Advanced Finite Element Analysis

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Lecture - 30

In the last class we had looked at two things. One is the updated and the total Lagrangian and one of the terms which we had not looked at seriously is the, what we called as the geometric stiffness term.

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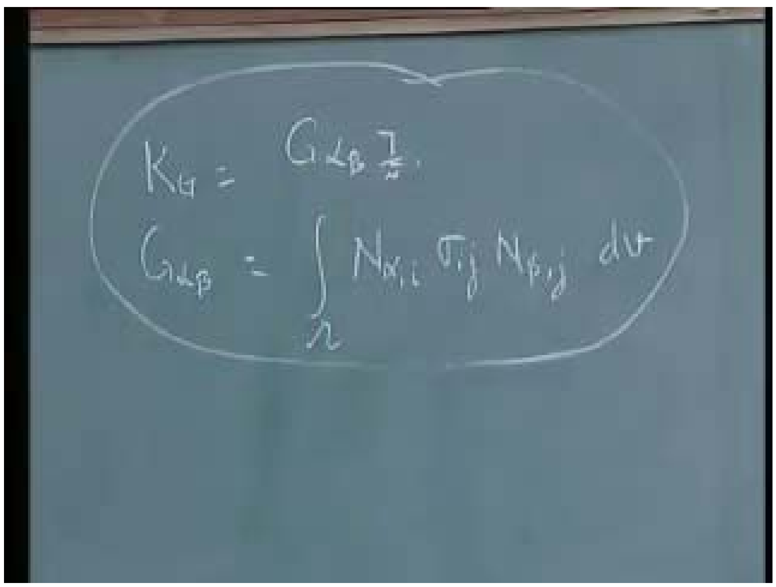

In fact, if you remember K had three terms in it. We recognize the first term to be K_M, the next is the geometric stiffness term which comes because of the variation of B with respect U and we call this is as K_G and of course we had another term, last term which we called as the term which is due to load or K_L term. We had three terms. I just want to say that this K_G term in fact can be easily written in this fashion for an updated Lagrangian formulation. You can derive this; I am not going to derive this, but you can ultimately say that or show that the stiffness K due to the geometry K_G can be written in this fashion.

Before we concluded in the last class, we saw that or we realized that there is one thing that is important with all these formulations; that these formulations are good as long as geometric non-linearities are concerned or as long as, as long as material non-linearity is concerned. But, once there are other constraints like incompressibility, this formulation, this displacement formulation has problems of what are called as locking. We had seen this before.

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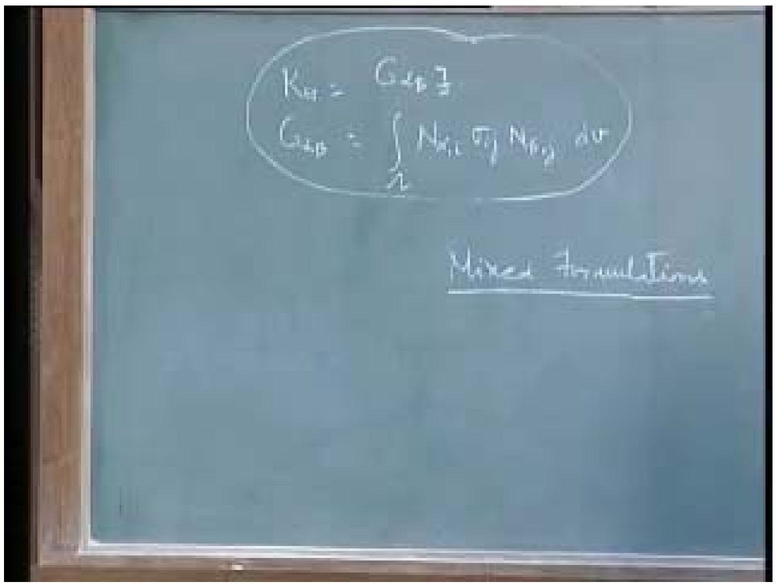

In fact when we looked at even the small strain case, we saw that we have to move to what are called as mixed formulations, in order to take care of, in order to take care of this problem of locking. In fact, I want to state that there are other issues also apart from locking. Before we go to that, let us look at what we mean by locking. Let us do a very simple problem. Let us look at the problem what is what now appears in the screen.

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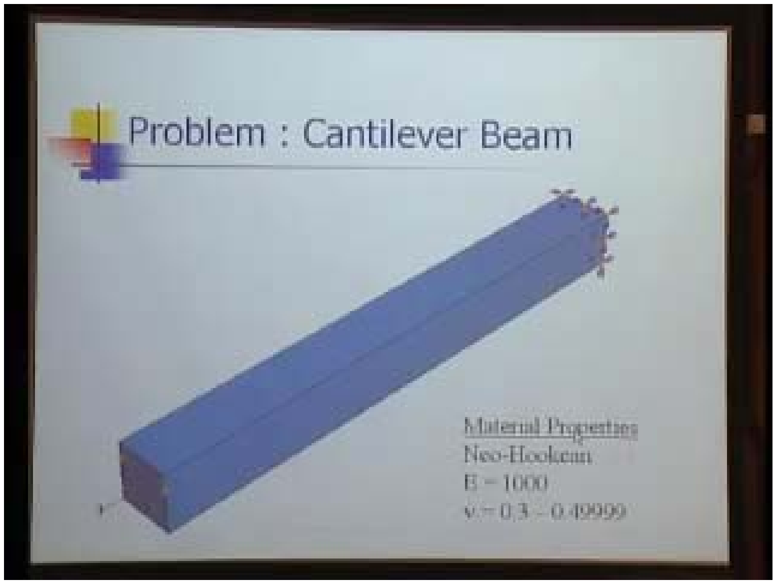

It is just a cantilever bending problem. It is very simple material properties. It has now nu varying from 0.3 to 0.499. Let us look at the results now.

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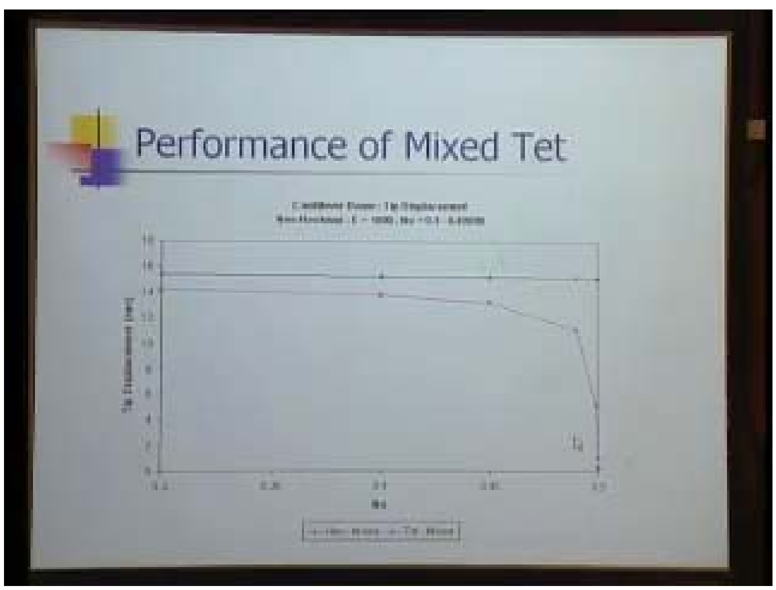

The results show the variation of the displacements with respect to nu. We know that as we approach 0.5, nu is equal to 0.5, we approach a condition of incompressibility. If you now, in fact these results are for even mixed problem, you know, mixed formulation, even mixed formulations have problems, but nevertheless let us understand what we mean by locking. As I now increase 0.5, nu to 0.5, see how the tet, tetrahedron mesh locks. In other words, the displacement drops drastically and in fact there is no displacement at all when it comes to nearly nu is equal to to 0.5. On the other hand, if you look at the hexahedron mesh, mixed hexahedron mesh, that is a straight line and it does not lock and the tip displacement which is plotted in the Y-axis is around 15.5. Let us look at the next result.

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This is very, this dramatically brings out what happens when you move towards 0.49999 that is incompressible limit. At 0.49, look at the difference between 0.49 and 999. At 0.49, the difference is not much; both of them are deforming, though there is a tendency already for the tet mesh, which is given to the right of your screen, right side of the screen, it tends to be slightly different, displacements are tending to be different. But, look at what happens as I increase that 0.499 to 0.49999; 4 nines. There is absolutely no displacement, everything being the same. Please note that load is the same; load is the same, geometry is the same, there is absolutely no displacement of the cantilever beam.

Can you see that? It almost becomes a, takes a straight line. You can see that. It is almost a straight line. On the other hand, on the other hand, look at the hexahedron mesh. The deformation is as good as what is 0.49. Again the hexahedron mesh is based on mixed formulation. In other words, even the tet mesh has some problems; we will comment on it later, but I want you to understand what is meant by locking. Locking does not mean, it will throw you out and say, I cannot do the problem. It simply means that the results will be wrong; the displacements will be very, very small. So, this is one issue.

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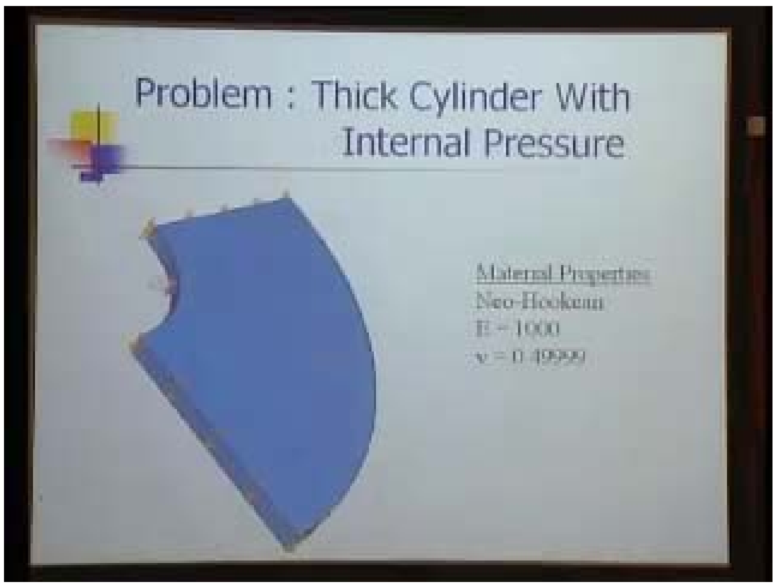

There are other ways of dealing with this problem. Say for example, if you look at the thick cylinder with internal pressure as is shown here and as we move towards the incompressibility limit, we will see that the tet mesh can be now or the tet formulation can be now modified using what is called as enhanced, mixed enhanced formulation and now you see that as the number of nodes increase, the mixed tet, mixed enhanced formulation gives as good a result as that of the hexahedron mixed formulations.

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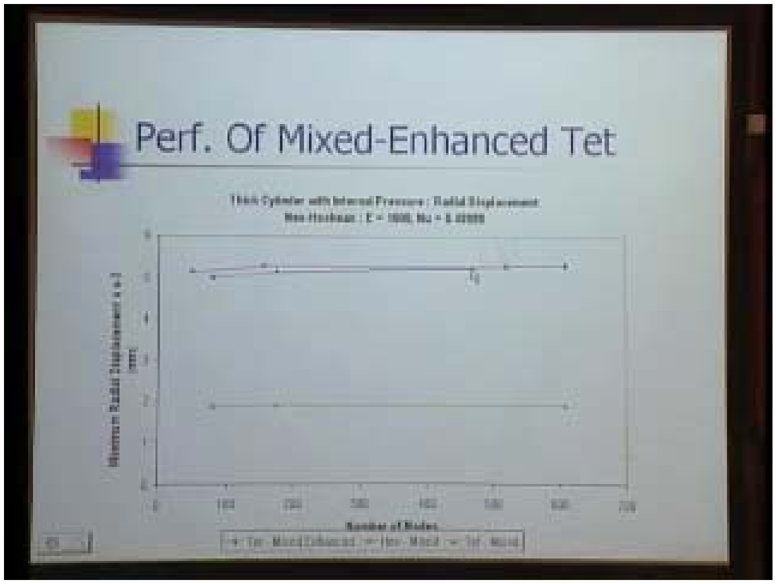

On the other hand, the tet mixed formulation is given by that green line and it completely locks or the displacements are very small. We have not shown here the displacement of, the displacement, of a displacement based formulation. We have not shown that here basically because, you cannot run the problem. It will be zero, it will lock straight away. At 0.49 this mesh will lock straight away. So, we will get back, we will see what this mixed formulations are and look at, I think most of you know this; we had already done it, but only thing is that we are now going to write this in the large deformation setting. If you remember, we had looked at the variations, variational principle to be or we saw that the variational principle that is used for the mixed formulations are different.

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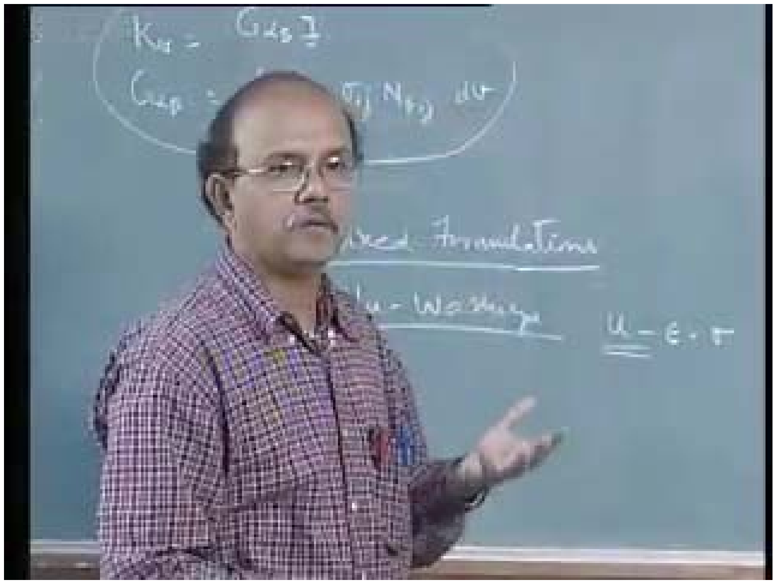

If you notice that it would be Hu-Washizu formulations, Hu-Washizu formulations. In other words, if you remember, what we said is that displacement alone is not, is not the variable. We had strain. Depending upon the formulation we said that one of the major formulations which are useful to us is the three field formulation where we said u, epsilon and sigma all of them are involved as variables. We will see how these mixed formulations are useful in this context of large deformation. Let us now write down the variational principle for the mixed or next? formulation.

Variational principle is actually straight forward. It is very elegantly written.

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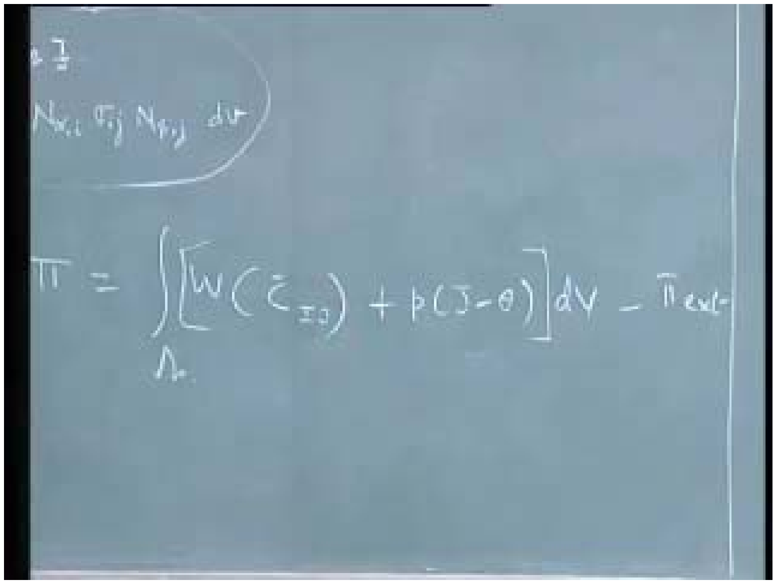

There is nothing, there is no difficulty about it. The first term remains the same which is the strain energy term, W. The only thing is that now, look at this; C is written as or W is written not in terms of C, but C bar, we will define C bar in a minute, plus p into J minus theta, p into J minus theta which is under integration dV; integration being taken with respect to omega$_0$. Now, note this term. We are familiar with such a, such a term in our previous, in some of our previous classes as well. Where did we put this kind of thing? If you remember we had put it for the strain energy term itself, because we wanted constraint.

Here in fact if we look at this, you would recognize that it is very similar to what we had written there where that Lagrangian parameter or Lagrangian multiplier is recognized with respect to p J. J, what is J? Determinant of F and the theta is the volume in the current configuration and p is actually the mixed or what is called mixed pressure in the current configuration. What we are essentially doing is to force the, in this, in this term what are we doing? We are trying to force the current volume to be equal to that of J and the difference is actually multiplied by a Lagrangian multiplier, typical Lagrangian multiplier formulation and is added to the, to the variational form. This is the starting point. Of course, there is the other term pi external term that is also involved. That is the second term.

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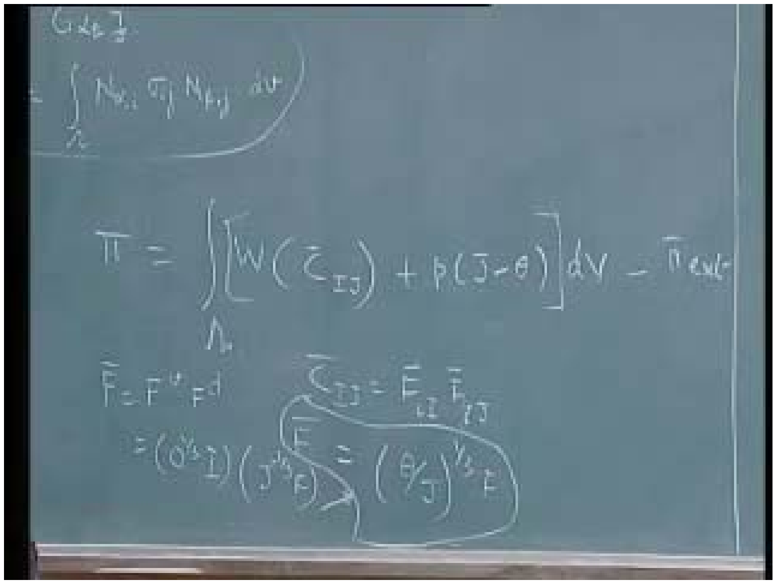

Now, what is C bar? C bar is defined in terms of F bar. So, that is F bar F transpose F which is written as F bar i I F bar j J or i J rather, sorry, i J F transpose F. So, i I, i J is what is transpose F C_{iJ}, where F bar is written as theta by J whole power 1 by 3 F; theta by J whole power 1 by 3 F. Actually, you can extend this further saying that F bar is split in terms of F v that is F bar is equal to F v F d, deviatoric and volume terms, where this is written as theta 1 by 3 I and F d is written as, we had written this before, J power minus 1 by 3; correct, exactly J power minus 1 by 3 F and that is what has led us to write like this - F bar is equal to theta by J whole power 1 by 3 F.

In other words, what it really means is that whenever I have F, expressions in terms of F, I send in F bar or I calculate instead of there being F, I send in F bar or in other words, I use F bar for this calculations. The procedure is exactly the same as we did before.

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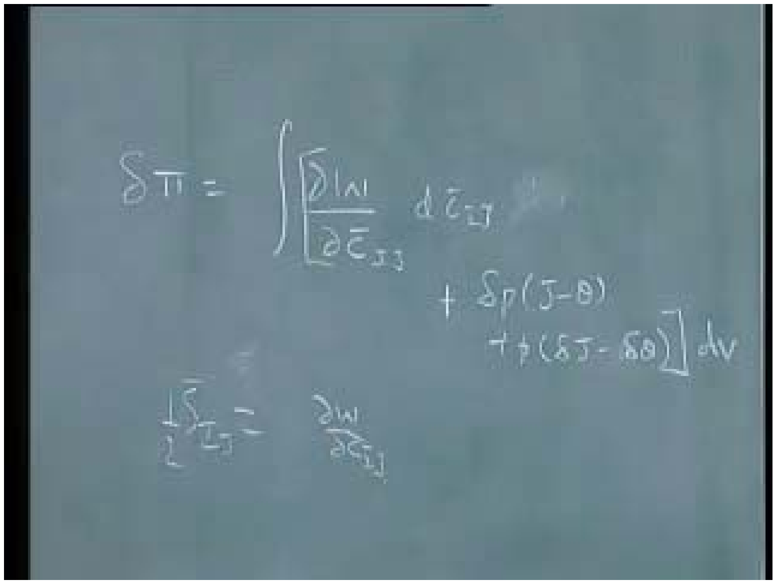

The only thing is that now I only have an additional term here or in other words, I have to have the first variation to be equal to zero and the first variation is calculated in the same fashion. That is in other words, dow W by dow C bar IJ dC bar IJ that is the first term dV plus the additional terms come from this one. The first one is p, so delta p J minus theta, first variation of p plus since both of them are also variables delta J minus delta theta, dV I will put it here. Now, I can express dow W by dow C bar IJ to be half S_{IJ}, because S_{IJ} is equal to two times, remember that S_{IJ} is equal to 2 dow W by dow C_{IJ}. From this expression, bring that to the left hand side and write it, because bar is there, I will write that as bar.

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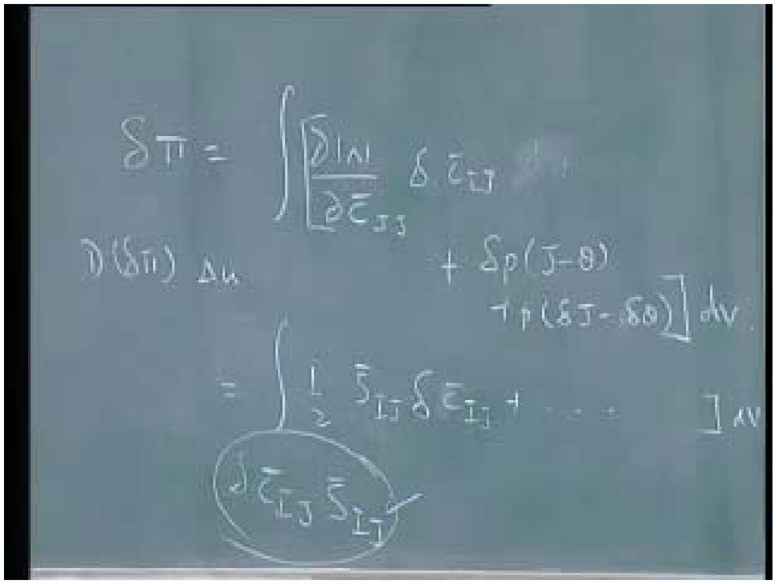

Substitute it back into that expression, so that I will get half of S bar IJ dC, sorry dC bar IJ plus all that terms dV. Now, oh sorry, delta C bar, not dC delta C bar. Now, the whole idea is to calculate the first term that is what is this term? Of course wherever C bar is there, be careful to put that. Our first job is to only calculate this. Calculate this, substitute it back into the expression, so that the first variation is equal zero would be complete. Now, once I do that, then it is very easy to do the next step. Let us summarize these steps. You know, the steps are the same; steps are no way different, only the algebra is different. Now what did we do? We took pi, now we added one more term, so this is the variational equation.

What did we do? We found out the first variation, first variation is equal to zero. Now, this will be my starting point for Newton-Raphson scheme. What is that I should do? I should linearize this; I should linearize this; that is the step. In other words, D of delta pi capital delta U is what I should find out. This step is the one which will linearize this. So, once I linearize, what is that I will get? Tangent stiffness matrix. These are the three steps. Even if you want to do your own formulations, your own finite element, that is all to it. Write down the first variation. If you do not have pi to be defining this kind of problem, for a, for a particular problem if pi cannot be defined, then go to the next step, this step, write this down. This, many times we would call this as virtual work terms and then start from there and then do a linearization and linearization results in K_T matrix.

The only problem will be, as you move towards more and more non-linearity, there will be so many terms that will come in. So, the expressions will become more and more complex. So, you have to be very patient in differentiating it very correctly; that is all to it. But, the procedure does not change, the procedure is the same. Now, let us calculate what is this term dow C bar IJ S bar IJ. That is the first term plus all these terms are there. That is the first term that we will calculate. Now, I think we did this similar exercise before. If you remember, we did that. In fact, what we did? How did we find out say, delta C_{IJ} S_{IJ}? If you, if you remember, very straight forward thing. In terms of delta F F, we wrote this in terms of delta F F.

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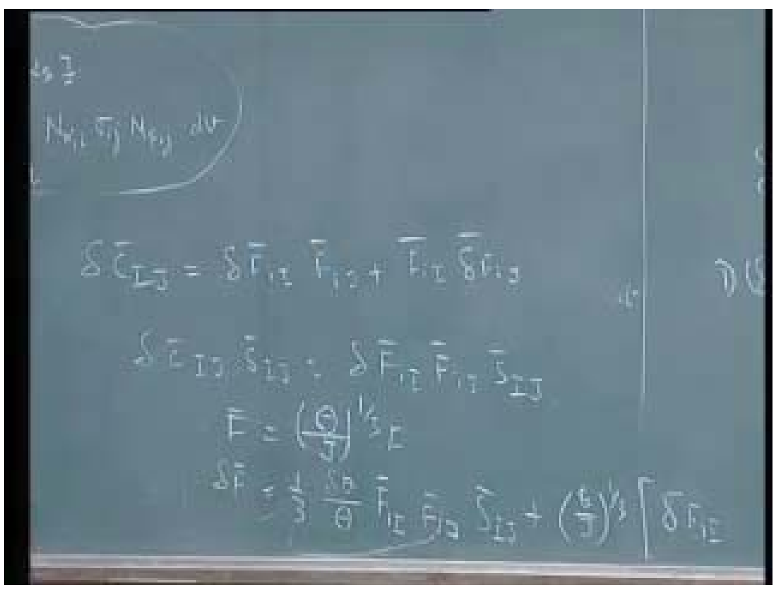

First we wrote delta C_{IJ}. We had two terms because of F. Those two terms are delta F_{iI}, then F F transpose F, so delta F transpose F that will be iJ plus F_{iI} delta F_{iJ}. This will be, am I right? Yeah, okay fine. Now the only difference is that there will be bar terms here, all of them will be bar. Then, we will multiply this by S bar IJ and recognizing the fact that S is symmetric, second P-K stress is symmetric that is the reason why we always use second P-K, you will see that this at least gets to a better shape and then you will write this as, it is equal to delta C bar IJ S bar IJ is equal to, multiplying this by S bar IJ, recognizing the fact S bar IJ is equal to S bar JI, you will get this as delta F bar iI F bar iJ S bar IJ. Now, this expression we will have to substitute it back into the major or this one here, but before that we have to see that how do we calculate or how do we get F delta F bar? This is delta F bar iI from my definition of F bar.

Remember that F bar we had written it in terms of theta and J; theta by J whole power 1 by 3 F. From this, I have to find the first variation of delta F bar. Why is that I have to, sorry, I have to write neatly here. Why is that I am worried about this, basically because there is a variation. I cannot, I cannot substitute here variation of delta F straight away, because this involves variation of theta as well as variation of J and of course the variation of F. This is like what? The differential, dF; dow F by dow theta into d theta plus dow F by dow J into dJ and so on. Writing like that, you will see that each of these terms has to be accounted for, if I have to look at the first variation of F bar. That is the problem and so I will have three terms - one corresponding to the first term theta. So, let us say that that is written as 1 by 3 delta theta. You will see, I mean there has been, some juggleries again have been made; theta F bar substituting it into this expression here, F bar iI F bar iJ then S bar IJ plus you can do it separately and then substitute it. You will get exactly the same expressions plus theta by J whole power 1 by 3 into delta FiI, oh sorry. That is this term that is the variation with respect to this term, this being constant, this term and then we will have a delta J terms, delta J term. That also has to be added, but before we do that let us see what this delta J term is. Delta J term I will just write that here.

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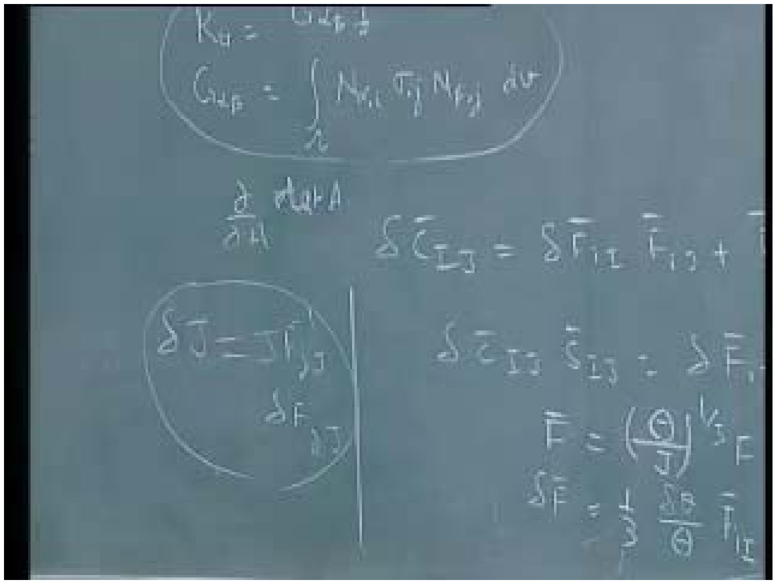

What is delta J? How do we calculate delta J? Can I calculate this in terms of F? This can be done, because J is what? Determinant of F; we know determinant of F, so we know that dow J by dow F, if you remember we had calculated this. Dow J by dow F is equal to what? J F inverse transpose, F inverse transpose. In the same fashion, you can find out delta J also. Delta J is equal to J F inverse transpose. Therefore F inverse jJ into delta F, so delta F should be this side; dow J by dow delta F, so, that is coming to other side, so that will be jJ. This is the delta J term. So, that delta J term will also go into this term here, so that I will write it down here minus, is this clear? This we got from my previous expression for dow by dow A. I think, if I remember right, we had dow by dow A of determinant of, determinant of A is what we wrote down before and from that we can write down this expression. It is a pretty long expression that here what you will get here.

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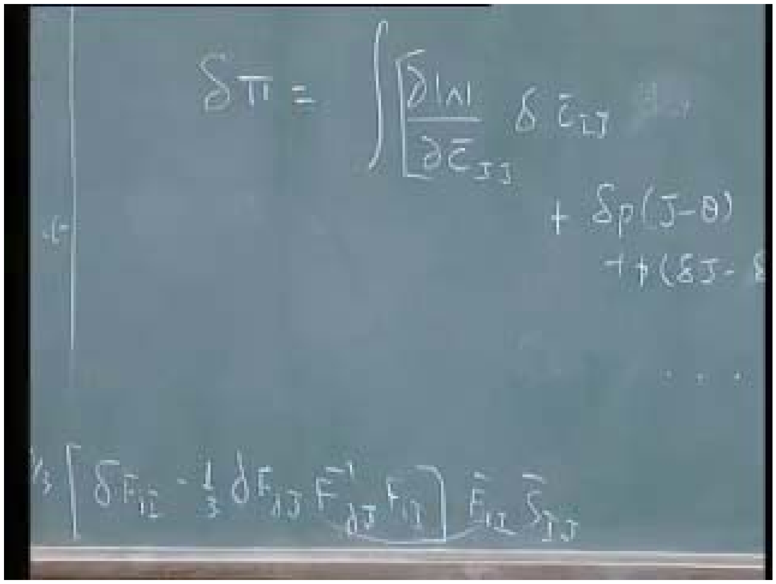

That will be minus, I am continuing from here or else we can write that here itself, minus 1 by 3 delta F_{jJ} F inverse jJ. We will summarize this in a minute; just you can write it down, into F_{iI} this whole thing multiplied by F bar iI S bar IJ. The whole idea here is to just replace this term, this term. Looking at this term very carefully, replace this into my first variation; replacing that into my first variation that is the whole idea here.

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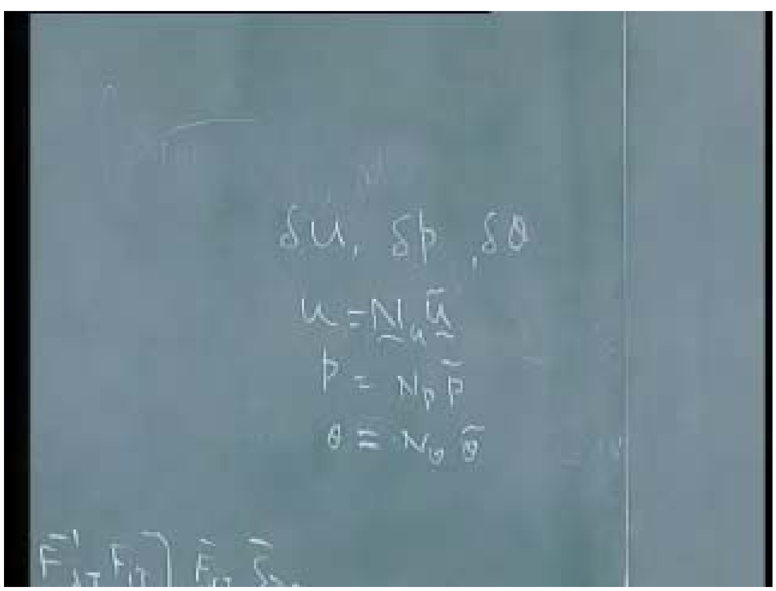

Now, note that when I do this, just to not to keep track you note that there are three things here - one is u, variation is now with respect to p also. So, the other variable is p which is a pressure term. The third is theta term. So, there are three variables. That is why it is called mixed formulation. You have three variables – u, p and theta. So, corresponding to these three variables, you write delta u, delta p, delta theta. In other words, in other words, what we mean to say is in usual finite element the only variable that you will solve for is displacement. You will have, these are called as displacement based formulation.

In mixed formulation, you have not only u, you have as variable p and theta; you have variables p as well as theta. In other words, as we did before, just to recapitulate what we did in our earlier classes, we have to write down the variation of p as well as or interpolate the values of p as well as that of theta. In other words, when I have N into u, u to be interpolated between of course sigma of that or in terms of N and u where this u is the degrees of the freedom at the nodes, I will have to write very similar expression for p as well as theta and call this as p now, N_p p and theta, N_{theta} theta. Do not remember, I mean, do not forget that; remember that there are other shape functions N_p and N_{theta}. It is all also in the whole game. Is that clear? That is why it is called as mixed formulation.

There may be questions again, you know, which we answer. Suppose I put a mixed formulation for a displacement based finite element, will I get a better result? Will I get a better result? Is it that I put mixed formulation to avoid locking or do I put or do I use this to get a better result for problems which do not lock; your regular say nu is equal to 0.3? Is the question clear? Because, I have now two more variables will I get better results? You do not get a better result as long as, as long as the P here or the theta here are interpolated in such a fashion that you can get these values from u itself. Suppose this is interpolated to be a constant; pressure or say, theta is interpolated to be a constant and that is what you will get or strain is interpolated to be a constant and that is what you will get if you go to u. If you go to u formulation alone, then your results will not be in any way better.

In other words, if these interpolations are not going to improve the values that can be obtained from the displacement formulation through, say a corresponding differentiation. Suppose you take a triangular element. If you have a triangular element, the displacements vary say, linearly. Now, look at the strains. There are say, constant strain triangles. So, the displacement formulations themselves give you constant strains. Now, if you now put here in this formulation such that you will get a constant strain, though you will introduce strain as one of the variables, one of the field variables, you will not get better results. But, that brings us to another question.

Can I keep on putting this kind of interpolations as and I mean how I like? Is it that I can vary these interpolations? Can I have an interpolation say, in an element with pressure varying in a cubic fashion, displacements varying in a linear fashion and so on? Can I put down this kind of things? Answer is no. That affects what is called as the stability of the problem or in other words, the problems becomes, the problem becomes unstable. Unstable in the sense that you cannot solve or there will be singularity in the ultimate solutions that you would be solving or in other words, there are certain conditions which define; there are certain conditions which define the stability of the problem, problems like this.

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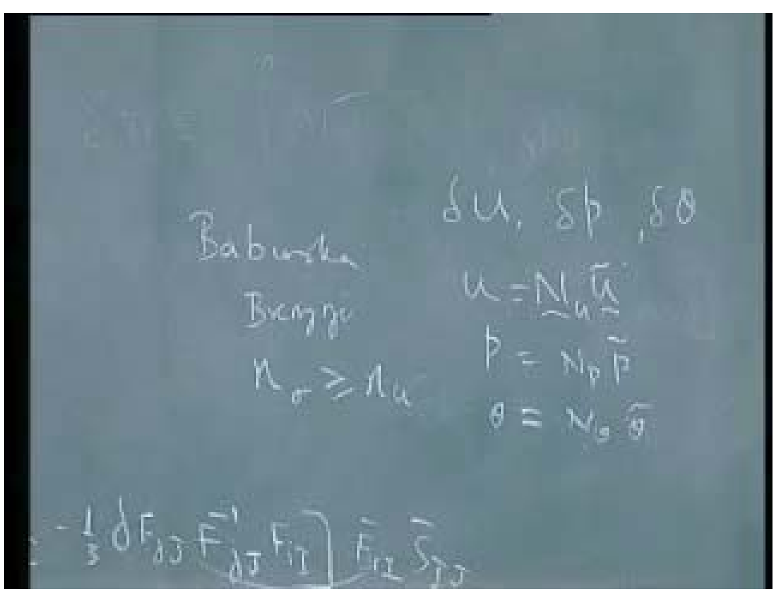

These conditions are called as Babuska Brezzi condition and of course, we are not going to cover these conditions, because they are highly mathematical. We are not going to cover that, but nevertheless you should know that these are Babuska Brezzi conditions which are the ones which control the stability of problems, which involve constraints like incompressibility. This can be stated in a very simple fashion. Many of the problems, for many of the problems this can be stated in a very simple fashion saying that per element or patch of element the degrees of freedom for say, stress and displacements are involved, the degree of freedom for stress should be greater than or equal to degree of freedom for displacement. This is called as constraint or this is called count condition; n sigma should be greater than equal to n_u.

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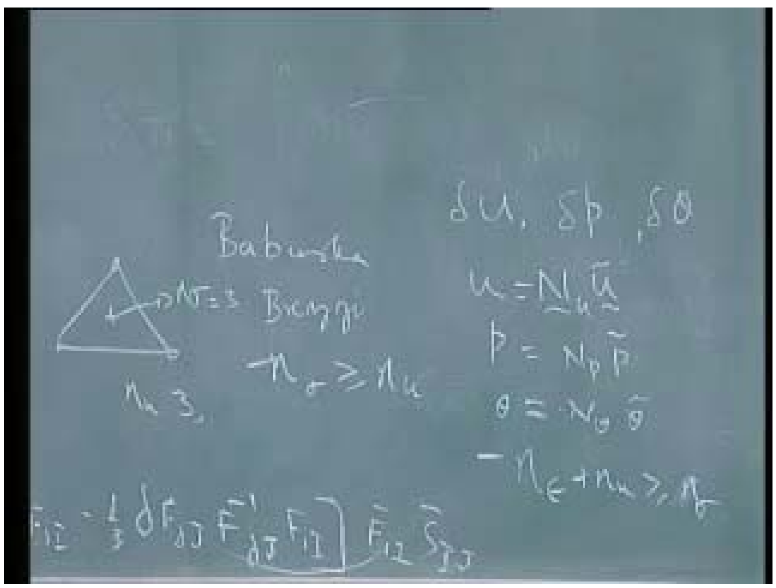

Let us say, take a triangle say for example, a triangle problem. You have displacements defined at the three nodes and you have pressure that is defined say, at the centre and this pressure in other words, the pressure is constant throughout the element. This we need it to tackle incompressibility. Now, if you look at this problem, two dimensional problem, let us say, sigma; not, not pressure, but sigma. So, the numbers of degrees of freedom for sigma is equal to 3. So, n_{sigma} is equal to 3. There are three degrees of freedom, because $sigma_{11}$ $sigma_{22}$ sigma $_{12}$, because it is a two dimensional problem. If you look at the displacement degrees of freedom, how do you calculate this, the displacement degrees of freedom? This is equal to 3 into 2 6, but we have to have three fixities for it not to become singular; displacement along x, displacement along y and one more rotation, so three is fixed. So, 6 minus 3, then it will become n_u is equal to 3, n_u is equal to 3. So, you will see n_{sigma} is equal to n_u. So this, if you have this condition, this interpolations to be in this fashion, then you will not have conditions of locking, oh sorry, conditions of stability. Is that clear? That is one way of of saying that I cannot, I mean this is one way of saying that I will see to it that my formulation is stable or else of course, I mean, this is not Babuska-Brezzi condition, but if Babuska-Brezzi condition is more mathematical than this, of course, you will come to a very similar conclusion even if you use a Babuska-Brezzi condition.

What happens if I now use strain, stress and displacement? Then in that case, I got to have a condition where $n_{epsilon}$, where n is the number of degrees of freedom for strain plus n_u should be greater than equal to n_{sigma}. That also, that condition should be satisfied. So, these are called as count conditions. Of course, apart from this, both of them should be satisfied. These are what are called as count conditions. What does this count condition tell us? It tells us that you cannot, you cannot have your own interpolation schemes as and when you like; you cannot change them also. They have to be controlled by several other factors. So, it is not very easy, in one line; it is not very easy to get a better finite element result using a mixed formulation, for a regular problem. So, mixed formulations are used only for incompressible problem. I just wanted to remove this notion. That is the reason why I had spent quite a few time. I hope you understand that. So, these conditions, count conditions control the stability of the problem. In fact, these, the mathematicians called this as saddle point problems and in fact, Babuska-Brezzi conditions are ones which are used in saddle point problems. We will not go into the details of it, but we can, you know, make use of these things.

We will get back to our delta pi. Now, what we are going to do is basically substitute this expression, this is what we had derived this into my expression for delta pi and you can do that and then you can, of course, again write this in terms of two things - one is what is called as the, our famous updated or the total Lagrangian.

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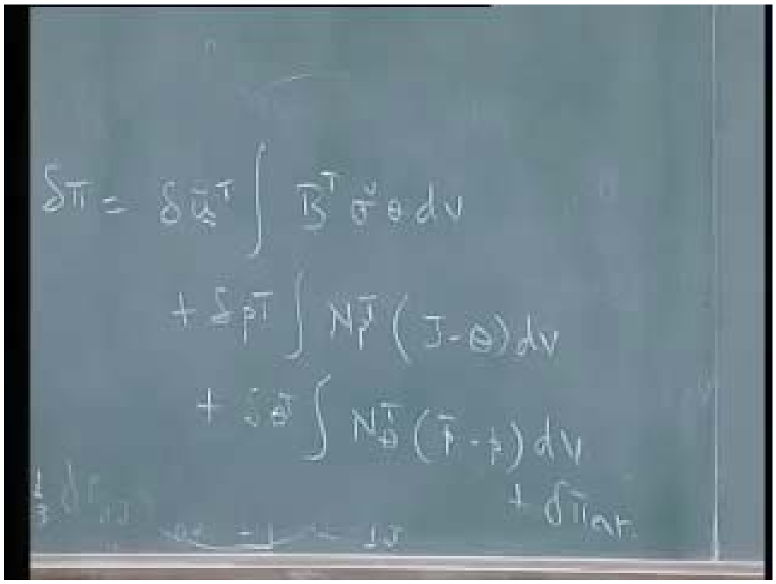

You can convert this into, at this stage itself you can convert them into a finite element form. I will write down the final finite element form for this. The whole formulation is given in Zinkevich's, Volume II. I am not going to write down the whole expression, because they are too long. It is not necessary for us to know every, you know, expression right now. But, though they are, as I told you, algebraic manipulations please refer to Zinkevich's, volume II, volume II, to get the complete expressions of all the terms that are involved here. So, this delta pi can be written in terms of, in fact, I will not be able to write down all the terms. It can be written as delta u transpose, all the terms that are involved there. Please refer to that book, B transpose sigma theta dV. It is very similar to what we wrote in one of our earlier classes plus delta p transpose N_p transpose. This is for my delta p term. Remember, we had a delta p term into Np transpose J minus theta dV. Then, we have our p term and a theta term delta theta term, so plus delta theta transpose integral N theta transpose p bar minus p dV plus the delta pi external. For explanations refer to that book.

Now of course, this will be the starting point for me, as I told you, for tangent stiffness matrix and let us see how the tangent stiffness matrix now looks like. The rest of the details, fill it up.

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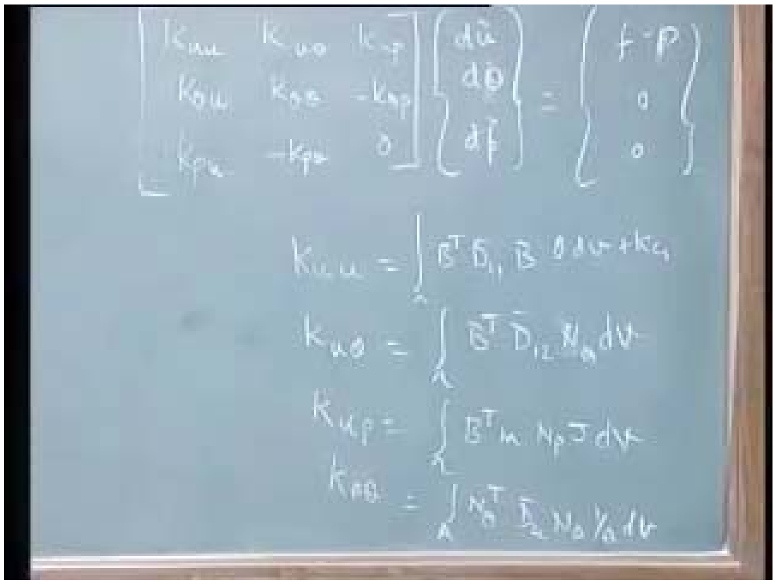

This is how it looks like. The tangent stiffness matrix looks something like this. You will see now that it has three variables u, theta and p and look at the right hand side. There are two zeroes corresponding to the second and the third equation. See how many K's are there. K's, K, it was only one K. K_T if you remember, we started with just K_T delta u is equal to delta F. Now, look at the K's that are available. You can keep on, you know, writing each, for each K you can keep on writing expressions like this. In other words, mixed formulations give rise to a K with three variables du, d theta and dp. But, fortunately for us the second and the third expressions have zeroes on the right hand side. What is it, what is the advantage? You can condense it out; you can condense d theta and dp out.

In other words, if you look at the first expression K_{uu} du plus $k_{u\ theta}$ d theta plus K_{up} dp is equal to f minus p. f is the external load; p is, you can get p from the initial Newton-Raphson scheme. Now, using the say, third expression you can write down dp. Using the third equation you can write down dp in terms of du. Substitute this from this back into the second expression. Write, write theta in terms of again du. Then, substitute that into this expression.

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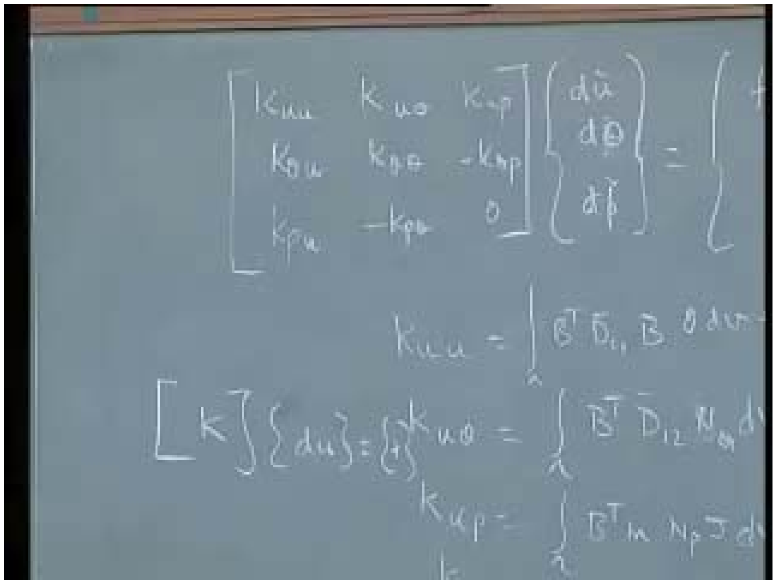

Then, ultimately what you will get interestingly is only of the form K du is equal to some f. What I want to emphasis is that in mixed formulation, you do not or good mixed formulations you do not solve for theta and p in a global level. In some of them, yes, you might, but most of the formulations you do not solve that at the global level. You condense it out. What is condensation? You write that in terms of du or in terms of the displacement, remove it. I hope you understand how I removed it. dK_{pu} du is equal to K_{ptheta} dp. So, dp is equal to K_p inverse K_u du. That is all, substitute this back into this expression. So, you have now, from here you can write down in terms of u du and so on. So, this is what is called as static condensation. These degrees of freedom are condensed out most of times at the element level itself and then they are not assembled. They are put into du and ultimately you solve an equation of this form. Is that clear? So, that in a nutshell is the mixed formulation. But again, the story does not end. That is the problem with many of these highly non-linear problems.

Mixed formulations do not guarantee, note this carefully, do not guarantee two things. One is, let us take the first thing - do not guarantee that incompressibility is maintained. With all the story, ultimately the bomb shell is that they do not guarantee incompressibility to be maintained. What they guarantee not even, not even guarantee what they try to do is not to lock. That is all, they do not lock. In fact, in the tet you saw that even mixed locks, we will come to that as the next step; one, let us finish this. So, it does not guarantee incompressibility condition. It just guarantees that there is no locking. It does not satisfy the constraint. For satisfaction of constraint, again you have to apply one more algorithm say, what we call as a Lagrangian approach or augmented Lagrangian, as it is called.

An augmented Lagrangian approach is applied on top of whatever we have done in order to ensure that incompressibility condition is also satisfied. Suppose you want, in other words, if you want exactly nu is equal to 0.5 and exactly the incompressibility condition to be satisfied, then it is not enough if you do only mixed formulation. You also have to do augmented Lagrangian formulation, but on the other hand, you are at 0.495; it is nearly incompressible, nearly incompressible, then, this algorithm will be good enough and we will see to it that the solutions do not lock. Now, that is the first thing.

The second is that the mixed formulations when they are used with tetrahedron elements, with tet elements are not again very good. Their performance is not very good. Why are we harping on tet, basically a tetrahedron element, basically because most of the mesh generators they generate only tetrahedron elements, nice tetrahedron elements. In fact, they have problems even for linear case. For non-linear case, you have severe problems. So, what do you do?

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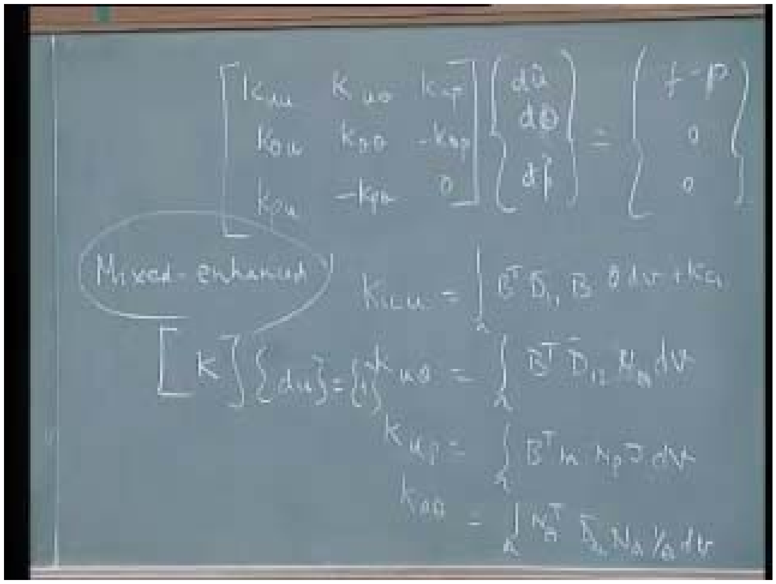

You have not only a mixed formulation, but you have what is called as mixed enhanced. Still worse, that means you are going to add one more enhanced strain; the strain is enhanced, another expressions are arrived at for enhanced. In fact, you can have a look at that result here in the screen.

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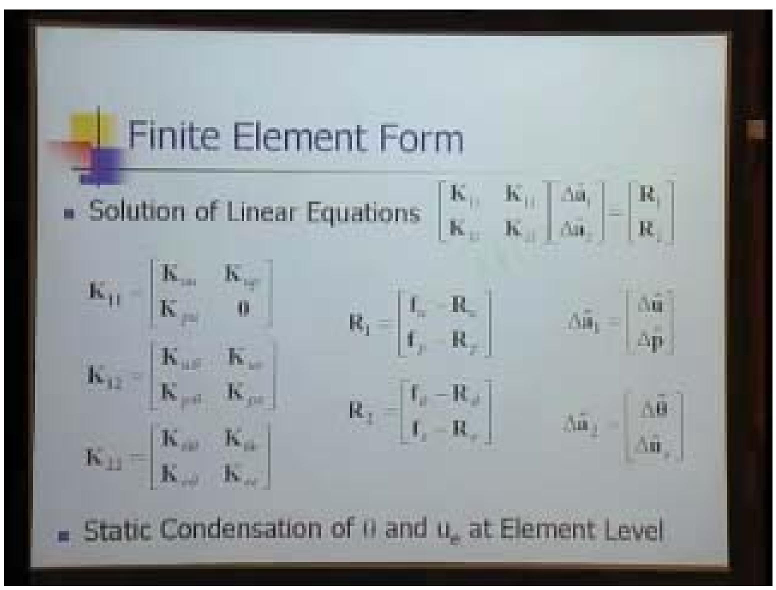

Yeah, you have such, you will see, you look at the number of expressions again for enhanced condition. Let us look at the results of the enhanced formulation.

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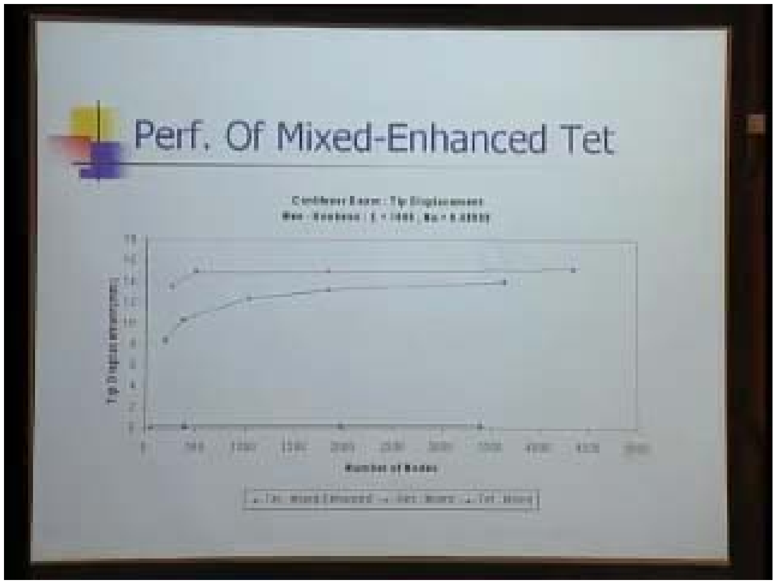

Yeah, you see that mixed enhanced, tet mixed enhanced which is given by that blue line there. That blue line there, it approaches of course, with number of nodes it approaches the hex mixed. On the other, on the other hand, if you have only a tet mixed, it still locks. So, you have a, you have another formulation on top of mixed formulation for tetrahedron elements to perform better in terms of this locking, in terms of incompressibility. These are the issues that are involved; we are not going to the details of mixed enhanced formulations. People who are interested in mixed enhanced again refer to Zinkevich's, volume II. All the latest papers are summarized in Zinkevich's, volume II; you can look at that as well.

So, that sort of gives you an overall picture of the mixed enhanced, mixed formulations. Though we have not derived each of these terms, they are available in text books. You can have a look at that. They are no more than some algebraic jugglery that has been done, but the concepts are what we have taught. Please have a look at this mixed enhanced formulation also. Just to close this topic, I want to tell that most of these softwares today come as a mixed formulation. In other words, most of the times hexahedron mesh or quad mesh performs well, but still tet mesh – though they call this as mixed formulation - its performance is not that good. There have been issues with respect to locking. We will close here and continue in the next class.