**Advanced Finite Element Analysis**

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**Lecture – 3**

In last class, we had a glimpse of a technique that is very useful for solving non-linear problems and you remember we called this as an incremental-iterative approach.

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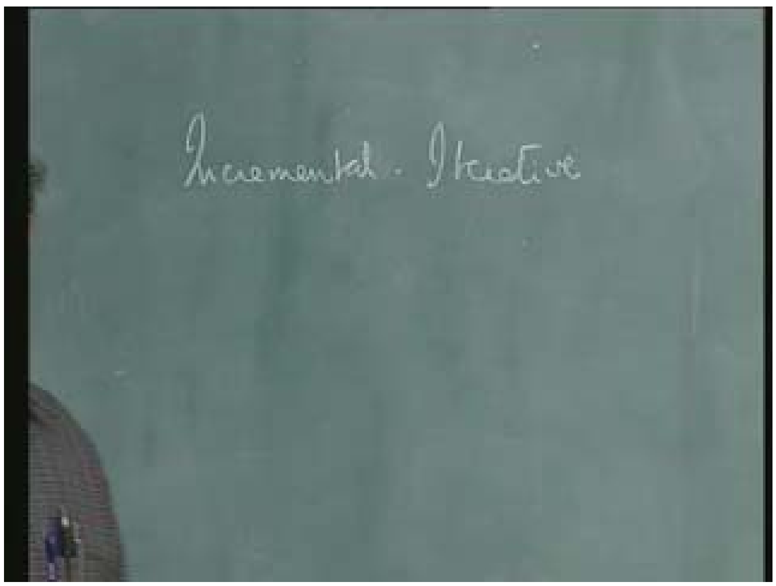


We said that Newton-Raphson scheme, which we ultimately derived an equation also, was one of the techniques for implementing this incremental iterative scheme. Let us do a small problem to understand what this scheme looks like. But, even before we do it, let us understand the whole technique through a graphical presentation.

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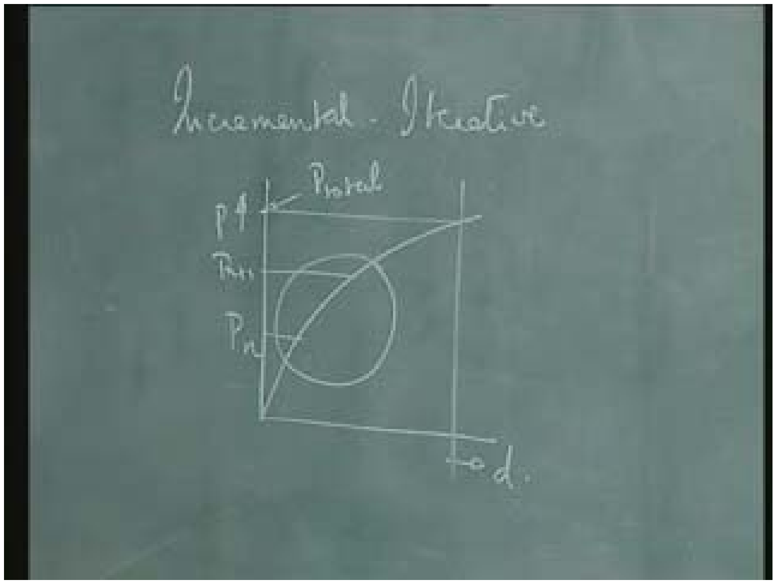


In other words, if you look at, say for example load deflection curve, non-linear load deflection curve, say deflection in the x-axis and essentially is a non-linear curve; so, let us say it goes on like this. Our whole idea in one dimensional problem, though strictly for one dimensional problem, it becomes slightly more complex when we go to multi dimensional problem, is to actually trace this curve closely. In other words, the whole idea here is given a P, how do I find out a displacement corresponding to this P, so that the load deflection curve automatically says that whatever is the internal forces, stresses which generate internal force equilibrates this P. Is that right?

What is that we essentially do? If this is the total P that I apply, $P_{total}$ that I apply, I divide this into smaller incremental loads say, $P_1$, $P_2$ and so on, until I reach the total load to be $P_{total}$. Now, let us understand how this process of Newton-Raphson scheme is applied to a particular nth step, as we move from nth step to say n plus 1th step. So, let us say that that is the nth step, so, I have load of $P_n$ and I have to get to equilibrium step to $P_{n+1}$.Do not tell me that I can just get this point and do it, because I do not know this curve I am trying to get this curve, I am trying to trace this curve. It is not that $P_{n+1}$ I know it, just go there; that is not the issue. I am at $P_n$. I know everything about point $P_n$. I do not know $P_{n+1}$. It is only an illustration that I have put this.

Let me draw that in a more, I would say a detailed fashion; zoom into this portion and see what I do here.

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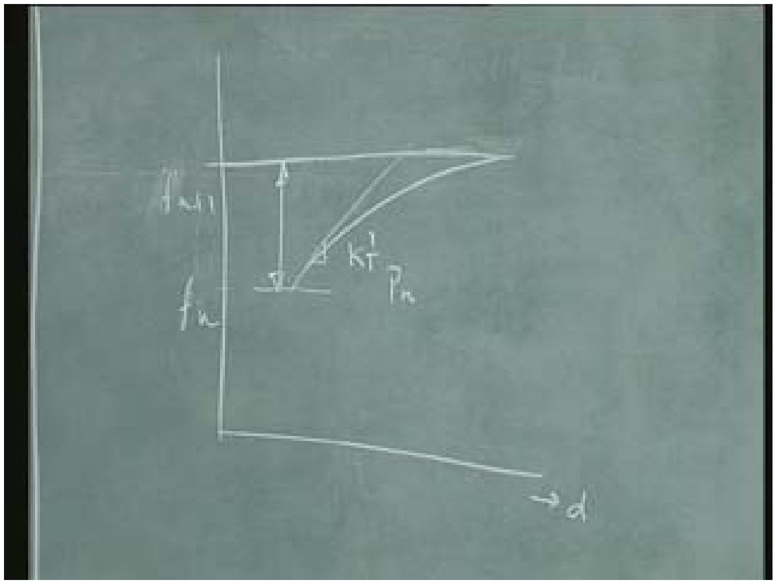


I am there like that and that is $P_n$; I have to go to $P_{n+1}$ or rather just a second, let me put it as $f_n$. I think last time I used so that there is no confusion. Let me put it as $f_{n+1}$. Though usually P is used for load, let us, let me say $f_n$ and $f_{n+1}$, so that will be clearer, because I think that is what we used last time. Now, so I know $f_n$ and what are the other things I know? I know how to calculate the tangent stiffness $K_T$ at this point; I know how to calculate the tangent stiffness $K_T$ at this point. Tangent stiffness is nothing but, what is it? Dow p by dow u, in which case, in this case it happens to be nothing but the slope of this curve. So, the initial slope of the curve at the start of my iterations or at the beginning of my journey towards n plus 1 is very well known. So that is the slope of the curve. Let me call me this as say $K_T$ and usually iterations are given here, the first iteration level; I know that.

Now, when I start this iteration what is the error? The error is, that is my error, because my internal forces have now equilibrated up to $f_n$. My internal forces if you remember, we had called this as $P_n$. Now, at that point of time when I start my journey towards n plus 1, what I know is $sigma_n$ and that $sigma_n$ gives rise to $P_n$.

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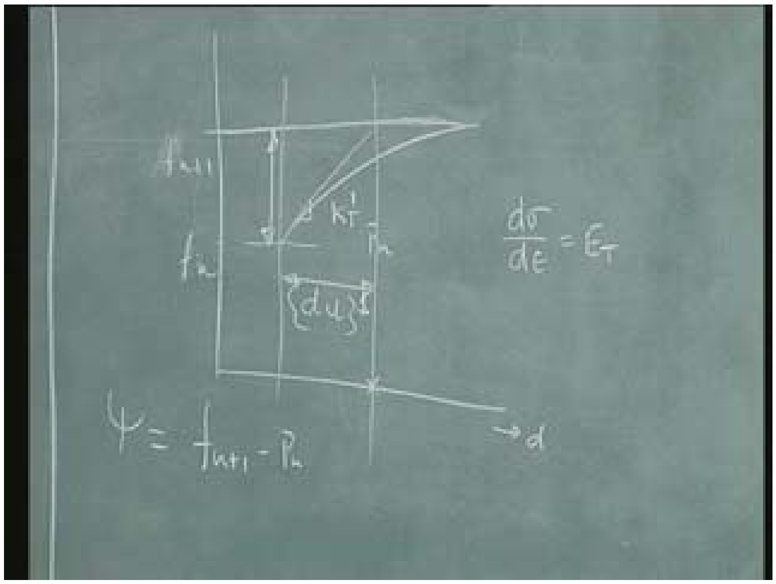


So, if I now calculate the internal forces at that point of time, what I will get is $P_n$ and my error if I remember and if you remember that it was the difference between the load that is applied and what you have got internally, so $f_{n+1}$ minus $P_n$. So, that is the error. So that would form the right hand side of my expression $K_T$ into delta u is equal to psi; right hand side of my expression for my first iteration. Is that clear? That means that I will solve this equation $K_T$ into delta u is equal to or du is equal to psi. So, I will get to a new d or new u. Let me call that as du, du at the end of my first, at the end of my first iteration. That means I am here. I calculate now epsilon with this. We will see the details in a minute. I calculate epsilon and from epsilon, I calculate sigma. $E_T$ into epsilon is equal to sigma. How I do that?

Note that d sigma by d epsilon is written as $E_T$ and I know how to calculate d sigma by d epsilon that is given. We will see an example in a minute. So, I know it, so I can calculate d sigma or in other words, I can calculate sigma. Knowing displacement, I can calculate sigma, knowing sigma I can calculate the internal forces or in other words, I will go to, this is the point that is given for this displacement; the force, the internal force is given by the point lying on the load deflection curve. So if this were to be the deflection, this will be the load. That is what it means. So, this will be my new internal forces.

Yes; so we had, in other words the question is what is the error now? Yes, correct; so, the last time we said that the error is to be calculated at the end of first iteration. I have not yet calculated the error now; I have not yet calculated. That is my next step. I used this error, first step error, because I am at $f_n$, what is the error, that I used. We can call this as zeroth error at the start; zeroth error at the start of the increment or sorry, iteration. Is that clear? So, use this. This is the error when I increase now my step. Is that clear?

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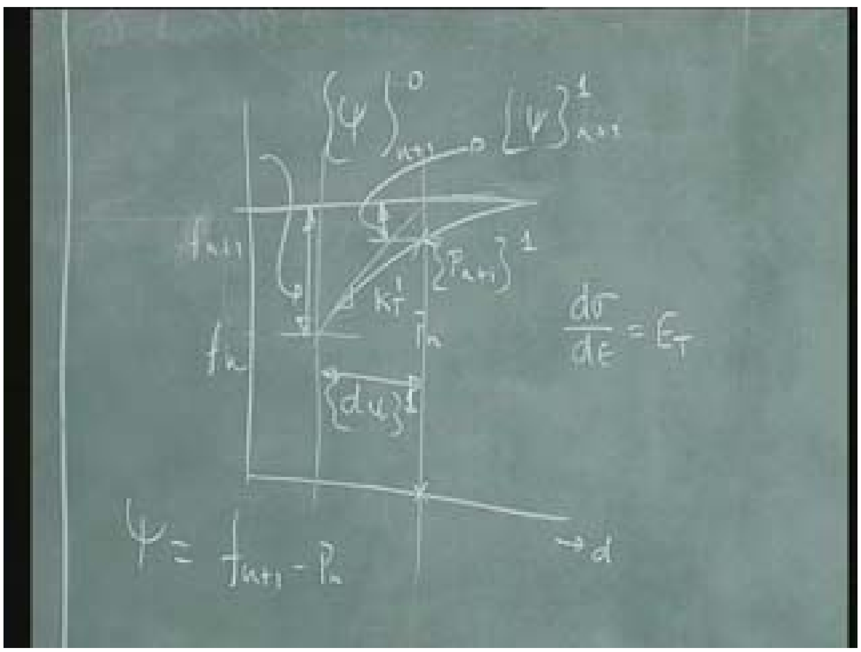


That is the, in other words, this error if you want you can call this as psi n plus 1 zero. That is the start which is equal to the error which I have to compensate. I have not yet calculated the error now. I have only calculated the internal force which is given by that point which, let me call this as say $P_{n+1}$; $P_{n+1}$ at the end of 1th iteration or $P_{n+1}$ if you want you can write it outside also. What is the error now? Error is between $f_{n+1}$ and $P_{n+1}$, that difference. So, the error has now reduced to this. That is my error at end of my first iteration, which I am going to use for second iteration as the input. So, that is my error. At the end of first iteration psi 1 say n plus 1. You can call this also as psi 2 n plus 1, I mean anyway but as long you understand it is fine. But, there may be a small confusion so that is why I am writing it as psi 1 n plus 1 meaning that that is the error at the end of first iteration. What do I do? I look at the error and see how large it is, how large it is I am still not happy with it. You know it looks like the error is quite large now. So, what do I do? I have to do one more iteration. What do I do? I find out again my good old friend the tangent stiffness matrix dow P by dow u at this point where I am standing now; that is at this point. So, what is that? That is nothing but the slope of this curve again.

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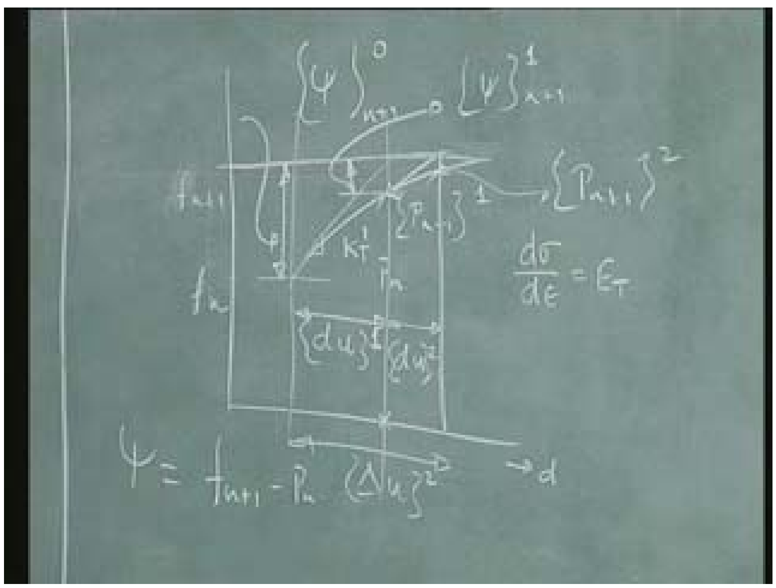

So, I calculate the slope there. Yes, exactly. The point it is calculated at different displacement point and hence the slope now will be different. That is called full Newton-Raphson method. So, for every iteration, you calculate the stiffness, what we call as tangent stiffness again. So, I come to that point, I calculate. That is what I made a comment if you remember in the last class itself, look at the enormity of the problem. $K_T$ in a large problem is like forming stiffness matrix. Again you form the stiffness matrix. That is again and again you are going to solve the same problem, it looks like that and so the time required for a non-linear problem is very high.

Your observation is very correct that we need to calculate the stiffness matrix again. So, with this error and this being the slope at that point, I again calculate my better approximation for the displacement. So, what do I do? That is my better approximation. Obviously this is the slope if this is the height and that is the width. So, I will get this, is the du 2. So, the sum of these two are, that is delta u at the end of the second increment. Is that clear? Now at that point, again I calculate what sigma is and then what are the internal forces? I hit the graph now at this point. Look, I have you know ……. the P, moving closer.

Again, I am here. So, that will be my internal forces at the end of the second iteration. That will be called as P n plus 1 2; very good, so 2. What is my error? Obviously, this is the error. See, the error has dropped. Initially it was so much, now this much and this much, still I am not satisfied. What do I do? I compare this with an allowable error which I say should be allowed say, 10 to the power of minus 6 into epsilon 1. We will discuss more about that later, but right now an allowable error which is say 10 to the power of minus 3 times, minus 4 times or minus 6 times, which is usually used of the original error, first error. The error cannot be an absolute quantity. The error that is allowed cannot be an absolute quantity, because it depends upon the units in which you are working. It is very important that it is a relative quantity, relative to this. I see it and I am not still happy with it. I go to one more iteration and I get that point.

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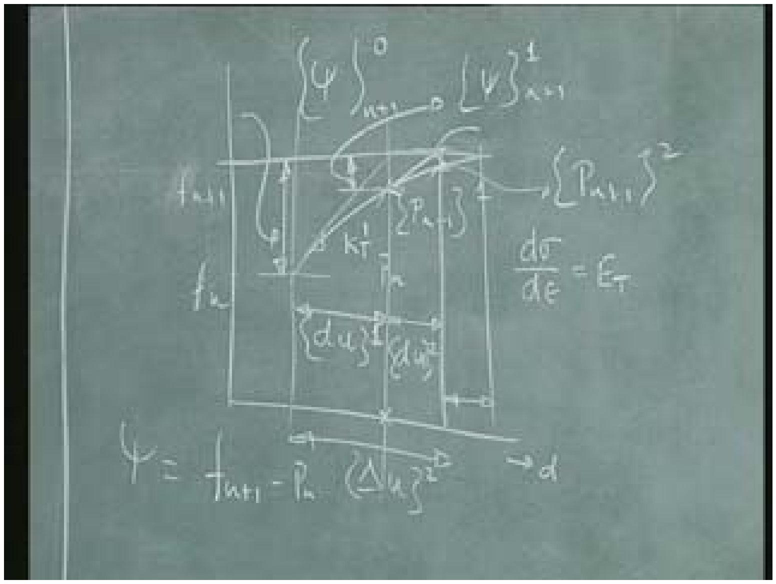
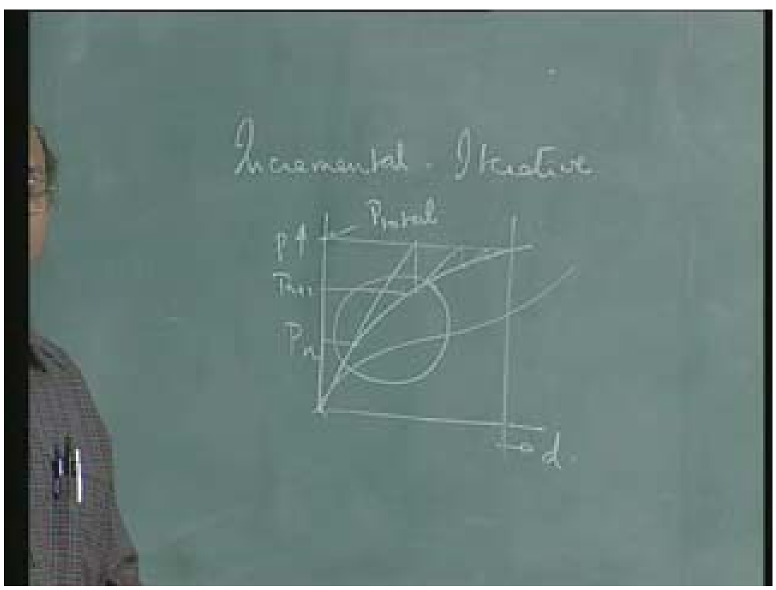


This is my better approximation of my displacement.Very good question. So, what would happen if I apply the full load? Only one stiffness, so what would, let us come back to this. What would happen if I apply the full load?




This curve is a nice curve, what I have drawn. So, you will start at this point and apply the whole load here. That means that you will find out the stiffness here, you will come back; stiffness here come back, stiffness here come back, stiffness here and so on. This may converge, but may be it may take more number of steps than that, but it may converge, there is a good chance. But, sometimes what happens is that if the curve is not so very nice, the curve  say, is something like that then you will not get convergence; you will not get convergence. You have to necessarily follow this curve by an incremental force approach. Is that clear? In other words, to answer your question, whatever be it, you know you can put the whole load here, do the problem, the procedure is the same. If you get convergence, you are lucky. If you do not get convergence, you have to come back and cut the steps and so on.

This is what most softwares . They put an increment, check whether there is convergence; if there is no convergence cut down the step, give smaller loads, see whether there is convergence and then if still there is no convergence they will cut down. So, usually many of the softwares also ask for a minimum time, what they call time step. I already said that time steps are pseudo times to carry the loads. What they mean by that statement that we want minimum time step, what they mean is what the minimum load is that we have to give. That is what they mean. Both of them are the same. You keep on cutting down the load and then you would see that things would follow the load deflection curve. Is it clear, any question?

Yeah, I am coming to that. What is the difference between, is there any other method for Newton-Raphson? There is, since I made a statement full Newton-Raphson method, there is also a method called modified Newton-Raphson method.

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There is a method called modified Newton-Raphson method. What is modified Newton-Raphson method? Here you do not calculate the stiffness for every iteration; you do not calculate it for every iteration. You calculate only once, may be at the start here and then use this itself in order to calculate further the error. This is an approximation.

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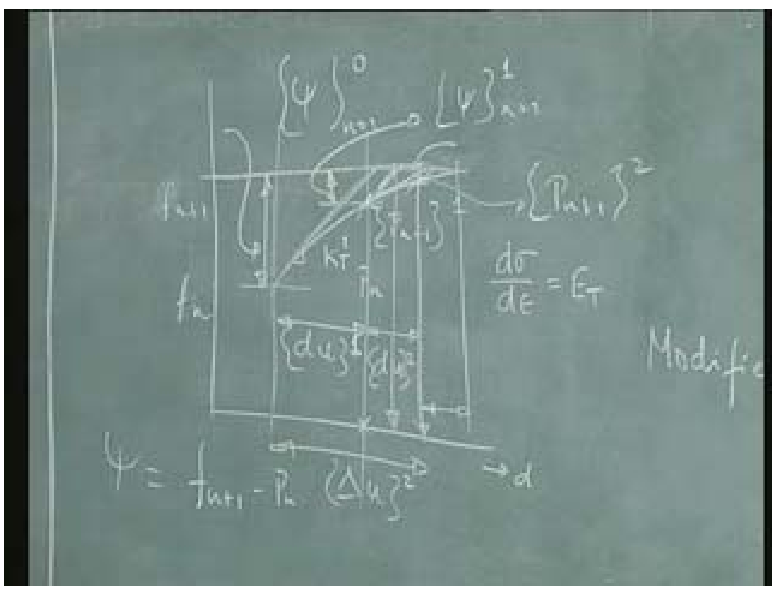


In other words instead of using this slope at each of those steps which I have merged to get to this point, I use only the first or the initial slope. Yes; this will be parallel lines. See, here the slope will be, you know, like this. This slope here will not be parallel to this, because that slope will be different. Now, if I use a modified Newton-Raphson method, then the slope will be parallel to that. The slope here, I should not say slope, but my tangent stiffness at that point would be parallel to it. Then, tangent stiffness here also would be parallel to it. Now, as it is you can see right now, when I use the tangent stiffness at that point to be parallel to this, that is, this and this are parallel, obviously, the next point I am going to hit is this point, instead of this point. In other words my claim or my climb towards the correct solution is going to be slightly less or may be more number of, correct, more number of iterations it may take. But now what is the advantage?

Yes, yes; you need not calculate the tangent stiffness again and again. So, once you calculate it, you can, if you want to do, whatever decomposition you want to do you do that to solve the equations, keep it ready. So, you need to keep substituting. You have to calculate $K_T$ only once and then decomposition you need to do only once and so you can solve much more faster or quickly when compared to the previous case. But the compromise is that you may have to go through, to a number of iterations. Is that clear? But, one bit of warning. It is not necessary that this method would give rise to convergence solution. It might also diverge; for this step, it might also diverge. T is, in other words, modified Newton-Raphson method is an excellent technique to reduce time under many circumstances, but has to also be used with a bit of a caution that your time step size also may be very small, because when you use Newton-Raphson and this depends upon the type of non-linearity that you are dealing with.

There are very many number of schemes; modified Newton-Raphson, Newton-Raphson is one of the methods. There are many number of schemes that are used, especially when the curves are crazy, you know; they are not as good curves like this. When the curves become or the non-linearies become quite complex, then I cannot use such schemes, I have to go for special techniques. Let us not worry about that right now, but I mean a small variant may be there. You yourself can think and say why not I calculate the tangents every two iterations? Yeah, you can do that. Instead of taking the tangent only at the beginning, I can take the tangent right at the beginning and then after third iteration, fifth and so on. That is also possible. These are gimmicks; no mathematical, mathematically rigorous, rigorous proofs are there. They are gimmicks to reduce the time and if you are, as long as you are able to reduce the time, well and good. With that background let us, yeah?

Yes, so that is why I said these are nice curves. What if the curve has an inflection point? What if the curve is something, we will get back to this curve.

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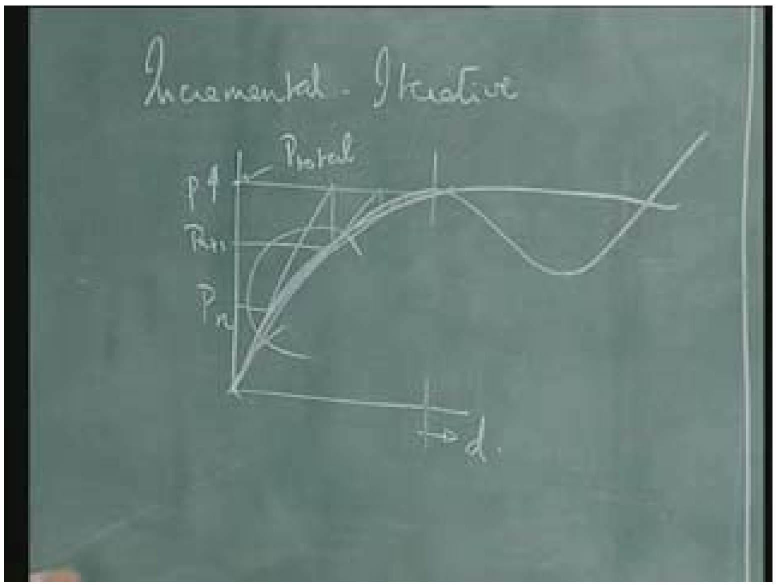
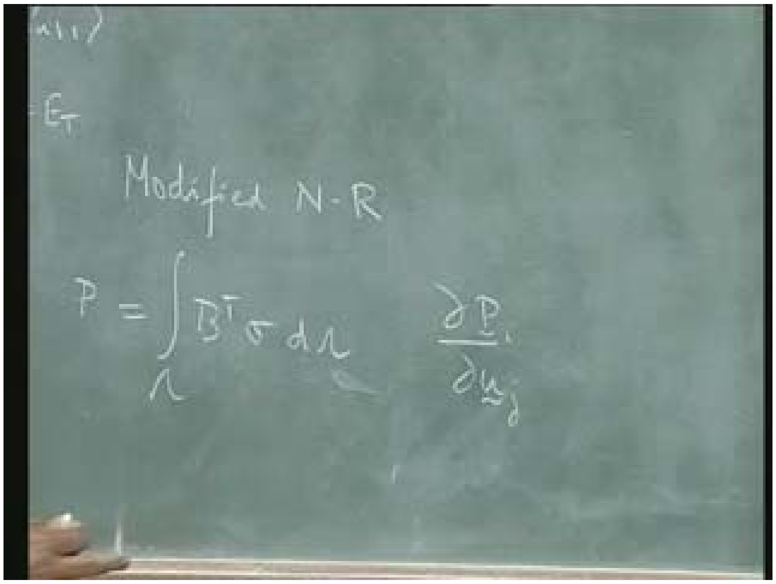
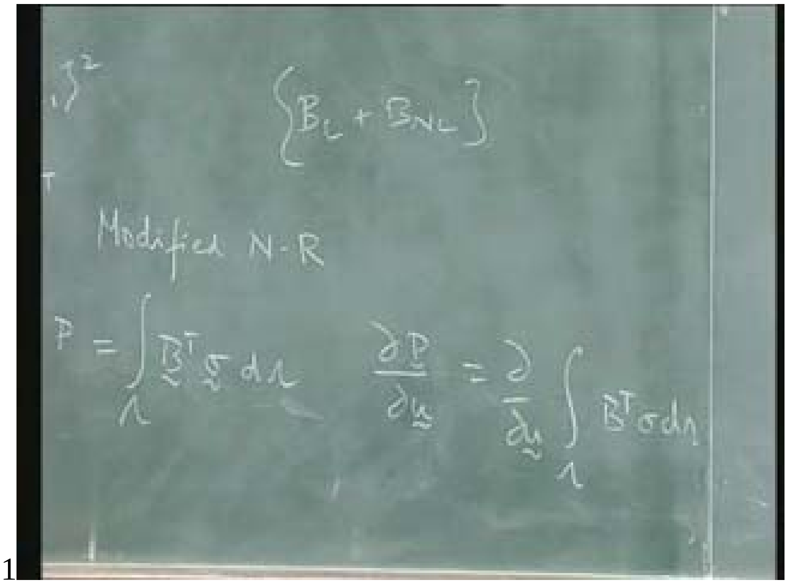



They are not nice curves like this. Say, the curve actually goes like this comes in and goes like that and what if the curve is like this and so on. Yes, these techniques, these require special techniques many times. Newton-Raphson may not work for many of these techniques and we will see that. If time permits, in this course we will cover it or at least we will see some references where these things are done. But, these are very special circumstances where there is a snap through and so on that would have happen where solution may jump or bifurcations and so on and solutions may jump from one point to the other and that gives rise to lot of troubles and that is why I made a statement that there are special techniques when things are not as nice as what we are seeing. These are within quotes “nice non-linearities”. Non-linearity itself is not very nice, but at least they are not that bad, they are nice non-linearities.

Before we go further, let us do a problem and before even that, let us look at this, our internal forces more carefully.




Remember that I said the internal force is written as B transpose sigma say d omega or dv and this would be call, though I am not going to give a very rigorous definition, I am not going to do virtual work, I just want to indicate a small thing before I go further and consolidate my whole, the whole procedure by doing a numerical problem. Look at that B transpose sigma. This is what is called as P. What we are interested in is what? Dow P by dow u; of course, these are vectors P and u. This results in a matrix $P_i$ $u_j$. k i j if I call this as k t i j, the resulting differentiation I call this as k t i j, then dow $P_i$ dow $u_j$ would define my k t i j and so on.

511 


We will see more details again later. Let us see what we have to do? Note that both of them are not scalars and that right now B matrix is our strain displacement matrix. Let us say it is a constant, strain displacement matrix and we do not have any other things inside it or in other words, we are looking at small deformations where my strain definition, which I had used in my earlier classes are valid.

In other words, I am right now putting a rider. Understand that rider that it is not that I can use that B which I had defined in my earlier classes that strain displacement matrix all the time. These are linear strain displacement. People call this as $B_L$. This B itself may undergo changes and B has to be written in terms of $B_L$ plus $B_{NL}$. People sometimes write this as $B_L$ plus $B_{NL}$. That is basically because the strain terms what you look at and strain displacement terms what you have are ones where only linear terms are considered, non-linear terms are not considered. So, we stick to this or in other words, for small deformation, not moving into finite deformations, for small deformation this $B_L$ is enough to define, though we may move into the non-linear regime of stress-strain curve.

Let us now look at, in a very simple fashion, small deformation. How this can be modified? It is very simple to modify a tangent stiffness. So, this would be dow by dow u integral B transpose sigma d omega. So, how would I write that?

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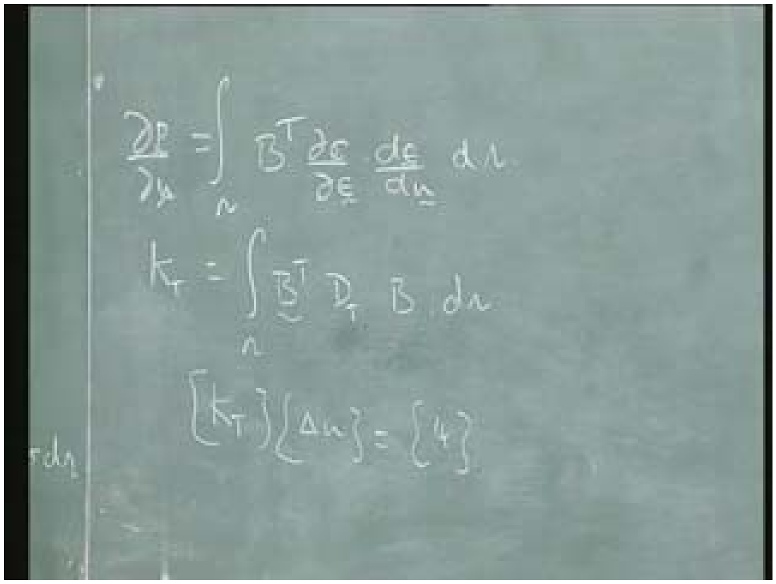

I would write this to be, so, dow P by dow u to be, B being, I said independent; it is B transpose, so what would be the next term? What  be the next term? Dow sigma by dow u, but sigma is actually a function of epsilon, epsilon is a function of u. So, you can write this as dow sigma by dow epsilon into, I am just using e chain rule d omega. So, d sigma by d epsilon is nothing but the slope of the stress strain curve, d sigma by d epsilon. People call this with different names. Suppose you are working with Abacus, sometimes this is called as Jacobian and so on. So this would be or a small modification of this may be called as Jacobian. Most important, this d sigma by d epsilon is the most important quantity that is useful in non-linear finite elements. Note that in a linear finite element, this guy straight away reduces to your well known stress strain relationship, I mean Hooke's law and it can be expressed in terms of e and u and so on; isotropic elasticity.

Look at that. This becomes very simple B transpose, of course, all of them are matrices, B transpose; let me call this d sigma by d epsilon say as $E_T$ or $D_T$, however you call it; if you had called previously E, you call it as E, previously D, you call it as D or $D_T$. I am going to interchangeably use, because people are familiar with either E or D matrix or E matrix, so, $D_T$. Then, d epsilon by du, what is this? Strain displacement matrix, so B. I have left out one of the things into, you remember in the previous case, into P is multiplied, dow P by dow u is multiplied by delta u. I have just left out that term. Remember what we had for our Newton-Raphson scheme? P plus dow P by dow u into delta u, yes and this is what we called as, if you remember what is that we called this as? A tangent stiffness matrix; so, this is what we called as $K_T$ or tangent stiffness matrix.

What is most important in this case is that we have to have this dow sigma by dow epsilon and this $D_T$ definition is what is important. In fact for any material, suppose you have new material and if you are able to define this dow sigma by dow epsilon and d sigma by d epsilon. Why I had put dow sigma by dow epsilon is basically because sigma can also be a function of say damage parameters and so on and so forth, in which case I cannot write it so very nicely. It is always important that you realize what you are writing and what are the assumptions that are involved? Sometimes this sigma may be a function of say epsilon and say a damage parameter d. Then, I have to extend these terms. Right now, I am not assuming all that. I am just saying that sigma is function of epsilon; most of the cases this is fine, sigma is a function of epsilon.

That is what is written as $D_T$ B d. Remember this $K_T$ into delta u was what we called as my error terms. That is what $K_T$ does. Is this clear? Maybe a few of these things we saw it in the first course itself, but it is always good to look at it more carefully and write down all the things before we start and move to further advanced material. Having said this, we will look at a small example or a small derivation rather, before we move to an example. Let us look at an elastoplastic system, just a one dimensional case to understand how things are before we move to a very concrete numerical example. I would like that you bring your calculator for the next class, so that this example can be worked out by you as we do it in the class.

Now, let us now look at these things for an elastoplastic system. Now, I am sure all of you know the stress strain curve quite thoroughly.

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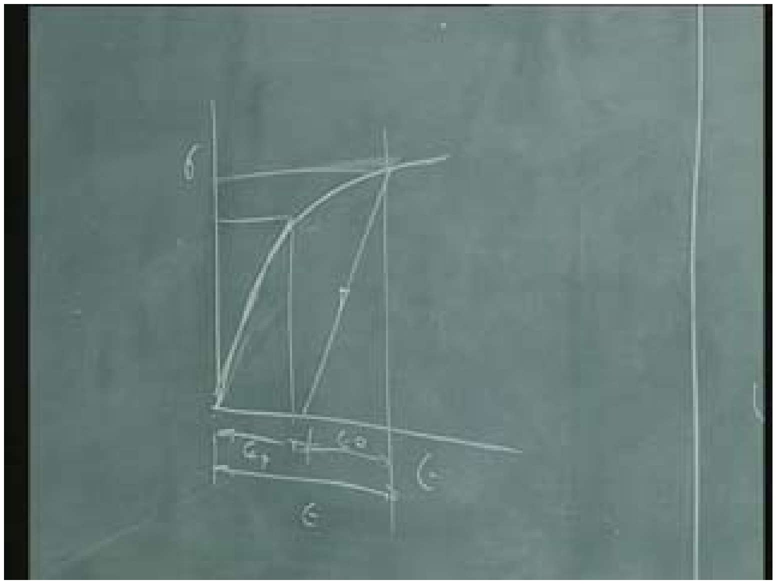

Few points which I am sure all of you remember that the stress strain part or the elastic part, I mean it is an exaggeration actually, it will not be so very away from the axis; usually these will be so very large that it will be very, very close to the axis, just for clarity, I am drawing it like that. So, this would be the stress-strain curve. Most of the times, why most of the times is basically because it is easier to understand it. This would also be treated as a bilinear system or in others words, the elastic part suppose that is the sigma y, the elastic part here may also be treated as a linear part; it is a linear part, I mean it is possible that you treat it like this. Anyway, does not matter whether you treat it as a bilinear system or you treat it as a pure linear followed by non-linear system.

There are few points in order if you look at the stress strain curve just to recapitulate that note that, when I unload from any point on the stress strain curve, from any point on the stress strain curve, I have an elastic part and a plastic part. If this is the strain from which I am unloading, f this is the strain from which I am unloading, then I have a recovered strain which I call this as epsilon$_e$ and a permanent strain which  call as epsilon$_p$; so, epsilon$_e$ plus epsilon$_p$. I think some of this we have done in previous course; it does not matter, for continuity, let us go ahead and do it.

The first thing that I want you to understand is that these strains consist of elastic and plastic and that the unloading is an elastic unloading and when I reload it, obviously you know that when I reload it from here, it goes and it catches the curve there and so, the yield point is now moved away from this position which I have called as the initial yield point to the current yield point which is at this point.

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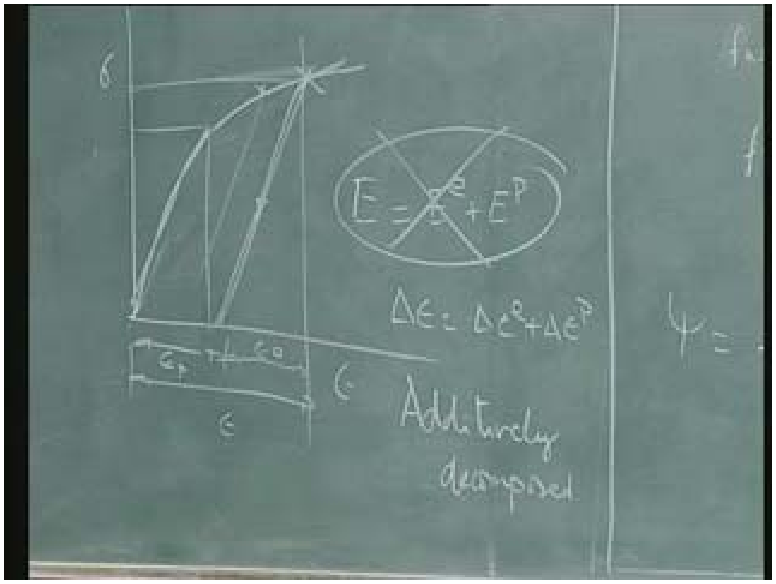

So, the most important point is that yield point is not a constant or yield strength is not a constant when we do an elastoplastic analysis. What is usually stated as yield strength of a material is I would say  yield; in fact, the stress strain curve is a locus of yield points; it is a locus of yield points. Every point here, after I move from the initial yield strength of the material are all yield points. Is that clear? So, I can, suppose I had unloaded it here, then that would be my yield point. So, it is a locus of yield point that we get and so, it defines the flow of the material, so called as flow curve as well. What are the other important points which I want you to remember is that this kind of nice splitting, epsilon is equal to epsilon e plus epsilon p (is it epsilon? it looks like E) is not valid, when the deformations are large.

Suppose I call the strains in finite deformations; these are infinitesimal deformations, smaller, when I use small e or epsilon, then we are talking about infinitesimal deformations. When I talk about finite deformations, I will use capital E and I cannot use a statement or put down a statement which says this statement is not right. So, though here I put this as an approximation, you will be surprised to know that in finite deformations I cannot put epsilon is equal to epsilon e plus epsilon p. To an approximation, I can say delta epsilon is equal to delta epsilon e plus delta epsilon p, but more correctly I would use some other notations for s. But, we are not into finite deformation, we are still trying to understand small deformations, keep that in mind, when we come to that state. Right now, let us see that I write now delta epsilon, let me write this as delta epsilon e plus delta epsilon p.

We call this yeah, yeah; delta epsilon is what I am going to use corresponding to delta u. So, it is the strain increment, delta epsilon is the strain increment. Why I have gone from epsilon to delta epsilon, because I am going to use an incremental iterative approach. Hence, I should know how to deal with delta epsilon. That is what I will get. So, delta epsilon is an increment in strain and that increment in strain can be classified into delta epsilon or can be additively decomposed. This is what the term additively decomposed into, that means that note this, that means that there is another way of decomposing. We will see that there is a multiplicative decomposition later in the course, towards the end of the course, but right now, I am using very specific word additively decomposed, delta epsilon is delta epsilon e plus delta epsilon p.

My whole idea if I have to form a tangent stiffness matrix, which is my ultimate goal, my whole idea is to do this and get what is called as the $D_T$ or $_T$ which we did or in other words d sigma by d epsilon. Is that clear? So, d sigma by d epsilon. Now, let me remove this. I hope you have understood these parts.

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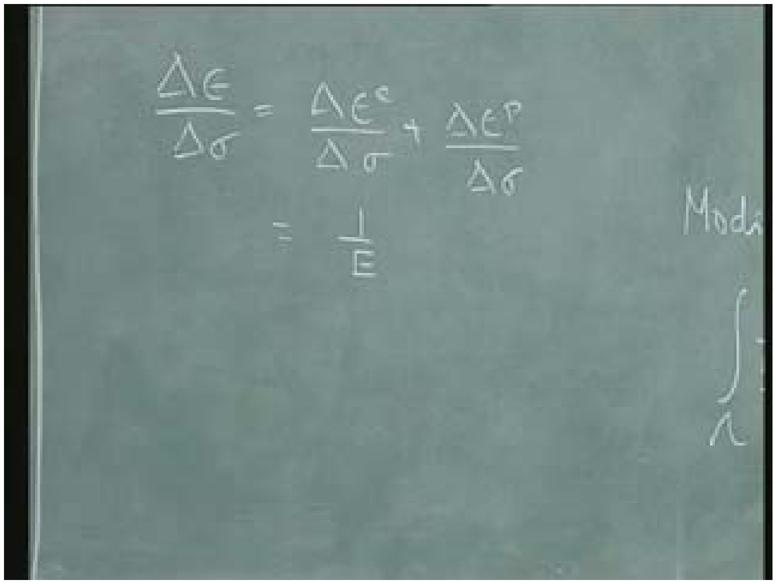


Now, let me divide this delta epsilon by d sigma or delta sigma so that write this as delta epsilon by delta sigma is equal to delta epsilon e by delta sigma plus delta epsilon P by delta sigma. Delta epsilon e by delta sigma is what? No; what is delta epsilon e by delta sigma or delta sigma by delta epsilon e, b? No; that is not at yield strength. What is delta? So, delta sigma by delta epsilon e, as you mean, right that is the 1 by E value. Yes, so this is the 1 by E value. The first thing that you have to notice is that E values are still valid. Obviously, from this graph you will see this is epsilon e. So, delta epsilon e, you can consider this as delta epsilon e. So, delta epsilon e because of the fact that I am going to unload through the elastic path, obviously the relationship between sigma and epsilon e even in the plastic region is through E, capital E and so what is this E?

Please note again the difference. This is my Young's modulus, this is not my strain. There is going to be a slight confusions on this, but the the context will tell you what we mean. This is not my finite strain; this is my Young's modulus. Please note that the slope of the unloading curve which is E for most circumstances are the same.

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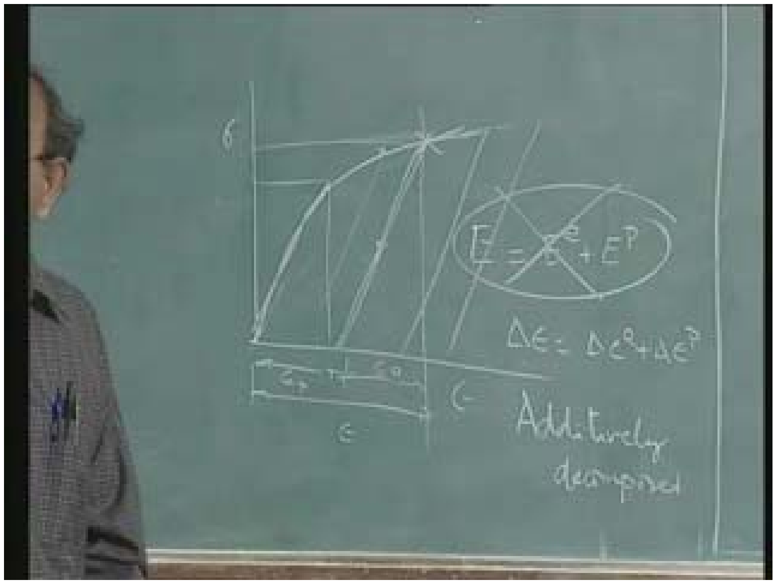

So this, if I unload it from here, I would be unloading it in a region or in a path which is parallel to the first one. When I  it from here, it would be again parallel. When would it change? When I have a damage parameter that is acting. These are again you know, look at the assumptions behind it. That is why non-linearity is very complex. A puritan may question; he may say that what are you doing? There may be damage, there may be void formation; there may be other micro cracks that are formed, so in which case e or E may change. You go into an entirely different regime, but for most of the applications again we abstract. So, we look at the assumption; that is where our engineering, you know, judgment is there. We look at the significance of the problem, of this assumption for that problem and say that okay fine; for this I am okay, if I neglect the damage and e or E is parallel. So, most of the times e or E is such that or e or E remains a constant or in other words the unloading curves are parallel. Is that clear?

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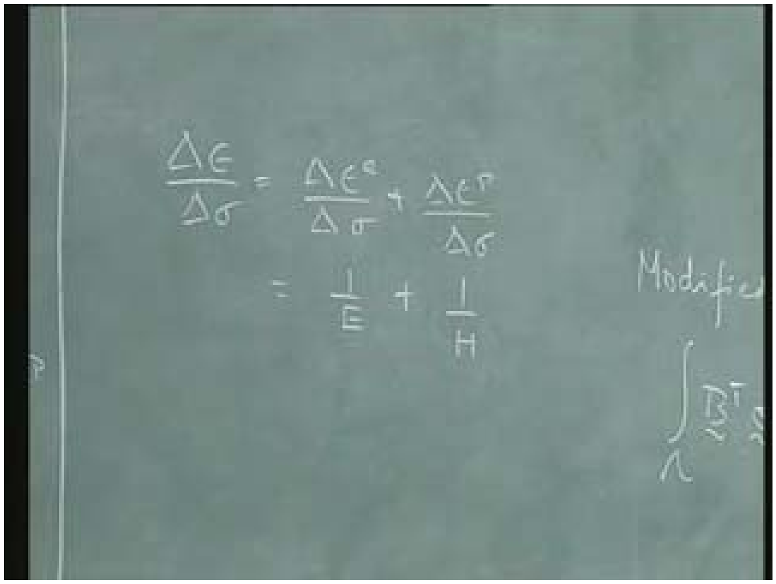


So, delta epsilon e by delta sigma is 1 by E plus this term. What is this term? These terms are called, this term is called by different names by different people. Interestingly, metallurgists are very interested in that kind of terms delta sigma by delta epsilon P; slope of the stress plastic strain curve. They would call this as plastic modulus. Many of them call this as plastic modulus and use the letter H to denote delta sigma by delta epsilon p. They would either put this as 1 by H or some people put this H prime as well. L now just stick to 1 by H, where H is equal to delta sigma by delta epsilon p.

Note that delta sigma by delta epsilon is the, what we call as tangent modulus; tangent modulus, which results in that $D_T$ term, $D_T$ is a matrix which has come to this place.

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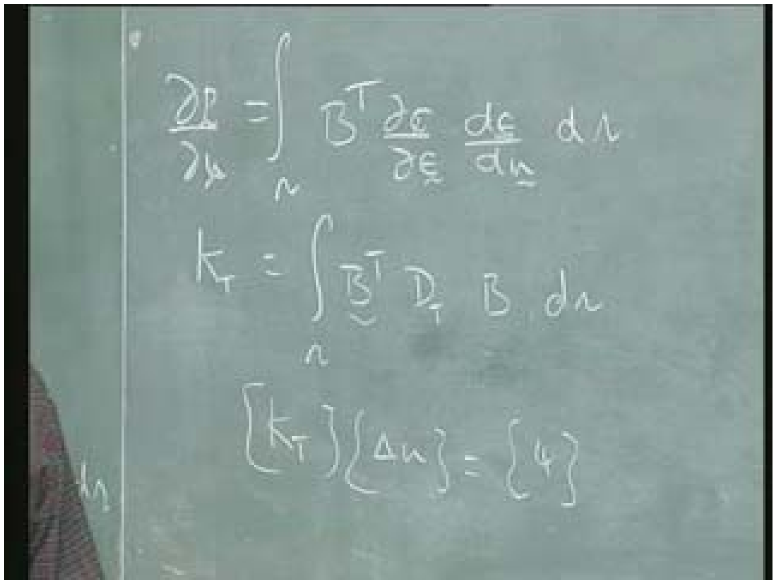
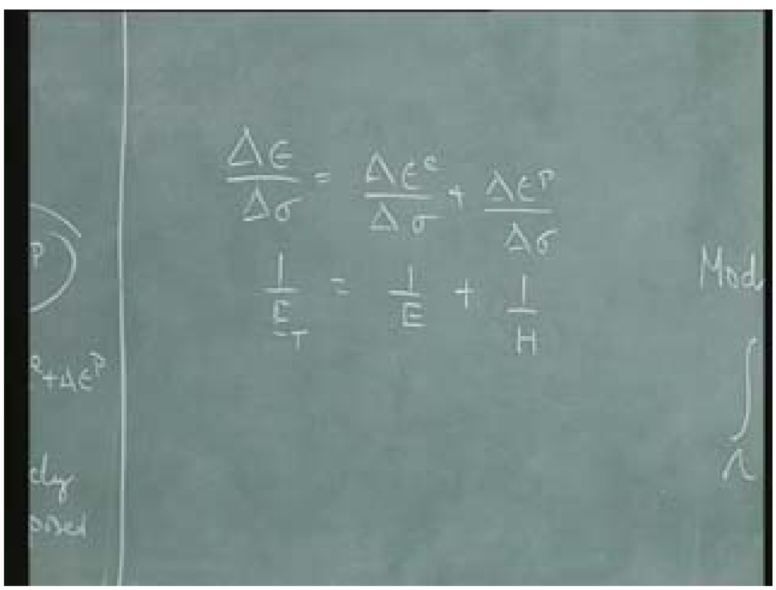


Note that this $D_T$ is a matrix. That is what we are going to derive now and that $D_T$ has d sigma by d epsilon and from a material point of view, though this is a matrix from a material point of view, what goes into $D_T$ is d sigma by d epsilon, let me call that as tangent modulus. This is tangent stiffness; that is tangent modulus which I call as $E_T$.



So 1 by $E_T$ is what is called tangent modulus. Yeah, $D_T$ is the matrix from $E_T$. Actually B is a matrix, $D_T$ is matrix and B is a matrix. As a material Like you have E matrix; D matrix and you have corresponding E to it. So, E divided by 1 minus 2 nu into such and such say, for example you would have written. That is what we mean by $D_T$ and the material property which enters into $D_T$ is what I call as $E_T$. Note again one more important thing that unlike my good old friend E, $E_T$ may not be a constant. $E_T$ may not be a constant. That is what we are going to see now and that this also, this part also may not be a constant.

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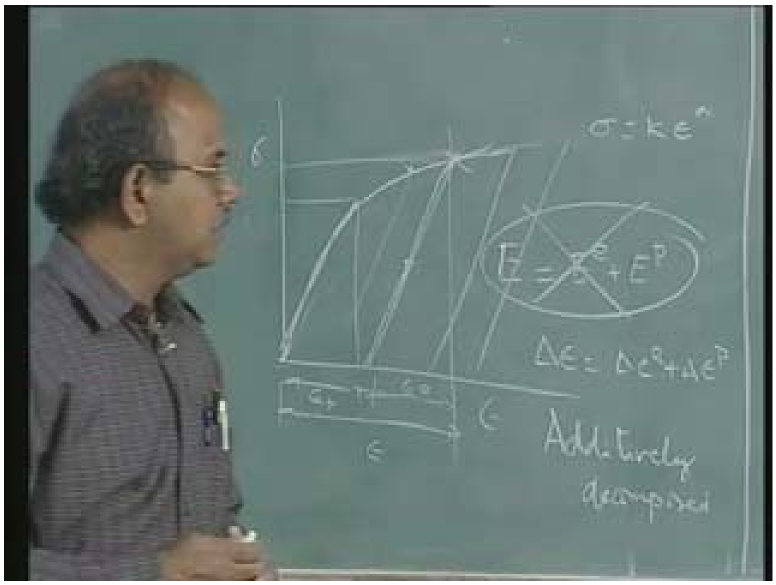


So, that is dictated by the curve here. If it happens to be bilinear, then I will have only two values. If it is not bilinear, if the curve is a non-linear curve expressed, for example, in terms of sigma is equal to K epsilon power n or something like that, then d sigma by d epsilon would not be the same. Is that clear?  would not be the same for the analysis.

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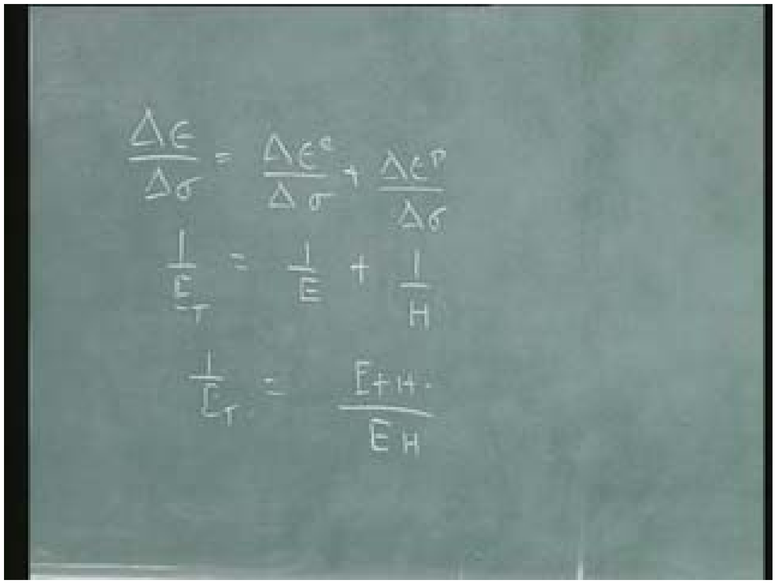

Now let us remove this reciprocal thing, so that I can write this as 1 by $E_T$ is equal to E plus H divided by EH, then let me add E squared, subtract E squared to, you will understand why I am doing that in a minute; subtract, so E plus H plus E squared minus E squared, just add and subtract that so that this can be written as E into yeah, let us just do , one more step to it, so E H plus, so that is what is the, let us look at this carefully. So E H, one second before we go further E H, 1 by $E_T$ is equal to EH divided by E plus H.

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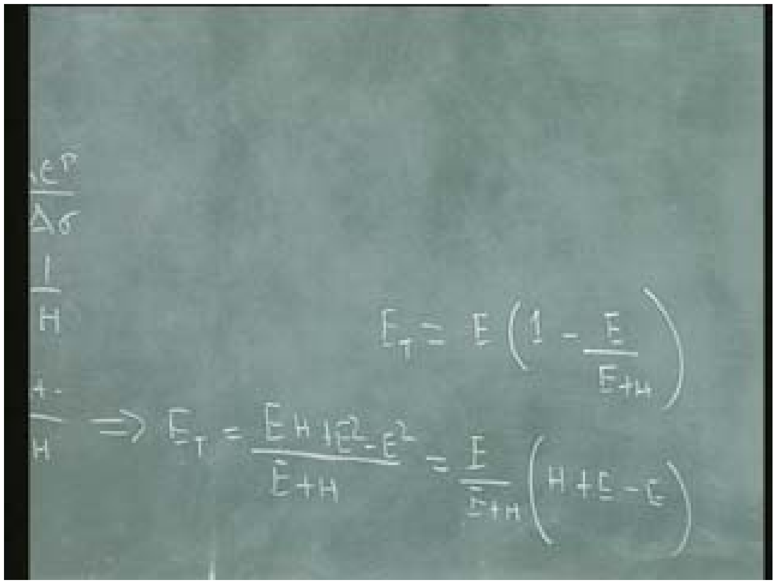


Let me write this in a slightly different fashion, so that from here we will go to $E_T$, so that it will be easier. E H divided by E plus H. I am, I am going to do a small manipulation with some purpose. So, now let me add E squared and minus E squared. So, that is equal to E divided by E plus H into H, so, the first term H here plus E minus E. That is what you get. Now, this can be written again, in a slightly modified fashion I am going to write it here. This can be written as E into 1 minus E divided by E plus H. Actually, I did this small jugglery to show you one important factor. I mean, same thing I could have retained this itself, but I just want to show you a small factor here that is why I did that.

Now, note this. So, as long as my E alone was there or in other words my body did not go to plastic strain, I did not have this term here. I had E alone. Now, when my body went into the plastic region, in other words, when H comes into picture, when H comes into picture, then you see that my body stiffness is going to drop by that term, by that term. My ,body stiffness is going to drop. The one thing that has happened because of plastic strain is that the stiffness of the body keeps dropping and that is what you see as the slope of this stress stain curve.

We will continue with this in the next class and see how to convert the or use $E_T$ in order to write down $K_T$ or the stiffness matrix. You can retain E H by E plus H itself; it does not matter, you can do that. Anyway, I need only the $E_T$. In other words, the lesson is that $E_T$ is derived from E and H. Even if H is not given, it doe o matter in a problem; $E_T$ is just d sigma by d epsilon. If it is expressed as K into epsilon power n, you can write this as, d sigma by d epsilon to be n K epsilon power n minus 1 and so on. So, in other words, $E_T$ is just a function of the strain as well. We will see in the next class.