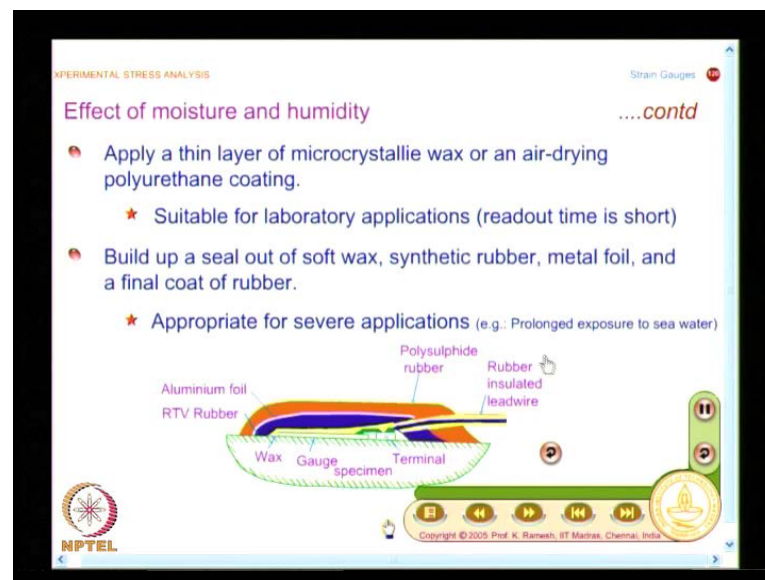


**Experimental Stress Analysis**  
**Prof. K. Ramesh**  
**Department of Applied Mechanics**  
**Indian Institute of Technology, Madras**

**Lecture No # 40**  
**Special Gauges**

We have been looking at finer aspects of strain gauge instrumentation system. And in the last class we looked at how to account for transfer sensitivity in a general case, then we looked at how you can effect these corrections in the case of a T rosette, Rectangular rosette, as well as Delta rosette. Then we moved on to how to make strain measurements under hydrostatic pressure, nuclear radiation, temperature extremes and when you have cyclical loading and finally, we looked at environmental effects. I said water is elixir of life, but water is very bad for strain gauge installation, we saw Strain gauge installation need to be protected against moisture.

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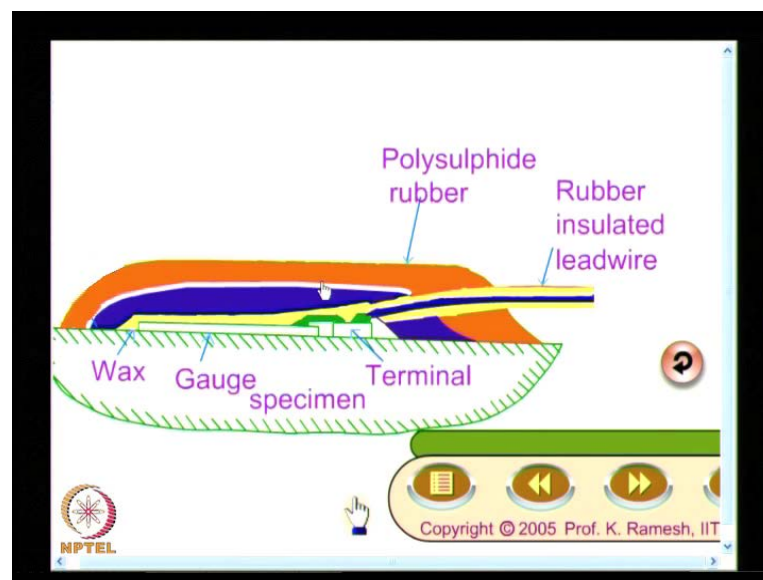


So, when you learn that it has to be protected against moisture, one of the simpler approaches is apply a thin layer of microcrystalline wax or an air drying polyurethane coating and this is good enough for laboratory measurements and your read out time is essentially short. On the other hand if we have to make strain measurements under

prolonged exposure to sea water you need to go for a very elaborate protection for the strain gauge installation.

So, you need to build up a seal out of soft wax, followed by synthetic rubber, then metal foil and finally, a coat of rubber, and you could see this animation which shows the specimen first, then you have the gauge, then you have a seal of soft wax, then you have this RTV rubber, you have aluminum foil and then followed by a coat of rubber. So, when you have to make measurements on underwater pipe line or half shows steel structures below the water, then you need to have such well developed protective arrangement for your strain gauge installation and I would like you to have a neat sketch of this a figurative sketch.

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So, what you essentially have is on the specimen you have the gauge, then you have a coat of wax, then you have a coat of rubber, then you have a aluminum foil and have a final coat of polysulphide rubber and this is also you can see it as animation.

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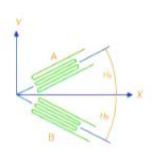
EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

### Plane-Shear or Torque Gauge

$$\epsilon_A = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2} \cos 2\theta_A + \frac{\gamma_{xy}}{2} \sin 2\theta_A$$
$$\epsilon_B = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2} \cos 2\theta_B + \frac{\gamma_{xy}}{2} \sin 2\theta_B$$

The shearing strain is

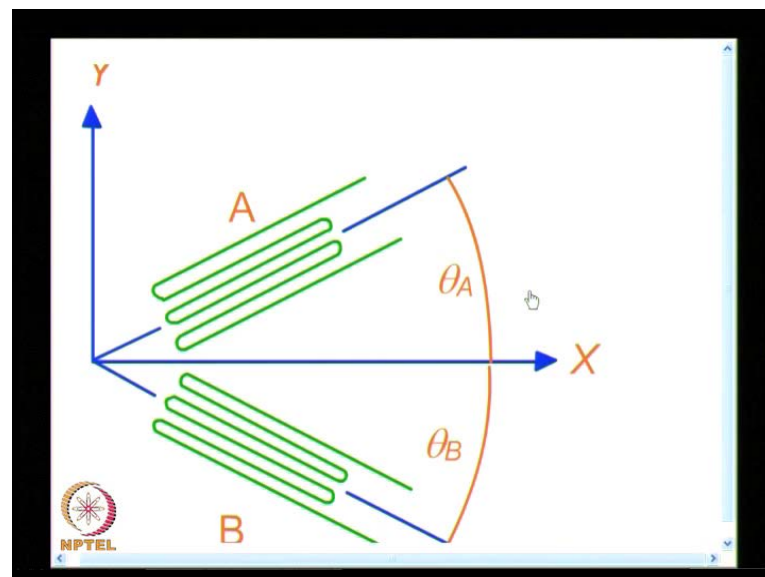
$$\gamma_{xy} = \frac{2(\epsilon_A - \epsilon_B) - (\epsilon_{xx} - \epsilon_{yy})(\cos 2\theta_A - \cos 2\theta_B)}{\sin 2\theta_A - \sin 2\theta_B}$$


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Where you see one layer after the other, you know now we move on to an interesting class of special gauges. First we take up a plane shear or torque gauge and what we are going to do here is we are going to look at a very generic arrangement and in this you have two strain gauge.

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Elements which are oriented at angles theta A and theta B, and you have the reference axis as X and Y. So, you have 1 strain Gauge element measuring strain a long theta equal to theta A and another strain gauge element which is measuring strain along theta equal

to theta B. We will develop equation for a very generic arrangement first, then we will substitute specific values of this theta A and theta B and simplify. And you know in all these approaches I have said you need to know the strain transformation law, if you know that analysis of strain gauge data is very simple. Here the strain transformation law is written down in terms of cos 2 theta and sin 2 theta, instead of cos square theta, sin square theta in that form, we have written it in another convenient fashion.

So, when I have a strain gauge oriented at theta equal to theta A, Epsilon A is given as Epsilon x x plus Epsilon y y divided by 2 plus Epsilon ,Epsilon x x minus Epsilon y y divided by 2 into cos 2 theta A, plus gamma x y divided by 2, sin 2 theta A and you simply replace theta A into theta B in the second expression. And my focus is to find out the shear strain. So, from these two expressions I can solve for the shear strain gamma x y and that is finally given as gamma x y equal to 2 into Epsilon A minus Epsilon B, minus Epsilon x x minus Epsilon y y, multiplied by cos 2 theta A minus cos 2 theta B, divided by sin 2 theta A minus sin 2 theta B. And what I am going to do is, I am going to take the first step I will make theta A equal to theta B in magnitude and if I have theta A equal to minus theta B then you will find cos 2 theta A minus cos 2 theta B will go to 0.

(Refer Slide Time: 07:15)

EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

Plane-Shear or Torque Gauge

....contd

- The gauges are oriented such that  $\theta_A = -\theta_B$

$$\cos 2\theta_A = \cos(-2\theta_B) = \cos 2\theta_B$$

$$\gamma_{xy} = \frac{(\epsilon_A - \epsilon_B)}{\sin 2\theta_A - \sin 2\theta_B} = \frac{(\epsilon_A - \epsilon_B)}{2 \sin 2\theta_A}$$

- Thus, the shearing strain is proportional to the difference between the normal strains experienced by the gauges A and B when they are oriented with respect to the x axis.

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So, you will have only 1 term in the expression for gamma x y. So, when I have theta A equal to minus theta B I get gamma x y as Epsilon A minus Epsilon B, divided by 2 sin 2 theta A. And you know in normal strain gauge measurements you connect 1 strain gauge

to a Wheat Stone bridge, only in special applications you may connect more than 1 strain gauge, suppose I use a delta Rosette or a rectangular Rosette, essentially I would connect each of the strain gauges in a Rosette to separate Wheat Stone bridges for strain measurement. Now, here what you are looking at is I will connect 2 strain gauges they may form a T Rosette and when I connect them on adjacent arms.

We have already seen adjacent arms cancel each other and if the angles are oriented at theta A equal to minus theta B, then you get by connecting 2 strain gauges on adjacent arms of the bridge, the bridge automatically gives you Epsilon A minus Epsilon B.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

Plane-Shear or Torque Gauge

....contd

- For the angle,  $\theta_A = 45^\circ$ ,

$$\gamma_{xy} = \frac{(\epsilon_A - \epsilon_B)}{2}$$

- The subtraction  $(\epsilon_A - \epsilon_B)$  will be performed automatically by the bridge and the output will be  $2\gamma_{xy}$  directly.
- Shearing strain can also be measured with a two-element rectangular rosette by orienting the gauges at  $45^\circ$  and  $-45^\circ$  with respect to the x axis.

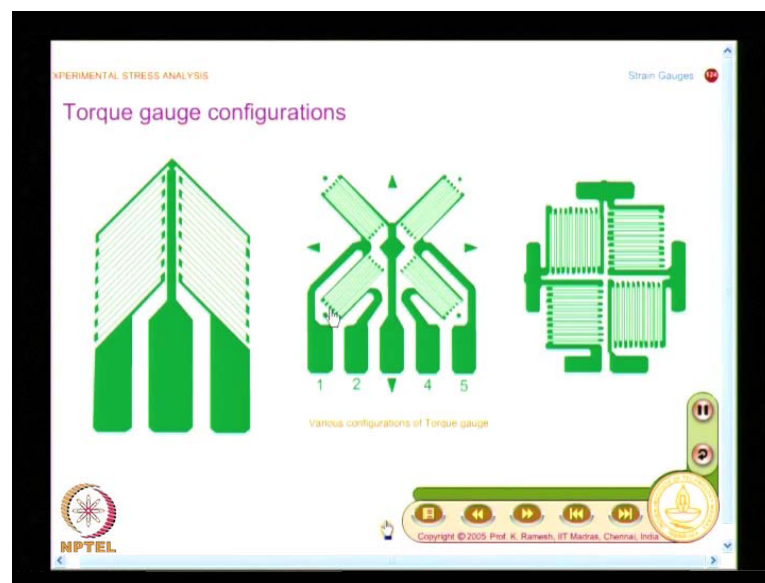
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So, what you essentially get is shearing strain at the point of interest and I can take a very special case when I keep theta A equal to 45 degrees, I get a much simpler expression and that is how you have most of these popularly available strain gauges are oriented at. So, what I have is I get essentially gamma x y equal to Epsilon A minus Epsilon B divided by 2. So, I have a T Rosette and I connect the 2 elements of this to 1 Wheat Stone bridge. See this is the special application here we are not measuring state of Strain at the point of interest. We are interested only in finding out shear strain we are not interested in anything else, in such a case you can connect elements of the Rosette to appropriate arms of the Wheat Stone Bridge. So, this is the special application it is not a general rule, in a generic case you will connect each strain gauge to Wheat Stone Bridge only in transducer application, where you design your spring element and you know the

state of strain on the spring element, your focus is to measure force in order to amplify the signal, you try to have minimum full bridge configuration. So, you have to distinguish in strain gauge instrumentation, how to connect the strain gauges and how many channels you require, you need to have a planning and you need to bring in concepts of strength of materials. So, that you paste the strain gauge properly on the structure as well as connect them correctly in your Wheat Stone Bridge.

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So, do not take it that in one class we saw the 2 strain gauges are connected to Wheat Stone Bridge and I will do that for every other application. And you also have different configurations of this available instead of just 2 strain gauges you have four strain gauges which are oriented at 45 minus 45 and 135 and so on. So, here you will get four times a signal and these are all special gauges which can be used to measure the value of torque, if you calibrated properly you can also measure the torque that is applied to the structure.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

### Stress Gauge – Principle

- Usually transverse sensitivity of a strain gauge is a nuisance in strain measurement.
- In stress gauge, the foil design aims to increase the transverse sensitivity as high as the Poisson's ratio of the base material!
  - In view of it, stress gauge will be different for different specimen material.
- This helps in simplifying the governing equation so that the strain gauge output could be directly related to stress.
- The output of any gauge is expressed by

$$\frac{\Delta R}{R} = S_a (\varepsilon_a + K_t \varepsilon_t) \quad (1)$$

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So, in these applications you will connect all the strain gauges appropriately in a single Wheat Stone Bridge. We have seen how to measure shear strain people also asked the question? In basic stresses analysis we are only looking at evaluation of stress components, so why not we also design a stress gauge instead of a Strain gauge. So, the focus is whatever the measurement I get should be proportional to Stress, then it becomes a stress gauge. See we have looked at transfer sensitivity effects or a nuisance and I said, what you considered as nuisance in one application can become beneficial in another application. That is a case with friction, without friction you cannot walk on the street and you need to minimize as much friction as possible in many of your rotating components. And if you do not have friction you cannot apply breaks.

So, friction is advantageous as well as disadvantageous, on similar vein we will see in the design of the stress gauge, how do we play with the transfer sensitivity? In fact, we would like to have maximum transfer sensitivity. So that a given strain gauge functions like a stress gauge. So, that is what is mentioned here, transfer sensitivity of a strain gauge is a nuisance in strain measurement. In stress gauge the foil design aims to increase the transfer sensitivity as high as the poissons ratio of the base material. So, what happens? In view of this stress gauge will be different for different specimen material, see that we have accepted in strain gauge technology. We have looked at self temperature compensated gauges all these self temperature compensated gauges are meant for a particular specimen material. So, we are accustom to this kind of a scenario

in strain gauge technology. So, you extend a similar argument for the development of the special gauges too, I have given the result in advance, but we will see how the result is arrived at.

So, let us look at the expression for  $\Delta R/R$ , what we looked at earlier we will recall here, that is given as  $S_a$  into  $\epsilon_a$  plus  $K_t$  into  $\epsilon_t$ . We are not written it in terms of  $S_G$ , we have written the generic Expression here. And now we are essentially trying to find out on the free surface, we are essentially finding out the state of strain or state of Stress on a free surface and we consider that as a plane Stress situation.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

Stress Gauge - Principle

....contd

- The relationship between stress and strain for plane stress is given by

$$\epsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_t) \quad \text{and} \quad \epsilon_t = \frac{1}{E}(\sigma_t - \nu\sigma_a) \quad \longrightarrow (2)$$

Substituting Eq. (2) in Eq. (1) yields

$$\frac{\Delta R}{R} = \frac{\sigma_a S_a}{E}(1 - \nu K_t) + \frac{\sigma_t S_a}{E}(K_t - \nu) \quad \longrightarrow (3)$$

- From Eq.(3) it is clear that if  $K_t = \nu$ , the output of the gauge  $\Delta R/R$  will be directly related to  $\sigma_a$ .

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So, I can replace the strain quantities in terms of Stress components by invoking stress strain relations that is what I am going to do. So, when you look at the relationship I have  $\epsilon_a$  equal to  $1$  by  $E$  into  $\sigma_a$  minus  $\nu$  times  $\sigma_t$  and  $\epsilon_t$  is given as  $1$  by Youngs modulus  $E$  into  $\sigma_t$  minus  $\nu$  times  $\sigma_a$ . And when I substitute in the previous equation I get  $\Delta R/R$  equal to  $\sigma_a$  into  $S_a$  divided by Youngs modulus into  $1$  minus  $\nu$   $K_t$  plus  $\sigma_t$   $S_a$  divided by Youngs modulus  $E$  into  $K_t$  minus  $\nu$ . So,  $\Delta R/R$  is a function of axial stress, as well as transverse stress, suppose I make  $K_t$  equal to  $\nu$  this term vanishes.

So, what you are really looking at is in a strain measurement system I essentially measure change in resistance, now we have looked at  $\Delta R/R$  could be related to Axial stress



provided the gauge configuration has a transfer sensitivity of a very high value equal to the Poisson's ratio, see we are not taking a wire and making a strain gauge now, we have the luxury of etching any pattern of my design on a metal foil, because I have the etching process I can design complicated patterns. In fact, the diaphragm gauge which you saw the very complex pattern which was use to measure pressure. So, now, we will have to go and investigate what type of grid pattern I should design which will have transfer sensitivity of the order of Poisson's ratio, that is what we are going to look at. So, look at this diagram very carefully.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

### Stress gauge construction

How to make a strain gauge such that its transverse sensitivity is as high as the Poisson's ratio of the test specimen?

The answer is simple. The gauge has two elements oriented at an angle and the angle of orientation decides the function of the gauge as a stress gauge.

This angle is

$$\theta = \tan^{-1} \sqrt{\nu}$$

Note the subtle difference between shear stress gauge and stress gauge.

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And there is a subtle difference between what we saw as a torque gauge and what we say as a configuration for the stress gauge here also I have 2 elements orientated at plus theta and minus theta, but what the subtle difference is these 2 elements are join they are not independent elements.

So, I have 1 strain gauge which is made in a fashion having 2 elements which are join. So, the whole thing is 1 strain gauge. So, I will have 1 tab here I will have another tab here this has to be connected to the Wheat Stone Bridge. And I have also caution earlier you must develop a familiarity by looking at a pattern, what this gauge stands for? In fact, you will also be tested whether you have been able to identify strain gauges patterns of various types, that will be tested in your examination and you should not get confused between T Rosette or 2 element strain gauge or a stress gauge, there is a subtle

difference, the subtle difference you should understand. The elements are joined here and this is how you have made a stress gauge and what is this angle.

Before looking into the proof this angle is found to be theta equal to tan inverse root of nu, that is a Poisson's ratio we will go and look at the mathematical steps and convince our self that, when I have theta equal to tan inverse root of nu, you will get correct values for our delta R by R as a function of only the stress. And what is indicated in this diagram is you have the X axis Y axis and in addition you have also located the principle stress direction at the point of interest. So, I have principle stress sigma 1 is oriented at an angle phi from the horizontal and when I do the mathematical development I will refer these two angles with respect to the principle stress as the reference.

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**EXPERIMENTAL STRESS ANALYSIS** Strain Gauges 01

**Proof**

- The strain along the top grid element is given by
 
$$\epsilon_{\phi-\theta} = \frac{1}{2}(\epsilon_1 + \epsilon_2) + \frac{1}{2}(\epsilon_1 - \epsilon_2)\cos 2(\phi - \theta) \quad \longrightarrow (1)$$
- The strain along the lower grid element is given by
 
$$\epsilon_{\phi+\theta} = \frac{1}{2}(\epsilon_1 + \epsilon_2) + \frac{1}{2}(\epsilon_1 - \epsilon_2)\cos 2(\phi + \theta) \quad \longrightarrow (2)$$

The diagram shows a coordinate system with X and Y axes. The principal stress direction  $\sigma_1$  is oriented at an angle  $\phi$  from the horizontal. The strain gauge elements are oriented at an angle  $\theta$  from the horizontal. The angle between the principal stress direction and the strain gauge element is  $\phi - \theta$ . The relationship  $\theta = \tan^{-1} \sqrt{\nu}$  is indicated.

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So, I will have one as pi minus theta other one as pi plus theta and again you have to go back and look at the strain transformation law, that is what I am going to do, these equations are not difficult, this equations you already know, only aspect is we are applying it for a different strain gauge configuration. So, I have two elements, what way we are going to look at is, because they are joined together essentially I am looking at Epsilon pi minus theta plus Epsilon pi plus theta that is what I am going to finally, do.

So, I will look at what is the expression for Epsilon pi minus theta, since I have taken reference as the principle stress axis I write it in terms of Principle strains. So, Epsilon pi minus theta equal to 1 by 2, Epsilon 1 plus Epsilon 2 plus 1 by 2, Epsilon1 minus

Epsilon 2 multiplied by cos 2 pi minus theta and this you can appreciate, because this is like a normal strain gauge which is sensitive to strain along the gauge length. So, that you will call that as Epsilon pi minus theta similarly this is like another strain gauge which will measure strain along the gauge length.

So, that is denoted as Epsilon pi plus theta. So, you change theta from minus theta to plus theta see in strain gauge instrumentation if you look at we design complicated gauges, but finally, we will also raise a question can I do the stress measurement with a single conventional strain gauge? If you look at the theoretical development what I have done I have cleverly taken to start with the principle stress axis, I have taken that axis as a reference. And you note one more aspect suppose I have pi equal to 0, the expression for Epsilon pi minus theta and Epsilon p plus theta will be exactly same. You keep this result in your mind we will use this result when I go and develop whether a single element strain gauge could be used to measure stress.

(Refer Slide Time: 23:27)

The screenshot shows a presentation slide titled "Proof" under the heading "EXPERIMENTAL STRESS ANALYSIS". The slide content includes the text "Summing up these two equations" followed by the mathematical equation: 
$$\epsilon_{\phi+\theta} + \epsilon_{\phi-\theta} = (\epsilon_1 + \epsilon_2) + (\epsilon_1 - \epsilon_2) \cos 2\phi \cos 2\theta \quad \rightarrow (3)$$
 The slide also features a "Back to main" button, a "Strain Gauges" link, and a "....contd" link. The NPTEL logo is visible in the bottom left corner, and a copyright notice "Copyright © 2005 Prof. K. Ramesh, IIT Madras, Chennai, India" is at the bottom.

So, we will develop a complicated gauge pattern to start with, in an unknown situation you will do this, but you will also find out whether you could use a simple normal strain gauge for stress measurement and when I add these 2 strain quantities I get this as Epsilon 1 plus Epsilon 2 Plus, Epsilon1 minus Epsilon 2, multiplied by cos 2 pi into cos 2 theta and I will further simplify my focus is finally to get the addition of these 2 strains

proportional to a single stress component. That is what I am going to show and we will not evaluate the angle theta, but we will say theta equal to.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

Proof ...contd

- From Mohr's strain circle,

$$\epsilon_{xx} + \epsilon_{yy} = \epsilon_1 + \epsilon_2 \quad \longrightarrow (4)$$

$$\epsilon_{xx} - \epsilon_{yy} = (\epsilon_1 - \epsilon_2) \cos 2\phi \cos 2\theta \quad \longrightarrow (5)$$

- Substituting eqs. (4) & (5) into eq.(3) gives

$$\begin{aligned} \epsilon_{\phi+\theta} + \epsilon_{\phi-\theta} &= (\epsilon_{xx} + \epsilon_{yy}) + (\epsilon_{xx} - \epsilon_{yy}) \cos 2\theta \\ &= 2(\epsilon_{xx} \cos^2 \theta + \epsilon_{yy} \sin^2 \theta) \\ &= 2 \cos^2 \theta (\epsilon_{xx} + \epsilon_{yy} \tan^2 \theta) \quad \longrightarrow (6) \end{aligned}$$

Back to main

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So, and then we will simplify the expression. Then we will say take theta equal to tan inverse root of nu is what we have said. So, that we will do it by induction and we also know from Mohr's strain circle Epsilon x x plus Epsilon y y, equal to Epsilon1 plus Epsilon 2 and Epsilon x x minus Epsilon y y as Epsilon1 minus Epsilon 2 into cos 2 pi into cos 2 theta. And we will replace it in terms of Epsilon x x and Epsilon y y, so when you substitute these into the earlier equation I get Epsilon pi plus thetaplus Epsilon pi minus theta as Epsilon x x plus Epsilon y y plus Epsilon x x minus Epsilon y y cos 2 theta and finally, it simplifies to 2 cos square theta Epsilon x x plus Epsilon y y tan square theta. See what we have done is we have simplified the expressions the way we want it, the focus is for what values of theta this expression get simplified and that is what we will take it up now and we start with the premise if the gauge is manufactured.

(Refer Slide Time: 25:45)

**EXPERIMENTAL STRESS ANALYSIS** Strain Gauges

**Proof** ....contd

- If the gauge is manufactured so that  $\theta = \tan^{-1} \sqrt{\nu}$

Then,  $\tan^2 \theta = \nu$      $\cos^2 \theta = \frac{1}{1 + \nu}$

- This simplifies Eq. (6) as

$$\epsilon_{\phi-\theta} + \epsilon_{\phi+\theta} = \frac{2}{1 + \nu} (\epsilon_{xx} + \nu \epsilon_{yy}) \quad (7)$$

$$\sigma_{xx} = \frac{E}{1 - \nu^2} (\epsilon_{xx} + \nu \epsilon_{yy})$$

$$\sigma_{xx} = \frac{E}{2(1 - \nu)} (\epsilon_{\phi-\theta} + \epsilon_{\phi+\theta}) \quad (8)$$

This is the last slide for this link. To go to next/other chapters navigate through the main menu button.

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So, that theta equal to tan inverse root of nu. I specify what is the expression for theta. So, when I take theta equal to Tan inverse nu, root of nu I get tan square theta equal to nu and cos square theta equal to 1 by 1 plus nu and this simplifies Epsilon pi minus theta plus Epsilon pi plus theta equal to 2 by 1 plus nu Epsilon x x plus nu times Epsilon y y. And I get expression for sigma x x as Youngs modulus e divided by 1 minus nu square multiplied by Epsilon x x plus nu times Epsilon y y.

So, I finally, get sigma x x is related to Epsilon pi minus theta plus Epsilon pi plus theta throughout this factor e by 2 into 1 minus nu. And what we have seen I have 2 elements these are join and when you connect this in one arm of the Wheat Stone Bridge, I measure essentially some of the strains. And now I have an expression to find out the normal Stress sigma x x directly from strain measurement, so I call it as a stress gauge, only the multiplication factor has to be reset in your instrumentation system. So, what you have is if you design a foil like this having 2 elements when you make the measurement it is possible for you to find out the normal stress component along the bisector.

Now I said Iam not going to stop here you cannot run for the stress gauge and we have also seen the stress gauge is dependent on the base material because of the factor the Poisson's ratio, because I have the factor as Poisson ratio it depends on the base material.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges 15

### Single element strain gauge as a stress gauge

$y$  ( $\sigma_2$  direction)

$x$  ( $\sigma_1$  direction)

$\theta = \tan^{-1} \sqrt{\nu}$

• This is possible when the principal stress directions are known.

$$\sigma_1 = \frac{E}{1 - \nu} \epsilon_\theta$$

• Locate the gauge along a line which makes an angle  $\theta = \tan^{-1} \sqrt{\nu}$  with respect to the principal axis.

$$\epsilon_{\phi-\theta} = \epsilon_{\phi+\theta} = \epsilon_\theta$$

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So, I need to go and develop how to use a single element strain gauge as a stress gauge. See in all this these are all you know once you have understood something you oriented at particular angle you get the result you want. For this I need to know the principle stress direction, without knowing the principle stress direction, I cannot do and use a single element strain gauge to measure stress. There are certain applications where you can minimize the number of channels by just pasting 1 strain gauge appropriately and if you have design demand of stress quantity along a particular direction then your job is done. So, what you have here is the prerequisite is I need to know the sigma 1 direction and I will have to align a simple normal strain gauge at theta and theta is given as tan.

Inverse root of nu and we have already noted when you have pi equal to 0, Epsilon pi minus theta equal to Epsilon pi plus theta that is equal to Epsilon theta. So, the final expression reduces to sigma 1 equal to E by 1 minus nu Epsilon theta. So, the trick what we have done is a normal strain gauge can be used as a stress gauge, but it can measure only when it is align with respect to one of the principle stress directions. And the angle at which you have to alien it is also specified and you know you do not to make a gauge which is dependent on the specimen material here. Here I have to alien appropriately for different specimen material that is all I have to do. So, now wehave a via media.

So, somebody ask you why are you doing only strain gauge you can also go back and say no I can also use and develop as stress gauge, it Is one of very common questions. In

interviews people can ask do you know what is stress gauge you should not see stars, you should say there is way to do it and these are all recent development. You know people play it with metal foil where you have to etch. If you have to live with wire probably they would not have proceeded in the direction, because you have the specialty to etch any pattern of your desire. Whatever the design that you have you are in a position to explore and in fine innovative methods to improve your measurement and what we.

Have done here if I look at stress gauge we have selected a particular parameter, so that one of the terms becomes 0. You have this idea borrowed even in fracture mechanics, my next focus is to find out how to get stress fringe factor use in strain gauge technology. First will see a very elaborate method, then we will simplify it, what we will do is elaborated to simplify we have to knock of terms and what we are going to is we have to knock out 2 terms in a series.

So, I will have to work with theta as well as alpha, in this case we have to knock out only 1 term, so we decided that we alien at angle theta. So, that idea as similar you know the people also played with Poisson ratio. So now, once you got a queue you know researchers try in the direction and find out what way this can be further exploited. So, you would see Poisson ratio is effectively utilized in the calculations and the orientation decides the functionality of the strain gauge and I have to knock of 2 terms.

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The slide is titled "Stress Intensity Factor" and is part of a presentation on "EXPERIMENTAL STRESS ANALYSIS". It features two diagrams of "Edge-cracked tension strips" under uniform stress  $\sigma_0$ . The left diagram shows a "Single-edge Crack" with a crack of length  $a$  on one side of a strip of width  $w$ . The right diagram shows a "Double-edge Crack" with cracks of length  $a$  on both sides of a strip of width  $w$ . To the right of the diagrams, there are two bullet points: "The stability of the crack is determined by the opening mode stress intensity factor,  $K_I$ ." and "The crack will be initiated when  $K_I > K_{Ic}$ ". The slide includes a navigation bar at the bottom with various icons and a copyright notice: "Copyright © 2005 Prof. K. Ramesh, IIT Madras, Chennai, India".

So, I will have theta as well as alpha. So, that is the way I am going to about, but before we going do it we will also recall what we have learnt in our course on fracture mechanics. What is a stress intensity factor? Essentially the stress field in the vicinity of the crack is possible to write in terms of the stress intensity factor and you may have multiple cracks in a system the crack that is critical is going to propagate. And fracture mechanics says that when  $K_I$  is greater than the fracture toughness that is  $K_{Ic}$  the crack will be initiated. So, the focus is for a given structure you need to find out what is the stress intensity factor, here we are essentially looking at simple mode I situations. So, the idea is there are occasions where you need to experimentally find out what is the stress intensity factor. There are many methods you can do it by numerical methods, you can do by method of Photoelasticity. You can do by method of photo elasticity, you can do by method of Moire Holography and we will learn how strain gauge technology could be used.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

### Strain Field in the Vicinity of a Crack (Mode I)

The three term representation of strain field is

$$E\epsilon_{xx} = A_0 r^{-1/2} \cos \frac{\theta}{2} \left[ (1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + 2B_0 + A_0 r^{1/2} \cos \frac{\theta}{2} \left[ (1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \right]$$

$$E\epsilon_{yy} = A_0 r^{-1/2} \cos \frac{\theta}{2} \left[ (1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \sin \frac{3\theta}{2} \right] - 2\nu B_0 + A_0 r^{1/2} \cos \frac{\theta}{2} \left[ (1-\nu) - (1+\nu) \sin^2 \frac{\theta}{2} \right]$$

$$2G\gamma_{xy} = A_0 r^{-1/2} \left[ \sin \theta \cos \frac{3\theta}{2} \right] - A_0 r^{1/2} \left[ \sin \theta \cos \frac{\theta}{2} \right]$$

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So, what I need to do is I need to look at strain field here and what is the strain field in the vicinity of a crack. And you know you just look at this expression, you need not copy down this expression, you can copy down later simpler form of this and what you have here is expression for a Epsilon x x, Epsilon y y and gamma x y is given what.

You need to look at is I have three coefficients, A naught, B naught as well as A and if you closely look at it is a first term which is related to the stress intensity factor and now



you all know I am going to paste a strain gauge at an arbitrary angle. So, I need find out for an arbitrary angle what is the expression for strain. So, if I know the expression for Epsilon x x, Epsilon y y and gamma x y I can write on the strain expression along any axis of interest that is not a difficult aspect. So, essentially you have look at this strain field and in order to have reasonable accuracy I am taking a three terms solution. In fact, if you look I can write infinite number of terms, but those terms are will not be in position to evaluate and you should also not stop at only the first term. See in order to enhance accuracy we want to take as many terms are possible and if we also look at the methodology should be simple enough to measure.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges 17

### SIF Evaluation by Strain Gauges

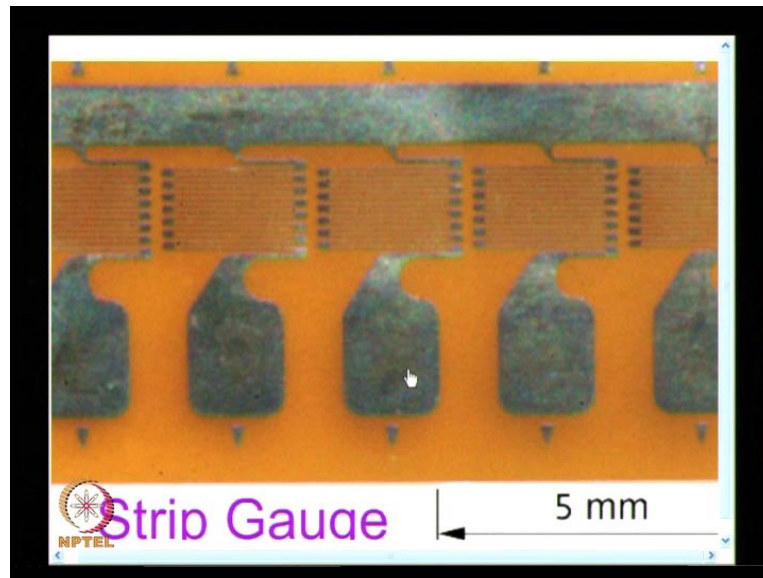
- The unknown coefficients  $A_0$ ,  $B_0$  and  $A_1$  can be determined if three or more strain gauges are placed at appropriate positions in the near field region.
- If  $A_0$  is known,  $K_1$  can be obtained from  $A_0 = \frac{K_1}{\sqrt{2\pi}}$
- Strip gauges have been developed in which, on one backing several closely placed strain gauge elements are etched.
- From these strain gauges, strain information over a field could be evaluated.
- If sufficient data is available, it is possible to extend the overdeterministic least squares method to strain gauge data too.

Strip Gauge 5 mm

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Courtesy: Vishay Micro-Measurements  
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So, my focus is bring in three terms, but knock of two terms intelligently, by selecting appropriate angles that is what I am going to do. And this is what you summarized here you have,  $A_0$ ,  $B_0$ ,  $A_1$  are unknown coefficients which depend on the geometry of the specimen and the loading and we will knock of other terms leaving only  $A_0$ . If  $A_0$  is known  $K_1$  can be obtained from  $A_0$  equal to  $K_1$  by root of  $2\pi$ , this is what I am going to aim at. And if you are with situation like fracture mechanics, what you would like to do is you would like to measure at several points in the vicinity of the crack tip, find out strain quantities and in such a case normally what you will have to do you. Have to paste a strain gauges at different locations and there aligned becomes difficult.

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So, in order to simplify for such applications you also have what are known as strip gauges that is what is shown here, I think I will enlarge it and then show you what is the strip gauge. So, what you have here is you have strain gauges pre aligned available in one backing very small strain gauge and even to solder and connect and paste, all these are very complex issues, but the advantage is you are able to get strain at short distances very comfortably.

So, I can essentially get the strain field and in fact, people have developed methodologies by using Strip Gauges, find out the strain field and solve this in an overdeterministic fashion using the principle of least squares and you can actually evaluate several terms in series and finally, evaluate only these **Stress in**. So, you could satisfy the strain field information by pasting several strip gauges in the near vicinity, but the very thought is.

So, difficult you know, because as available as one backing pasting may be simpler, but soldering is going to be a very tricky issue, that is where you will find that hold field techniques are better, when I have to get a field information hold field techniques are a lot more better, but you also need to have a via media when I use a strain gauge is it possible to get the strain field? If you ask that kind of a question, in principle it is possible, but you have to take a lot of effort to do that. And this is where you know researchers have stepped in and then played with an equation and found a very simple approach where a

common general purpose strain gauge, a single strain gauge that too could be use to find out the value of  $K$  how elevated it is you will have look at.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges 17

### SIF Evaluation by Strain Gauges

- The number of strain gages required for the determination of  $K_1$  can be reduced to one by appropriate choice of strain gauge location and its orientation.
- The principle is to make the governing equation only as a function of  $A_0$ .

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So, the idea here is the number of strain gauges required for the determination of  $K_1$  can be reduced to 1, by appropriate choice of strain gauge location and its orientation. See essentially my interest is to knock of  $B_0$  and  $A_1$ . So, I want to knock of  $B_0$  and  $A_1$  terms.

So, I will look for two orientations. So, you are trying to find out strain along the particular location, which is certain an angular locations, at that locations you will measure the strain along the particular orientation. So, I have two parameters to play with, so I can have the governing equation only as a function of  $A_0$ . See people have done it out of research, you have result readily available and we are going to discuss in class within 10 minutes, in your 10 minutes we will find out what researches are done in 6 months or almost a year to arrive at implementation of this idea. The idea is I should have the governing equation only as a function of  $A_0$  and you will look at here look at the animation carefully.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

### SIF Evaluation Using a Single Strain Gauge

Commonly available strain gauge can be used to measure SIF provided it is put at a carefully selected point and at a suitable orientation.

Poisson's ratio of the base material dictates the determination of  $\theta$  and  $\alpha$ .

These are so selected that some terms in the strain field go to zero such that the strain reading could be easily related to the SIF.

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So, what I am going to do is I want use the common strain gauges, so I will have the orientation theta, but at orientation theta I will have the strain gauge align that angle alpha. So, for this value only I would find out the strain along this direction and now question is how to select the theta and alpha? That is what you will look at it, you make a needs sketch of this diagram that illustrates how to use a single strain.

Gauge for stress intensity factor measurement I have this as the crack tip and crack tip was taken as a origin and you will locate the point P which is at an angle theta from the crack axis. At point P you measure strain along the direction alpha, that is very important and I have already given you the clue. You know we have the Poisson's ratio of the base material which was used in designing the stress gauge, we will exploit those ideas and we will use the Poisson's ratio of the base material to find out appropriate values of theta and alpha, so that some terms in the strain field go to 0. So, you have to evaluate the theta, alpha intelligently to knock of the terms containing  $B_0$  and  $A_1$ .

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**Strain field**

For the rotated coordinates shown in the figure, the strain  $\epsilon_{x'x'}$  is

$$2G\epsilon_{x'x'} = A_0 r^{-1/2} \left[ k \cos \frac{\theta}{2} - \frac{1}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \cos 2\alpha \right. \\ \left. + \frac{1}{2} \sin \theta \cos \frac{3\theta}{2} \sin 2\alpha \right] + 2B_0 (k + \cos 2\alpha) \\ + A_1 r^{1/2} \cos \frac{\theta}{2} \left[ k + \sin^2 \frac{\theta}{2} \cos 2\alpha - \frac{1}{2} \sin \theta \sin 2\alpha \right]$$

where  $k = \frac{1-\nu}{1+\nu}$      $G$  – shear modulus

The values of  $\theta$  and  $\alpha$  are so selected that some terms in the strain field go to zero such that the strain reading could be easily related to the SIF.

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So, I will have expression only in terms of  $A_0$ , then I can find out Stress intensive factor from that. And now for this you need to know the strain along any arbitrary direction, this expression you can write it now. So, what I have here is  $2G$  into  $\epsilon_{x'x'}$  equal to  $A_0 r^{-1/2}$ , multiplied by  $k \cos \frac{\theta}{2} - \frac{1}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \cos 2\alpha + \frac{1}{2} \sin \theta \cos \frac{3\theta}{2} \sin 2\alpha$ , plus  $2B_0 (k + \cos 2\alpha)$ . So, now, you have a clue how to find out  $\alpha$ , this should be minus  $k$  if I have this is minus  $k$  this term will go to 0.

So, find out  $\alpha$  and then find out  $\theta$ , that is how you play with the equations. So, I have this as plus  $A_1 r^{1/2} \cos \frac{\theta}{2} [k + \sin^2 \frac{\theta}{2} \cos 2\alpha - \frac{1}{2} \sin \theta \sin 2\alpha]$ . So, once you look at the expression for the strain gauge which we have used, we have it in terms of  $\theta$  and  $\alpha$ . And you get a clue how to get the values of  $\alpha$  as well as  $\theta$  and  $k$  is given as  $\frac{1-\nu}{1+\nu}$  and  $G$  is the shear modulus. See whatever the development that you have done in stress gauge playing with the Poisson's ratio and orientating the strain gauge appropriately, that kind of an approach is extrapolated in the measurement of stress intensive factor also, once you strike an idea you have also will find out whether this idea can work in other situations.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges

### Selection of alpha and theta

- The coefficient of  $B_0$  term is eliminated by selecting the angle  $\alpha$  as
$$\cos 2\alpha = -k = -\frac{1-\nu}{1+\nu}$$
- Next, coefficient of  $A_1$  vanishes if angle  $\theta$  is selected as
$$\tan \frac{\theta}{2} = -\cot 2\alpha$$

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So, now I have an expression I can go and find out what is an expression for alpha and theta. So, we have  $\cos 2\alpha = -k$ , so you have  $-\frac{1-\nu}{1+\nu}$ . So, I can find out what is value of alpha. And if we look at the expression for  $A_1$  that vanishes if angle theta is selected such that  $\tan \frac{\theta}{2} = -\cot 2\alpha$ . See you have the strain expression and when you substitute these values of alpha and theta you will find it is only a function of  $A_0$ , the terms containing the  $B_0$  and  $A_1$  or cleverly not of. So, you play with the angles and you are able to do it. So, these are all final aspects of strain gauge instrumentation, when you normally paste a strain gauge you are only measuring strain along a particular direction, whatever the strain that you measure is intelligently related to the stress intensive factor by your understanding of the strain field expression. And you know if I am going to have alpha and theta come as 22.32 degrees and 32.5 degrees nobody will use it, see people find this as a great use mainly.

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EXPERIMENTAL STRESS ANALYSIS

Strain Gauges 10

### SIF evaluation using a single strain gauge – Summary

$$\cos 2\alpha = -\frac{(1-\nu)}{(1+\nu)}$$
$$\tan \frac{\theta}{2} = -\cot 2\alpha$$

For aluminium ( $\nu = 1/3$ )  $\theta = \alpha = 60^\circ$

$$K_I = E \sqrt{\frac{8}{3}} \pi r \epsilon_g$$

Where,  $r$  is the radial distance and  $\epsilon_g$  is the measured strain.

This is the last slide for this chapter. To go to next/other chapters navigate through the main menu button.

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Because most popular aerospace material is aluminum, for aluminum both angles reduce to 60 degrees, it makes your life very simple. So, that is what you will see now, when I have aluminum  $\nu$  equal to 1 by 3 and  $\theta$  and  $\alpha$  goes to 60 degrees, see the angle is very important, when the angle is very important if the angle is not possible to achieve, this technique would not get taken off. And an aluminum is the one which is used in space structure, all aerospace structures that is where maximum amount of experimentation is done, because safety is the very important aspect. And you find fortunately the values of  $\theta$  and  $\alpha$  is as simple as 60 degrees and you have to look at  $\theta$  as well as  $\alpha$  in a generic situations and when I have this the final expression is also available.

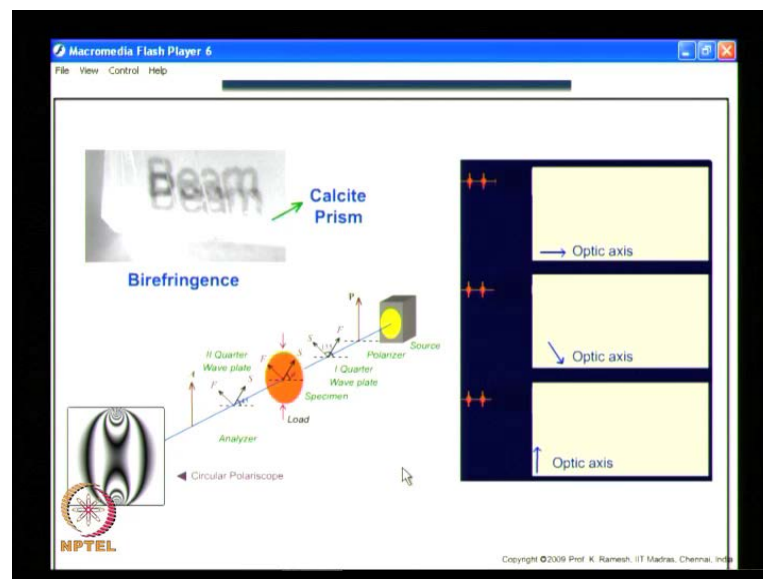
So, what I have here is  $K_I$  equal to Young's modulus  $E$  into root of  $\frac{8}{3} \pi r \epsilon_g$ . So,  $\epsilon_g$  is the strain along the particular direction for the case of aluminum, this is measured for the values of  $\theta$  as well as  $\alpha$  equal to 60 degrees, for steel you will have to find out  $\theta$  and  $\alpha$  because you know the Poisson's ratio you can find out what is  $\theta$  and  $\alpha$ .

So, if you orient this then you have an expression for Stress intensive factor. See fracture mechanics is becoming very important, earlier it was primarily used for aerospace structures, then people started using it for designing nuclear installation. Now, even automobile manufacturers want to look at fracture mechanics aspects, some aspects of the

design and in future it may into many of the day to day activities of design. So, knowing how to measure fracture parameters is also very important.

So, because strain gauge is a very versatile technique it is also desirable that you have a method to find out stress intensive factor, approximately even if it is not very accurate. See if you look at in this problem how close the strain gauge should be pasted? It is problem depending what should be the value of R what should be the value of the gauge length, your accuracy depends on all that because when you have a crack you are going to have a very steep strain gradient. So, those aspects are there, but from a designing point of view even if you know what range the value exits that can give a direction on which way to take. And you know people also have a extended these methodologies to composites, given for composites how to find out stress intensive factor from a single strain gauge you have papers available.

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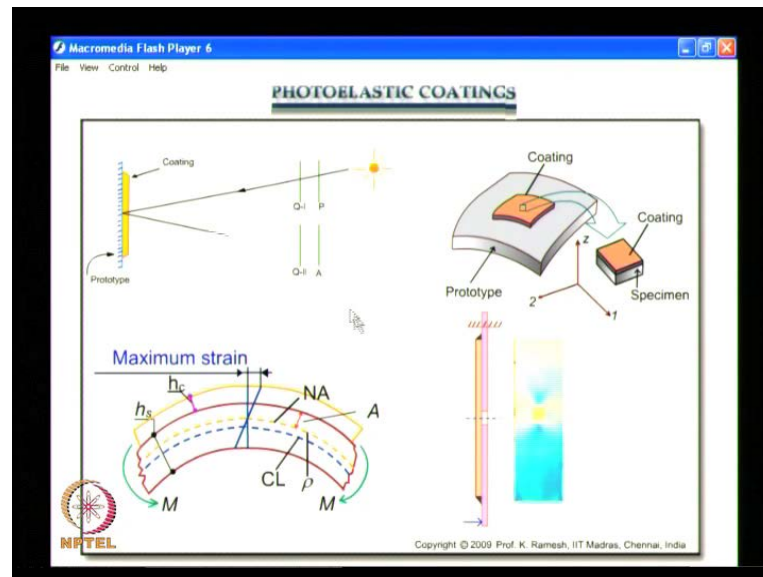


So, with this we come to the close of this course and we will also have a look at a brief overview of what we have been discussing in various chapters. We started with transmission photo elasticity I said the key concept was when you have model that is loaded it behaves like a crystal, when it is loaded when the loads are removed it behaves like a normal material. And crystal exhibits a property of Birefringence and this is exploited in Photo Elasticity and you are able to view fringe patterns in different optical arrangements, here you have a circular polar scope, you have both the dark field as well



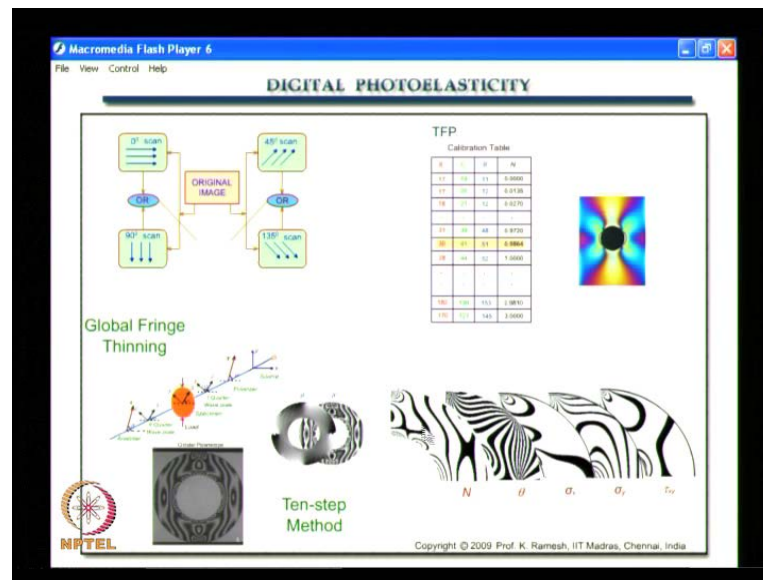
as bright field possible. And we also discussed what way the incident light should be oriented with respect to the optic axis of the crystal? When it is perpendicular it is useful in photo elasticity and when you have incident ray, you have 2 refracted rays travel with different elasticity.

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So, that is the basic physics in transmission photo elasticity and we also need to have methodology is to analysis prototypes, that was discussed in photo elastic coatings and I said all the coating techniques share a commonality, we have photo elastic coatings, we have brittle coating and to a extend you can also think of strain gauge as a coating applied on the material. And one of the key concept here is thus strain in the prototype is faithfully transfer to the coating and in the case of photo elastic coating you have thickness of border of 3 millimeter is pasted, what are known as correction factors. Not only this being a industry friendly technique, I have also mention that engineering is approximation and approximation starts right at the data collection stage in the reflection photo elasticity, you do not have a normal incidents, you have only a oblique incidents and you want to minimize this angle of oblique.

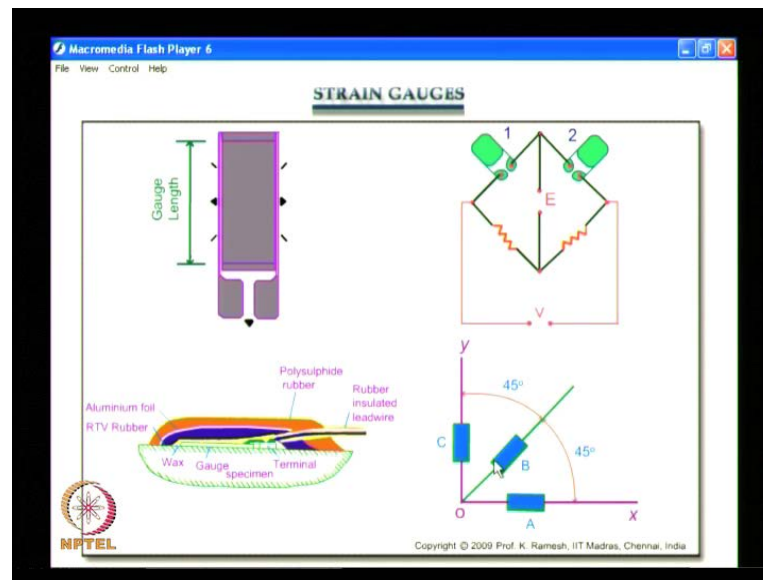
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So, you keep the polar scope at least 2 to 3 meters away from this specimen and you see rich fringe patterns and these are possible to interpret for stress information. Then we moved on to another aspect of photo elasticity which is becoming very popular these days, known as Digital Photo elasticity. And initial stages they had developed algorithms to thin the fringes, that is what you have as the global fringe thinning algorithm.

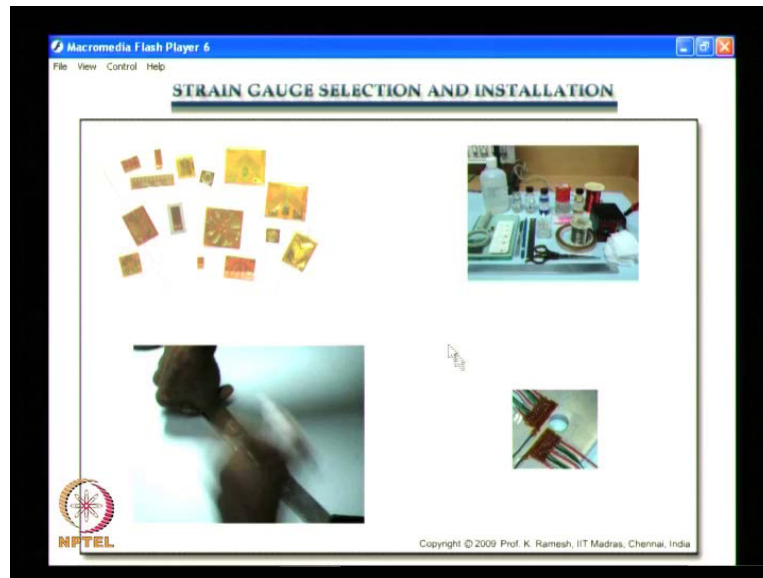
So, it employed scan at 0 degree, 45 degree, 90 degree and 135 degree. And logical operators were use to get this skeleton free of noise, then I said that you also have extension of color codefor quantitative measurement in digital photo elasticity, which is called as three fringe Photo elasticity were you are able to find out the R, G and D values and find out the fringe order at the point of interest. And when you want to do Whole field analysis, what is possible now is the employment of what is known as a 10 Step Method, which assures you high quality Isoclinic data, as well as is Chromatic data. And this high accurate values are needed for stress separation status, we have not looked at in detail how to separate the stresses, but if you know N and theta you have methodologies available to get sigma x, sigma y as well as  $\epsilon_x$ . So, what you have here is the modern methodologies not only stops at finding out N and theta.

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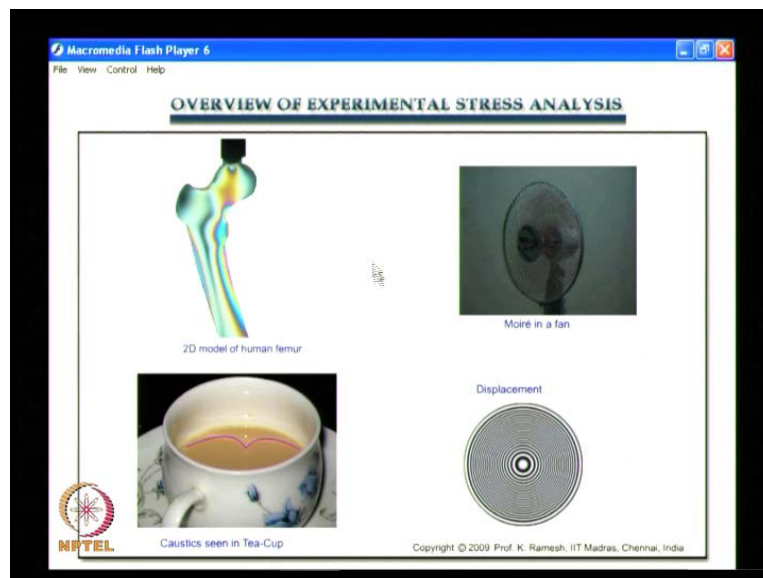
Accurate at every point in domain, you can also post process them and even go to the extent of separating out the stresses. And what we have found is among various phase shifting techniques the 10 Step Method is found to be robust and a commercial polar scope could be use to record 10 images. The normal do not have much influence in the 10 step method. When we looked at the method of strain gauges and I said a strain gauge measures strain along the gauge length, it is only measuring a component of strain and for measuring the change in resistance the popular instrumentation is a Whit Stone Bridge. And if you have to measure sate of strain at a point you need three strain gauges, they are three aligned available in one carrier known as a Rosette. And we discussed various final aspect of strain gauge instrumentation and we also finally looked at moisture is very bad and this need to be protected. So, you have a multiple layer form with the layer of wax, then layer of rubber. Then aluminum foil and finally another layer of rubber and I said for any one of this strain gauge.

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Instrumentation, strain gauge selection and installation is very crucial, we saw that variety of patterns exist and there is a very elaborate kit available. And there is a procedure how to apply the bonding and a simple **signoculte** cement gets cured by a thump pressure, that is what is illustrated here. And you press it with a gauges panks and put it with a thump pressure and after about 2 minutes the bonding is completed and this shows use of strip gauges and finally.

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We come and see what we learned in the first slide, the focus is I can get stress information, I can get displacement, slope as well as curvature, when I say what is stress analysis. And what this slide shows is for every experimental technique there is a physics behind it and this physics needs to be understood for you to interpret the result, so that is the focus. So, I see a plastic cup here and I see beautiful play of Moire in the case of the fan. So, you need to have physics understanding experimental method. Whenever they come across any new physical phenomena they try to develop a new experimental technique. Thank you.