Rocket Propulsion

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Lecture No. # 32

Efficiencies due to Mixture Ratio Distribution and Incomplete Vaporization

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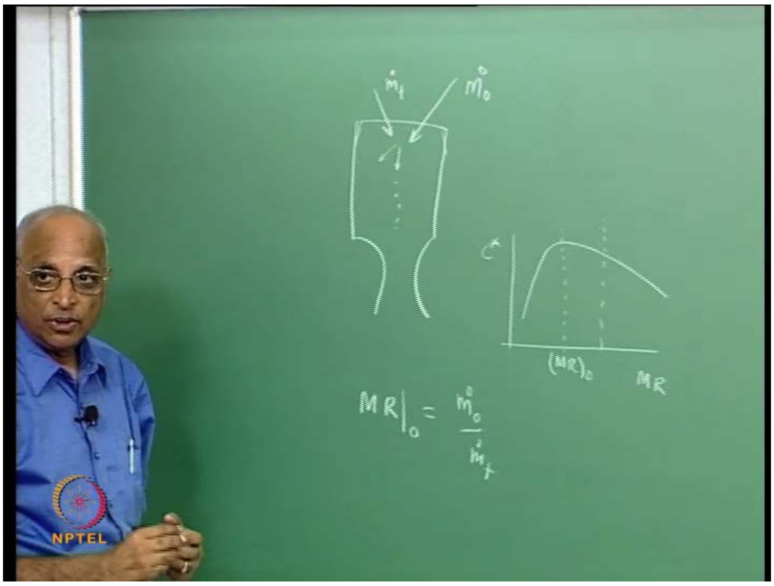

Well good morning. In the last class, you will recall I told you in a thrust chamber of a liquid propellant rocket, the mixture ratio is not distributed uniformly, why was it we told this? We said maybe fuel is used for cooling or you could have barrier cooling, we said in some injectors like shower head, the droplets travel straight into the chamber from the injector; whereas, in some cases it is given an orientation therefore, you know it is really difficult to determine the performance of a chamber, for the simple reason; in the earlier classes, we also told if I say something like c star as a function of mixture ratio, we always operate it in the fuel rich region, let us say this is stoichiometry we have maximum c star in the fuel rich region, maybe I would choose a injection mixture ratio; that means, I inject some mass of fuel maybe flow rate in kilograms per second.

I inject some oxidizer m dot o into it, and the mixture ratio I get at injection let me call it at nominal or injection at o, let us say is equal to m dot o divided by m dot f, and I would like the c star to be maximum therefore, maybe I adopt this as the mixture ratio I call this as mixture ratio corresponding to injection. But what is it we found? We found that the mixture ratio here would be different; mixture ratio here would be different; and it would be different at the different places. Now how do you compute the c star? If I want to calculate the c star for this chamber and from c star multiplied by the coefficient of the nozzle I get the I s p, well the I s p and c star is going to be different from this.

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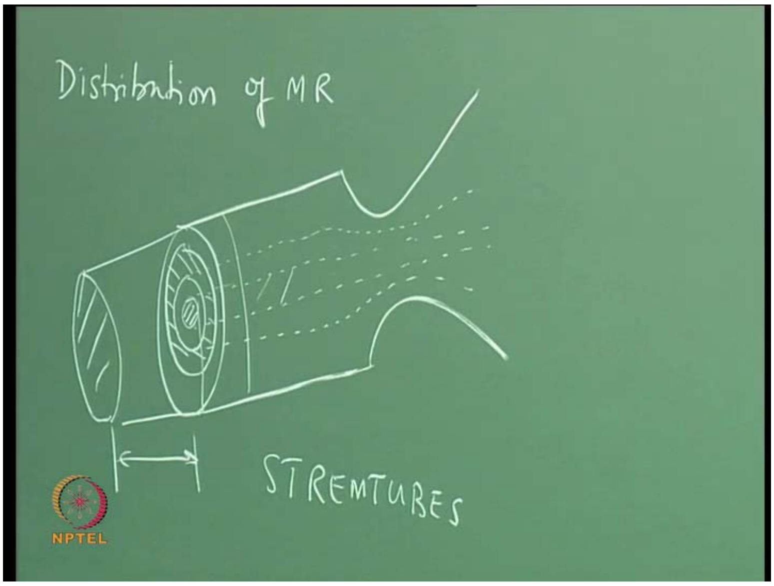

Therefore, we must find out what is the effect of variation of mixture ration in the chamber? I think that is the one thing which we have to do without it we will not be calculating the efficiency like I s p and the c star correctly. Therefore, first I would like to say well I need something like a distribution parameter distribution of mixture ratio in the chamber, and some injectors we said are so fine I can have the fans here and there and distribute it quite well, whereas, in some cases it is not that well distributed, when I have film cooling I have fuel rich zone near the wall and maybe the core might be different, therefore, I need to have some method of characterizing the distribution of mixture ratio. How can I do it? Well I tell myself well why not I compartmentalize maybe I look at a zone here maybe I look at a zone surrounding this, maybe I look at a zone still surrounding this, and then I find the mixture ratio at the different zones and then have something like a distribution parameter.

Let me explain it through another figure, maybe this is my chamber and now I have the chamber like this; I have the nozzle like this, all what we are saying is maybe if I were to say I am looking at the distribution of mixture ratio; maybe in this zone at the core I have a particular mixture ratio and let us say it goes straight like this, maybe a surrounding core over here has another mixture ratio and then it evolves maybe like this, maybe the nozzle goes like this it evolves like this and so on I have different regions of mixture ratios.

Maybe this is the third region I could have, maybe I have an a central core; I have an annulus of a different mixture ratio; I have annulus of different mixture ratios and so on, and what would happen but I know as soon as I inject something it is highly three dimensional like for instance.

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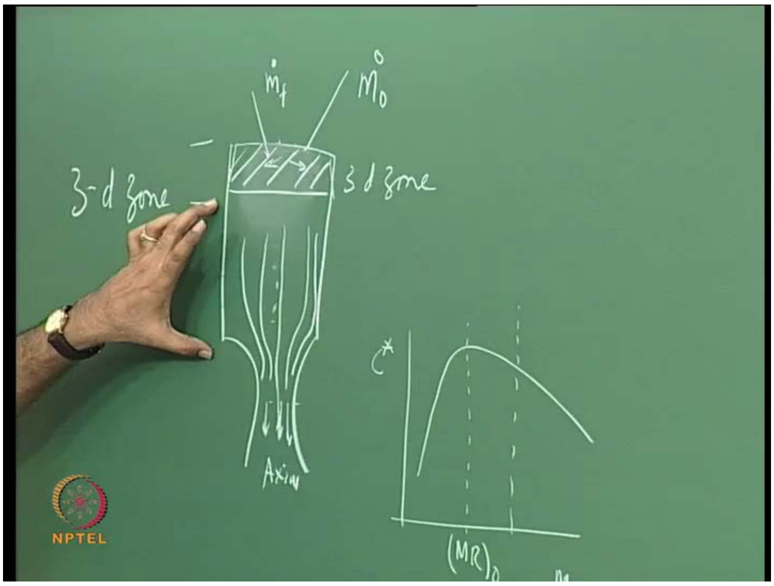

Let me correct this figure now for you I inject the fuel and oxidizer into it, initially the droplets are travelling at all in all direction therefore, I have a very strong three dimensional zone, and what do you mean by a three dimensional zone? Droplets are travelling like this maybe some vaporization is taking place here, and therefore, to quantify this zone is little difficult therefore, I say immediately downstream of injection because I inject in different directions I have let us say a three dimensional zone. But I also know when I come to the throat the flow is sort of normal at the throat flow is axial at the throat, right and if it is going to be axial at the throat somewhere upstream the flow should have become axial, in other words if the flow is straight over here maybe the flow should have come like this; flow should have come like this; flow should have come like this; flow should have come like this.

In other words even though in the zone downstream of injection I have a highly turbulent three dimensional zone, somewhere along I should have started getting flow along the axis because at the throat it is anyway axial. Therefore I say yes initially I have a three dimensional zone, but subsequently I can think in terms of tubes or stream tubes which are going axially, now I slightly extend this figure I say this is my injector over here, I have initially a zone wherein the three dimensional flow takes place, and thereafter since the flow at the throat is one dimensional I have something like tubes or something like stream tubes in which maybe combustion takes place.

Now a rocket chamber is not that long therefore, I do not have sufficient length for one stream tube to get effected by the second stream tube, because it takes some time for fully developed turbulent to effect itself, and therefore, I can say in each of these small stream tubes I can construct as many stream tubes as I like, in each of the stream tubes I could have something like a laminar combustion taking place and the stream tubes do not mix with each other, why they do not mix? The chamber is short I do not have sufficient time for turbulence to develop and mixing to take place. Therefore I make these assumptions and these are quite valid because of the short length of the combustion chamber.

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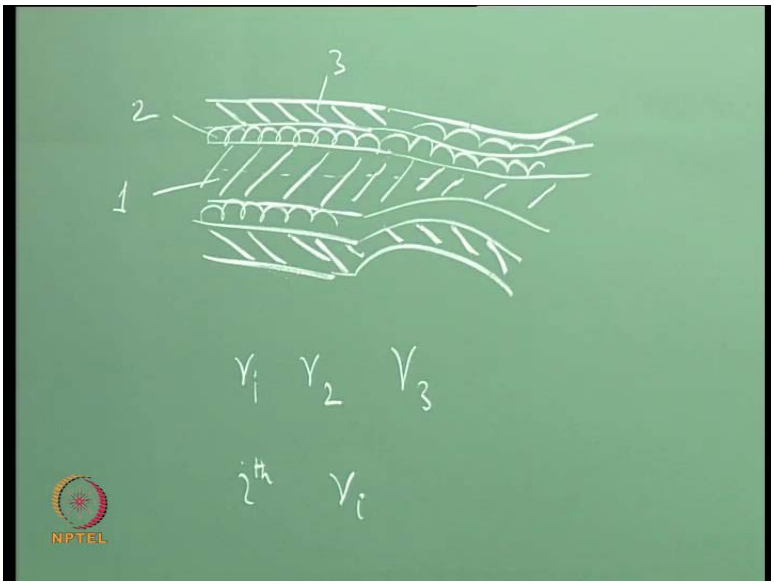

And now I can think in terms of the total thing as something like a series of stream tubes, I say this is the centerline of my chamber I have initially one central core let us say it has mixture ratio 1, surrounding it I have another stream tube this is now an annular stream tube with mixture ratio 2, surrounding it I have another stream tube with mixture ratio 3 and so on, and this when it comes to the nozzle it goes like this and comes out, maybe it goes like this and comes out you have a series of stream tubes which burn, therefore, this is how it burns and gets through the nozzle, maybe this goes like this, and therefore, now I say maybe the core has a mixture ratio r 1, the annular surrounding it has a mixture ratio r 2, the next one has a mixture ratio r 3, or rather otherwise the ith stream tube which I consider has a mixture ratio let us say r I which is local.

Now, I want the net composite performance of a series of stream tubes, and now what is it I am saying let us come back to this figure, maybe the core has a mixture ratio 1 here let us say this is r 1, maybe the outer most near the wall has a mixture ratio let us say r 5, maybe someone has a mixture ratio r 3 and so on. Therefore, now I want to put this in terms of a single parameter say a distribution parameter, and what is my mixture ratio at injection it is M R 0, this is the value which I am injecting which is the ratio of mass of oxidizer to mass of fuel.

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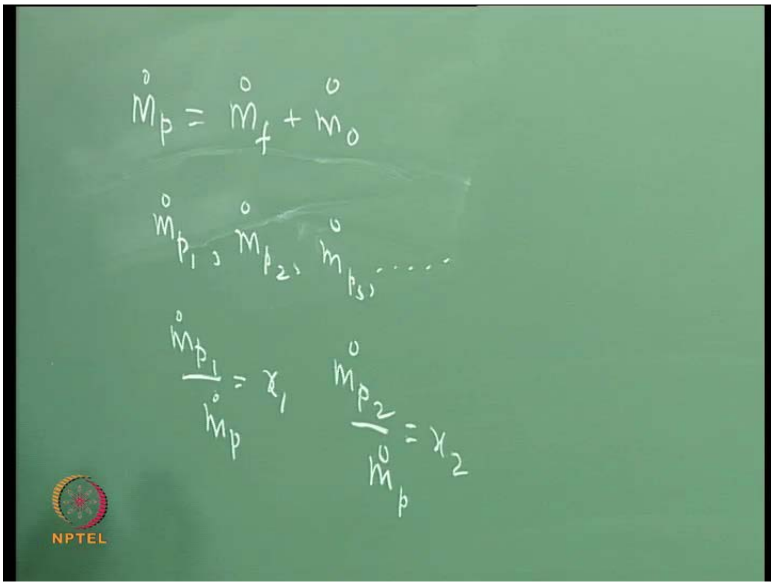

$$\dot{m}_p = \dot{m}_f + \dot{m}_o$$

$$\dot{m}_{p_1}, \dot{m}_{p_2}, \dot{m}_{p_3}, \ldots$$

$$\frac{\dot{m}_{p_1}}{\dot{m}_p} = x_1 \qquad \frac{\dot{m}_{p_2}}{\dot{m}_p} = x_2$$

Therefore now, I can say well my mean mixture ratio at injection is equal to let us say M R 0 or let us say M R 0 over here, whereas, each of the stream tubes has a value r I, is this part clear if this is clear I ask myself another question, the total propellant which is injected is equal to m p dot so much kilograms per second is equal to m of fuel plus m of oxidizer, now the total quantity of propellant which is injected is shared between the different stream tubes, let us say the stream tube have one has a mass of propellant m p 1, the stream tube two has a m p dot 1, m p dot 2 second stream tube, m p dot 3 is the third stream tube and so on, and therefore, I can say I can divide all these things by the total propellant m p dot 1 divided by the total propellant, I call as the value of the fraction; that means, x 1 I call m p dot two divided by the mass flow rate of propellant as x 2 and so on, I have x 1, x 2, x 3 that is the fraction of propellant flowing in stream tube one is x 1, flowing in 2, is x 2 flowing in 3 is x 3 and so on.

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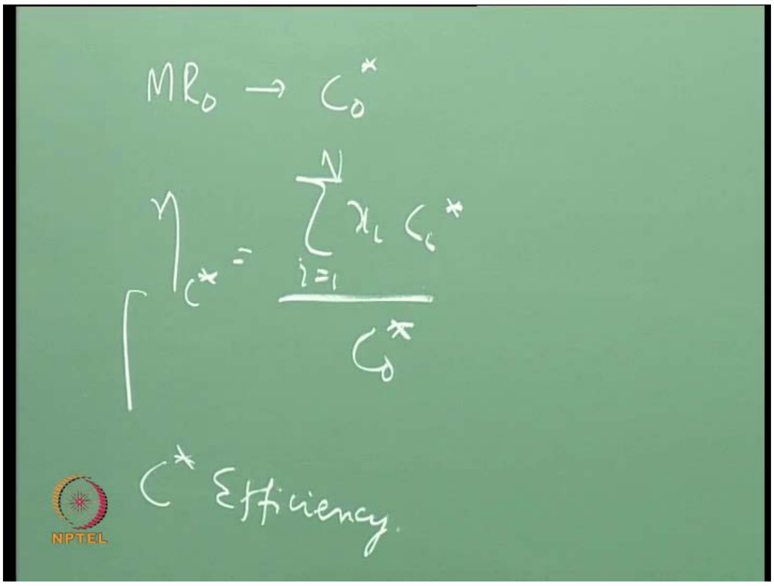

$$MR_0 \rightarrow C_0^*$$

$$\eta_{c^*} = \frac{\sum_{i=1}^{N} x_i c_i^*}{C_0^*}$$

C^* Efficiency.

Therefore, the summation of x would be what? Summation of the mass fraction of mass flow rate going from let us say stream tube 1 and let us say I have n stream tubes will be equal to what? Unity very good that that is what it is, therefore, this is my first equation.

And my second equation is maybe when I have mixture ratio over here corresponding to a lower mixture ratio or corresponding to a different values of mixture ratio at each mixture ratio let us say r i at mixture ratio r i the c star value would be c i star, it is not going to be the optimum value, but it is going to be a different value. Now, therefore, now I say I want to find out the mass weighted c star for the chamber and therefore, what is it I do? I say I have x i in the i s stream tube, in the i s stream tube I have x i c star and therefore, the net mass weighted value of c star goes from 1 to n, where I x i is the mass fraction in stream tube I c i is the value c i star is a value of the characteristic velocity in that particular stream tube.

Now, this is going to be instead of getting the value of the value corresponding to this let us say the value which I get corresponding to M R o that is the mean mixture ratio, let us say the value as per the figure is c naught star then the value of efficiency what I get is I get a value equal to x i into c i star as I goes from 1 to n divided by the value of c naught star, and this gives me my efficiency due to the mal distribution or distribution of mixture ratio. What is it we do? We calculate the performance of each stream tube as if it were an individual rocket, we calculate the fraction of the propellant going and that gives me the mass weighted c star, and that mass weighted c star divided by the conditions at injection is what gives me the value of c star efficiency.

I think this concept is important, you know first time we tried it around 10, 12 years back and we found it works and we can have an idea on how to distribute things such that I can get the optimum value of c star, because we have to distribute the propellants and when you distribute it you get variable mixture ratio, and the assumption is individual stream tubes do not mix with each other, and why they do not mix? Because the length is short the turbulence still does not develop totally and therefore, mixing between stream tubes is not there. And now how do I distribute see we have done the problem, but can I characterize a distribution parameter.

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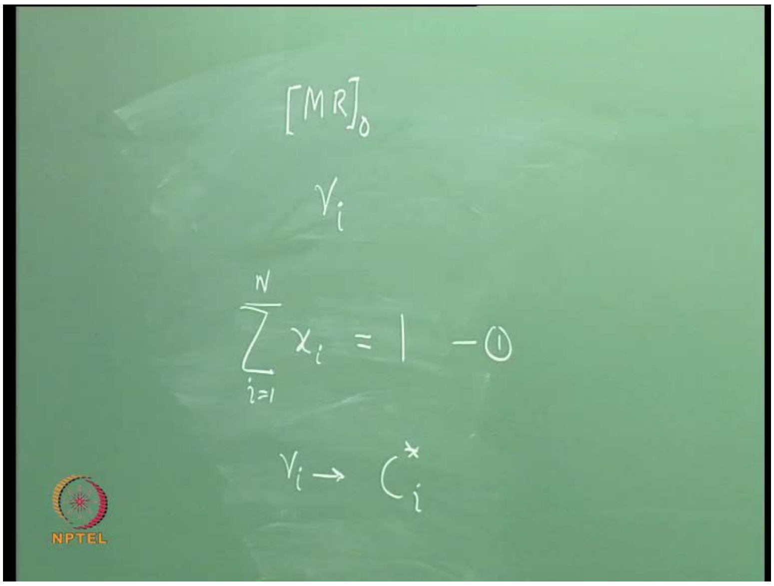

What should the distribution parameter consist of? Let us erase this out and put that down, we tell ourselves again come on.

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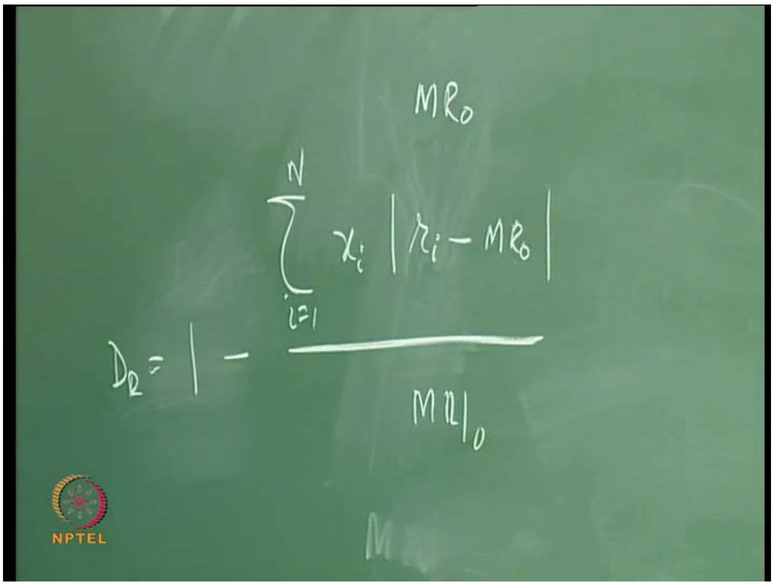

In the stream tube i, the value of the mixture ratio is let us say r i, the overall mixture ratio is let us say M R o we still look at this therefore, the change from the mean is equal to r I minus M R o, this is the change in mixture ratio in the ith tube, it could be greater than 1; it could be greater than M R o; it could be less than this, because I could have either side, therefore, or what is it I am talking I should not have erased this figure maybe I could have again I plot it, this is the value, this is the value of your M R o; this is the scale which is mixture ratio, it could either be here or here, therefore, I say well the change from the mean is the modulus of this value, and what is the value? The fraction is going as x I and therefore, I can put something like a distribution parameter dr as equal to the fraction of the propellant in the particular stream tube.

And I sum it up over all the stream tube going from 1 to n, and I divide it by the mean value of the mixture ratio, in other words I get the distribution parameter dr as equal to the dispersion multiplied by the fraction divided by the mean mixture ratio, and what is it I find? If I have a beautiful mixture ratio distribution everywhere I get the same value, well my value comes out to be 0, therefore, I have to make some changes; that means, I am looking at a distribution therefore, dr should be equal to one minus this value, if everywhere I have the same mixture ratio well my distribution parameter is unity, if I have at different places different mixture ratios well the deviation from unity will tell me what is the distribution index which is known as a mal distribution or distribution parameter.

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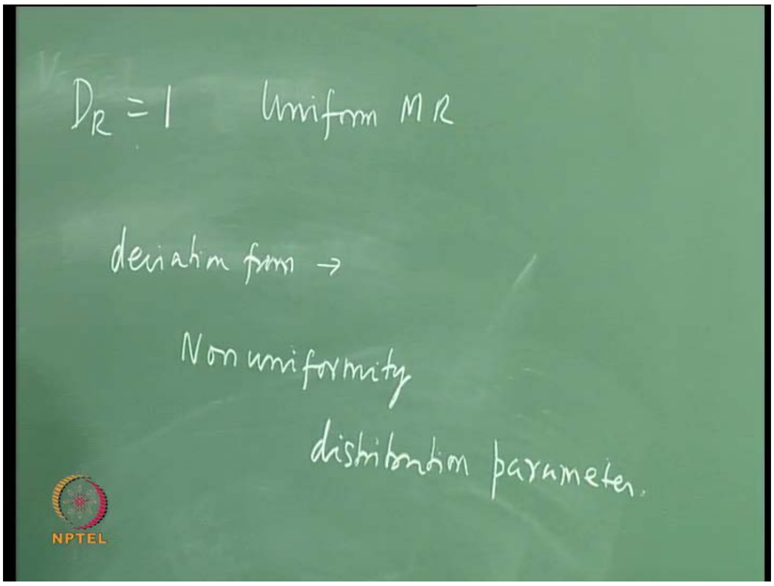

$D_R = 1$ Uniform MR

deviation from →

Non uniformity

distribution parameter.

Therefore, this tells us let us put it together it is something which we must learn to use, we tell ourselves well the distribution parameter dr if it is 1 it tells us I have uniform mixture ratio, whereas, the deviation from 1 if it is always going to be less than 1, and the and the amount it is less than 1 tells us how much it is deviating from a uniform distribution, and therefore, we say it is a non uniformity parameter. right Now, I would like to again revisit the problem what we had, we told ourselves eta c star is equal to some of the x I's into c i's star and this value we expect it to be less than c not star, and therefore, eta c star will be less than 1, therefore, there is no way it can be greater unless your mean is something wrong you have decided the mean for a lower value and then if you operate this I could have a higher value otherwise it is not possible, how does eta star depend on dr? Well, I have let us let us plot it out.

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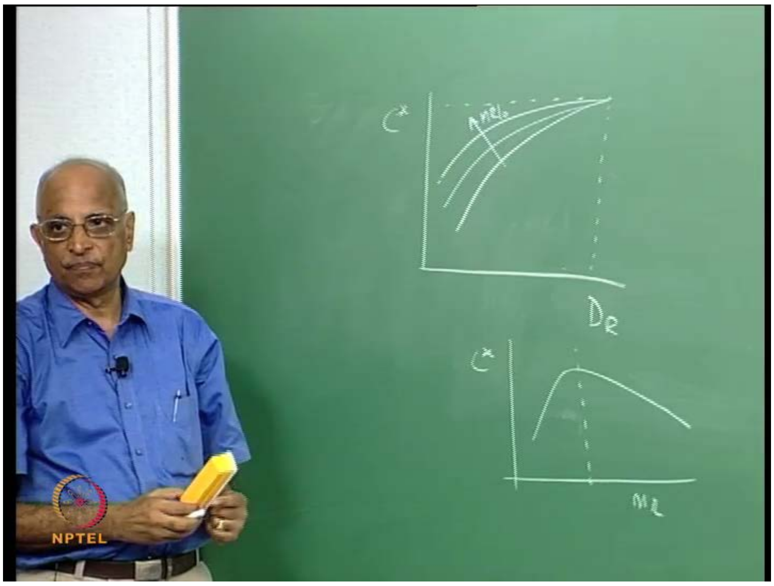

I have the value of c star as a function of dr, when the distribution parameter is unity I get the value corresponding to the maximum, as the distribution parameter decreases my value falls down, but when we looked at this particular figure which I keep erasing out each time and even though I have to use it, what was the shape like? It is like this. This is the nominal value of let us say M R o which I choose, I find that the slope of this in the fuel rich region is somewhat steeper, in the oxidizer rich region or the above the thing it somewhat shallower, and therefore, if I increase the value of mixture ratio I will get curves which go like this, this we say as a function of mixture ratio nominal.

All what I am saying is if I were to have the nominal here then the change in c star is little bit less, whereas, if I choose a value here I am in this gradient and therefore, it will fall, and this is how c star variously distribution parameter had with mixture ratios, this is all about distribution which causes a penalty; that means, what is we are telling.

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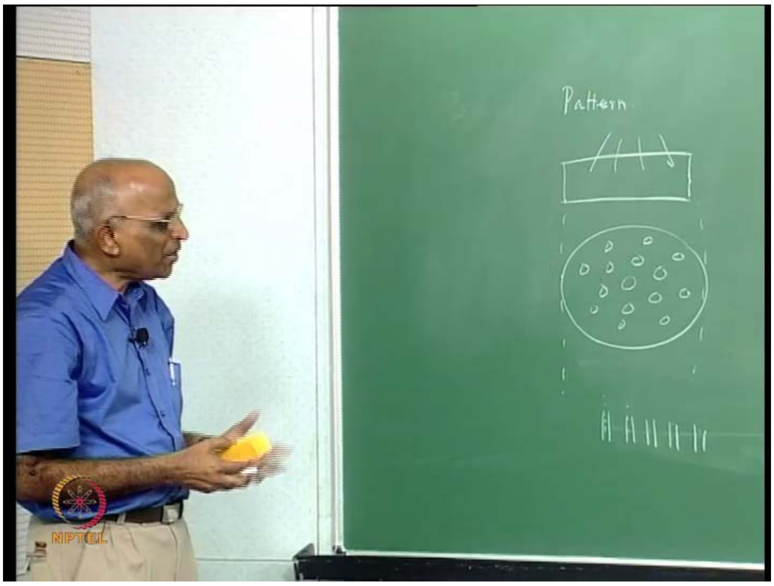

The mal distribution or distribution of mixture ratio in the combustion chamber or thrust chamber is of cause of concern and leads to a penalty, and how do you qualify this penalty? In terms of efficiency which is the real value of c star due to distribution divided by the value of c star at the nominal injection value. And how do you get this? Let us put it as c star net or c star equivalent that you calculate from the c star corresponding to the individual stream tubes and the mass of propellant flowing through the stream tubes. I think I hope this is clear are there any questions on this, because this is something which you will not find in text books, but his is something which we must understand right.

(( )).

Can I restate your question; you want to know how to determine the distribution of mixture ratio? Well you know the injector type of injector you use, you know how it is forming droplets, you know the distribution of droplets at the head, you know what type of droplets of oxidizer and fuel are coming, because your concern is how do I estimate the value of let us say x i for each of the stream tubes and the value of c i, c i you know anyway because you know how c star varies with mixture ratio, the only thing you need is what is the value of x i? You know you can think in terms of a simple experiment, let us think of it, and such experiments are known as patternization; that means, you would like to study the pattern of the injector.

What you do is you have an injector over here let us say the injector is like this, I have a series of these injector holes which are there let say it is either coaxial; it is either impinging jet whatever it is that you have a series of holes, and what is it I do? I sort of simulate the oxidizer by a particular fluid simulate the fuel by another fluid and now I pass the fluid through this injector may be at ambient conditions or hold the chamber under pressurized condition, I put a series of test tubes here; I put a series of test tubes all along a little bit away in the three dimensional zone, and now I collect the mass of the fuel mass of the oxidizer at the different places; that means, I can study the pattern formed by the injector, and this is what is used to calculate the x i dot, and once you know x i dot I can find out the non uniformity parameter and the efficiency due to the non uniformity.

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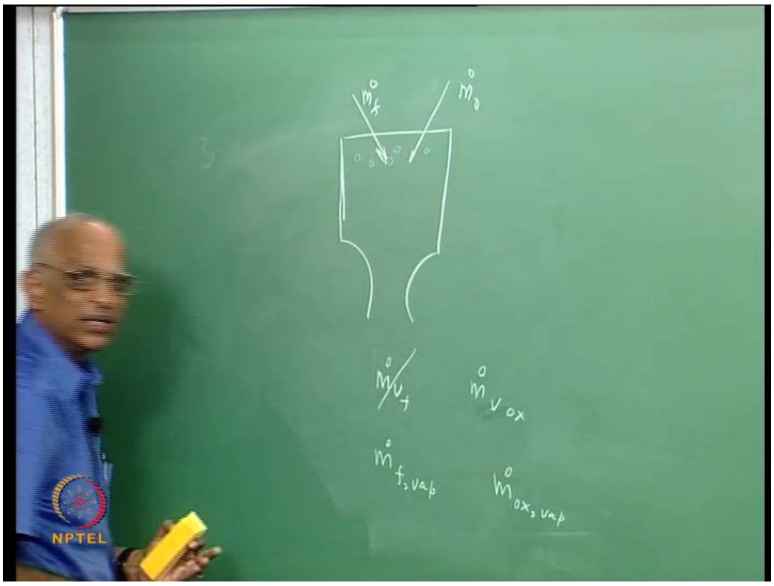

Now, I go to the next you know are there any other problems we could have lets quickly go through the next one and next one, is somewhat little more challenging what is it? Again I pose the same problem again, I have the chamber thrust chamber I inject let us say mass of fuel at a particular flow like kg per second, mass of oxidizer at a certain rate into it, and these are both as liquid propellants I am injecting, but what really reacts is not liquid and react liquid does not react with liquid, what happens? I form droplets the droplets have to evaporate, the vapor has to mix and burn therefore, what will react is mass of the vapor formed from the liquid fuel let us say m v F and we say m dot of oxidizer m dot vapor of the oxidizer.

Let us make this even clearer, instead of using this symbol I say out of liquid fuel which is sent over here may be the chamber is such that not everything evaporates, and everything if it comes and evaporates here you are losing something, may be it must evaporate in the zone for combustion, we say m dot F of the liquid fuel which is injected only a certain portion evaporates instead of using this symbol I used this symbol, I say out of the oxidizer liquid oxidizer only a certain fraction evaporates, because my chamber I cannot keep on increasing the chamber to infinity to make sure everything evaporates and everything burns.

Therefore, it is quite possible that not all the fuel which is injected vaporizes, not all the oxidizer which is injected, may be just after this I cover this I will go and tell you what is that time taken for things to evaporate, let us do that exercise also, now if I have to calculate that how do I do it, how do I calculate what is the penalty I pay? If all the liquid fuel and all the liquid oxidizer, which I inject into the chamber does not evaporate and mix and burn.

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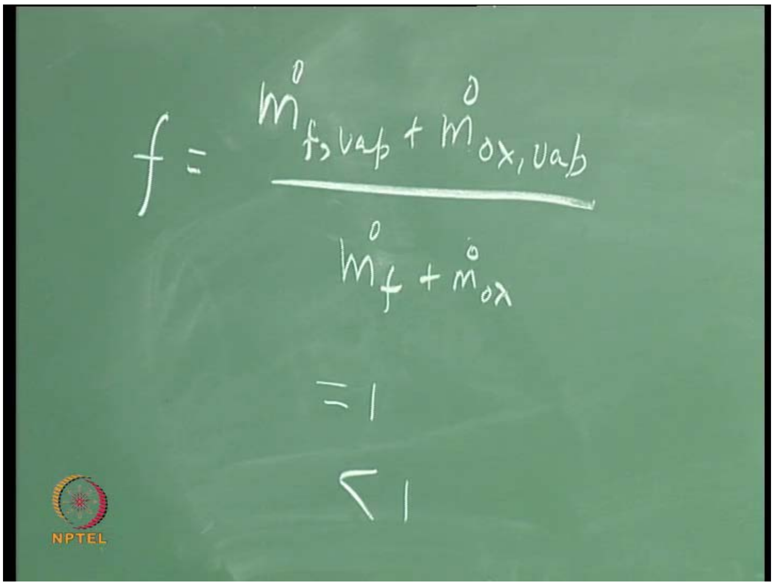

$$f = \frac{\dot{m}_{f,vap} + \dot{m}_{ox,vab}}{\dot{m}_f + \dot{m}_{ox}}$$

$$= 1$$

$$< 1$$

Therefore, now I say out of the total propellant which is m dot F plus m dot o, the thing which gets converted to vapor is equal to m dot of fuel which vaporizes plus m dot of the oxidizer let us say oxidizer which vaporizes, therefore the fraction of the propellant which really contributes to combustion is m dot fuel which is vaporized, plus m dot of the oxidizer which is vaporized divided by the total liquid which gets injected over here, is it all right; that means, now I have to say this fraction will anyway be maximum it will be equal to 1 or it will be less than 1, now I add another twist to the whole thing I tell myself well what is the mixture ratio corresponding to evaporation?

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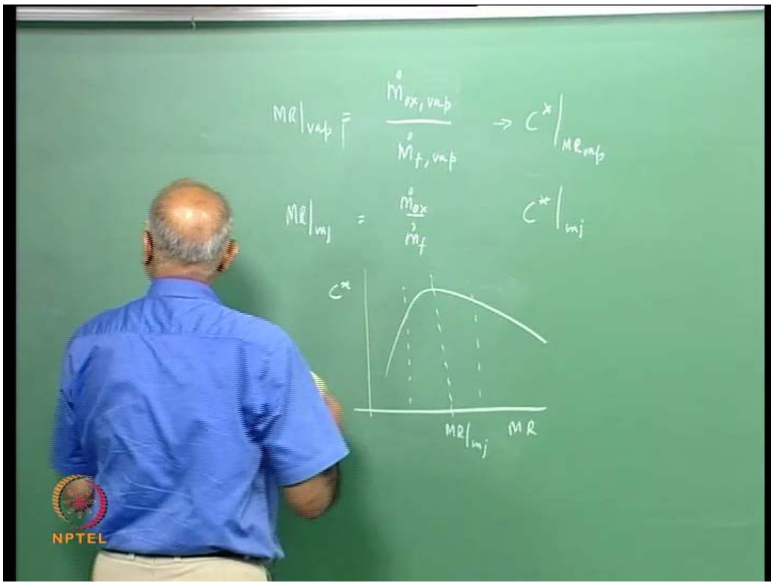

The value of mixture ratio corresponding to evaporation is equal to m dot oxidizer which has vaporized divided by m dot fuel which has vaporized. Now, what is the value of mixture ratio at injection? Is equal to m dot oxidizer which is injected over here whatever is injected now I say oxidizer divided by m dot fuel which is injected. And this is going to be different from this, and what is going to happen? Let me re-plot that figure this figure is a very crucial figure, because I keep on using it, we say well it is like this and goes like this, this corresponds to may by injection I say M R corresponding to injection, which is what is over here; maybe the question whether oxidizer evaporates faster, fuel evaporates faster depending on which evaporates faster I could have vaporization here; or vaporization here, in other words I could have a c star corresponding to mixture ratio corresponding to vaporization I will have a c star corresponding to injection which is the nominal value. And now, what is the efficiency due to non evaporation, how do I put it together? Well I have the fraction here which is playing havoc, and I have the ratio of what is evaporated to this.

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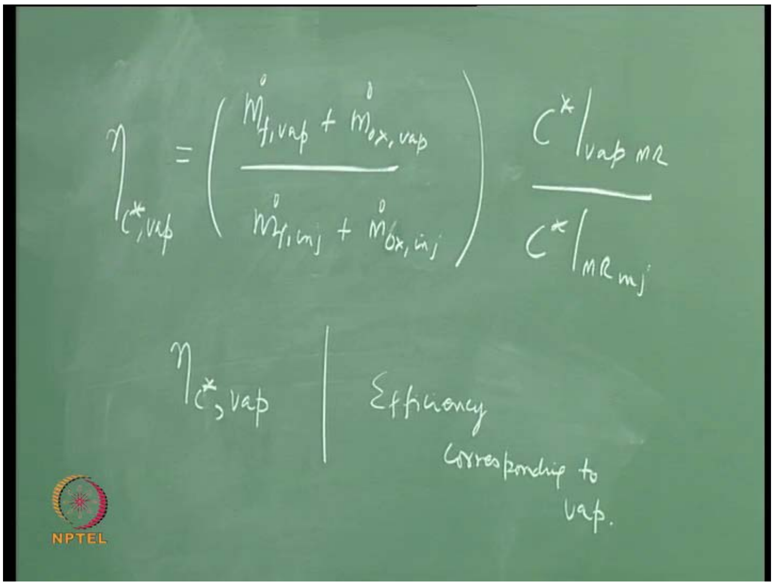

$$\eta_{C^*,vap} = \left(\frac{\dot{m}_{f,vap} + \dot{m}_{ox,vap}}{\dot{m}_{f,inj} + \dot{m}_{ox,inj}}\right) \frac{C^*|_{vap\,MR}}{C^*|_{MR\,inj}}$$

$\eta_{C^*,vap}$ | Efficiency corresponding to vap.

Therefore, I tell myself the c star due to evaporation being limited is can be written as fraction coming over here, which is equal to m dot fuel which is vaporized; plus m dot oxidizer which is vaporized, divided by m dot fuel corresponding to injection; plus m dot fuel m dot oxidizer corresponding to injection, into the value here which is c star corresponding to what is being vaporized mixture ratio; and this is c star corresponding to mixture ratio corresponding to injection, and this gives me what? c star mass weighted c star over here and therefore, this gives me the efficiency due to vaporization.

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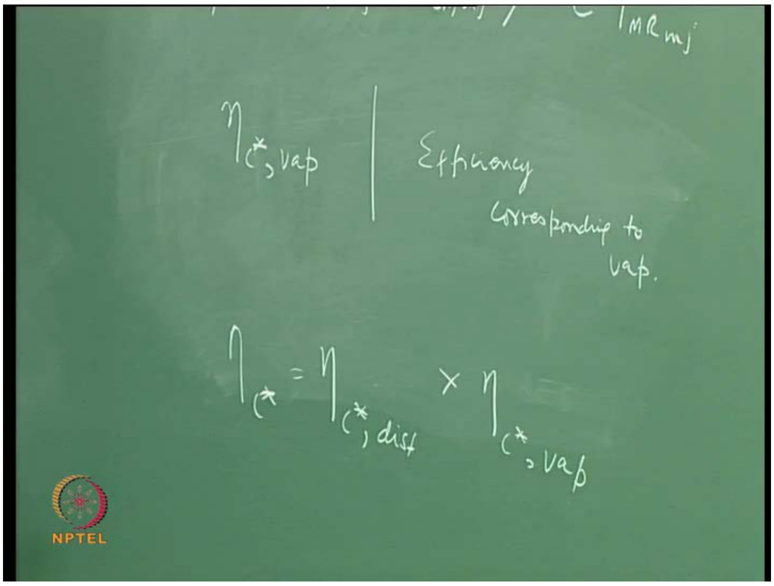

$\eta_{C^*,vap}$ | Efficiency corresponding to vap.

$$\eta_{C^*} = \eta_{C^*,dist} \times \eta_{C^*,vap}$$

In other words, this was the value of c star modulated by the total mass of propellants which are evaporated, this is the value of c star modulated by the total quantity and therefore, the you have a value of c star corresponding to vaporization over here, and in this term I find this will be always less than 1, this depending on the choice of my injection could be either less than 1 or greater than 1, but in general the c star vaporization will be less than 1, and therefore, I get an efficiency corresponding to vaporization. Therefore, what will be my total efficiency? The total efficiency of my chamber eta c star would be equal to eta c star due to distribution and that is what we had here.

Therefore, this was due to distribution maybe we could also write it as due to distribution, because we are also getting the second one due to vaporization, and the next one due to eta c star due to vaporization and this is my net value of c star in the chamber. See I did not consider diffusion mixing and I did not consider chemical reactions, because chemical reactions and diffusion are somewhat much faster than evaporation and therefore, evaporation tends to be the controlling thing, and that is why we say the total efficiency of c star is equal to that due to distribution and that due to vaporization. I think this we must keep in mind. See I just made a case saying that well we need vaporization we would like vaporization to be completed.

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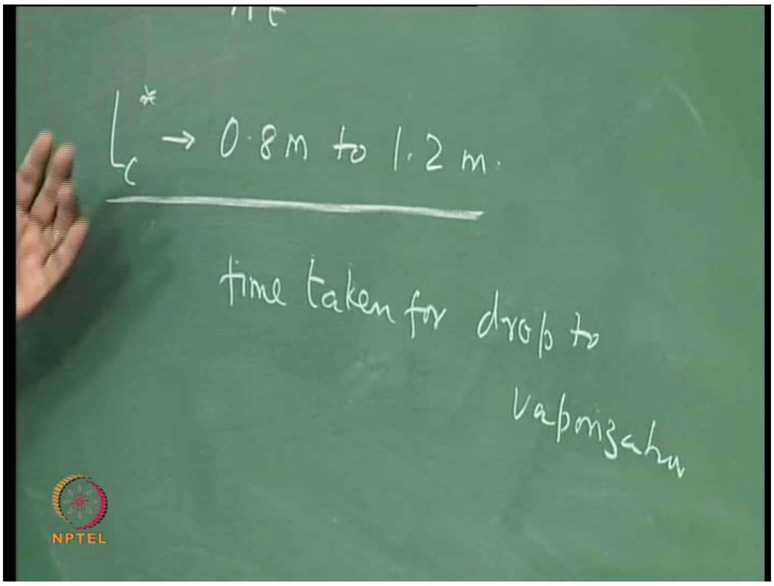

And normally we choose l star of the chamber, and what did we say was l star? l star was equal to the volume of the chamber divided by the throat area which has units of length meter cube by meter square, and this length is something which tells us whether vaporization could be completed normally rockets are so designed that l star varies between 0.8 meters to 1.2 meters. 0.8 meters is a hydrogen oxygen 1.2 is nearer to things which are hypergolic, but I do not think this is a real thing maybe we should really go back and find out the time taken for a drop to vaporize, I think that is more reasonable right, but these are thumb rules and these are based on experience and there is no way this could be wrong, and what is l star? l star tells me the distance taken for vaporization and combustion to be completed and therefore, this l star tells is given between these two values for it to be completed, an indication of residence time during which the combustion events could be completed.

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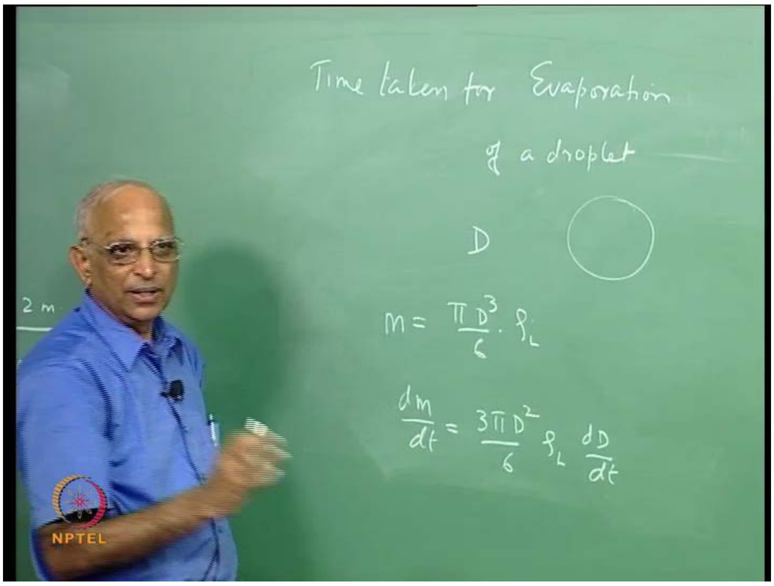

Therefore, let us do the last part which wherein we would also like to find out how do I estimate the time taken for burning to occur or time taken for evaporation to occur? Now, I introduce the word droplet, I have a series of droplets which are produced by the injector I say that the mean droplets size is let us say d, I want to find out what is the time taken to for that droplet to vaporize? So, immediately I tell myself that the mass of the droplet is equal to let us assume the droplet is spherical, therefore, I have pipe into D cube by 6, 4 upon 3 pi r cube or pi D cubed by 6 is the volume, let us say D is in meters this is so much in meter cube, and multiplied by the density of the liquid that is liquid fuel or liquid oxidizer is the mass of the droplet right is it All right, volume multiplied by the density.

Now, I want to find out the rate of evaporation I say dm by dt is due to density is a constant, the diameter of the droplet changes therefore, I get 3 pi D square divided by 6 into the density into d D by dt, thus this tells me how the droplet diameter changes with time, but we must also keep in mind you know let us keep the physics clear, the droplet vaporizes at the surface therefore, you should have something to do with the surface area, and what is surface area of a sphere of diameter D? It is something like pi D square, therefore, I rather look at pi D squared that is d D squared by dt rather than d D by dt, I think for anything to evaporate since its happening at the surface I am more interested in the variation of D square rather than d, therefore, lets convert it to this parameter D square parameter.

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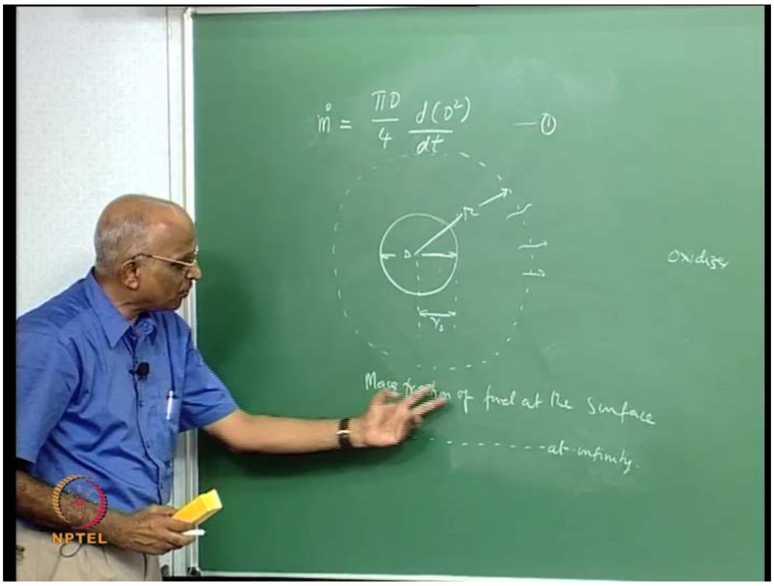

I can write therefore, dm by dt which is equal to m dot of the droplet is equal to I have pi D divided by 4 into d of D square divided by dt, what did I do? I brought one d here it becomes d D square, but d of d D square is 2 d therefore, another two should come here therefore, I should divide it by 2 and I get the value of pi D by 4 d by dt square.

Now, let us do this 2 d D square comes here, and I get 2; 2 and two get canceled and I get back the value of 2 over here, let us say this is my equation 1, but I want to find out what this value is therefore, let us picture in our mind I have a droplet let us say of diameter d fuel droplet of diameter d, let say the radius of the droplet also we will keep in mind that is r is the value maybe r s at the distance of the surface, and how does a droplet evaporate why does why should it evaporate in the first place? At this surface of the droplet maybe I have high value of concentration of the fuel far from the droplet I have only oxidizer, because what is it I am considering. I am considering the evaporation of a droplet in an oxidizing medium, let us say I have a diesel droplet in air therefore, why should diesel droplet evaporate? I have a higher concentration of diesel here, I have a very low concentration of here, because of the concentration gradient I have the fuel migrating.

Therefore, in other words I am now interested in something like a mass fraction of fuel at the surface, maybe I am also interested in mass fraction of fuel at infinity, these are the two things I am interested, let us consider the following let us take an area a let us consider that the droplet is evaporating and as the droplet is evaporating mass of the fuel is getting vaporize, let us consider a radius r over here some diffusion is happening something is moving out, now what is it I am saying mass faction of the fuel at the surface how do I how do I write an equation for the mass fraction.

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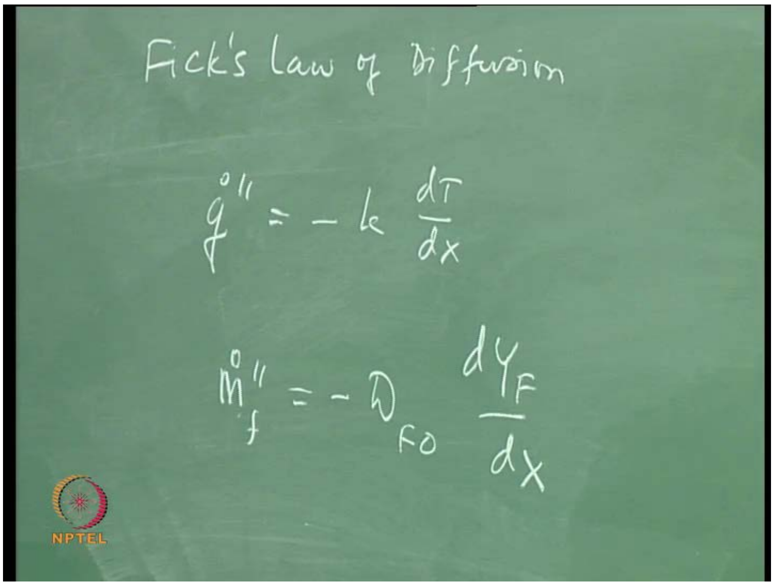

Fick's Law of Diffusion

$$\dot{q}'' = -k \frac{dT}{dx}$$

$$\dot{m}_f'' = -\mathcal{D}_{FO} \frac{dY_F}{dx}$$

We say mass fraction and mole fraction we told ourselves mole fraction is denoted by X, mass fraction is denoted by y capital Y or small y, and what is the mass fraction it is equal to mass of the fuel I am interested in, mass of the fuel divided by the mass of the fuel I do not even need this I am just writing this mass of the oxidizer this is the fraction of the mass of the thing which is getting, which is that is the total mass is this; this is the fraction of the fuel this is this.

At infinity the value could be 0 or a small number, at the surface the value could be a large value, let us assume that the value at the surface I have to calculate I say it is Y F of s at infinity it is equal to Y F of 0, because of the variation in the mass fraction; that means, I have a density in mass fraction, I have the mass which is migrating out, and what is the law which tells us how the what is the mass flux which goes? We say that is the flick's law of diffusion, in case some of you are little little bit rusted on this all what we say is we have heat conduction equation, what does heat conduction equation tell you? That the heat flux is equal to something like a Fourier law k into the temperature gradient dt by d x, and similarly if I were to write an equation for mass flux it is equal to may be the diffusion coefficient of the particular medium which is going in this.

Let us say fuel, diffusion coefficient of fuel in oxidizer or fuel in oxygen or fuel in oxidizer into let us say the value of the gradient over here, but you will also recall that the equation which is given for diffusion is given in terms of concentration gradient, now I am writing as a terms of mass fraction therefore, let us clarify what the unit should be, the diffusion coefficient since its concentration we use the diffusion coefficient and the unit is meter square by second, the mass flux is equal to kilogram per meter square second.

Well concentration has no units, because it is a fraction no here you have mass fraction as no units and therefore, the unit is 1 over meter, and therefore, now if I have to have the same units I must multiply it by the density which has units of kilogram per meter cube, and therefore, now what happens is I have meter, meter square, kilogram per meter square second is what I get over here, therefore, in terms of concentration flick's law of diffusion says mass flux at any distance is equal to the diffusion coefficient of the particular substance in the medium multiplied by the density of the medium at that particular point divided by this, this is the flick's law. I have to use this law because I want to find out the rate at which it is going.

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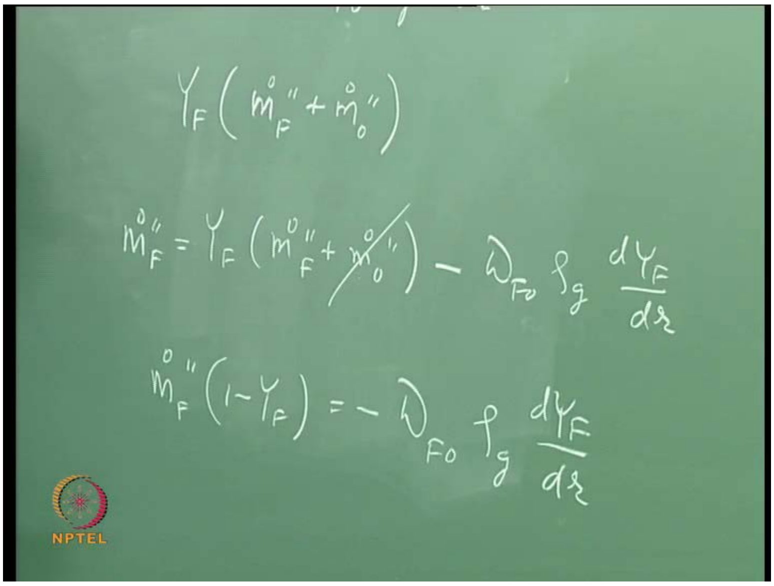

$$Y_F\left(\dot{m}_F'' + \dot{m}_o''\right)$$

$$\dot{m}_F'' = Y_F\left(\dot{m}_F'' + \cancel{\dot{m}_o''}\right) - D_{Fo}\,\rho_g \frac{dY_F}{dr}$$

$$\dot{m}_F''\left(1 - Y_F\right) = -D_{Fo}\,\rho_g \frac{dY_F}{dr}$$

Therefore let us now write out the equation, I say because of diffusion due to the gradient I have minus diffusion coefficient of fuel in oxygen multiplied by the density, and what is the density? We are looking at the gas density lets be very clear, over here I should have multiplied this we all did not remind me rho L was missing, because we had m dot is equal to pi D cube into rho L was here, whereas, here it is the density of the gas which I am considering let us put it density of the medium gas over here, and what is it I had rho g into the value of dY F by dr, because our coordinate is r instead of x I am considering a droplet and I am considering a symmetric therefore, my dimension is r over here.

Now, I have to solve this, but now I realize I have a concentration Y F of let us say m dot of the fuel plus m dot of the oxidizer which is available at any particular point, and the fraction of that is also available, and therefore, I must say that the mass of the mass flux of the fuel which is leaving is equal to Y F into this is due to the masses which are available of the oxidizer which is available, plus now I get the value due to diffusion fuel oxidizer into rho g into dY F by dr, why did this term come? See we talked in terms of due to the gradient you have this, but at any point you have m dot o x and m dot F, and the fraction of that is also available to you.

Therefore, the total mass flux which is available is the sum of these two, and again we note that F is what is migrating o is not migrating therefore, this can be dropped this is equal to 0, and therefore, I can write m dot of the flux of fuel which is migrating into 1 minus Y F is equal to minus diffusion coefficient of fuel in oxygen into rho g multiplied by dY F by dr.

Now, I find yes I am looking at the mass flux, I would like to convert it into the mass, and what is the value between relation between mass flux and this? We immediately tell ourselves well the mass flux of fuel is equal to the amount of fuel which is migrating divided by 4 pi r square, this is the flux; this is the total area; this is the quantity kilogram per second; this is kilogram per second per meter square therefore, this is the value, and therefore, from that equation if I were to substitute it let lets finish it off on the extreme right side.

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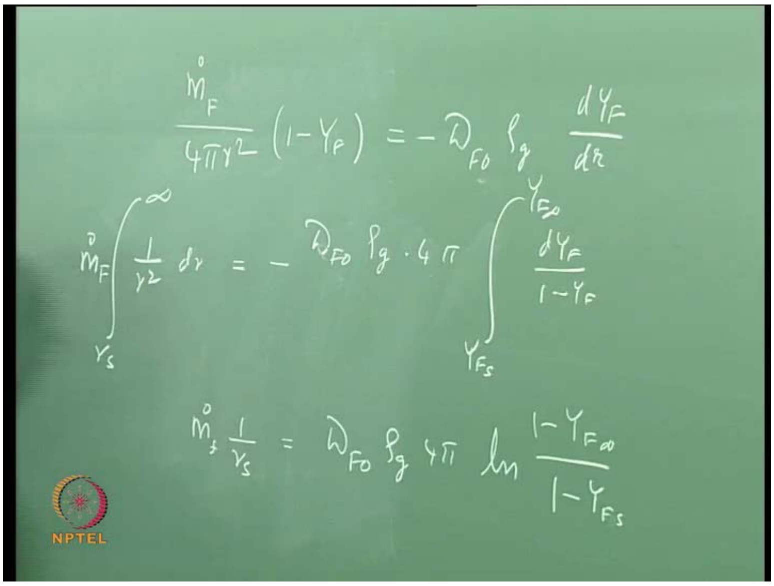

$$\frac{\dot{m}_F}{4\pi r^2}(1-Y_F) = -D_{FO}\,\rho_g\,\frac{dY_F}{dr}$$

$$\dot{m}_F\int_{r_s}^{\infty}\frac{1}{r^2}\,dr = -D_{FO}\,\rho_g\cdot 4\pi\int_{Y_{Fs}}^{Y_{F\infty}}\frac{dY_F}{1-Y_F}$$

$$\dot{m}_f\,\frac{1}{r_s} = D_{FO}\,\rho_g\,4\pi\,\ln\frac{1-Y_{F\infty}}{1-Y_{Fs}}$$

I now get the value of m dot of the fuel which is leaving into 4 pi r square, no divide it by 4 pi r square into 1 minus Y F is equal to minus diffusion coefficient of the fuel in oxygen multiplied by the density of the gases into dY F by dr, I play with this equation a little bit I simplify it and now I say dY F by dr is equal to minus if you please be careful.

Let us make sure we are doing things correctly, d F naught into rho g into 4 pi r square, no dY F means everything should come at the denominator, therefore, I should get if I have to keep it in the numerate d F naught rho g into 4 pi r square into I get the value of dY F by 1 minus Y F should be equal to what on this side? Should be equal to 1 over r square I take I remove the r square here I bring it on this side, into dr into the value of m dot F. What is it I have done? m dot F into 1 over r square, I just transpose 4 F 4 pi over here d F naught rho g into 4 pi I still retain this sign and this is what I get.

I want to what is happening? The droplet changes from the concentration Y F s at the surface to Y F at infinity, and therefore, I integrate this equation between let us say the surface of the droplet at r s to infinity, and therefore, what is it I have the corresponding thing here? I have Y F at the surface corresponding to Y F at infinity and this is what should give me the answer, and therefore, now I simplify it and get the value of mass which is going is equal to what? This is equal to minus 1 over r; minus 1 over r infinity is 0, minus minus is plus therefore, I get r is I have dropped the value of F may be we could retain it is this integration, now I integrate on the right hand side d F naught I have a negative sign dY F by 1 minus 1 Y F, I have minus sign minus log of 1 minus 1 F, minus and minus sign cancels into rho g into 4 pi into logarithm of, what logarithm of 1 minus Y F at infinity divided by 1 minus Y F at the surface.

Is it all right, this is the value of the mass which is leaving, but the mass which is leaving we always saw we already saw it is equal to pi D by 4 into d D square by dt, and therefore, I can substitute it in that particular relation and if I were to do so what is is I get?

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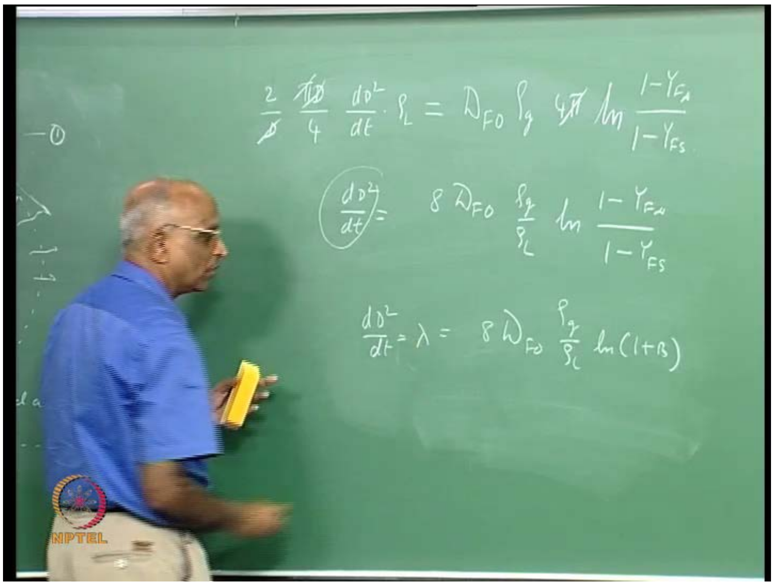

Let us substitute the value of m dot F is equal to pi D divided by 4 into d of D square divided by dt into rho l is now equal to, I have to get the value of r s over here, is equal to I have whatever I have written on the right hand side diffusion coefficient due to the fuel in the oxidizer into the density of the gas at that particular point into 4 pi into what is it I get? lon of 1 minus Y F at far away the concentration or the mass fraction divided by the mass fraction at the surface.

What is it I am interested? I am interested in solving for the rate of evaporation which is d D square by dt, is therefore, equal to or else I can put in terms of diameter which is equal to d by 2, and therefore, what is it I get? I get this d and this d cancels, pi and pi gets cancels I get 4into4into is equal to 16, so therefore, it becomes 8, diffusion coefficient of fuel and oxygen into rho g by rho l into I get lon of 1 minus Y F at infinity divided by 1 of the mass fraction at the surface, all other terms have gone and this particular value d D square.

Now, when I look at this expression well that diffusion coefficient depends on the type of fuel type of oxidizer and therefore, for a given combination at a given mixture ratio this is a constant, density cannot change much, density of the liquid is constant, I am just talking of these two parameters let us put this together, I have a term 1 minus the mass fraction of fuel at far away, minus 1 minus y of mass fraction at the surface, supposing I were to define a term b I just say a term b which I call as equal to mass fraction at the surface minus mass fraction at infinity divided by 1 minus the mass fraction at the surface, and I call this as something you know at this surface minus infinity is something which transported, I call this as a transport number.

Then immediately I say 1 minus F infinity into 1 minus F s in terms of b becomes equal to 1 plus b, because if I add 1 to it I get Y F s minus 1 over here, I therefore, can write the expression as equal to the value of I can write the this particular expression as equal to 8 diffusion coefficient of fuel in oxygen rho g by rho l into ln of 1 plus the transport number, and this is the rate of evaporation I find it really is a function of fuel and oxidizer which I say is equal to d D square by dt which is a constant for a given combination, and this defines my evaporation rate constant. Therefore, I find evaporation rate is a constant and therefore, the time taken for a droplet of size d naught.

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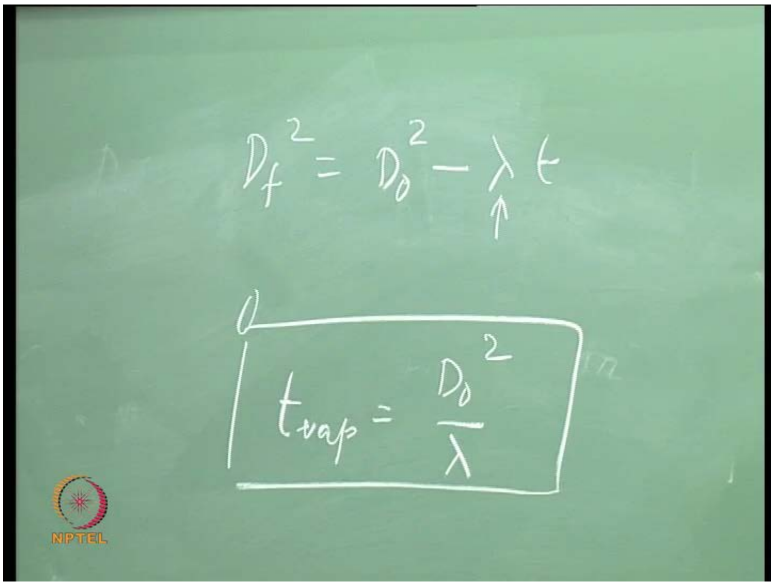

Let us say the droplet size is d not, the time taken for it to evaporate will go as d naught square; that means, I am looking at the final value d F square is equal to d naught square minus lambda t, lambda is known from this equation then my final value is 0, therefore, the time taken for evaporation is equal to d naught square by lambda, and therefore, I know my time taken for evaporation; I know the droplet therefore, I can find out the length of time taken or the fraction of the fuel evaporated, and this is all about evaporation and time taken maybe we should learn to play with these equations the transport equations find out this, and therefore, before I stop this let me quickly summarize what we have been doing in this class.

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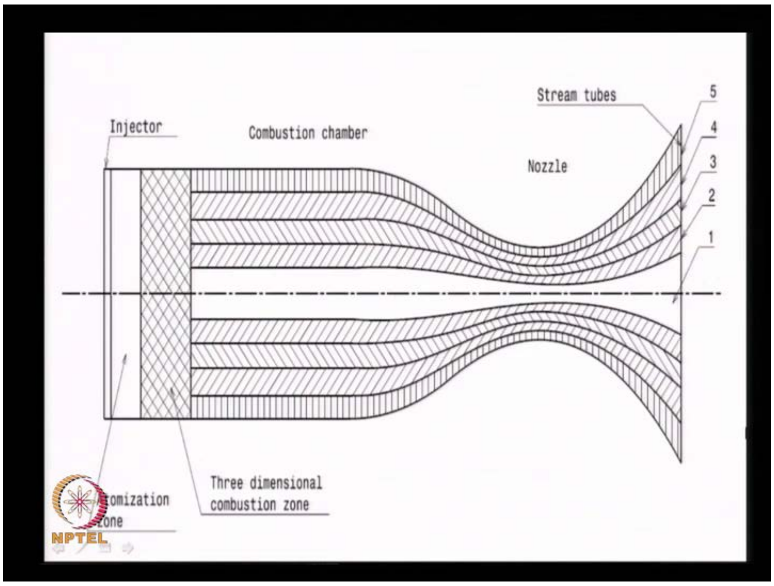

We started with a stream tube model, we said we have something like several stream tubes here all of them mix.

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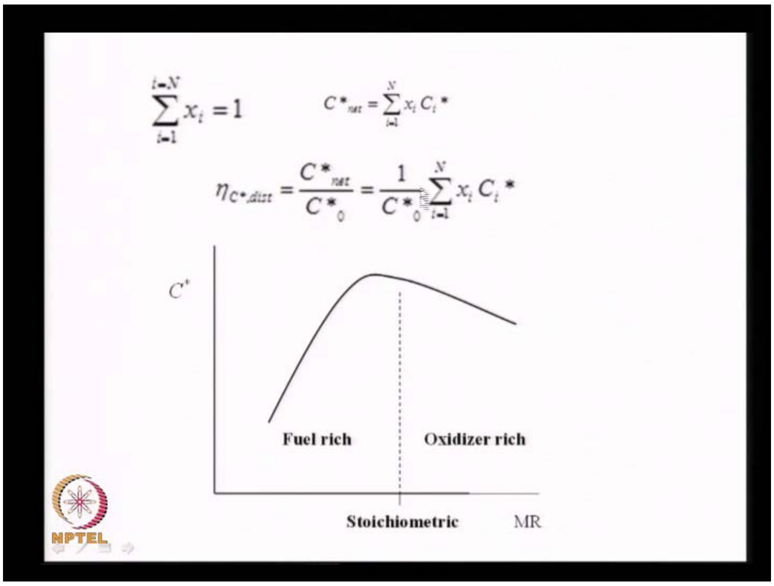

$$\sum_{i=1}^{i=N} x_i = 1 \qquad C^*_{net} = \sum_{i=1}^{N} x_i C_i^*$$

$$\eta_{C^*,dist} = \frac{C^*_{net}}{C^*_0} = \frac{1}{C^*_0}\sum_{i=1}^{N} x_i C_i^*$$

And give a value of eta C star due to distribution is equal to the net volume, as a function of the summation of the product of concentration and the C i value.

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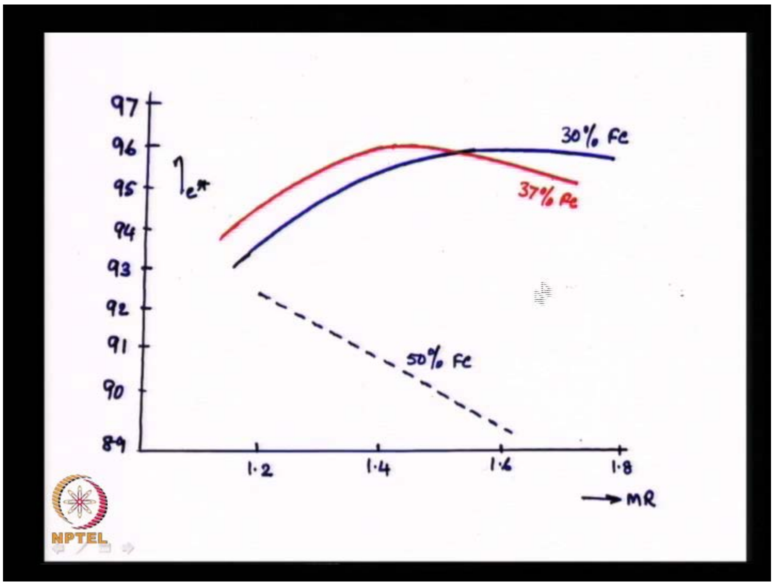

We also you remember I showed you this figure, I said I will revisit it later; when you have more or less different values, different distributions you find that the C star if the distribution is very erratic it falls rapidly, otherwise it mix it changes to different values of mixture ratios. In this case, it is so happened in this experimental engine the peak value was about the same, but normally we would expect a lower value of C star as the percentage of film cooling increases.

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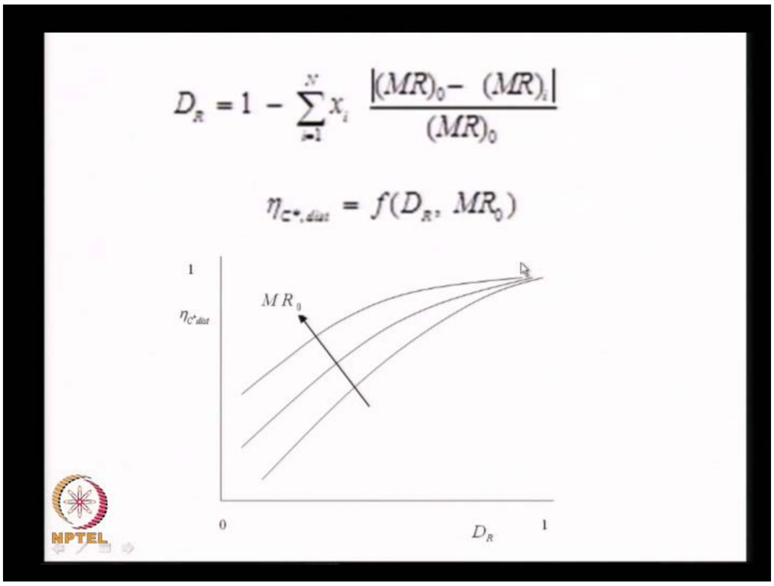

$$D_R = 1 - \sum_{i=1}^{N} x_i \frac{|(MR)_0 - (MR)_i|}{(MR)_0}$$

$$\eta_{c^*,dist} = f(D_R, MR_0)$$

Well this was the distribution parameter this is what I showed on the board, we also said due to evaporation this is the value.

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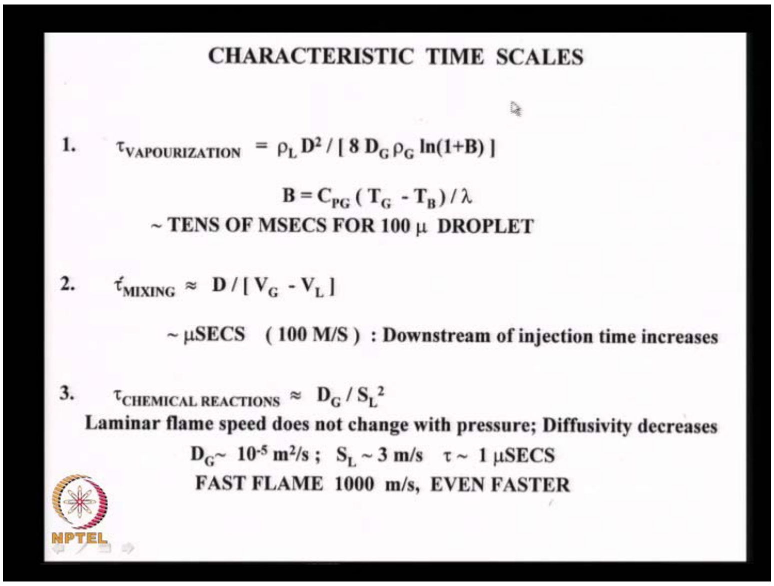

CHARACTERISTIC TIME SCALES

1. $\tau_{VAPOURIZATION} = \rho_L D^2 / [\, 8\, D_G \rho_G \ln(1+B)\,]$

   $B = C_{PG}\,(T_G - T_B) / \lambda$

   ~ TENS OF MSECS FOR 100 μ DROPLET

2. $\tau_{MIXING} \approx D / [\, V_G - V_L\,]$

   ~ μSECS (100 M/S) : Downstream of injection time increases

3. $\tau_{CHEMICAL\ REACTIONS} \approx D_G / S_L^2$

   Laminar flame speed does not change with pressure; Diffusivity decreases

   $D_G \sim 10^{-5}$ m²/s ; $S_L \sim 3$ m/s $\tau \sim 1$ μSECS

   FAST FLAME 1000 m/s, EVEN FASTER

And now I summarize with this, we found that the time for evaporation in terms of the transport number B; you will recall B was defined in terms of mass fraction, it is also possible to define it in terms of enthalpy divided by the latent heat of vaporization. The enthalpy we define it as the difference in enthalpy between the hot gas temperature and the boiling temperature of the liquid, and therefore, B becomes this enthalpy difference divided by the latent heat of vaporization of the liquid fuel.

In this expression lambda is the latent heat of vaporization and not the evaporation rate constant, and the time taken for vaporization typically for a 100 micron droplet is around 10 of milliseconds, we can talk in terms of mixing time as equal to the diffusion coefficient divided by velocity, meter square by second by meter per second this is much lower than vaporization time, and chemical reaction time is something like the characteristic time like let us say the diffusion term divided by the laminar flame velocity square, which is again much smaller than this and therefore, vaporization essentially controls this and we know how to take care of vaporization to get the c star value.

I think, I will complete the remaining portions in the next class. But this is what we did in the class today. Thank you then.