Advanced Finite Element Analysis

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Lecture - 21

Yeah, let us continue with what we were doing in the last class. First let us clarify certain doubts which were asked at the end, so, may be things are clear. First is that there is a confusion on that e.

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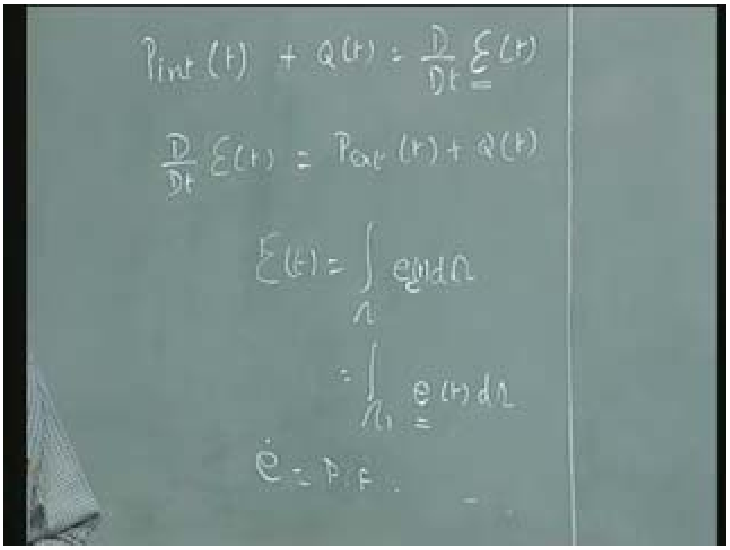

$$P_{int}(t) + Q(t) = \frac{D}{Dt}\underline{\underline{\mathcal{E}}}(t)$$

$$\frac{D}{Dt}\mathcal{E}(t) = P_{ext}(t) + Q(t)$$

$$\mathcal{E}(t) = \int_{\Omega} e(t)\,d\Omega$$

$$= \int_{\Omega_0} \underline{\underline{e}}(t)\,d\Omega$$

$$\dot{e} = P : F$$

Let me change that to script e, so that you do not have confusion with our green strain, green, so that you are clear that the script e is what we call as the internal energy. That is one; so, that change you can make and the other one is, again there is with respect to what we defined as small e. That, there again we will make a small change. For a current, we had used the same e, saying that the situations will tell you what it is, but we will not do it. So, we will use e_c for the current specific internal work and internal energy and then just the e for the reference configurations, when we refer to reference configuration, so that our last equation e dot is equal to P colon F dot, all those things are valid. So, we will make just these notational changes, is that clear, so that things are easier to understand. Though the situation would have warranted it to say what it is, but we will maintain different notations. One of the major problems in continuum mechanics is notation. You are going to have so many variables, people use all sorts of notations, yeah, that is the thing; so, anyway we will do that.

The next question which I was asked, I said I will answer today, is that now look at these two equations. One of the questions is that does it mean P internal is a mechanical work, internal work is equal to P external. Yes, as far as the mechanical work is concerned P internal is equal to P external. Q indicates the thermal work; Q indicates the thermal work which means that it indicates it has two parts to it. One is the internal heat generation and the other is the flux that crosses across the boundary, so, these are the two things; we had split it up, please remember that.

Point number 2 - if I have P internal, say for example, in plasticity, in plasticity we have a condition where the stress and the work done by the stress or the rate of work done by the stress that is sigma, remember, we had written this as sigma colon D, that gets converted into heat. Is it that in that case P external is equal to Q? No, because this P external goes to increase this P internal. It is not that when I apply a load in a plasticity problem, say for example, forging problem, it is not that that force does not raise the stress and goes directly to heat the material, it is not like that. It is the dissipation which results in some sort of a source term which is present all over the material and that gives rise to the heat. So, they are two different things, so, that goes as a source term, so that dissipation so they are, they are to certain extent decouple, if you look at it, rather. In other words, if there is no dissipation you will not have any heat that is generated. We are going to see that later in the course. Is there any other question?

Yeah, say, what? Here I just made a notational change. Yes, this, this instead of, yeah, this one and this, I think we had used the same e. I would say now that we will put e_c for the current configuration and e for the reference configuration. So, D by Dt of e which is now in the material co-ordinate will directly yield you small e dot. So, there is not going to be any change. I hope everything is in order. So, now we will get to what is called as the second law of thermodynamics and we have just started with entropy principle. We will follow these symbols like c and without c to indicate the current and the reference co-ordinates terms.

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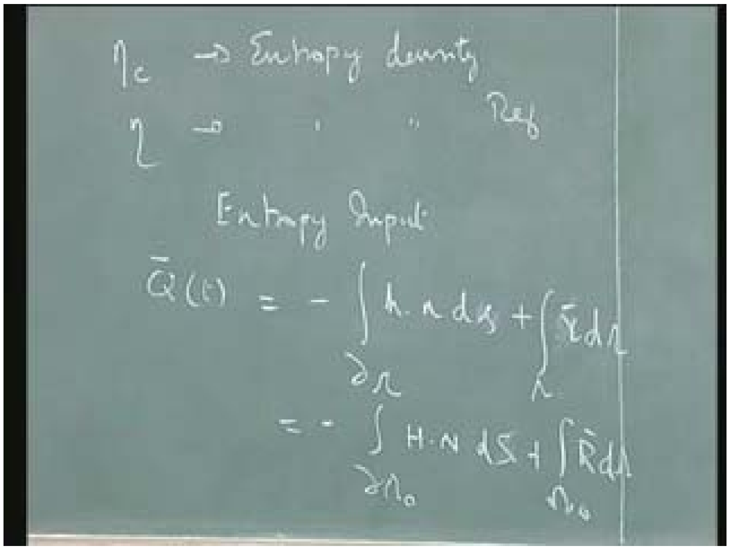

$\eta_c \rightarrow$ Entropy density

$\eta \rightarrow$ " " Ref

Entropy Input

$$\bar{Q}(t) = -\int_{\partial\Omega} h\cdot n\, ds + \int_{\Omega} \bar{r}\, d\Omega$$

$$= -\int_{\partial\Omega_0} H\cdot N\, dS + \int_{\Omega_0} \bar{R}\, d\Omega$$

Let me now define what is called as eta$_c$, the entropy per unit volume or you can say entropy density at the current configuration. Unit element volume, so, that is called as the entropy density and let eta be the entropy density in the reference configuration, in the reference configuration. Now, we can also talk about entropy input, entropy input. Entropy input is again due to two reasons. What are they? The entropy can be, input can be through an entropy flux, which enters at the surface as well as internal generation of entropy. These two terms, say for example, let me call that as Q bar t, I will tell you, I will, we will see how, why I am putting a bar there that show so much of a close relationship between temperature or, sorry, thermal and the entropy parts of it. This is equal to the corresponding terms which we had used for the thermal parts will now go into here.

One is we had used minus, if I remember right, this dow omega. In the previous case we had used it as q n, so, we will use that as, say, h n d omega plus the internal entropy generated, we had used Q there or ….. what is that I used? r, right, r; yes, so, we will use that same r, r bar d omega. This is at the current configuration. The reference configuration can be written as minus dow omega$_0$, now capital H dot capital N d omega, sorry, it should not be d omega ds and d capital S, we will retain that S there, plus omega R bar d omega or dv. I said we will use this interchangeably, omega and v, just to get familiarised with, as I said, notations are very important.

Now, we will give similar terms or similar names for small h and capital H. Small h is always associated with Cauchy entropy flux and capital H which is at the reference configuration is called as the Piola-Kirchhoff entropy flux. So, whenever we refer to the reference configuration, we will talk about the Piola-Kirchhoff entropy flux.

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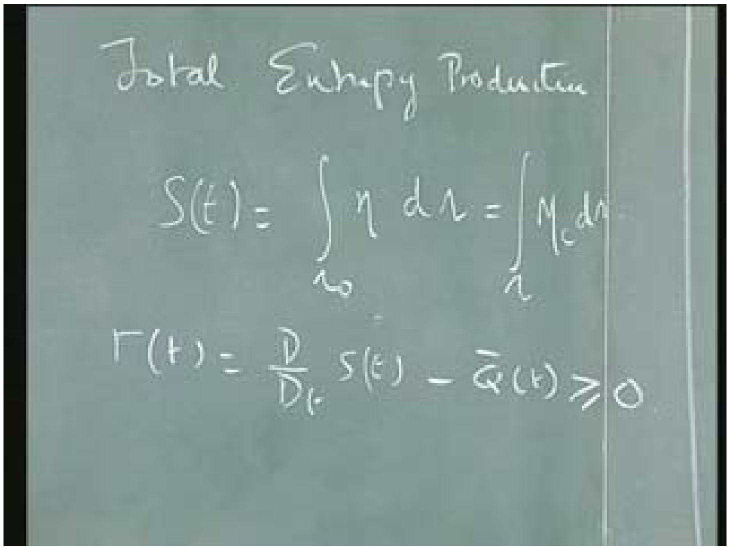

Total Entropy Production

$$S(t) = \int_{\Omega_0} \eta \, d\Omega = \int_{\Omega} \eta_c \, d\Omega$$

$$\Gamma(t) = \frac{D}{Dt} S(t) - \bar{Q}(t) \geqslant 0$$

Having said that we talk about what is called as the total entropy production as the difference between, we will call this as gamma t is equal to D by Dt, let me, before that let me define one more term which we would call as the total entropy, S(t). We will call that as, this is similar to script e that is internal energy that we get, as eta d omega or in other words, this is equal to eta$_c$ d omega depending upon whether you are the current that is the total entropy that is available, that is in the system similar to that of the internal energy. So, the total entropy production, which I would indicate as gamma t, is equal to D by Dt of S(t) minus whatever has been input into the system that is Q bar t minus Q bar t is, look at this very carefully, greater than or equal to zero. This is what is called as the entropy inequality principle, entropy inequality principle.

Now, look at the difference between what this equation says and what the previous equation says. The previous equation said that whatever you have given as an input was there to raise the levels of the internal variables. In other words, the previous equation was only a balance equation, balance of energy. The entropy production is not a balance equation. There is an inequality there; there is an inequality there. In other words, the entropy production is more than what has been given to it. This pushes us into the realm of what is called as irreversible thermodynamics. This is what we call as the, some people call this as second law or inequality principle; all of them are the same. This gives us the time tested notion that the entropy of an irreversible process keeps increasing, so, note that carefully. When does it become equal to zero?

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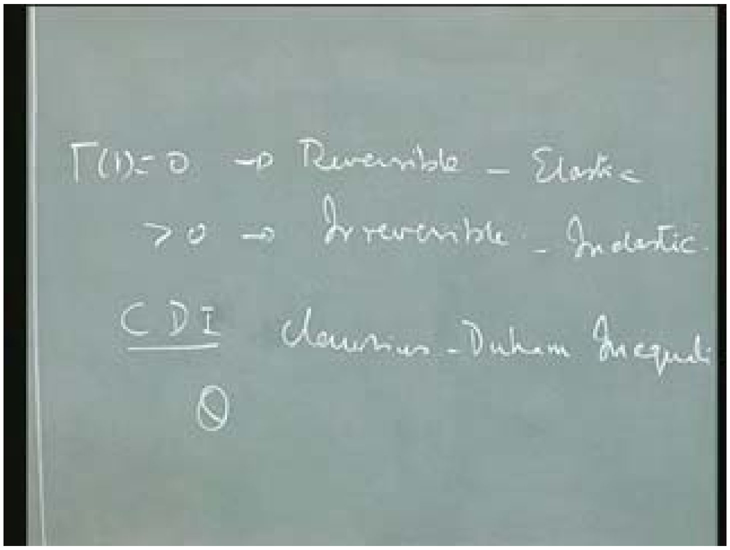

$\Gamma(t) = 0 \rightarrow$ Reversible – Elastic

$> 0 \rightarrow$ Irreversible – Inelastic

<u>CDI</u> Clausius – Duhem Inequali

$\oslash$

When it is equal to zero, in other words, when this is equal to zero, then we call this as a reversible, the process to be reversible or in other words, we talk about equilibrium thermodynamics or reversible thermodynamics. When inequality holds, then we talk about irreversible process or irreversible thermodynamics. Most of the process in nature, of course, is irreversible, but we make certain assumptions when we go along to say that certain process though is irreversible, we would, to make it easier to understand we would, call that as a reversible process.

Before we even go further, what is the practical aspect of this? That means that if a material is reversible that means that the material does not undergo any dissipation or in other words, it does not dissipate energy, so, you can get back what you have given to the material. This is typically what we are going to later define as the elastic material. In fact, look at the way we are coming to an elastic material. Elastic materials are ones which are basically reversible in nature. Reversible in what? Reversible, such that the entropy production, here this equation, happens to be zero. So, that is the strict elastic material. Inelastic materials - materials which are not elastic are ones where this entropy inequality holds good or this is greater than zero. Is that clear?

I want you to see the tie up between, say for example, second law and material behaviour. Most of these things are elastic materials and many of them are inelastic. Though, strictly speaking, strictly speaking we may argue that we are or solid mechanics uses irreversible thermodynamics, that is the first statement any book makes, but slowly you would see that as you turn the pages, they make assumptions that these processes are reversible, because it is much easier to treat with elastic material. So, it is this dissipation which we are talking about gets converted into what we call as heat. Is that clear? So, please do not talk about dissipation in an elastic material, very important. It is very important that we do not talk about dissipation in an elastic material.

Let us develop now, with this background, what is called as Clausius-Duham inequality. Clausius-Duham inequality takes off from this inequality principle of the second law and puts it down in a form which will be very useful to us later when we study material behaviour. So, Clausius-Duham inequality is also called as CDI; we will refer to this as Clausius-Duham inequality or CDI. In order to convert this particular equation into CDI, we introduce the concept of temperature. Look at that carefully; we are introducing the concept of temperature.

In fact, I am going to define what the temperature is. It is not that in many, in many of your earlier classes you would have defined temperature and from there you would have probably defined heat. But now what we are going to do in thermodynamics is that entropy is treated as a fundamental quantity and that the temperature is defined by means of entropy or from entropy.

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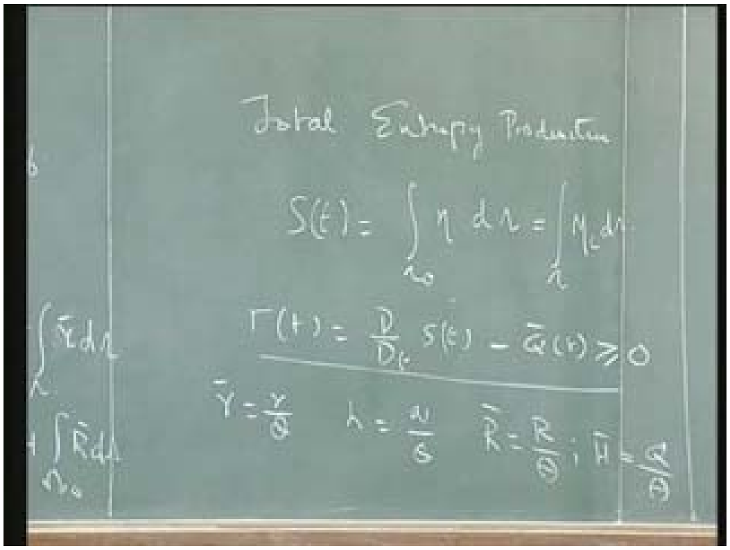

Total Entropy Production

$S(t) = \int_{\Omega_0} \eta \, d\Omega = \int_{\Omega} \eta_c \, d\Omega$

$\Gamma(t) = \frac{D}{Dt} S(t) - \bar{Q}(t) \geq 0$

$\bar{\gamma} = \frac{\gamma}{\theta} \quad h = \frac{q}{\theta} \quad \bar{R} = \frac{R}{\Theta} ; \bar{H} = \frac{Q}{\Theta}$

For example, the temperature is defined in such a fashion that R bar is equal to R by theta or in other words, theta gives the ratio of the heat source generated R, by R bar entropy that is internally generated or in other words, one minute, let me finish this or h is equal to q by theta and the same thing can be applied also for R bar is equal to R by theta and H bar is equal to Q by theta. So, theta, the temperature, of course, the temperature is defined in Kelvin, absolute temperature. Absolute temperature is defined like this as the ratio of the entropy flux, sorry, heat flux to entropy flux to heat generation to entropy generation whether it is in the current co-ordinate system or whether in the reference co-ordinate system. So, entropy is treated as a fundamental quantity and temperature is derived from entropy. But, there are other approaches to thermodynamics where temperature is treated as a fundamental quantity and the entropy is derived from temperature. But, in solid mechanics, thanks to Trusdal and his group came this kind of approach, which is followed throughout.

Now, let us work out the Clausius-Duham inequality. Yeah, is there any question, is that clear? So, theta is defined. How did I get it? Do not worry about that, because I am defining theta like this. Let me get back to my previous expression. That means that previous expression in the sense that this expression here and then replace some of the terms there with this equation for Q bar t and for S in terms of what we had defined as eta and so on. Please do that, let me see what you get. I will write that here.

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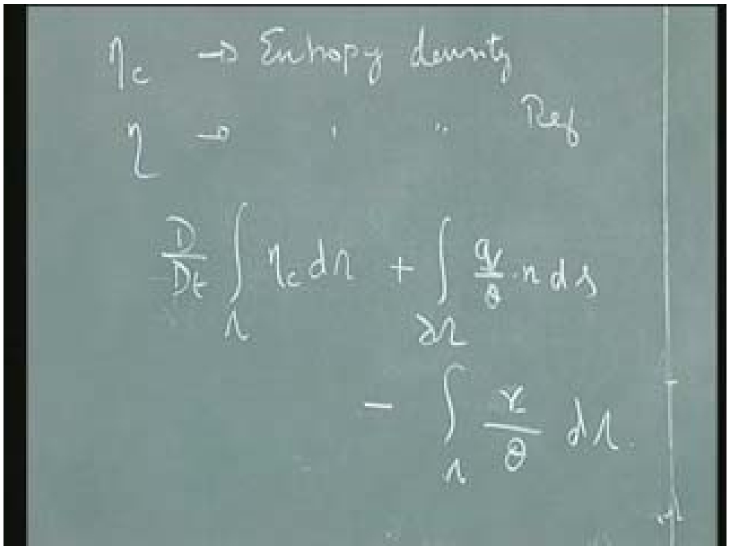

$\eta_c \rightarrow$ Entropy density

$\eta \rightarrow$ " " Ref

$$\frac{D}{Dt}\int_{\Omega} \eta_c \, d\Omega + \int_{\partial\Omega} \frac{q}{\theta} \cdot n \, ds - \int_{\Omega} \frac{r}{\theta} \, d\Omega$$

Now that I have defined all the quantities, the first term there, say for example, can be written as D by Dt of omega eta$_c$ d omega; that is the first term there, if you want to write it in current coordinate. Similarly, you can write in the reference coordinate also. The second term there is minus q bar t, minus q bar t, what was the first term? Minus of h by n, yeah, sorry h dot n. Now, what is h, what is h in terms of q? q by theta; so, the second term can be written as q by theta dot n ds and the third term is minus the internal energy term. We had r bar, if you remember we had r bar there. So, very logical there is nothing much there; r, let me use this r, r by theta into d omega. You can write this also in the current co-ordinate system. Just replace this by eta, replace q by capital Q. So, I will just do that.

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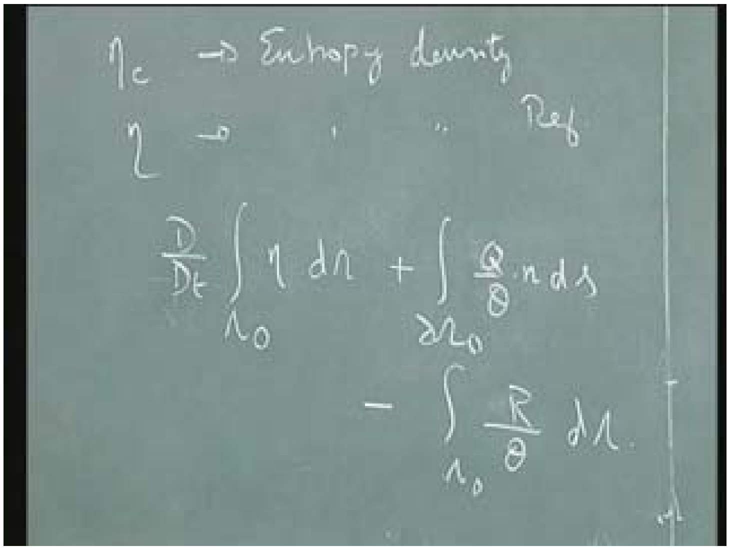

$\eta_c \rightarrow$ Entropy density

$\eta \rightarrow$ " " Ref

$$\frac{D}{Dt}\int_{\Omega_0} \eta \, d\Omega + \int_{\partial\Omega_0} \frac{Q}{\theta}\cdot n \, dS - \int_{\Omega_0} \frac{R}{\theta} \, d\Omega .$$

So, Q by theta d omega$_0$ and that is replaced, R.

Student: is it same theta or capital theta?

No, no; same theta. The point is the same at every element we are defining, so

Student: we are considering there is a change in r?

Yeah, yes; please note, I know the confusion. Please note that we are considering these equations at an instant of time. I made that statement yesterday also and making the statement today. Please note that we are considering the same element at the same state, whether that element is referred to the current or the element is referred to the reference configuration. What do I mean? Let me repeat that; it does not matter, let me repeat that particular part. It is not that we are considering the temperature when the body was in the reference state, no.

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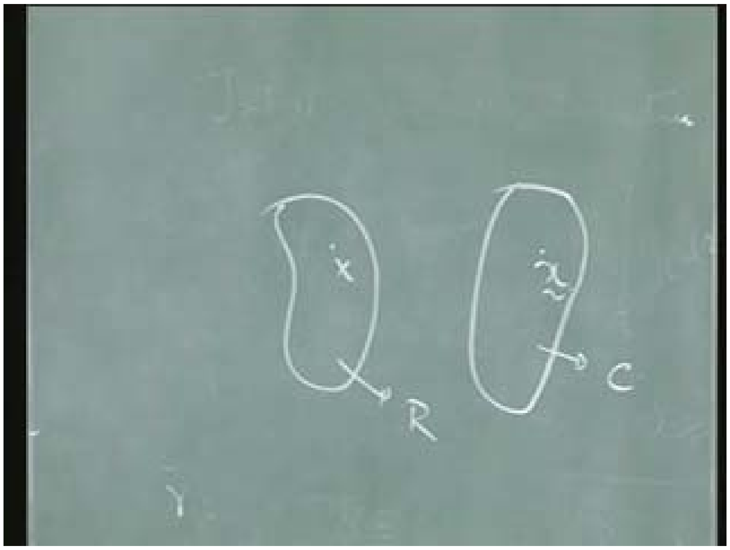

Suppose this is the body and the body after deformation becomes like that. So, this is my current and that is my reference. I am looking all the time at this body. So, when I take this point, the whole idea is whether I am calling this as x, small x dot or capital X. It is something like this. Say, suppose you have a name; you are Rama and everyday you wear different shirts. Today you wear a yellow shirt, tomorrow you wear a blue shirt and so on. So, today, I can call you the person with the blue shirt; that means your current configuration I am calling or tomorrow I can call you person with the red shirt. So, the point is the same, you are the same; but you have a reference name called Rama.

So today, if I say today, if you are wearing a particular shirt, whether I call you as Rama or whether I call you a person with yellow shirt, you are today what you are or what state you are in is what I am referring to. When I say Rama it was not that when you are born. If I say Rama answer your question or yellow shirt answer the question, it does not mean that Rama when you are born your name, you know, that is not, it does not mean anything like that. Similar fashion, when I say that I am in the current configuration, I am referring to the same body when it was there or when it is here; you know, that same point. But, the temperature or whatever, it is what is there today, now. It is not that theta becomes theta_0 or something else.

No, no. The theta is the same; the theta what we are referring to is the temperature at that point x.

Student: is that the small r and capital R may be different.

Yes; small r and capital R may be different, because the volume may be different, because per unit volume is what we are referring to, all those things may be different. No, no; but, temperature is referred to the same point. We are saying that the temperature is referred to the same point. That is why we said reference configuration or the current configuration. Let me take this thing. Let us look at what theta is. That is why we will not say, if there is confusion, we will not worry about both, we will only talk about theta; we will talk only about the reference configuration. So, it is not that. Please note that, we will follow this equation - capital eta Q and capital R. When I convert it, then I will get small q. This is basically because of the change in volume, change in surface, all those things that is what would induce these changes or else there is no change.

Now, let me apply divergence theorem to this quantity. How do I apply divergence theorem to this quantity?

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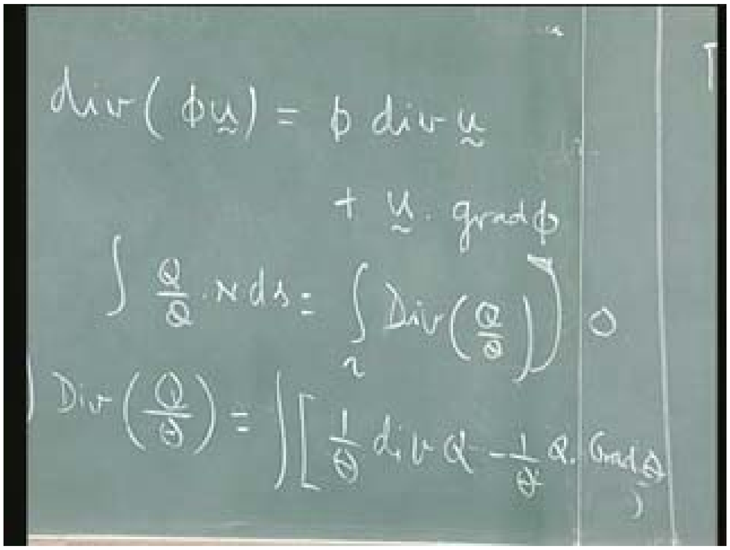

$$\mathrm{div}(\phi \underset{\sim}{u}) = \phi\, \mathrm{div}\, \underset{\sim}{u} + \underset{\sim}{u} \cdot \mathrm{grad}\, \phi$$

$$\int \frac{Q}{\theta} \cdot n\, ds = \int_{\Omega} \mathrm{Div}\left(\frac{Q}{\theta}\right) \bigcirc$$

$$\int \mathrm{Div}\left(\frac{Q}{\theta}\right) = \int \left[\frac{1}{\theta} \mathrm{div}\, Q - \frac{1}{\theta^2} Q \cdot \mathrm{Grad}\, \theta \right)$$

Let me write down an equation for divergence theorem of say, phi u. That means that this is like differentiating u v; u dv plus v du. Like that, suppose I have a scalar quantity phi multiplying a vector u, then how do I write this? This is equal to the, first is phi divergent of u plus, then I have to get to the first term. So, that is written as u dot Grad phi. So, this is how you write it. You can verify it by using simple rules of differentiation. Using that how do I write the divergence theorem of Q by theta dot n? Anyway, Q by theta dot n can be written as divergence of Q by theta and then use this expression in order to write the complete expressions for, that is in other words, can you write this?

In other words, sorry, I should write capital D, because divergence is with respect to the reference configuration. So, divergence of Q by theta, of course, that is a vector; Q is a vector, so, in this case u becomes Q, phi becomes 1 by theta and so, you can write that down using what we have. So, the first term now will become phi 1 by theta. So, this is equal to integral 1 by theta into divergence of u that means divergence of Q. Let us see how you write that down, the second one. So, I will have, theta I have to define; that is minus 1 by theta squared, then Q dot Grad theta. So, theta squared comes because of the differentiation of theta. Because, theta is 1 by theta, obviously 1 by theta I have to differentiate it, I mean, chain ruling is what you will get minus theta squared Q dot Grad theta.

Why did I do this? Very simple; this is a standard technique in continuum mechanics. I want to bring everything under one volume, so that, you remember this is what we did even in the equilibrium equation. When I do that, I would bring everything under one volume, so that I can write that what is under one volume or under one integration should be valid at every point throughout the body. You go back and refer to our equilibrium equation derivation; this is exactly what we did. Let us see what happens when I do that.

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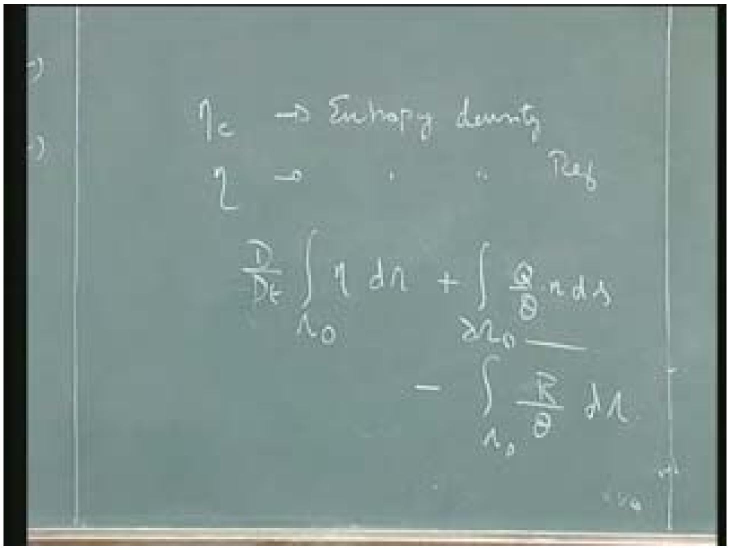

$\eta_c \rightarrow$ Entropy density

$\eta \rightarrow$ " " Ref

$$\frac{D}{Dt}\int_{\Omega_0} \eta \, d\Omega + \int_{\partial\Omega_0} \frac{Q}{\theta} \cdot n \, ds - \int_{\Omega_0} \frac{R}{\theta} \, d\Omega$$

In other words, here I am going to replace this by now omega$_0$ by all those expressions and let us see and now I am very happy with this because, I have to write only eta dot. Substituting now from here this equation, this equation into this equation here, now please write down what would be the ultimate equation we will get. Very straight forward; first term will become, what will be the first term? Eta dot, because eta is in the material derivative. So, I have no problem, D by Dt of eta will become eta dot.

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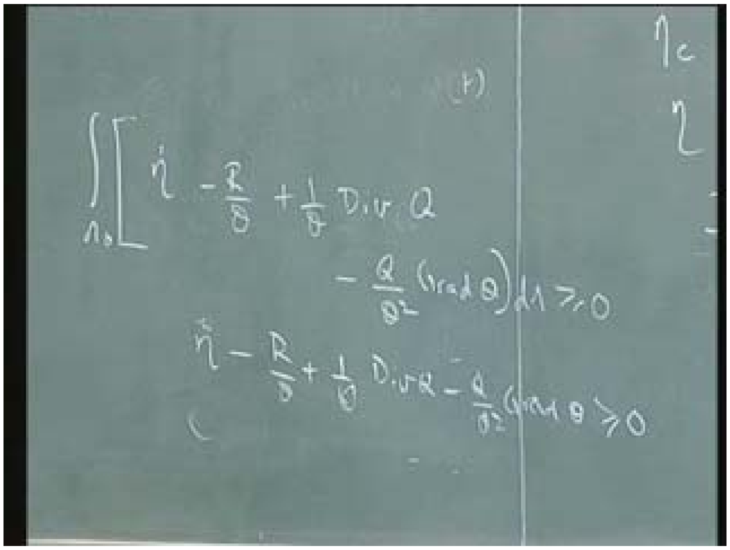

$$\int_{\Omega_0}\Big[\dot{\eta} - \frac{R}{\theta} + \frac{1}{\theta}\,\mathrm{Div}\,Q - \frac{Q}{\theta^2}(\mathrm{Grad}\,\theta)\Big]d\Omega \geq 0$$

$$\dot{\eta} - \frac{R}{\theta} + \frac{1}{\theta}\,\mathrm{Div}\,Q - \frac{Q}{\theta^2}\,\mathrm{Grad}\,\theta \geq 0$$

If you want, I can put this down in one integral completely under one heading or one say, bracket; the first term will be eta dot. Second, let us write that, minus R by theta; so, the third will be the first term there. That is this term here, which we had given. So, that will be plus 1 by theta divergence of Q minus Q by theta squared into Grad of theta, the whole thing omega, d omega. I hope I have not left out any term, yes. Please note that this is, this is N, capital N, because we had converted it into the current configuration, so, this should be what is the equation? Do not forget the equation, greater than or equal to zero. Now, develop a logic for this. The logic is very simple. Since this has to hold good for entire volume, I can take an arbitrary volume and this should be again valid and so, what is inside should be equal to zero at every point or greater than equal to zero rather, at every point, so that from here I can write eta dot minus R by theta plus 1 by theta divergence of Q minus Q by theta squared Grad of theta should be greater than or equal to zero.

Now, we had in the last course, may be even in the beginning of this course we had introduced a term called strain energy density function. We had called this as psi.

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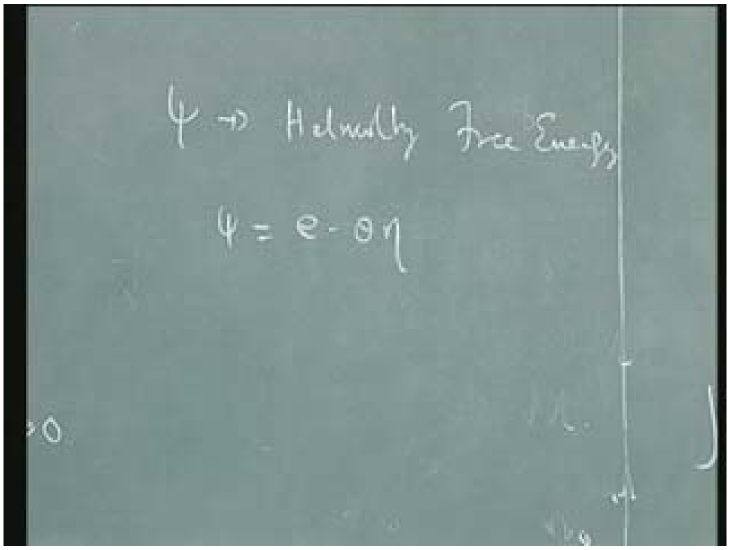

$\psi \rightarrow$ Helmoltz Free Energy

$$\psi = e - \theta\eta$$

Under the thermodynamics setting when we have entropy and temperatures and so on, the strain energy density function actually is replaced by what is called as Helmoltz free energy defined as, again, look at that word defined as, psi is equal to e minus theta eta. Helmoltz free energy is defined as like this or in other words, actually Helmoltz free energy is introduced in order to do a transformation of variables. These are, these are very common in many subjects including mechanics where you do a transformation of one variable to another variable. So, strictly speaking Helmoltz free energy is a transformation from one to the other. It is just, it is defined like this. Please do not try to start interpreting this. Like is it that kinetic energy has velocity, potential energy is, you know, somewhere there. If it is there, it is high. So, what is Helmoltz free energy? How do I, you know imagine this? You will not get anywhere. So, you please do not start interpreting every term that you put.

Yes, entropy to a certain extent you may, you may interpret. From your earlier classes you may say that it is just the order or disorder that is introduced, as disorder the entropy increases; you can have that. But, that kind of interpretation is okay to start with, but once they start putting down these equations it is very difficult to interpret every term; do not start interpreting every term. Many of the things that we do are more because of the mathematical ease of doing things. It is not just we are looking at every equation and every definition from a physical perspective, so, do not have that and then, now you substitute. Let us see, what you do? You substitute this into my expression, what I have.

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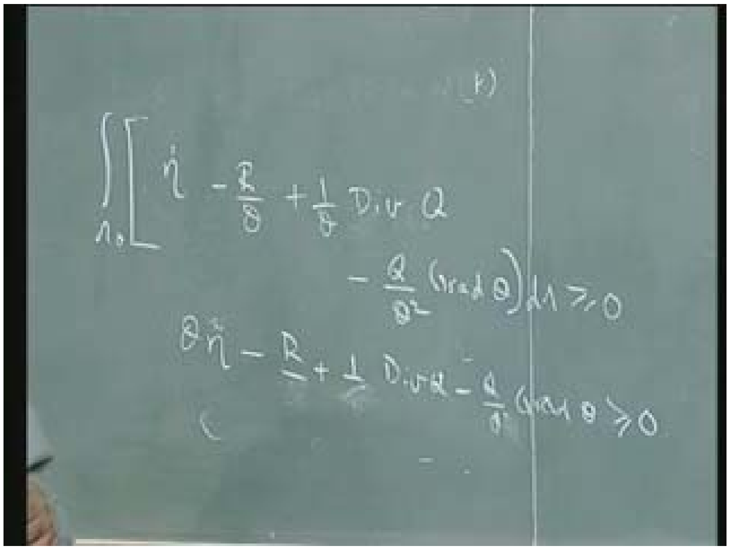

$$\int_{\Omega_0}\Big[\dot{\eta} - \frac{R}{\theta} + \frac{1}{\theta}\,\mathrm{Div}\,Q - \frac{Q}{\theta^2}(\mathrm{Grad}\,\theta)\Big]d\Omega \geq 0$$

$$\theta\dot{\eta} - R + \frac{1}{\theta}\,\mathrm{Div}\,Q - \frac{Q}{\theta^2}\,\mathrm{Grad}\,\theta \geq 0$$

One of the things I do before substituting is to multiply this equation by theta, so that theta eta dot I will remove that, I will remove this and I will remove this multiplied by theta; I had already defined that theta is an absolute temperature which means that theta is greater than zero. So, I am quite at ease when I just multiply it by theta. Let me see. I will give you exercise, two minutes, substitute in this, in this equation substitute this transformation and let us see what equation you get. This is just to make you familiar with this equation that I want to substitute.

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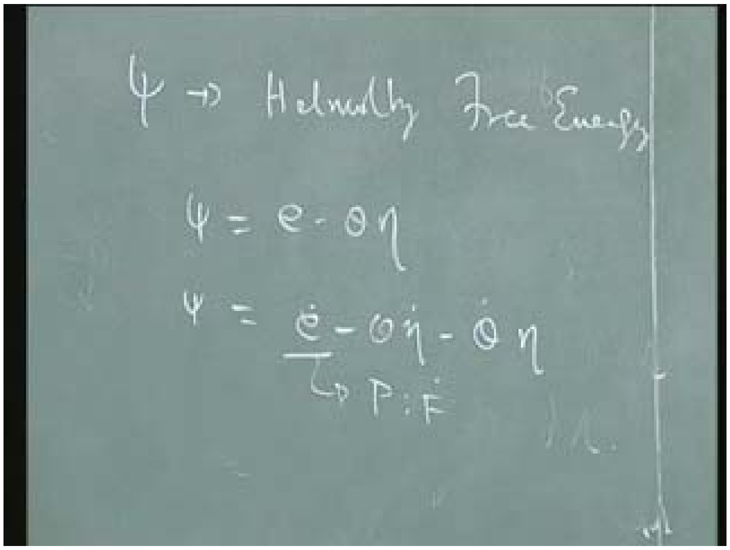

ψ → Helmholtz Free Energy

$$\psi = e - \theta\eta$$

$$\psi = \underbrace{\dot{e}}_{\hookrightarrow P:\dot{F}} - \theta\dot{\eta} - \dot{\theta}\eta$$

In other words, let me put this first psi dot is equal to eta dot minus theta eta dot minus theta dot eta. I want you to replace theta eta dot by means of psi and e dot. So, please substitute that. What is e dot? Simply, that it is reference configuration P colon F dot minus, yeah, but now we are only looking at the, if you look at, let me not put it now, I will do that in the next stage. Please, now substitute that; we will write it down in terms of e dot itself. Have you done that? Having done that, note that there is a minus R here plus divergence Q here, then substitute for P colon F dot. Yeah, shall I do it. Can you, can someone tell the result?

So, this theta eta dot, so, I will get psi dot, e dot will come to the other side minus e dot plus theta dot eta minus R plus divergent Q minus, one minute, let me finish this; theta, Grad theta greater than equal to zero. Yes, so, I have one term here, I have one term here, minus eta dot. This will be minus, am I right? Yeah, that we, no, no, but psi dot minus eta minus e dot, yeah correct, minus e dot, so, that term again comes the other side, so, this will become plus. Wait, wait, wait, wait, one second; let me do it from here.

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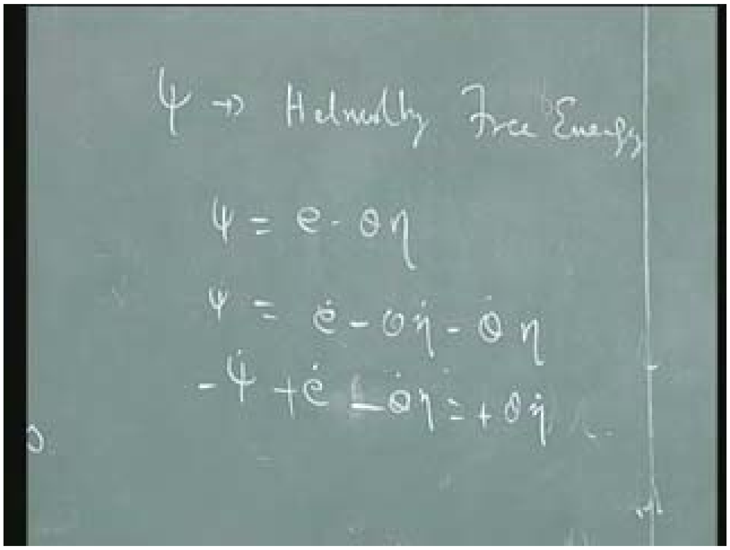

$\psi \rightarrow$ Helmoltz Free Energy

$\psi = e - \theta\eta$

$\dot{\psi} = \dot{e} - \theta\dot{\eta} - \dot{\theta}\eta$

$-\dot{\psi} + \dot{e} - \dot{\theta}\eta = +\theta\dot{\eta}$

So, that is psi dot minus eta dot minus, sorry, plus theta dot eta is equal to minus theta eta dot. This I want plus here, so, this will become minus, plus, minus; so, that, yeah correct, you are correct. So, I have to introduce that here.

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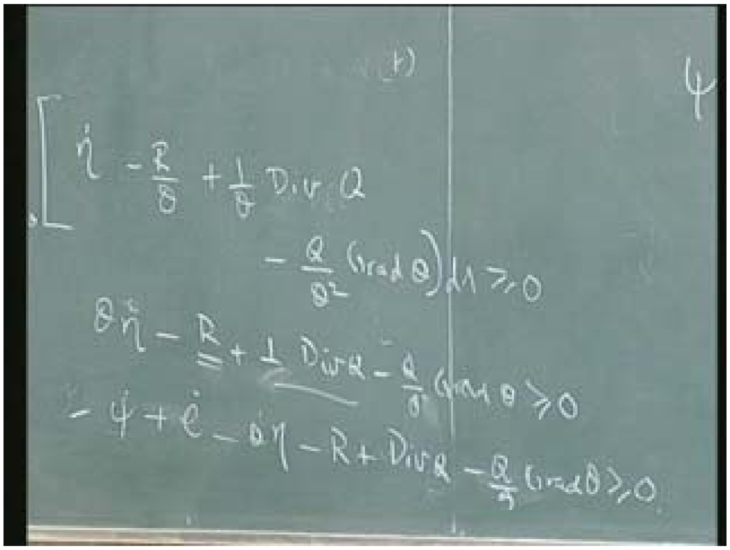

(+)

ψ

$\left[\dot{\eta} - \frac{R}{\theta} + \frac{1}{\theta} \operatorname{Div} Q - \frac{Q}{\theta^2} (\operatorname{Grad} \theta)\right] dA \geq 0$

$\theta\dot{\eta} - \underline{R} + \underline{1} \operatorname{Div} Q - \frac{Q}{\theta} \operatorname{Grad} \theta \geq 0$

$-\dot{\psi} + \dot{e} - \dot{\theta}\eta - R + \operatorname{Div} Q - \frac{Q}{\theta} (\operatorname{Grad} \theta) \geq 0$

So, minus psi dot plus e dot minus theta dot eta minus R plus divergent Q minus Q by theta, Grad theta greater than equal to zero. Now, substitute my previous expression for e dot P colon F dot and tell me what will be the result. Note that there are three terms here. This term, this term and this term - there are three terms which go into that e dot definition.

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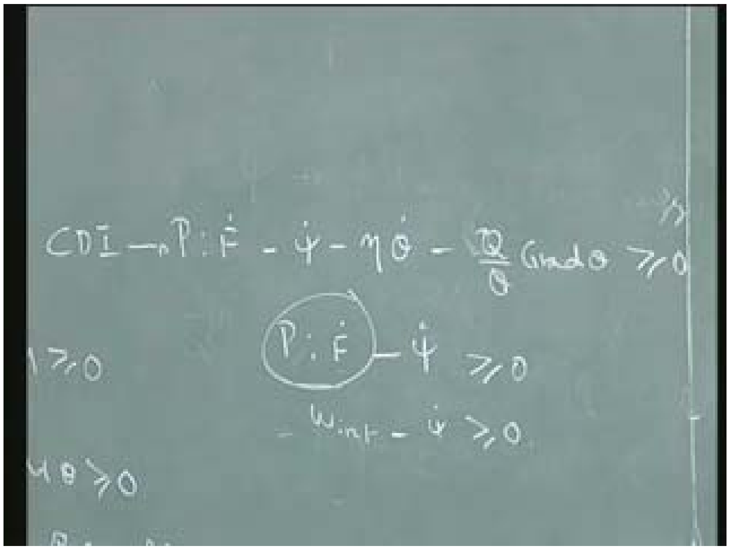

So, that will be P colon F dot minus psi dot, psi dot; then, one more term is there minus eta theta dot minus eta theta dot that is this term, divergence Q goes off, minus Q by theta Grad theta, minus Q by theta Grad theta is greater than equal to zero. This is a form of, most important form of what is called as Clausius-Duham inequality, form of Clausius-Duham inequality. Some of the text books may include rho here, density here in the definitions of eta, because please you have to be careful whether eta is defined in terms of per volume, unit volume or mass, in which case you may be introducing a rho or not introducing a rho. But nevertheless, since we had defined it by unit volume, so, this is the type of equations. This is the famous Clausius-Duham inequality.

We are going to use this repeatedly later, but we may make some small thing. We are not right now interested in temperature terms, so, we may remove whatever is there as far as the temperature that is theta dot and Grad theta is concerned. I mean these two terms we may not be interested, may not be; if you are interested, yes, you have to include that. Eta theta dot, this is theta dot eta theta dot or theta dot eta; you know, that is, does not make any difference. I know there are so many terms there, theta dot eta, eta dot theta, so, it is eta theta dot. When we are not interested in temperature, then what we have is only the first two terms F dot minus psi dot greater than equal to zero. In other words, this is the internal dissipation that you get. It is the internal dissipation that is or w internal; sometime people call this as w internal minus psi dot greater than equal to zero.

When it is equal to zero, then obviously we have the reversible process and when it is greater than zero we have the irreversible process. But, look at this term. I just want to comment on this a minute before we move to the next topic. Please note that P. What is P? First Piola-Kirchhoff stress is the work conjugate of F dot, F dot. So, P and F dot go together, so, you have to, we had done that before. You have to look at this carefully as to what terms that you use in your analysis. You cannot pick and choose your own terms and this is very important when we define what is called as the constitutive equations, the relationship between or in other words, the relationship between stress and strain. Is that clear? Fine.

With that background of the thermodynamic principles, we move over to the constitutive equations. But before we do that, let, let me just write down. I am not going to go into the details of the governing laws which we had already done in the first course, but may be we can summarise them a bit before we go into the details of the constitutive equations. But, I want to just finish this topic saying that any constitutive equation which you put down is very important that they follow the laws of thermodynamics. They are inconsistent with the laws of thermodynamics. That is why continuum mechanics emphasizes on constitutive equation and says that constitutive equation is not curve fitting. In fact, I am going to state two or three more things that are important for constitutive equations.

So, constitutive equations are not just curve fitting between stress and strain. You just do not take a specimen, just pull it and then say that stress is, the strain, stress is equal to some log times epsilon or whatever it is, because you have to follow certain other principles that we are going to put down. We will quickly summarise; let us see if you can do that on the three equations. What are the three equations that are of interest to us? Simple; if you want, you can add the fourth equation as the, what we did just now, the first law, one or in other words, we call this as balance principles.

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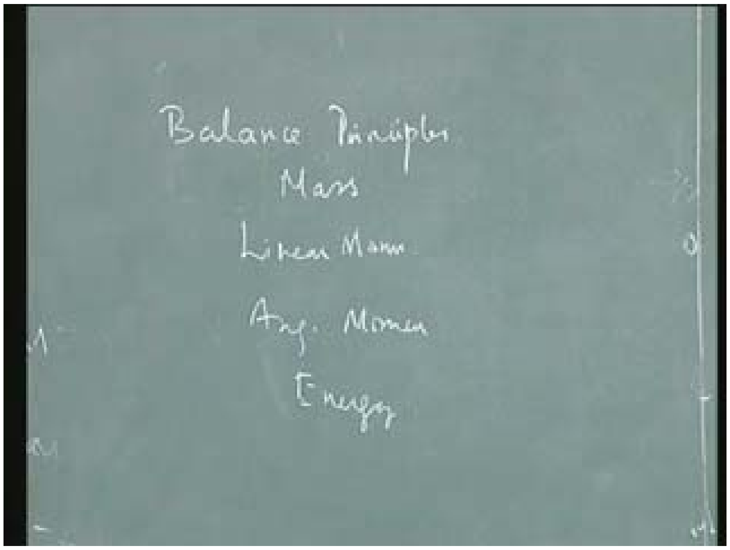

Balance Principles
Mass
Linear Mom
Ang. Momen
Energy

What are the balance principles that we are interested in? Mass, linear momentum, angular momentum and of course, the first law, which is the balance of energy. We can state these three equations in one simple equation, in one equation and then, replace certain things in that equation depending upon, whether we are interested in mass balance or momentum balance or energy balance. So, what is that?

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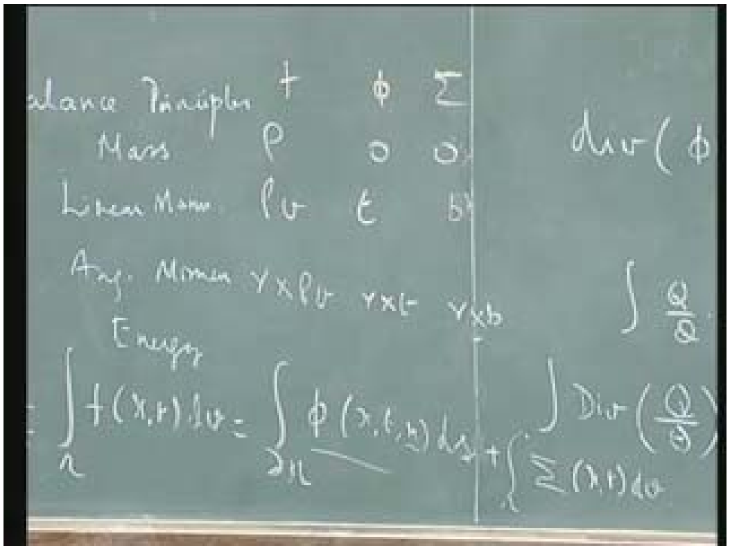

alance Principles	f	ϕ	Σ
Mass	ρ	0	0
Linear Mom	ρv	t	b
Ang. Momen	$r \times \rho v$	$r \times t$	$r \times b$
Energy			

$$\int_{\Omega} f(x,t)\,dv = \int_{\partial\Omega} \phi(x,t,n)\,ds + \int \Sigma(x,t)\,dv$$

$\mathrm{div}(\phi$

$\int \frac{Q}{\theta}$

$\int \mathrm{Div}\left(\frac{Q}{\theta}\right)$

Let me call this as D by Dt of a certain quantity f, which depends upon or which varies with respect to position and time dv. That quantity is equal to d omega phi which depends upon x t and n ds plus sigma, some quantity say sigma, which you can right now immediately say that, it is something like a generation term x comma t comma t and dv.

What I mean to say is this is a general balance law. Now, if I want mass, conservation of mass, I replace say for example, this f by means of density rho, density rho and then these two - phi and sigma has no meaning there. So, I will say, if I say this is the first one say, f phi and sigma, then if I substitute rho for f, phi for zero and sigma is zero or in other words D by Dt of rho dv, integral rho dv is equal to zero is what will give me the mass balance equation. So, for angular momentum what we have is rho v. Obviously, this is t, so, t dot n, so that you will, you will replace this phi by t dot n and sigma becomes b or since n is, dot n is common, so, you can say that this phi is nothing but rho v, rho v, velocity, t and v are the three terms which will go into the conservation equation and lastly we have r cross rho v, angular momentum equation r cross rho v, r cross t, because we are taking momentum about say, the origin and r cross b is what will go in to these three terms. You can, for energy you can write it down in a similar fashion.

This is the summary of the equation. You now know the tricks of the trade as to how you can convert this into current co-ordinate, sorry, to reference co-ordinate and all that. So, I am not going to repeat this. We will quickly move over to the constitutive equation part. Next three to four classes we will spend sometime on the constitutive equations. We will stop here and continue in the next class.