as a function of epsilon bar P, also in terms of plastic work. In other words, they, metallurgists call this function where $sigma_y$ function of epsilon bar P to be what is called as strain hardening and $sigma_y$, function of W P to be work hardening. But, fortunately for the Mises criteria materials, which we are going to concentrate extensively in this course, both of them happen to be the same. The results that we are going to obtain with either strain hardening or work hardening, they are equivalent.

Having studied that, having now completed all the things that are required except that a few comments again we are coming back to the Drucker's postulate which should be in order, in order to study further, I am not going to go into the details of Drucker's postulates, but I just want to state only the consequence of this. Because of these postulates we find that for materials which are stable, stable materials where we have stress strain curves, note, note that what we are talking about is true stress, true strain curve.

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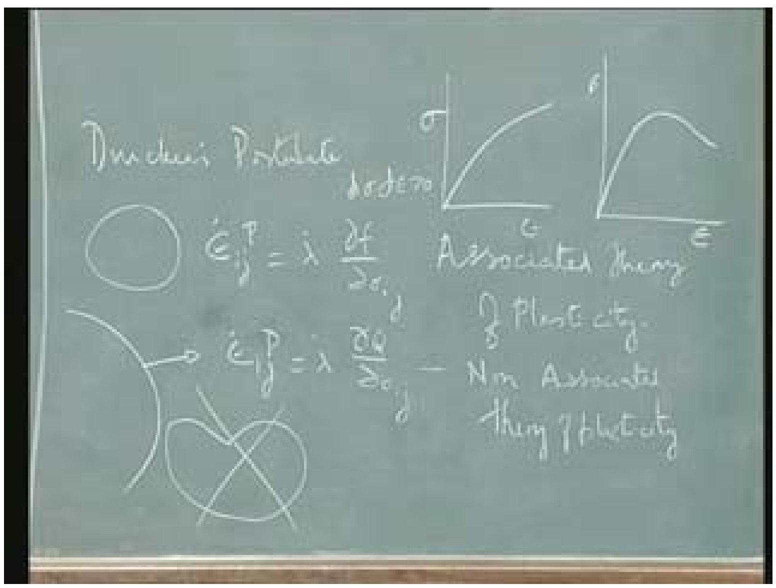

When it increases, when d sigma d epsilon is greater than zero, when d sigma d epsilon greater than zero in this case, as against materials where the curve can drop like that, sigma epsilon curves which are softening curves, sigma epsilon, these materials are called as stable materials or Drucker material. These materials are not stable materials. So, for stable materials, for which Drucker's postulate is valid and where this associated flow rule is valid, results in a convex yield surface. In other words, the yield surface will look like this, but cannot have, cannot have shapes say like that and so on. So, that shape is ruled out. It is not that these are the only materials that are available. There are materials like this. Most of the civil engineering materials would not follow this theory of, associated theory of plasticity and hence the Drucker's postulates and hence may not follow many of the consequences of this. So these are what I would call, within quotes, unstable materials. So, in this course we are going to concentrate only on associated theory of plasticity and stable materials defined by the Drucker's postulates.

In other words, the techniques for soils have to be changed a bit. Not that we are far away from it, we have to change a bit. In a programming language, it is just cut and paste; replace a few of the equations that we are going to develop for non-associated theory of plasticity. In fact, the theory will almost parallel that of the associated theory of plasticity.

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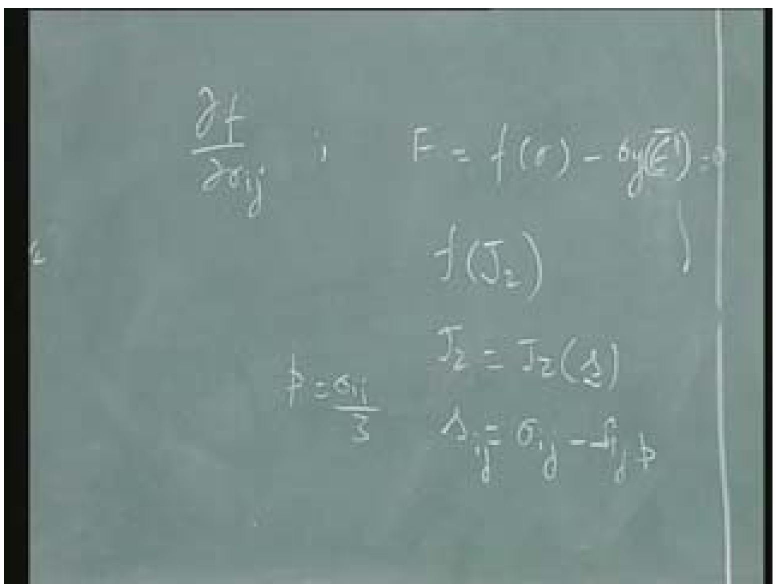

Having done all these let us now look at what is dow f by dow $sigma_{ij}$, dow f by dow $sigma_{ij}$ and that is an important quantity and you will find a very interesting result when you look at what dow f by dow $sigma_{ij}$ is. Now, how do I calculate this? Of course, I now F is equal to some function of sigma minus say $sigma_y$ epsilon bar P is equal to zero. I said normality flow rule actually should take this f, because there is only one f; so, that F has been replaced by small f. In practice if you see, that small f is actually a function of J_2

and if you look at that, J_2 is a function of the deviatoric stress and deviatoric stress is a function of, is written in terms of sigma. So, for example, deviatoric stress s_{ij} is equal to $sigma_{ij}$ minus delta ij P, where P is equal to $sigma_{ii}$ by 3; all these things you know already.

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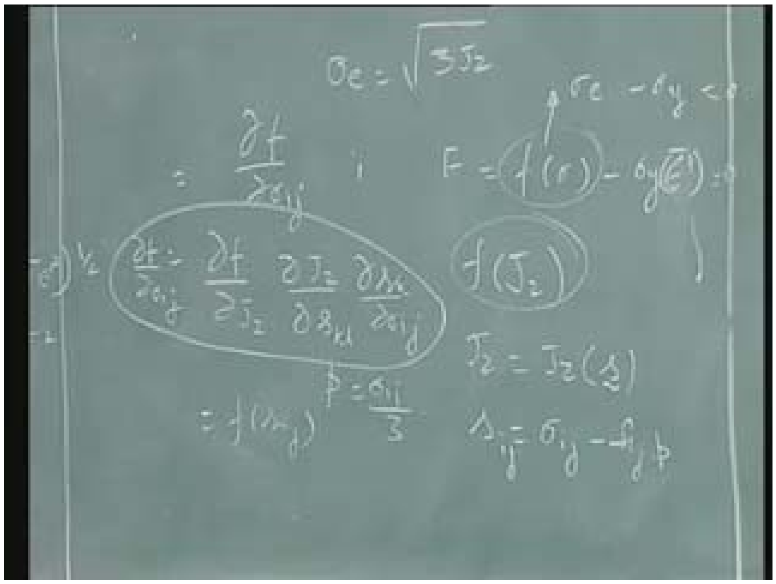

When I want to write dow f by dow $sigma_{ij}$, then I write this as dow f by dow J_2 into dow J_2 by dow s_{kl} dow s_{kl} by dow $sigma_{ij}$. This is what would now give me what is called as the dow f by dow $sigma_{ij}$. So, my first job is to express this f sigma in terms f J_2. Remember that Mises criteria can be written in this case as sigma e minus sigma y very nicely we wrote that; $sigma_e$ minus $sigma_y$ is equal to zero. So, first thing is that I have to replace $sigma_e$ by J_2. Of course, I can write it in terms of $sigma_e$ also. It is easier to work out the relationship.

By the way, what is the relationship between $sigma_e$ and J_2? $Sigma_e$ is equal to root 3 by 2 s_{ij} s_{ij}, which means that this is equal to root 3 by J_2. Is it clear? J_2 is the second invariant of the deviatoric stress. Any question? So, I have to now differentiate carefully between all these three things and then put them together. We will do that in the next class, this derivation, but you will be surprised to find out that this dow f by dow $sigma_{ij}$ will be a function of s_{ij} or in other words, when I substitute it in this form here, I will see that the plastic strain and stress, this would be replaced by stress quantity, plastic strain and stress are coaxial or in other words they are in the same direction.

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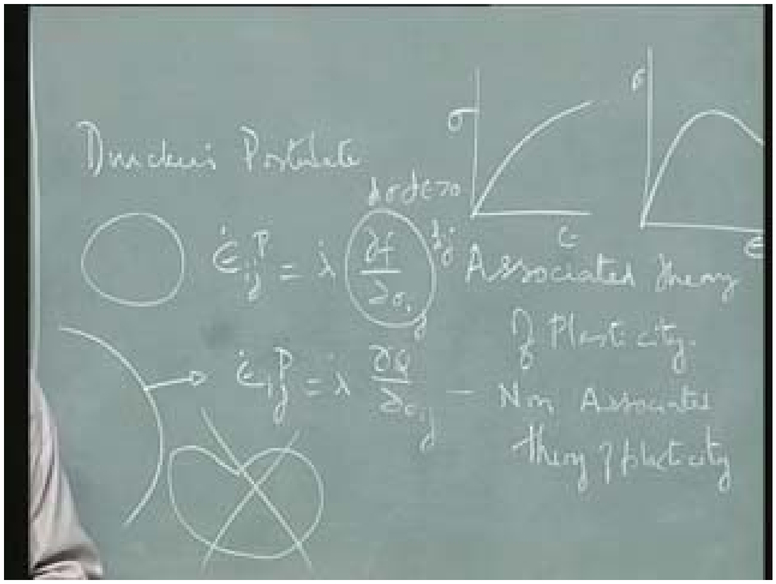

The incremental plastic strain or in other words, sorry incremental plastic strain stress would be coaxial. That is a very important say, result that we will use later. With this we will stop and we will continue this derivation in the next class.