Advanced Finite Element Analysis

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Lecture - 11

In the last class we had seen, looked at the mixed formulation.

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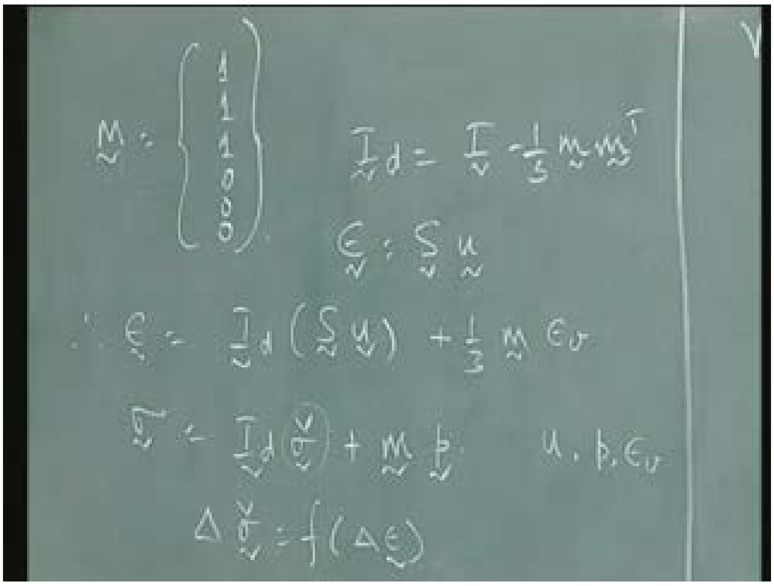

We will just quickly summarise what all we did in the last class, so that we can follow the derivations further. We started with the series of definitions and for example we started with what is M, then what is I_d and then we said that epsilon can be split into two parts - the volumetric part and the deviatoric part; same way sigma can also be split. But, one of the things which we emphasized is the fact that we are now going to look at not the irreducible form; that means displacement alone, but we said we will look at both - displacement, pressure and epsilon$_v$ as three of the variables that we will be interested in and hence we are stating sigma, I mean this is the common thing what you know. The only thing is that we defined delta sigma with that inverted hat to be one, which you will get from delta epsilon through the constitutive equation. So, this is the deviatoric part that is the pressure part, so that that together will define us the sigma.

In other words, this is the fundamental definitions which all of you know and then we wrote down the variational form.

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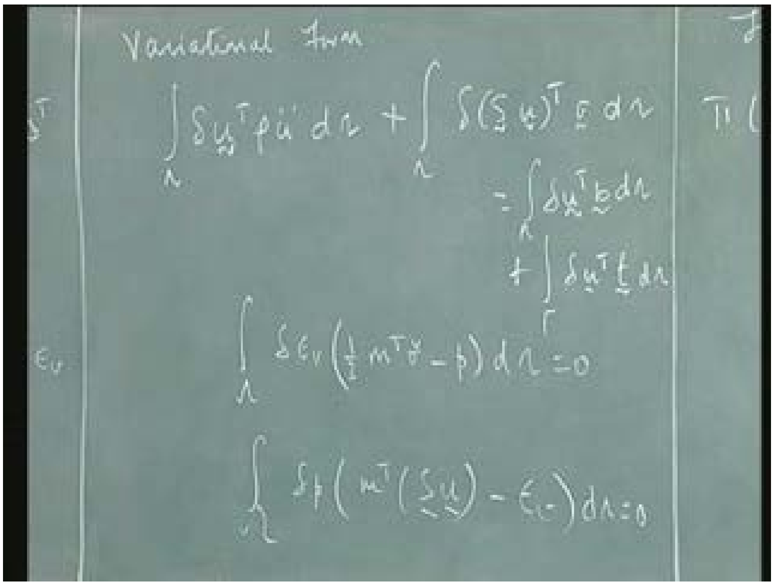

The variational form is very straight forward and the first variational form is exactly the same as that of the virtual work form which you had encountered in the previous course. There is no difference between them, between this and what you had done before and the only thing is that, to that we add two more. If you really look at it closely what essentially we are doing is that what happens or what should be zero, we are multiplying it by weight function delta epsilon_v and delta p. These three are quite easy to understand and then we combine these three things. In other words, we have three equations; note that we have now three equations.

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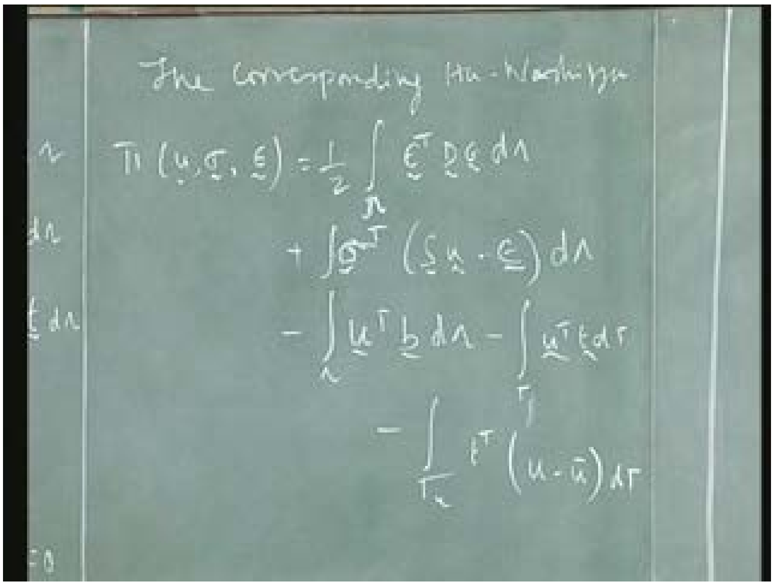

We can combine them together to write Hu-Washizu principle. In fact, you can come the other way as well. You can define the Hu-Washizu function and from here, recognising that we have displacement, p and sigma or rather sigma and epsilon$_v$, you can rewrite that in terms of p and epsilon$_v$ as well and then come down from Hu-Washizu principle in the same fashion, as you do for the irreducible form, for displacement form. Of course, this is written in terms of u, sigma and epsilon. You can re write this in terms of p and epsilon$_v$ as well and so from here you can come to this place or from here you can go to the other side, in a very similar fashion as we had done before.

We will not worry about the Hu-Washizu principle right now, but what we are interested in is, here it is much easier to understand from the formulation, from this place. So we will start from here, we will come to Hu-Washizu principle more in an elaborate fashion, when we talk about the large deformation case.

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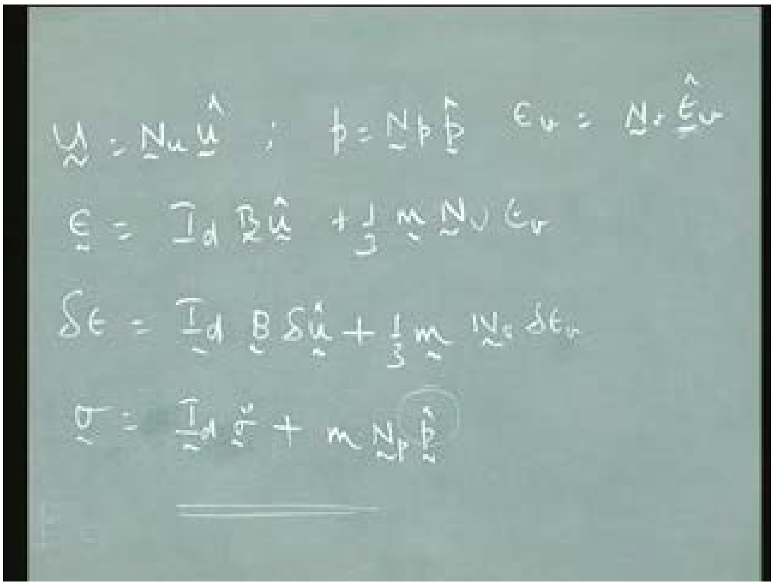

In order to convert this into a finite element formulation, what we need is things which are again familiar to us and these are the things that we need and in other words, what we have done is to now define the continuum displacement in terms of the displacements of the nodes and look at these two quantities. Now, we have pressures as well defined in a very analogous fashion to that of displacement and then we have what are called as the shape functions for the pressure and the volumetric term as well; that is N_p and N_v. Both of them defined these pressures, which we have substituted or which can be useful for us to substitute it back into the variational form.

Having said that, having written these things I can, do write epsilon and delta epsilon and sigma in terms which are familiar to us, now in terms of shape functions and the variations, now variations defined with respect to the discretized form and our job is quite simple. Look at the logic. It is very, very simple. Having defined this, we will go and plug these equations back into my variational forms and then write down a series or set of equations which I will use in order to solve for u, p and epsilon_v. It may not be necessary for us to solve all these things together and it may be at most instances possible to eliminate say, p and epsilon_v at an element level. We will see whether it is possible to do that.

So, as the first step, what I do is to substitute all the discretized form, written in terms of shape functions, into this mixed form and write down the corresponding equations and let us see how they look like. I think we stopped at that point in the class and if there are no questions, we will progress further. Let us see what happens to the first equation. I told you that you can yourself substitute it; you can see what happens.

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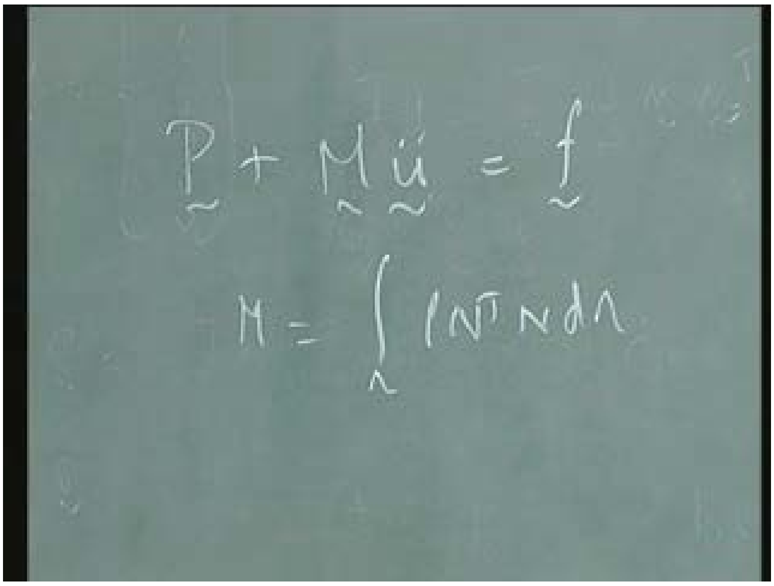

First equation can be written say, in terms of M; M u double dot is equal to f. Look at that equation and look at this. It is very obvious that the first term which is the inertia term goes as the M term. It is again a very familiar matrix, M matrix, which is the mass matrix that you would have called in your earlier classes. So, the mass matrix you substitute that. You can see that delta u transpose you can substitute in terms of N. So, this is again N, so N transpose, row N transpose N is what the mass matrix is; so you can define that N d omega.

You can, if you want for further analysis you can, either remove it or you can keep it as it is. If you are going to do a static problem, then you can remove that particular term. If you are going to do a dynamic problem, then you can keep that straight away there. So, next term is the P term, which is the internal force term. Remember that the internal force term comes from here, the second term here. Remember that S u is replaced in terms of b u, so you will, b u hat; you will get actually delta u transpose hat B transpose sigma. So, B transpose sigma is what was coming into the picture as the internal forces. Remember that that is what we used. So, that is the second term that I have. So p is equal to integral B transpose sigma d omega. So, that talks about our first equation.

Now, let us look at the second equation. Let me write this down, the second equation.

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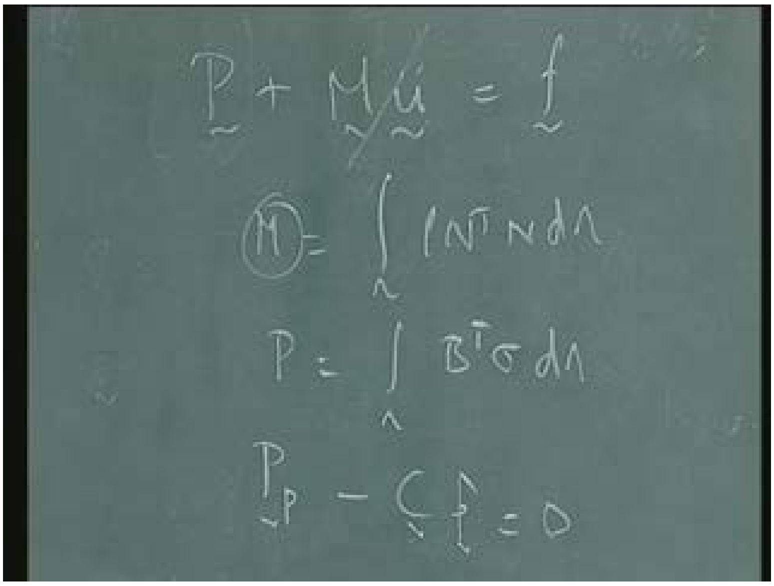

What is the second equation? How do I get it? I substitute the discretized form into my second equation. Look at these two terms; they are very, very easy to understand again. So, you substitute for delta epsilon$_v$ and p from the previous case and let me write this down as P_p minus C say, p hat is equal to zero. Can we repeat M? Yes; M is what? M, I had already defined 1 1 1 0 0 0. I mean just I had, why I had M? M I had put here, basically because it is easy to write it down.

Yeah, mass; this capital M, what is capital M? Capital M is nothing but the mass matrix. If I write it down as it is, it is consistent mass matrix, very familiar term in the structural dynamics. If you are, as I told you if you are, not interested you can remove that and you will get back your original equation. So, let us see what or how you get term? term comes from this first term and what is that term will be? So, delta epsilon$_v$, you are going to substitute from there; obviously, from this term here and so substitute that and see how we can write down that term.

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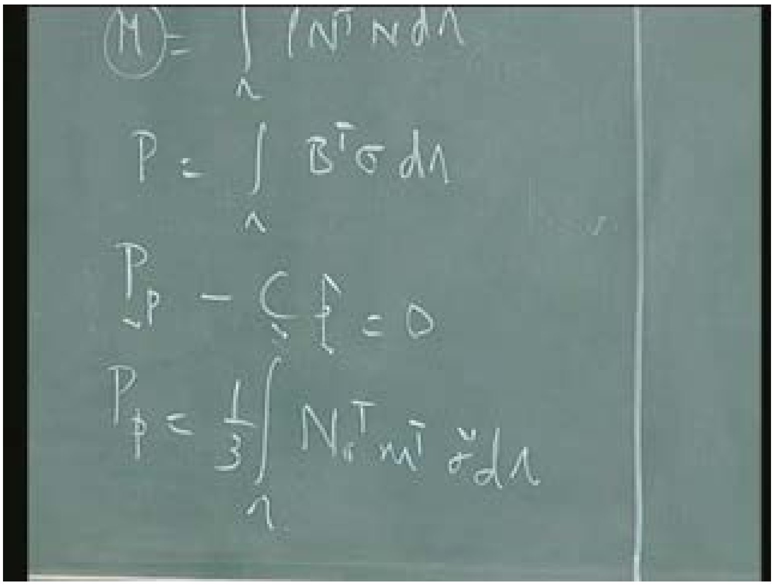

Let us see, what you get? , substitute it here; so, let us see what you get? Simple; integral omega delta epsilon$_v$. If I want to write, this is a number; but, if you want to write it in a matrix notation, you can write it as transpose into this in a matrix notation, because it is easier to follow matrix notation. So, you can convert this and write it, write this down in matrix notation. You can write it down as delta epsilon$_v$ transpose, the other terms there. So, you can say that 1 by 3, I will keep that out; this 1 by 3 here I will keep that out. So, delta epsilon$_v$ I am going to substitute in terms of N$_v$. So, N$_v$ transpose is my first term there. Then of course, I have my M transpose term; M transpose term, then sigma term there, d omega is my first term.

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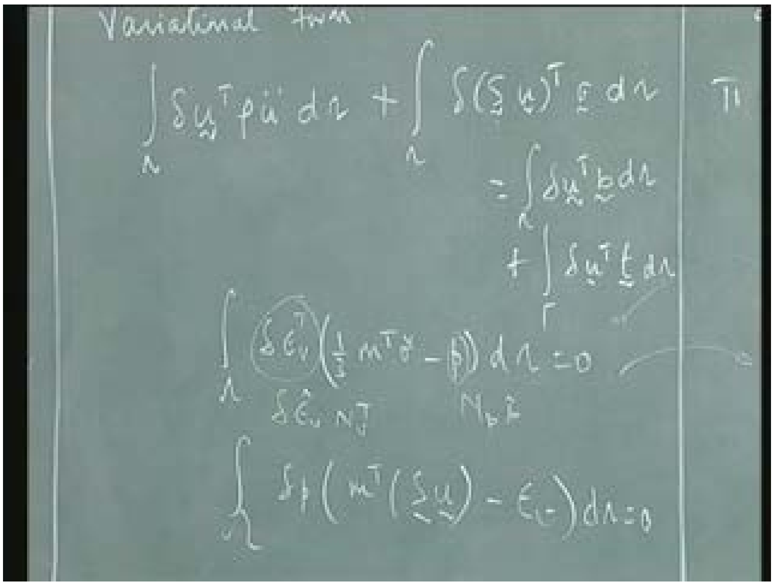

Then, my second term comes from this term here, again delta epsilon$_v$ transpose. My second term is what I call as C. So, it is obvious that now, it is very simple; now I substitute this in terms of N_p p hat and this again, the same way delta epsilon$_v$ hat N_v transpose. So, together I get N_v transpose N_p d omega, p being taken outside.

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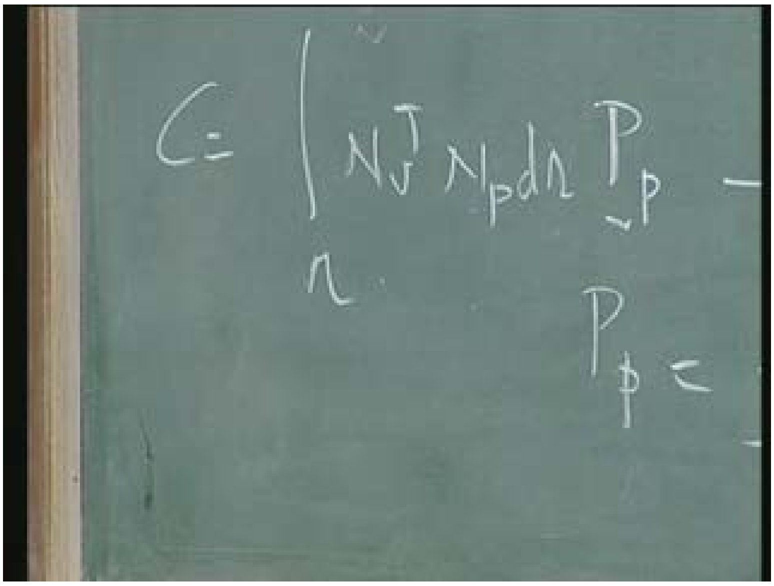

So, C is equal to omega into N_v transpose N_p d omega. I mean this is what we did also in last class. I hope things are clear.

One of points I want to state is that once you, it is so simple; finite element formulation is extremely simple. Once you have this kind of formulation, look at what we are doing? Very, very mechanical; I defined discretized form. Till now, till this point, it is not finite element per say. Hu-Washizu principle comes into variational forms; you know, they are not finite element. I enter finite element once I write these things. So, once I know the physics, once I know the formulation, I just, what I have to do is only take that out, substitute it into my variational form. In fact, if you want you can branch off from here, you can do some other technique; you can solve using some other technique as well. But we are, once we are into finite element we just substitute it into that expression. Is that clear? So, we go to, yes, any question?

Delta epsilon$_v$ transpose, yes. Yes, let me write that down clearly. If you have any doubts we will, so, you write this down. So, this is the procedure. The doubt is how do I get this? Still not very clear; it is very simple.

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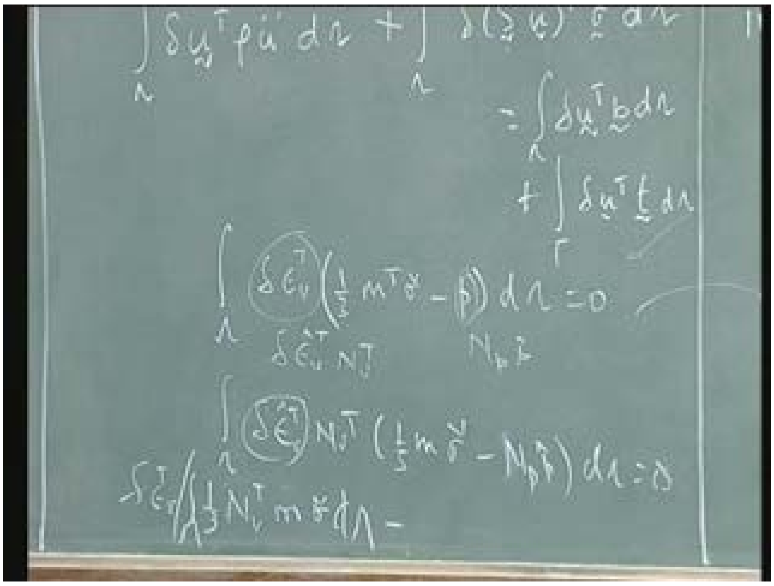

You see that I can write this down into delta v transpose N_v transpose into one third of m sigma minus p, instead of p I am substituting it N_p p hat, d omega is equal to zero. Delta epsilon$_v$ transpose is that which belongs to the nodes, because actually I should put B …., what we got from discretization. So, this is similar to delta u hat transpose that you would have had when we defined B transpose db as K. Take that out; take that out, so, I can write this down as delta epsilon$_v$ transpose omega, integrating throughout omega, N_v transpose, so, one third m N_v transpose, sorry, N_v transpose m, N_v transpose m sigma d omega. That is the, that is my first term, because this delta v hat is the same for both the terms, minus the second term N_v transpose N_p into p hat. So, the first term is what I go or I call this as because, this being common, this has to be satisfied at every point; so, this being common, we can take that out. So, minus C_p hat and C is what is the second term N_v transpose N_p p hat. So, that is what you get here in this term.

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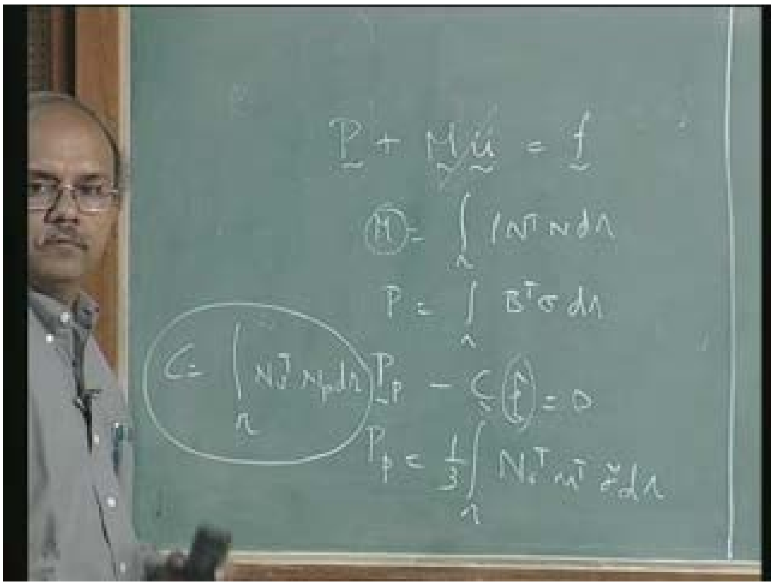

This is, as I said, I deliberately did that because that is the standard way of writing that equation. Now, let us come back to the third equation.

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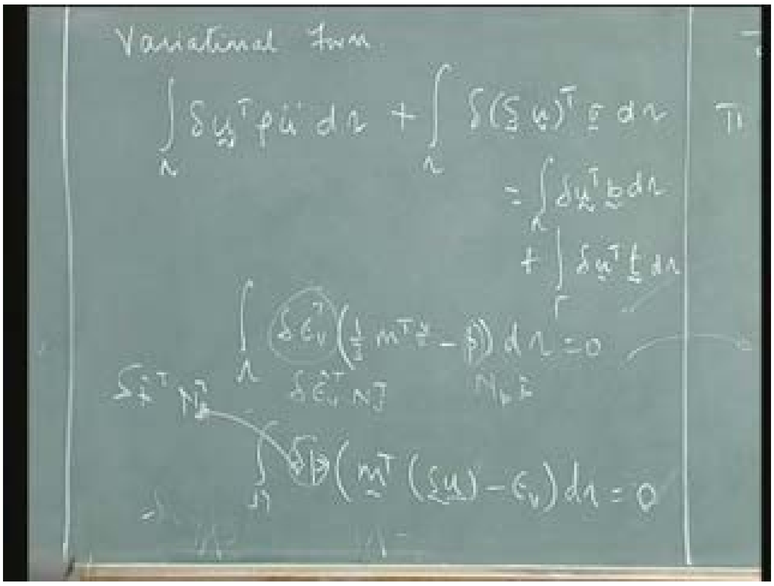

Third equation is delta p; I think I will just remove that and write that out again; integral delta p, into what is that we wrote? m transpose S u; m transpose S u minus epsilon$_v$. Strictly speaking, they should be the same and they should go to zero. So, you can view it, in a slightly crude fashion we can say that this is zero; zero multiplied by some quantity is equal to zero. Now, you can look at it, in order to understand it, you can look at it in that fashion as well.

There are, we are going to get three equations I know the first equation; that is the first equation and I know the second equation and let us now develop the third equation. What is the third equation? Again, the procedure is exactly the same. Substitute for these quantities from the discretized quantity and what do you get? Let us see what you will get as the first term.

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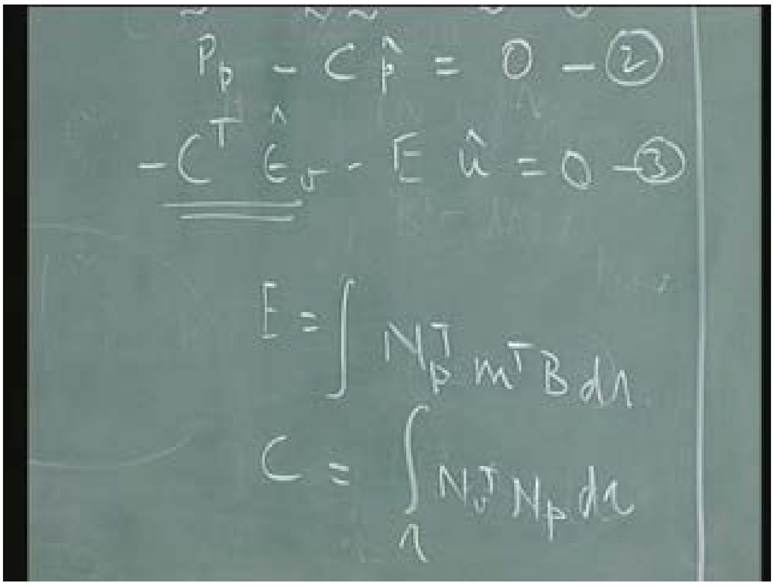

Delta p, again same way delta p transpose I have to put, so, let me call this as, you will see that I will get C transpose out of it; C transpose or minus in fact, epsilon, for the second term epsilon hat v minus E u hat is equal to zero. This is the second term that you will get here from this term; it is very simple. Delta p, now, I am going to replace it with delta p hat. This will be replaced by transpose, so delta p hat transpose N_p transpose. If you look at the second term, let us look at the second term first. I will get N_p transpose N_v N_v into epsilon$_v$ hat. So, N_p transpose N_v d omega is nothing but C transpose and so I get the first term to be minus C transpose epsilon$_v$ hat.

Look at the second term; the second term is here. Substitute in terms of S u, in terms of B, so that E will be, have a look at this term now; have a look at this term here. Then, you can write down E to be N_p transpose, that is the first term that comes here, N_p transpose. That N_p transpose comes here; then m transpose that is the m transpose comes here. Then, B; S u is written in terms of B u hat. So, B d omega is what you will get as the three things. So, you get that. That is the third equation and that is the definition of E. Let me complete the picture by writing the second equation as well here which we just removed. We will write the second equation minus C p hat is equal to zero. So, that is the first equation, the second equation, this is the third equation. Remember what we wrote for C? C is N transpose, N_v transpose N_p d omega is what we wrote. These are the three equations that we have and we have to solve these three equations.

Now, look at them; let us see how we can solve them, solve these three equations, because it is, by the way, what is f? How do I get f? External forces and that comes from this, obviously. I mean, though it is standard, we know that delta u transpose I substitute; so, N transpose B, then N transpose t d omega. Now, you can solve these equations straight away. For example, look at that equation. It is so nice; it is very simple to solve them.

I can write p hat, p hat to be, I can bring this guy to the other side. So, I can write p hat in terms of C inverse; I can write p hat and so, the first thing I can do is to solve for, straight away from these two equations without using the other guy can solve for this equation. Let us see how I am going to do that?

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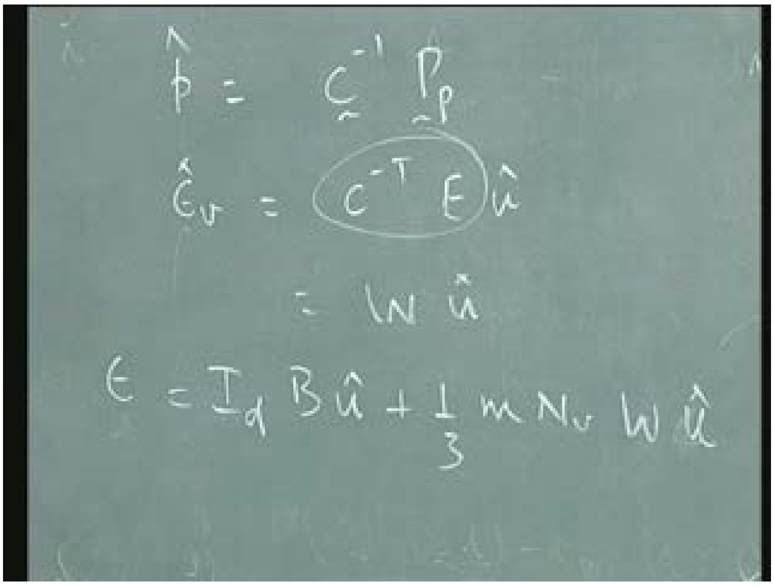

So, p hat, what is p hat from here? So, C inverse P_p. I mean of course, they are matrix; just for clarity, I have written them and epsilon$_v$ hat can be obtained from the second equation; from the second equation. Let us see, but there is going to be a small issue there; that is equal to C inverse E u hat; C inverse E, so, C inverse transpose E u hat which can be slightly re written as say W u hat, where W is C inverse transpose E. Of course, when N_v is equal to N_p, which is the case most of the times we take, then C becomes C transpose. So, this C inverse transpose can be written as C inverse. Is that clear? Yeah; no, there is a minus here. No, this should be plus here, first term; because, the first term was a plus, so we get, that i a plus there; where W is that quantity. Is it clear?

So, now substituting for epsilon, let us now do a small jugglery; substituting for epsilon, I can write epsilon to be I_d from the, look at this expression here. I can write epsilon to be I_d into B u hat. Look at what I am going to do. It is very interesting, so, plus one third m N_v epsilon$_v$ hat. Now, for epsilon$_v$ hat, I am going to substitute in terms of u hat, W u hat. So, I can write this down as W into, sorry, u hat. Look at this expression now. Does it strike anything, does it?

Look at that expression; this expression. What we have done is epsilon$_v$ we have written in terms of u hat, then re substituted back into epsilon. Look at this expression here. Absolutely; so, there is a relationship. Looks like this relationship is very similar to my old relationship of B. The B is now, looks as if the B is now modified. The B looks like, B has two terms in it; B has two terms in it - one term, which is the dilatational term and the other is the deviatoric term. B has split this; I mean it looks as if now B is split.

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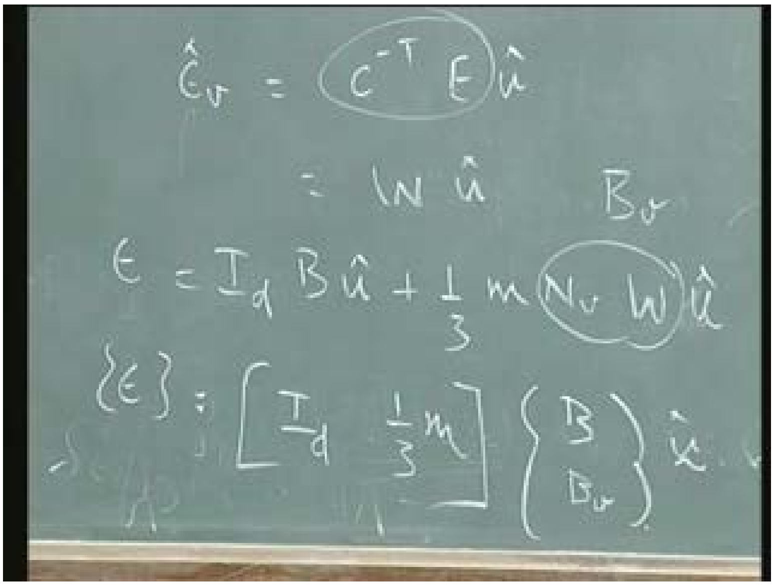

Let me define N_v W as say, B_v, B_v and write that down in a matrix form, so that epsilon is now written as I_d one third m B and B_v. Yes, B B_v; yes of course, u hat is common to this. So, I can write u hat ultimately. You can immediately foresee what I am going to get. Note that this is very similar to my splitting B into B deviatoric B dilatational in our B bar method, but we came through an entirely different route, a variational route; much more meaningful mathematically, but still arrive at the same result.

Let us now proceed and see what would be my tangent stiffness matrix. You can foresee what we are going to get, the procedure is again standard; there is no difference in the procedure. Let us see what we are going to do. What we are going to do is to take my first equation. As I said, let me remove this from further considerations, we have already considered these two guys here. If you want to add, of course, the second term that is m u double dot term, you can keep adding it; it is not going to change any of the things that I am going to do, you can keep adding it. But, only thing I am going to do is to now take this B transpose sigma term and then see what we can do about it. That is equal to, remember what we wrote for sigma, I_d sigma hat; look at this term here.

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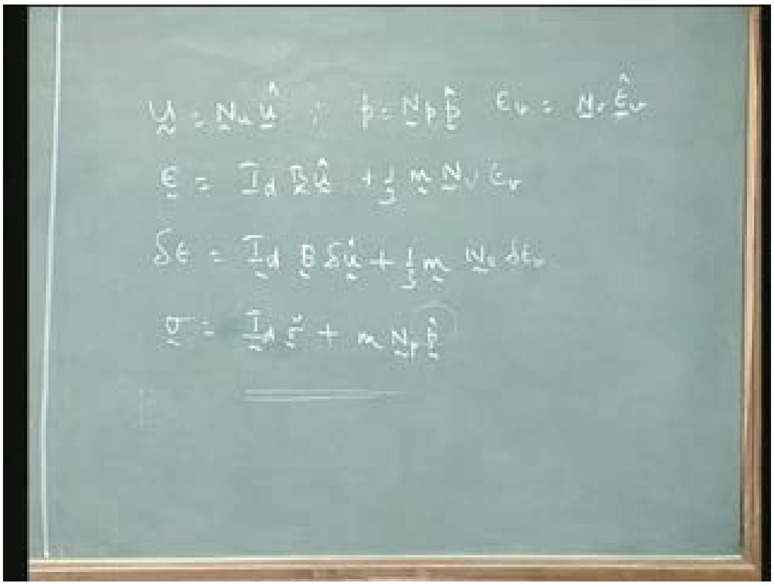

I am just keeping that there; please look at the term there and then, substitute that term here instead of this sigma, so that I will get B transpose, can you read that out?

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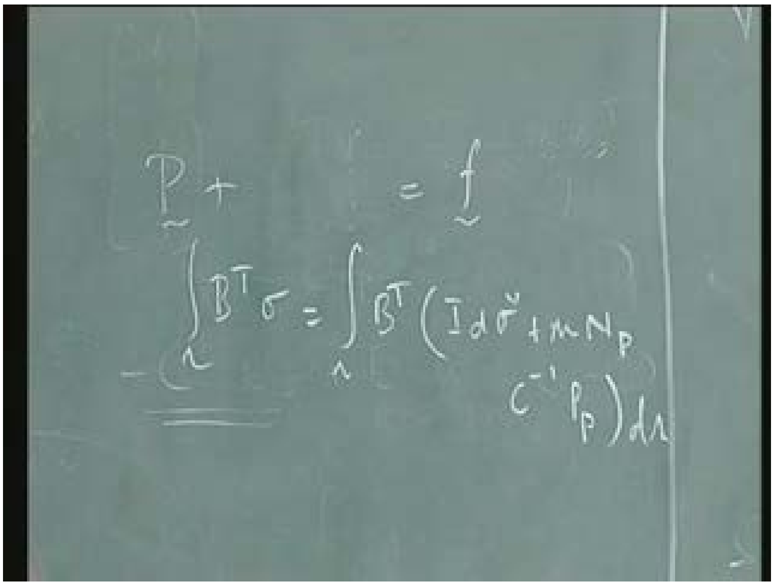

I_d sigma plus m N_p into p hat. But what is p hat? We just now derived what is p hat. C inverse ,so, I can write that down as C inverse .I hope it is clear that the two terms I_d sigma this inverted hat m N_p C inverse d omega is what is my term. Now, what I am going to do is to take this and then do a small jugglery; nothing else. If you want to do it, you can do it or else you can do it in any other fashion you want. But just to make things easier to implement, we are going to go through this step.

I hope you are not lost out in what we are doing. The two equations we have taken. Essentially I am able to, from the second equation I am able to get p. Third equation, I am able to eliminate epsilon$_v$ in favour of u hat; re substitute it back. So, all my equations, ultimately when I solve it, all my equations are in terms of nodal displacement only. But, only thing is that the K_T matrix what I am going to get is not my original B transpose dt B, but that is going to change. That is all, nothing else. So, the second term, the two terms to this, let me write that down.

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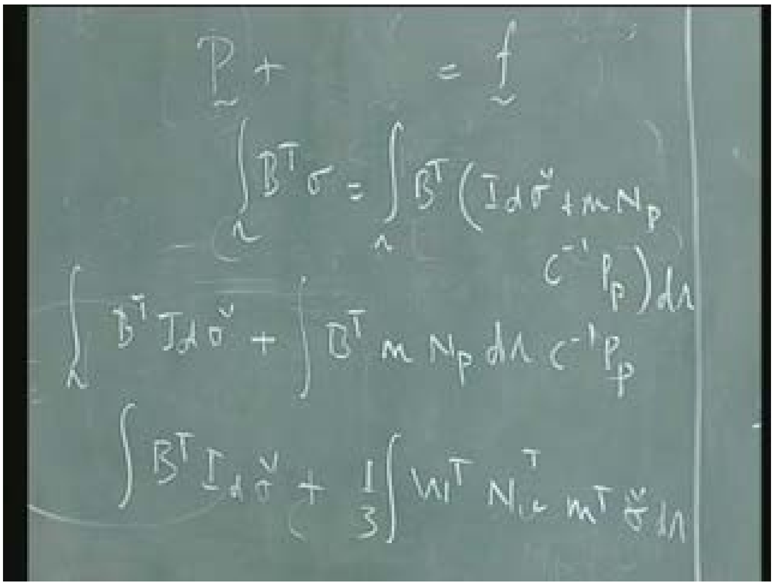

Omega B transpose I_d sigma hat plus B transpose m N_p C inverse d omega; so that would be the two terms. Now this term, the second term I am going to re write it a bit differently, because my, remember what my is; has an integral term in it. So, I am going to write this down in a slightly different fashion and I am going to write that as N_p d omega C inverse and then substitute for W from my previous expression and then write this down as B transpose I_d. We will verify that what I am writing, straight forward substitution. I do not have all the equations here, but you can verify that what you have; plus, yeah, integral I think one third will come into picture because of definition of W and so on.

So, W transpose N_v transpose m transpose N, yeah, N transpose, not P, but sigma hat d omega. I am substituting back all the equations which I know, which I d and so, I am going to write that down something like that.

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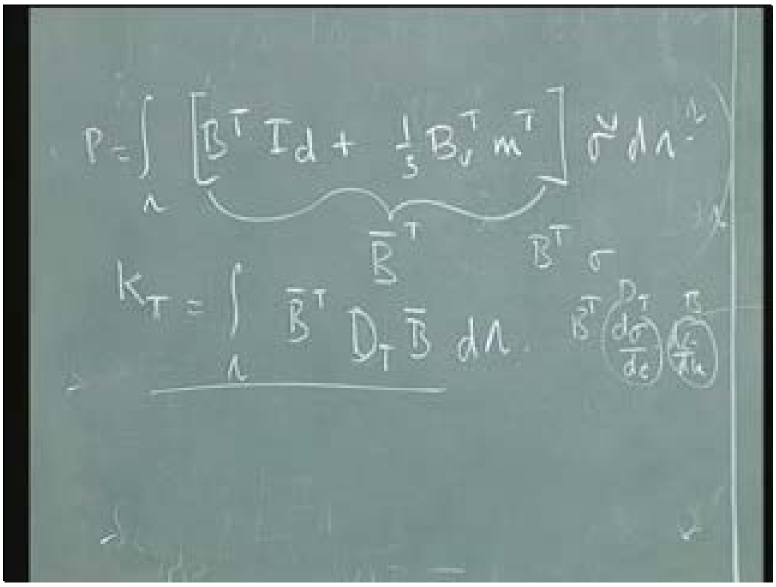

Now, again I am doing some juggleries here. Re substituting it, all expressions, so that I will write this down as B transpose I_d plus one third B_v transpose m sigma hat or m transpose rather m transpose sigma hat d omega is what I will have. So, this is what we actually arrived at as B transpose; so, let me call this as my new B and so that, I will call this as say B bar, a new B which is B bar.

Now the next step; this is p. The next step is my calculation of K_T; my calculation of K_T would lead to my introducing D_T here; omega B bar transpose, that is, this is rather B bar transpose, not B bar. So, d sigma hat by d epsilon into d epsilon by du hat which from here would again give rise to another B or B bar, so that I can write this down as B bar transpose, sorry, D_T; D_T is T sigma hat by d epsilon which is or delta epsilon delta sigma hat by delta epsilon, that is what we introduced right in the beginning, into B bar into d omega.

So, this is, the step is standard. Again, p is what? We remember that this is what we did for the Newton-Raphson scheme. If you remember, we had B transpose sigma instead of this and we defined this as B transpose d sigma by d epsilon into d epsilon by d u. Remember that we had this as D_T and remember that we had here B. So, B transpose D_T B is what we defined. In this case, it so happens that that B is not there, we have B bar. B bar is the very thing that we had looked at; modified B in our earlier approach. So, that is the K_T now you are going to use to solve the equations, the Newton-Raphson scheme.

Sometimes what people do is to combine these terms here into D_T and define a new D_T hat. There is nothing very different from what we are doing; just D_T is slightly rearranged. So, here you have, in this formulation, please note that there was a question on cross terms. In this formulation, you have all the cross terms intact. It is a very rigorous formulation. In this formulation, you will have all the cross terms intact. This whole thing is here and then this would be written like this. So, you will actually have four terms, all the cross terms, which will go into the implementation of the finite element analysis.

Now, coming back to where we started from, we started with elastoplastic analysis. What we have essentially done is to write down a small strain formulation for an incompressible or merely incompressible material. That is, we have diversified or we digressed from our elastoplastic analysis. We wrote down a small strain formulation, a mixed formulation for the analysis of nearly incompressible material and what I want to state is that, now if you go and substitute at this D_T your elastoplastic D_T, then this formulation can be directly used for an elastoplastic analysis. If you are not interested in elastoplastic, but if you are interested in any other material, still in the realm of small deformation and if you are interested that means that mu is equal to say 0.49, 495 and so on, but the deformations are not very large, then you can use the same formulation; no difference, same formulation, but a corresponding D_T can be used.

This is what I wanted to tell you that most of these algorithms are plug and play algorithms. It is not that I will repeat this, if I have to do non-linear elasticity; the procedure is the same. So, for solving a small deformation problem, this is the procedure. That is all, but I will add one more to this; one more to this. What is it that I will add? I will add stress update algorithm, if I have to solve plasticity problem. Now, the comment about these elements is that we should understand that most of the non-linear analysis is done with, not with higher order elements. It is not, it is not usually recommended to do due to various reasons, theoretical reasons. Right now, we are not in a position to see that, but usually most often we use only lower order elements.

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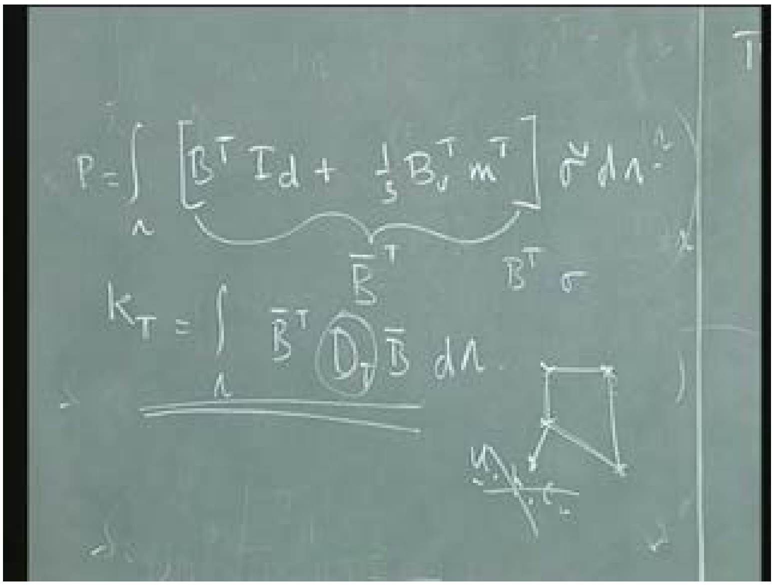

Say, for example one of the elements which you may use here is a say, quadrilateral element in which case you will define N in the same fashion as you had done for your earlier classes, but you would define N_p, since Np ….. the dow Np. Look at this B. What enters here is only the gradient that is dow N_u or the first differential of N is what appears there, but the first differential of p what we had defined, there is no corresponding p for N_p; there is no B_p term. That o enter that place, so, we do not have problem of defining p in the same fashion as that of N. So, usual method, usually what is done is that you will use the ….. and also note what is B_v term? B_v term is what we define here. This is the B_v term. It does not come from m_v term, it comes from this B_v term.

So, it is usual practice to define the u in this kind of element to be that at the nodes and define p and epsilon_v to be a constant inside an element. In other words, it is usual practice to distinguish between the degrees of freedom u and the other two degrees of freedom and you please note that it is not all the time that you treat u, p and epsilon_v to be as if they are degrees of freedom at the node. There are certain places where you will put degree of freedom at the node as well, p as a degree of freedom, but it is not the usual practice to define at the node, degree of freedom u, p epsilon$_v$. It is a very important point, because that is where there is usually confusion. It is not that here I have u, p and epsilon$_v$; no, that is not what we are talking about. p and epsilon v are defined such that say, for example for this element, N_p and N_v are constants or unity. That is as if it is defined for this element, so, it is discontinuous across the next element and so on. Same way, you can extend this concept to a solid element as well. Is that clear?

We will may be later in the course, we will define or rederive equations for tetrahedron elements. But, what I want to again point out is that, this is a general formulation and can be say, can be used or information can be extracted from this to derive formulations or derive K_T for plane stress or plane strain or axisymmetric and so on; 3D solid and so on. So, the corresponding terms are now replaced when we go to say, plane strain and axisymmetric. Of course plane stress, there are certain issues; I may not have time to cover every element formulation; every element how it looks like it may not possible for us to cover in this course. The whole idea is to give you a background. If I recommend Zinkevichs volume II, if you look at say, Zinkevichs volume II, you will see how this particular element or this procedure is now used for deriving plane stress element. There are some changes; there are going to be some changes, because of the way in which we operate for plane stress, but the procedure is the same; the concept behind it is the same. We are not going to specialise everything, because we have lot more to cover; so that we are not going to do. But as an exercise, I would like you to have a look at the plane stress formulation in Zinkevichs book.

Having done that, we will get back to our elastoplastic analysis and close that part. Before we go to the continuum mechanics aspects, let us now look at what the things are. We will motivate the cases now and then write down the equations and other things in the next class. Is it clear, this formulation is clear?

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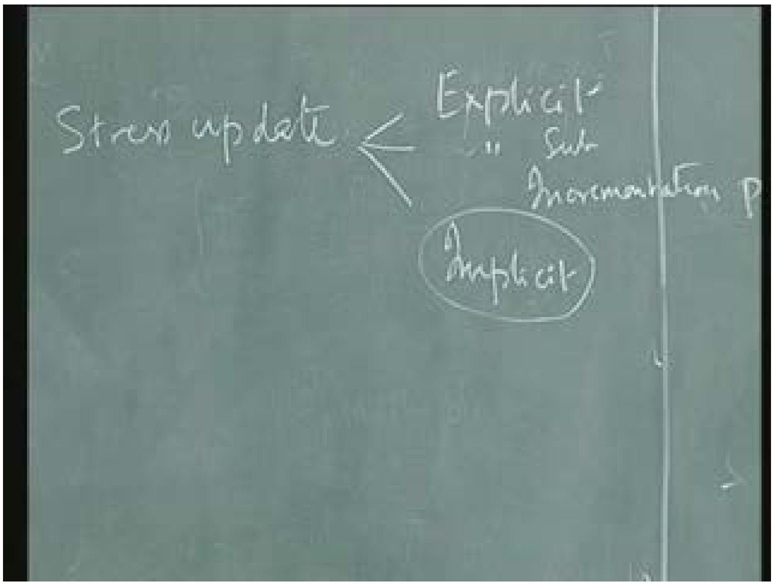

Now, we will now look at the stress update algorithm. What we are going to do is to follow a very similar procedure as we did for one dimensional case. Stress update algorithms are broadly classified into what are called as explicit algorithms, people call this as forward Euler method; sub incrementation, explicit with sub incrementation, we will define what sub incrementation is as we go along and then implicit algorithms. All of you by now know that all explicit schemes have conditional stability or in other words you cannot have a large delta epsilon, then the algorithm is not stable. We know that intuitively, because we know explicit finite element that is used in structural dynamics has some conditions attached to it called Courant condition. So, same way here also you have some problems with explicit finite element or explicit rather scheme. So, you may wonder why not we go to implicit scheme. There are very interesting things that would happen if the type of constitutive equations are different from that of Mises constitutive equations. So it is important that we understand explicit schemes - explicit with sub integration or sub incrementation, rather which is used extensively and we will also look at in a broad sense what this implicit scheme is as well.

Now, let us define what this is. I have, I have already done, lot of times I have stated what explicit is and what is implicit in this case.

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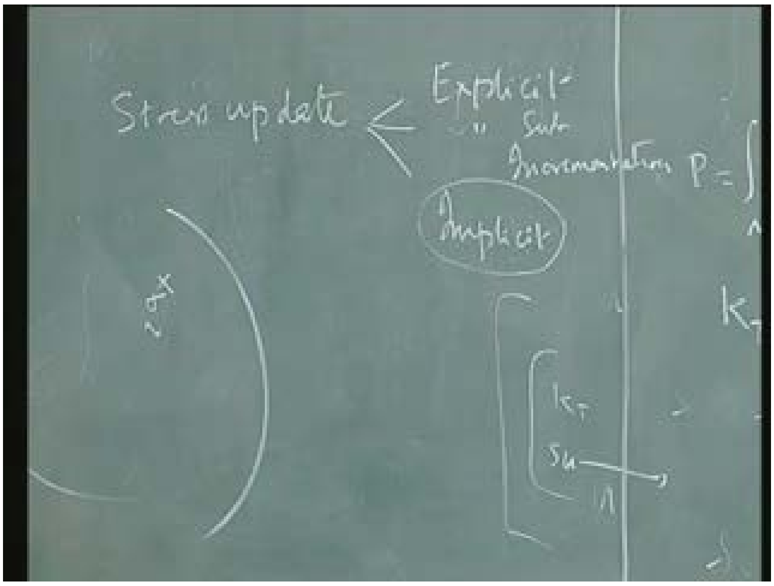

As we go along we will see that as well. That is the say, a part of the yield surface; my whole idea is that we are say, at this point. Remember where we are. We are in the, that is the big loop, the incremental loop we are solving. We are inside the iteration loop. Now I know how to form tangent stiffness matrix; now I know how to solve for u, I know or I have to learn only how to do stress update. So, we are right here, stress update; we are going out and looking at the issues that are involved in stress update. So, at the end of this, you will know all aspects of small deformation elastoplastic analysis.

We will complete the whole show there and on the way you also learn how to do small deformation elastic analysis also. Though it has not much meaning to it, puritans will not agree with you when you say I am going to do small deformation elastic analysis with the or in other words, if I say that I will do using Hooke's law, I will make nu is equal to 0.49 and do certain analysis, people may not agree with you. But, for many practical problems, this will be a good approach where nu is equal to 0.45 or 0.49 rather, you can use this formulation.

So, let us quickly look at what we mean by the stress update algorithm. The issues are very similar to what we did in the previous class.

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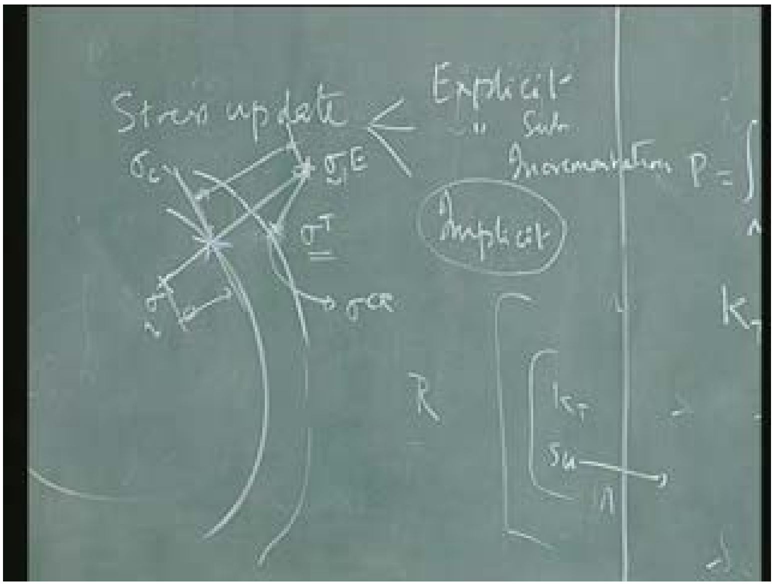

So, what I have to do is to now take this point, this point, the stress point which is say inside the yield curve or the yield surface, which means that we are at the elastic region, to the plastic region. So, I may have to go from this point to this point and as I move from these two points, what I also have to do is to update the yield surface. So, two things I am doing; one is to update my stress as well as update the yield surface. So, what is the issue? As I update the stress and as I update the yield surface, it should be in such a fashion that ultimately I will lie on the updated yield surface; that is the issue. That is the only thing that poses a lot of problems.

I will state in a nut shell what we are going to do and then expand this in the next class. I am going to follow our elastic predictor plastic corrector approach. So, from here I am going to predict in an elastic fashion. I am going to go out of the yield surface and reach a point which I will call as sigma elastic.

Obviously, you are going to pounce on me ….you are going to say that it is not going to lie outside it; yes, I know that. Then, I will correct it, I will come back and lie on, I will come back to a point which is corrected point, which I will call as sigma T. As I correct it and come back, I will also update the yield surface by calculating epsilon_p or delta epsilon_p. From here my yield surface will go out. So, that will be my new yield surface. When I do that explicitly, I will find that obviously, I may not lie on the yield surface. Then, I will correct this trial sigma by pulling it back in a simple fashion to the yield surface, the updated yield surface and I will get what is called as the sigma corrected.

Now in order to do this, I realize that I need one important factor. What is it? That R, which I did in the one dimensional case; how did I calculate R? I know where I the yield point. In the same fashion, I have to now find out as I move from this point to this point where I will contact the yield surface. That point I need to know. So, that point is called as the sigma say, contact; that point where I contact the yield surface is called sigma_C. Note again the concept is very similar. So, this is what I need not correct and this is the part that is the part which I have to correct. How I am going to do that and how I am going to ultimately calculate sigma CR by means of an explicit scheme is what will be the focus of next lecture. So, we will stop here. Is there any question? We will stop here and we will continue in the next class.