

Experimental Stress Analysis
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Module No. # 01

Lecture No. # 03

Stress, Strain and Displacement Fields

Let us continue the discussion on overview of experimental stress analysis, we have seen in the last class what the experimental techniques give directly. We have seen photo elastic give information of σ_1 minus σ_2 and the orientation of the principle stress direction θ . When we go moiré you get displacements when you go to holography you get the displacement vector and so on. So, essentially what we saw was each of the experimental techniques give you a particular kind of information based on the physics that we exploit in getting the information. And the other aspect is in a first level course in strength of materials, you try to understand, what is stress?

And you have to understand stress is a tensor when you say stress is a tensor you would like to know state of stress at a point of interest. At a point of interest you are focusing on what happens on all the infinite planes passing through the point of interest, this is what you get in a study of Mohr's circle. So, you understand stress is a tensor although many times, you get only the stress component there is a danger that you could contour stress as a scalar. The moment you come to any of the optical techniques you have the advantage of get in the information on the entire field.

So, you get the whole field information and you get the whole field information in the form of contours. So, when we go further what we are going to do this how to do the experiment, what physics employed and how to interpret we will study in detail later. To start with it would be desirable to see these contours for a class of problems from known situations to unknown situations.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Typical Results for Various Problems

The problems considered are

- ★ Beam under four point bending
 - Closed form solution by Strength of Materials is possible
- ★ Cantiliver Beam
 - Engineering analysis possible by Strength of Materials.
- ★ Disc under diametral compression
 - Only Theory of Elasticity can provide closed form solution.
- ★ Clamped circular disc with a central load
 - $w = \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y^2}$ obtainable from theory of elasticity
- ★ Spanner tightening a nut
 - Due to complex nature of the geometry only a numerical solution is possible

In all these cases relevant experimental results (recorded or simulated) are shown to appreciate the nature of fringe contours.

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Thus what we are going to see and we have discuss the last class the problem consider are beam under four point bending, cantilever beam, disc under diametral compression, clamped circular disc with a central load and finally, spanner tightening a nut. So, this is based on increasing degree of complexity, beam under four point bending easy to solve from strength of materials approach. And you have the closed form expression, the moment you come to cantilever beam you have Shear, but you do a engineering analysis by strength of materials. And when we come to disc, you cannot do by strength of materials, but theory of elasticity can provide closed form solution. And clamped circular disc you could find out the displacement, slope and curvature from study of theory of elasticity theory of plates. And finally, spanner tightening a nut, we have seen earlier that it is a down earth problem surprisingly; you cannot solve it by analytical methods. You have to solve it either by numerical or by experimental methods.

(Refer Slide Time: 03:41)

EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Typical results for various problems

Beam under pure bending – Analytical solution

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$
$$\sigma_x = -\frac{M_b y}{I_z}$$

Stress tensor

$$[\sigma] = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The slide also features a diagram of a beam on two supports with a central load, and a circular stress distribution plot showing compression on the top and tension on the bottom. The NPTEL logo is visible in the bottom left corner, and a copyright notice for Prof. K. Ramesh, IIT Madras, Chennai, India is at the bottom.

Alone and now we take up the problem of beam under four point bending. And what you have here is I have a beam and this is shown as a x axis, this is y axis and you have taken a central portion of the beam which is subjected to bending moment M_b . And in this you have the stress variation of the depth as like this, which shows the variation is linear and the central core does not contribute to load sharing. And you have a famous flexure formula here and which is since slightly different form then what you might have seen in some other books. Many times people simply say M by I equal to f by y equal to E by r it is written in a different fashion know the difference, this is much more precise mathematically, we show that this is the bending moment indicated by subscript b .

And the moment of inertia is indicated as I_z , depending on the axis that is chosen for the problem. And when we come to second term you also have a sign attach to σ_x , this is minus σ_x divided by y . Y this is show is the sign convention on a positive surface anti clockwise moment is taken as positive. And for a loading like this that top fiber of the beam is subjected to compression to indicate that you have an negative sign. And once you have this it is easy to find out, what is the value of stress component? You can directly write it styles play, please write down that expression and this term out to be minus $M_b y$ by I_z . And what you get here? You get as a function of y σ_x changes linearly and you also fine the σ_x is not a function of x .

So, it is constant or the entire length of the beam which is taken slightly away from the points of load. And this is what you get in a first level course expression for σ_x when you look at it is only a component. It is only a component, that you are looking at and what you have to understand is, you have to understand that this is just not a number you have to understand this as a tensor. So, when you say tensor I have to plug it in a three by three matrix, appropriately this component and fill the other components. In this particular example what you have is, you have the stress tensor only the component σ_{xx} exist all other components are 0.

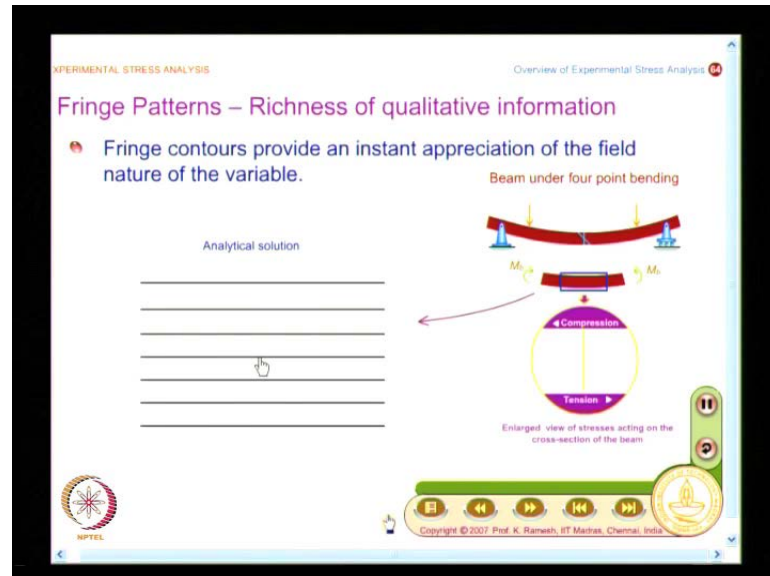
And what you have to important is the zeros are very important. And you know, if you look at the life history of Ramanujam, when he was young student, you are sitting in a class of mathematics. And he was observing zeros when you put the zero before number, the value of the number decreases when you put zero after the number, the value of the number is increases. You know, this is the very very important and pertinent observation zeros play every important role. And if you look at the history of science Indians are credited with giving the concept of zero to the entire world. So, though in this case you have zero present, you have to respect to those zeros. They serve a very important purpose and when I have a failure analysis then those zeros also play a very important role.

The failure planes could be different depending on the material on hand. And once you have this I have the analytical solution and it is possible for you to plot the value of the type of contours that you could get. And we have seen earlier that we would like to we get only σ_1 minus σ_2 in the case of photo elasticity, since you are the stress tensor you know how to find out the principle stresses. Then, you can find out what is σ_1 minus σ_2 and you can comfortably plot it. When you say contour what is that you mean, along a contour the value of the variable is same.

So, you plot discrete contours and would like you to makes some effort, because the problem is very simple. And find out how you can plot this contour what way will look like few minutes of your time, take few minutes of your time and then try to see how you can plot this contour. This is fairly straight forward, because if you know how to plot contours then you will know how to interpret the experimental results. Because, you need to have a visual appreciation of the variable over the domain and if you have done that what I will do is, I will show you some of you must have done it, I see some of you

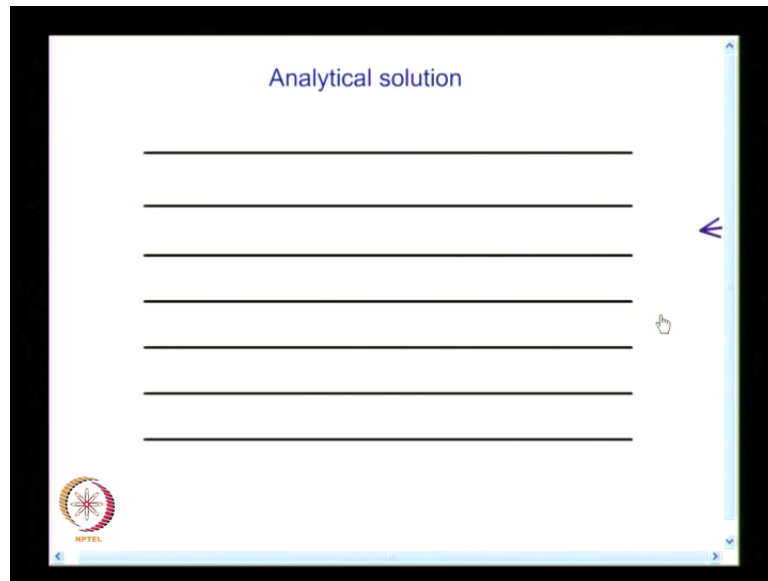
must have done it already. And if you look at I have in this zone the contours are essentially horizontal lines.

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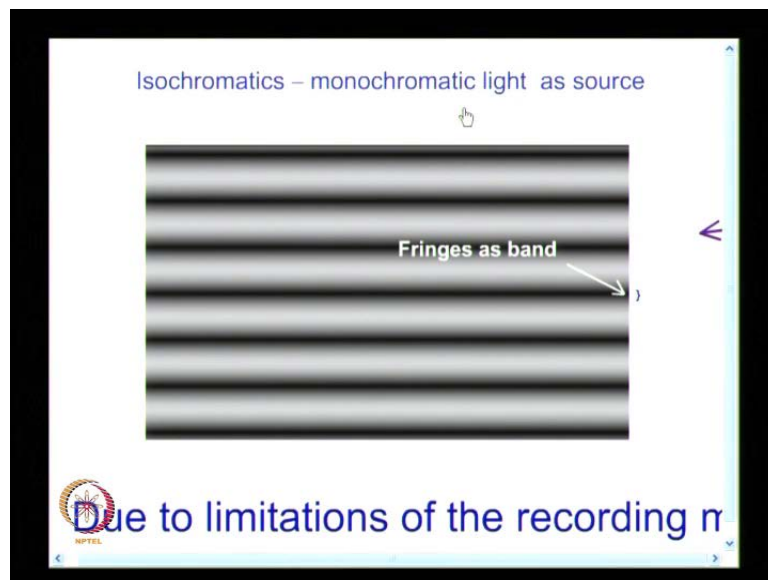
And in fact, if you go and look at many of the engineering problems, you get very interesting contours in some cases straight lines, some cases circles concentric circles, circles touching a edge. So, they give beautiful patterns in fact has written an article an art and science and various shown how the fringe patterns would be related to piece of art. So, fringe pattern gives the lot of information so, if you are an artist you look at artistic value of it if you are stress analysis, you look at the value pertinent your requirement.

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And now what I do is, let we look at an experiment what does it give, I will also magnify this picture and you have this as an analytical solution.

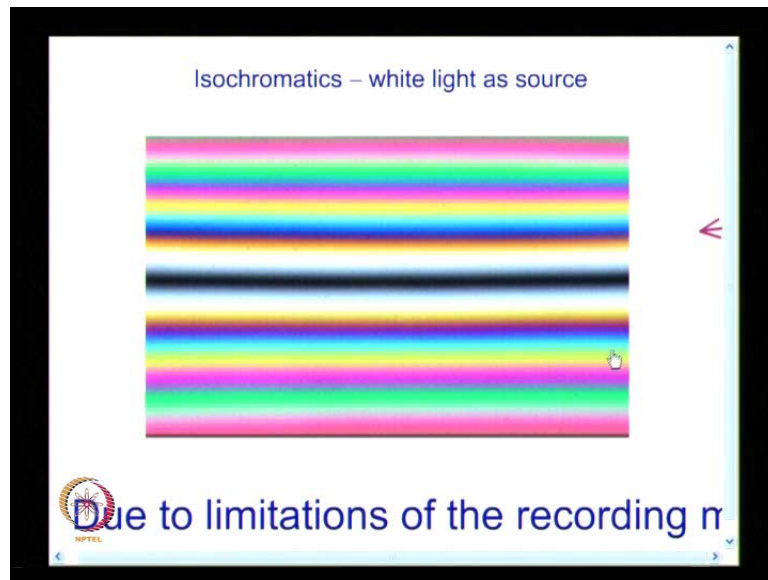
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Now, what I do is I take a model put it in an optics and find out, what the contours look like I do not get straight lines as thin as possible, I get only a band I get a essentially straight bands. So, this result matches with what you have seen in an analytical solution with the difference. The difference is I see the fringes as a band rather than a single line and I see this as black white contours mainly because I have used monochromatic light

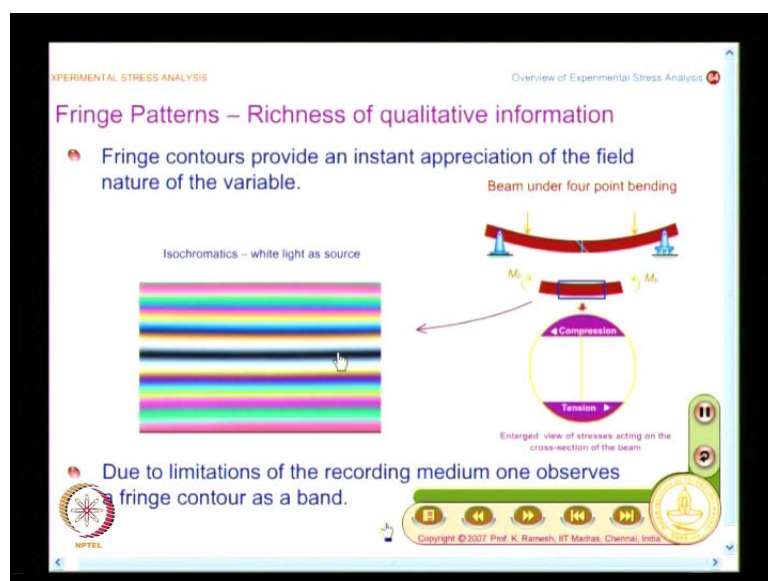
source for elimination. In most of the optical techniques it is desirable use monochromatic light source, because you have a single wavelength and data interpretation is for more simple. On the other hand we used white light which is the uniqueness of photo elasticity.

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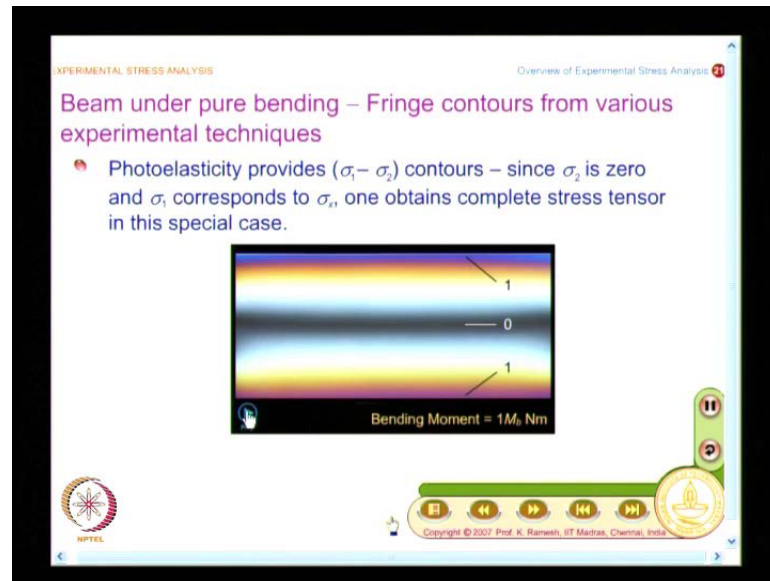
I get the same fringes appearing as beautiful play of colors and that advantage with color is using the color it is possible into label and all other fringes.

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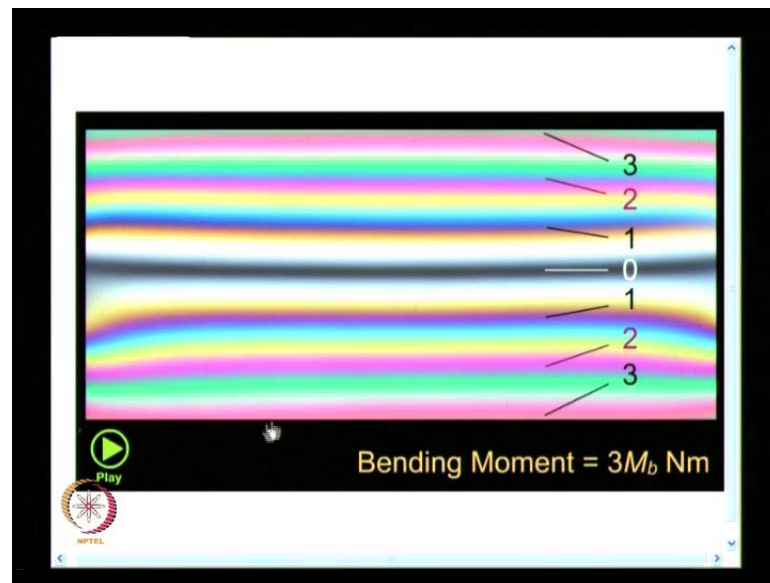
And why do we see the contour as bands, you see contour as band mainly, because of the deficiency in the recording medium. You observe this as the band, because of the deficiency in the recording medium. If you use a very fine digital camera, you would be able to find out the variation in the gray scale and pick out minimum intensity points as the fringe cylinder.

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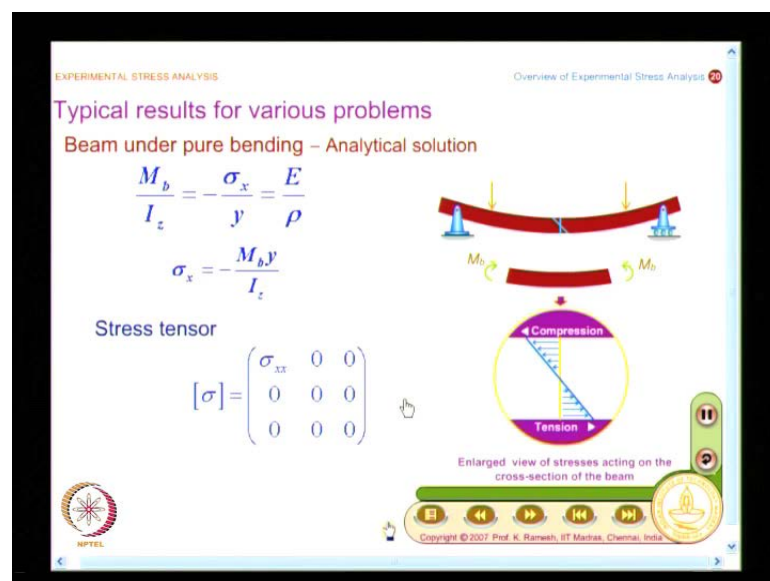
And now, what will do is it is also desirable that we go and look at how to number these fringes. And what we do is we now go back and see what photo elastic gives? It gives you contours are $\sigma_1 - \sigma_2$ in the bottom half of the beam. You have σ_1 corresponds to σ_x and σ_2 is zero and $\sigma_1 - \sigma_2$ contours appear horizontal. And you see some deviation from straight line in this zone, because this is closer to the load application point in the experiment. And what you see here is I have increase the load in steps of one and you find that this is fringe order 0 fringe order 1, fringe order 1 at bottom and when I double the load I see the fringes double. So, it is a function of load and I can also label them and when I triple load the fringes are increasing from 0 to 3 on either side.

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And what you can do is by look at the color it is possible for you to identify the number associate the fringe in photo elastic reasonably well. With I and with developments in digital image processing you have techniques, like three fringe photo elasticity has been develop were you could pick out this value very comfortably and then label these numbers. And one of the challenge task in any optical technique is how do I label these number, this is not something heavier and this is where many of the automated techniques people want to have where they would like to minimize the human.

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Now, we come back to stress field analytical approach. Now, we are given the stress field you are also see the stress tensor, with your knowledge of mechanism solids it is possible for you to find out the strain tensor. It is not difficult at all can I request you to look at brush your old memories and find out, how do you find out the strain tensor, because you have the stress tensor is available from stress tensor, we can easily find out the strain tensor. And you are very careful while applying this step and what you find here is stress tensor you have only one component it is a uniaxial state of stress.

(Refer Slide Time: 14:52)

EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis 20

Typical results for various problems

Beam under pure bending – Analytical solution

Using stress-strain relation one can find the strain tensor

Strain tensor

$$[\varepsilon] = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

$$\varepsilon_x = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -\nu \frac{\sigma_{xx}}{E}$$

Enlarged view of stresses acting on the cross-section of the beam

Compression
Tension

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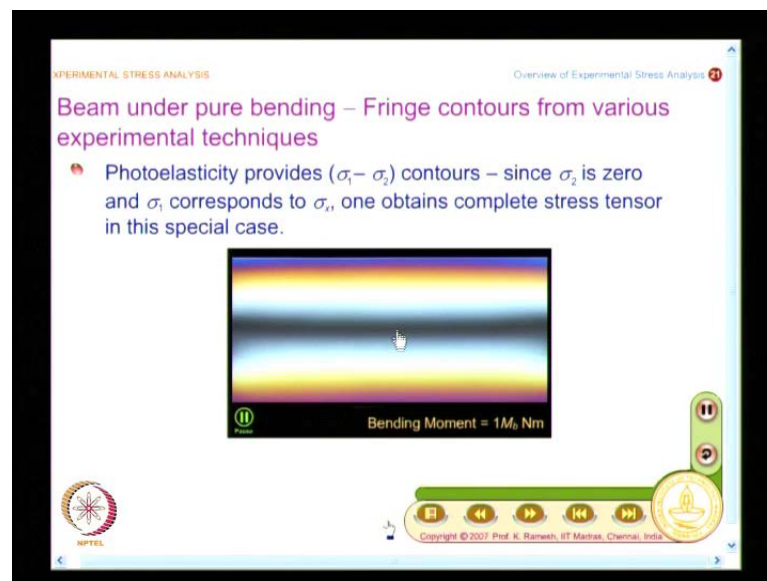
The moment I go to strain tensor, I find something different. I find strain tensor is no longer uniaxial, but it is axial. And can you say what is the value of epsilon x x epsilon y y and epsilon z z and epsilon x x is the does no problem you would be able to directly write from your understanding of even simple tension test. And what many people may ignore this; the effect of Poisson ratio. They may ignore the presence of epsilon y y and epsilon z z, you have be very careful about that there is a possibility that you may ignore their; these two strain components. So, what you have is, you have a stress tensor which gives you uniaxial state of stress.

On the another hand when I go to strain tensor, I have a axial state of strain you may ask, suppose I make a mistake and I gone interrupt the experiments will the experiments understand this or not. I always been saying experiment is truth, you may make approximations in your formulation to simplify your mathematics. And in some cases if

the value are reasonably small we may also ignore it, but the moment you come to experiments they always revile truth. And what is role of this strain components epsilon y y and epsilon z z, you have a Poisson ratio here and you have; when you have the beam under bending, I would like to showing the model. If I take the model and bend it and what I have here is the beam bends, the beam bends here and this is one curvature you have and this is what you do it in your simple strength of materials approach.

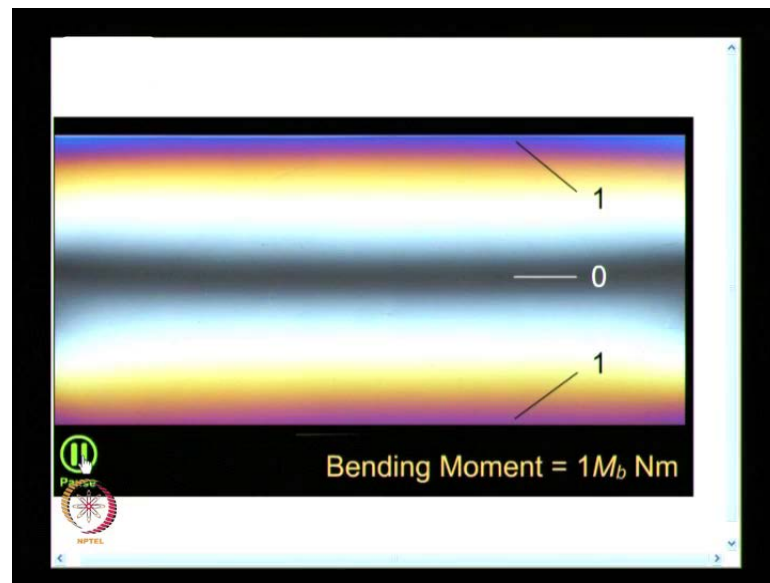
I have said that in addition to this you also have strain because of Poisson effect and what you find? Look at side face, you look at it side face what I find this I have a bulge at the bottom and it is not bulge out in the top. Why this is happens this in the wave I have bend this fiber is subjected to tension and the inner fiber is subjected to the compression. And what happens because of Poisson effect the bottom portion bulges out and top portion narrow. And you also see one more interesting thing, you have additional curvature here because of this and this curvature is known as anti classic curvature. And none of this in your first level in strength of materials.

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If people have not gone deeper into the analysis of beams, only when we go deeper into the analysis of beams you will find that these concepts you have to look at it. And what happens? The experiment looks at it, experiments revile this and that will see very closely when you see the fringe pattern will see the fringe pattern take of second for it to come.

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And what you will see is when I magnify it, you will see in a very circle fashion what you have on the top fiber, in this particular example the top fiber is subjected to compression and bottom fiber is subjected to tension. So, you will have the fringe order slightly greater than what is there at the bottom so, it is not perfect symmetry. So, the experiment captures this you may call it and call it as anti classic curvature, but I experiment reveals this information. So, you will not see perfect symmetric there will be small deviation and you should not come back and say I have done the experiment, experiment is wrong, is it not so.

In your analytical evaluation of stresses if you also bring in the effect of Poisson ratio effect then the stress patterns will be like this. So, experiment always gives you truth and that you should never forget it. Now, what we are going to do is will again go back to the slide and what we will do is? We will; we have seen the stress tensor, we have seen (()) strain tensor. Now, with that information using mechanics of solids that is using the equation of theory of elasticity, you could integrate the strain quantities and get you the displacement values. And I do not think you could have done it in a first level course, because you are happy with stress. You are happy with that the in bending central core does not contribute to load sharing, that is all you focus on that.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis 20

Typical results for various problems

Beam under pure bending – Analytical solution

From the strain field, displacement field is obtained as

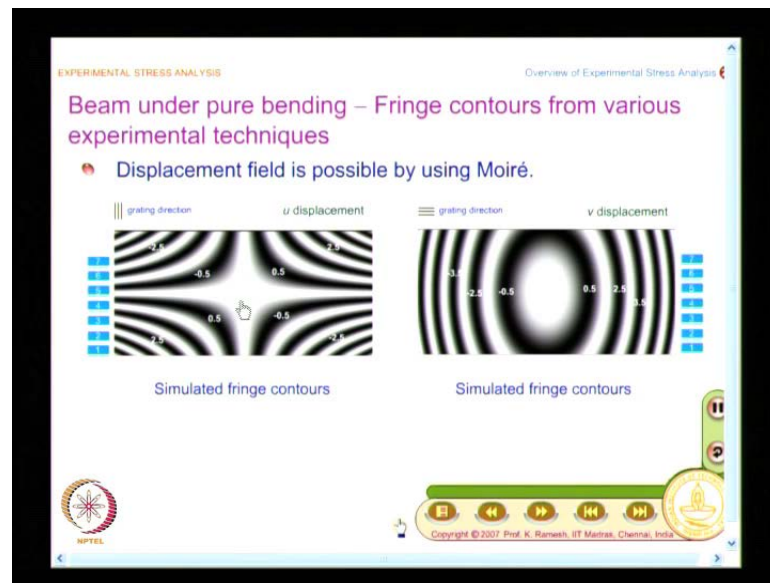
$$u = \frac{M}{EI} xy$$
$$v = \frac{M}{2EI} (x^2 + \nu y^2)$$

Enlarged view of stresses acting on the cross-section of the beam

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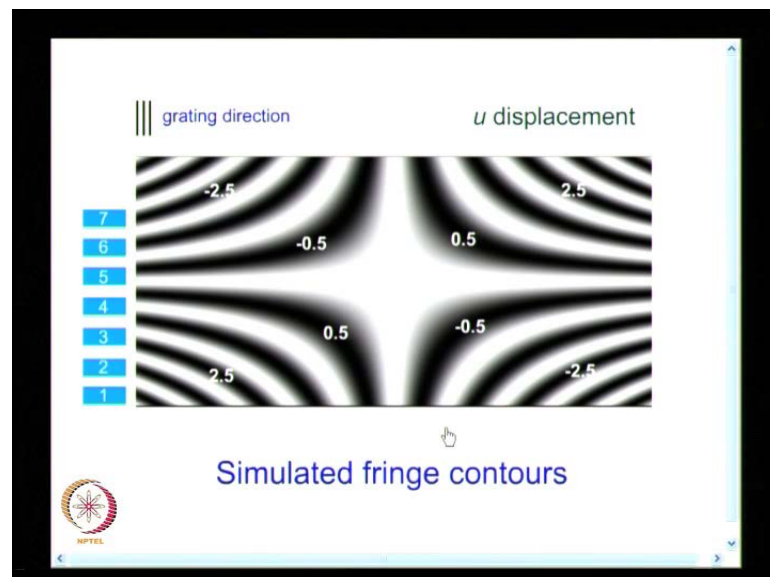
And you do some exercise on finding out beam deflection, beyond that you do not go and studied deeper. Now, when I look at this I can get the displacement field and displacement field is like this, it is fairly simple I have this u equal to M by $E I$ into $x y$ and this is nothing but an equation of an hyperbola. So, when you plot the contours you will have the contours looking like hyperbola. And you also have an expression for the displacement which is given as M by $2 E I$ into x square plus ν times y square. And in your deflection calculation, you may not have calculated ν displacement at all. You could have only found out the v displacement, you not found out the v displacement, u displacement you could have determined only the v displacement. Now, what will do is we have seen the fringe pattern from photo elasticity.

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Which gives you essentially contours of sigma one minus sigma two. Now, our interest is to see other contours, what I can do is I can do a contour on I can look at geomantic moiré and find out how the displacement field is? And this is what the u displacement and this is what v displacement and we would see this little closely.

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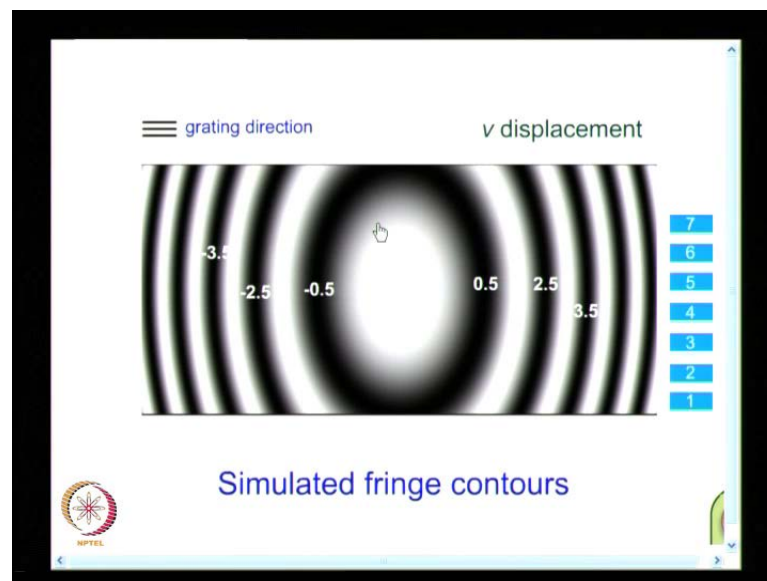


And what will see is; what you see here I have the fringes label and you have a 0.5 1.5 2.5 like this. And you have a minus 0.5 minus 2.5 so, when is I go to displacement I have both positive and negative numbering are the fringes, this is first observation. The second

observation is the fringe pattern comes with an indication on what is the grating direction that has been used. When we go to moiré for me to revile the displacement information I have to use a grating and grating direction dictates what component of displacement I do get. And I would like you to have a reasonable sketch of this. And when you do this, you will have an idea how the whole fill pattern looks like.

The many indirect leaning you learn, one is suppose I change load and you see this is how the fringes are develop, as the load is increase more and more fringes are develop. And this is the very key information in most of the experiment techniques to label the fringe orders, as an method to do. And other one what you find is the fringe thickness is different, this also carry some information, this will see later part of the course. So, you have a familiarity when I want to look at a displacement field in the case of beam under four point bending it would look like this. And we also have look at v displacement have been able to make a reasonable sketch of this. So, what I will do is I will see the v displacement and v displacement like this.

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And here, you find I have the grating direction horizontal. So, by comparing these two figures you can easily understand, what I say a displacement is the component perpendicular to the grating direction. And what I want to note down is when I have the grating direction one way, I need to have one optical arrangement to record it, when I have the grating direction in another way; I need another optical arrangement to do it.

And this is what I have said earlier experiments will revile information of a particular kind they may have the capacity to revile more than one information, but they may required to different measurements scenario. And when I have two different measurement scenarios, what I need to do is? I need to have there are also methods.

Where they combined and try to get more than one information in one core, when we look at moiré we will find out how you can get both u and v displacement simultaneously. And what you should keep in mind is if I get two information together it is always nuisance. And here again we can see as a function of load increase how the contours appear. And this is very important this will give you a short of understanding you also gives a feeling of doing experiment write in the class. When I increase the load, I see fringes are form and here again the labeling I have positive and negative quantities. If you have notice very carefully in photo elasticity, we have not labeled negative fringe orders, we have label all of them as positive. And this is an important point that have to note with, suppose I want to find out the strain, what do it?

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Beam under pure bending – Fringe contours from various experimental techniques

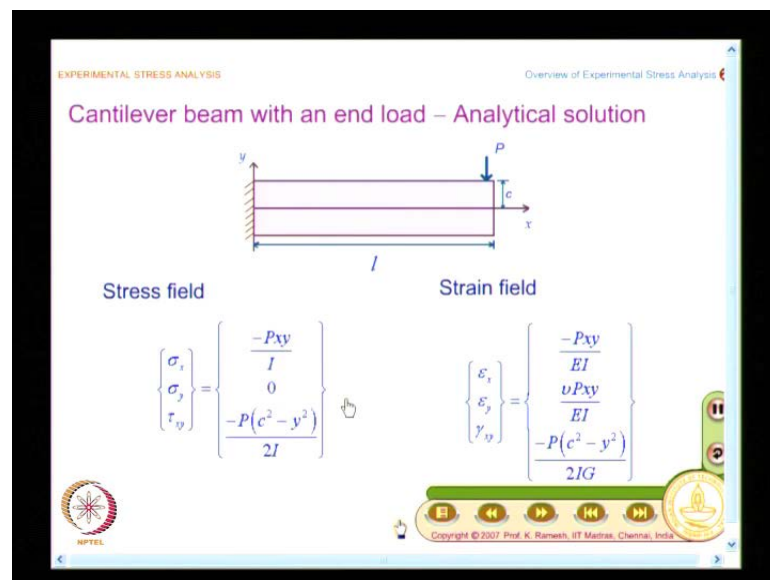
- Strain field is obtainable from the displacement fields – however differentiation in experimentation is prone to error.
- Use of Moiré interferometry is suggested to get the strain field.
- Strain at discrete points could be evaluated by strain gauges and along lines of interest by strip gauges.

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Now, I have the displacement I differential the displacement it is possible for weight find out the strain. But I may not want to do that if I have record from geometric moiré, because geometric moiré gives with less accuracy. And I have to go to moiré interferometry and then do the differentiation and get the strain field. Strain and discrete points with also be evaluated by strain gauges, suppose I want for a series of points then

I have what is on a strip gauge in one backing several strain gauges are available at fix lengths, given by the manufacture this is done by at the time of manufacture itself one grate containing several elements of strain gauge. So, I can get the strain from strain gauges, from point to point or along a line of interest.

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So, we have seen reasonably various contours that is possible from experiment for the problem of beam under four point bending. Now, let us move on to cantilever beam and what is the difference, because of the load here you have bending moment varying across the length of the beam. That is; what is seen in the first on here you do not have on bending moment, but the bending moment is the functional of x that wants to find of here and in addition you also have a shear stress. And for convenient the stress tensor is simply given as stress field where I given only the component sigma x sigma y and tau x y fairly straight forward this you have a solution from strength of materials.

What you may not done is may not calculate at strains and strain field look like this. I have minus P x y divided by E I epsilon y is new times P x y divided by E I gamma x y is minus P times c square minus y square divided by 2 I G on basely here the stress field is little more complex, strain field is little complex. And you also have anticipate the displacement is going to be complex than what you be seen in the case of beam. And we will have the look at it and the point to here note it when an do an analytical method I am able to get the stress field I am able to get the strain field.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Cantilever beam with an end load – Analytical solution

Displacement field

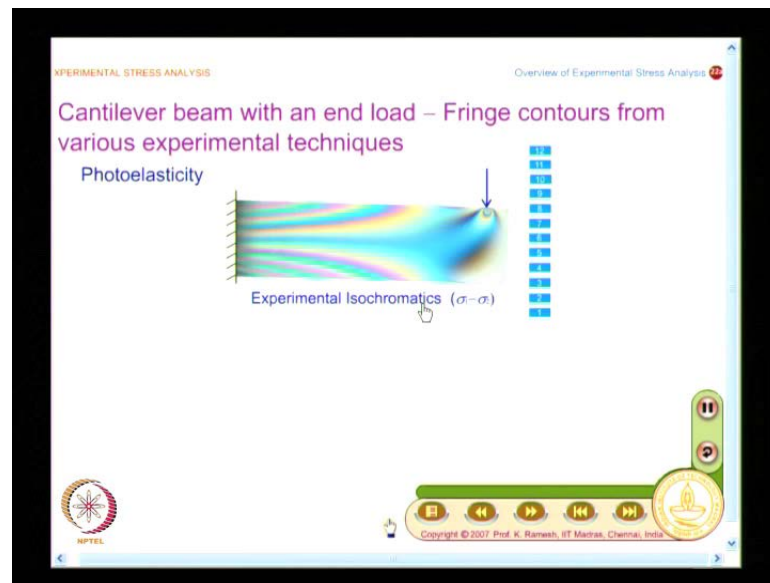
$$u = -\frac{Px^2y}{2EI} - \frac{vPy^3}{6EI} + \frac{Py^3}{6IG} + \left(\frac{Pl^2}{2EI} - \frac{Pc^2}{2IG} \right) y$$
$$v = \frac{vPxy^2}{2EI} + \frac{Px^3}{6EI} - \frac{Pl^2x}{2EI} + \frac{Pl^3}{3EI}$$

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I am also able to get the displacement field and that is what you see here, displacement field is give slightly involve more and more. You would worry it in your basic course in strength of materials only on the v displacement and when x is zero P l cube by 3 E I and you do not find out what happens to the displacement as the function of y. And what need to keep in this mind is a simply taken two dimensional beam a simply line and analyst. You do not consider this as beam and analyst, when you consider at a beam you also have to have a variation over the depth y.

And that is why you have a u expression you look at minus P x y square by 2 E I minus nu times P y cube divided by 6 E I plus P y cube divided by 6 I G plus with in bracket P square by 2 E I minus P c square by 2 I G multiplied by y. That is the interesting thing about in expression, when y is zero u is zero, you do not find out u displacement in strength of materials mainly, because you are considering at a two dimensional problem as one dimensional problem. So when they come theory elasticity you have made in improvement, an obviously now we will look at what the experimental method give as contours.

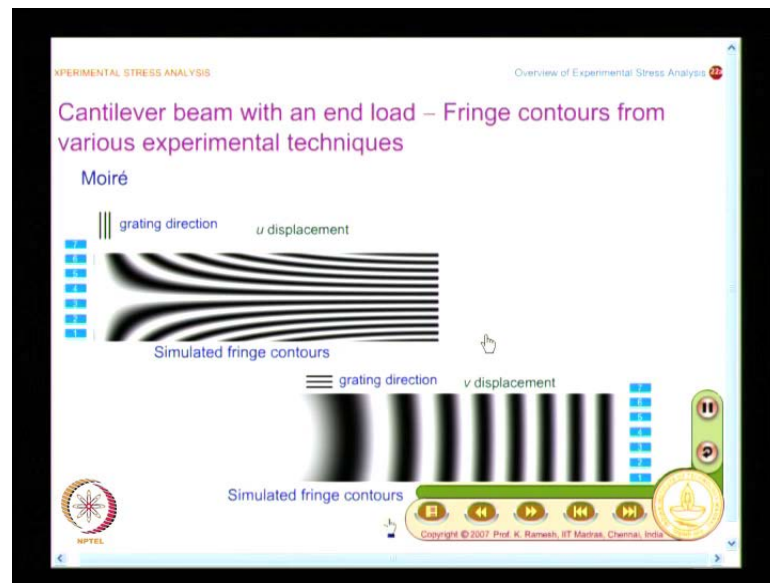
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And first thing you always doing is here photo elasticity we all know that give it contours of $\sigma_1 - \sigma_2$. And this contours not be able to calculate write at the class, you need better that you have computer software evaluated also have post processing where it can go and plot it. As before I would like to have a reasonable sketch of fringe fix patterns and what you find here this fringe patterns are definitely different from what you have seen in beam and four point bending. So, what you find is, the stress field is different which a contours directly bring out you have the effect of shear which is embattled in stress field.

And another important point note is, on the center of the beam you have light shade of blue, it is not black light what you have a four point bending. And this is load application point where you have fringes concentrates and this is the; this is the clamp end and this is experimental isochromatics. So, experimental isochromatic are different from, what you have see in the case of beam and four point bending and naturally displacement and contours is going to be still more complicate.

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So, you have u displacement, you have v displacement and what you have here is I again having grating direction. And moving this grating direction possible for to you to say this u displacement, moving this grating direction possible for to you to say this v displacement and you also have the feel of during the experiment by I see the function of load. I gradually increase the load more and more fringes form. And you could make a simple sketch of a try to get an one skeleton have to draw the band you get the geomantic shape and a few fringes give an idea how do I get fringes in the case of cantilever beam subjected to a end load. And I can increase the load, more and more fringes get found then I then also look at the v displacement, this also I can look at as the function of applying load.

And you will see as the load is increase you note is this zone, more and more fringes develop and they become tensor in appearance, I increase the load 2, I increase load 3, I increase to 4 5 6 and 7. So, what you find here is becomes almost parallel line more fringes are seen this zone. And when you draw the sketch necessary for draw all the lines you just draw a few lines indicating that you have fringes like this and slightly changes curvature as the close to the clamp end. And these are simulated fringe contours as you said some of you who have an exposure to computer graphics can make an attempt have those equations. You could go and make a contour plot and bringing to me and you will have doubt how get fringe bit, let me keep secrete of the moment and we will see count moment to get this band also when do the plotting.

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The slide is titled "EXPERIMENTAL STRESS ANALYSIS" and "Overview of Experimental Stress Analysis". The main heading is "Disc under diametral compression – Analytical solution".

- The famous assumption of plane sections remain plane before and after loading is not possible and one has to resort to the method of theory of elasticity to solve the problem.
- Closed form solution for stress field is as follows

The diagram shows a circular disc with a vertical diameter. Two downward-pointing arrows labeled 'P' are positioned at the top and bottom of the diameter, representing diametral compression. A coordinate system is centered at the origin of the disc, with the x-axis pointing to the right and the y-axis pointing upwards. A radius 'R' is shown as a line from the origin to the right edge of the disc.

At the bottom of the slide, there is a navigation bar with various icons and the text "Copyright © 2007 Prof. K. Ramesh, IIT Madras, Chennai, India".

Now, the next problem is the move on to disc and diametral compression. So, that is what shown in here disc and center of disc is taken as origin, I have x and y coordinates and R is the radius of the disc. I have diametral load P which is acting on it the will call diameter is either capital D or small d and for this problem you have close form solution from theory of elasticity. You do not; you will not able to approach and solve the problem from strength of materials, but you have able to solve the problem only the theory of elasticity, because plane section do not remain plane before and after loading. You also have the problem in the case of cantilever beam when you have a shear the plane should not remain plane they have a warping, fortunately there is no coupling between normal stress and shear stress. So, you could leave with flexure formula that is why called engineering analysis.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = -\frac{2P}{\pi t} \begin{Bmatrix} \frac{(R-y)x^2}{r_1^4} + \frac{(R+y)x^2}{r_2^4} - \frac{1}{D} \\ \frac{(R-y)^3}{r_1^4} + \frac{(R+y)^3}{r_2^4} - \frac{1}{D} \\ \frac{(R+y)^2 x}{r_2^4} - \frac{(R-y)^2 x}{r_1^4} \end{Bmatrix}$$

$r_1^2 = x^2 + (R-y)^2$ and $r_2^2 = x^2 + (R+y)^2$, R denotes the radius of the disc, D represents its diameter, t is the thickness of the disc and P is the compressive load applied.

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The moment called become to the circular called as you have to depend on theory of elasticity, fortunately theory of elasticity that is provide this solution. And here again given in convenient form of sigma x sigma y dou x y and you have minus 2 P by pi t. And I request all of you take down this equation though the equation are very long and very valuable information. When you develop photo elasticity, we could directly use this equation for our interpretation. And you have sigma x is R minus y into x square divided by r 1 power 4 and R 1 is defined r 1 square equal to x square plus R minus y hole square. And the second term is R plus y into x square divided by r 2 power 4 and the r 2 is defined as x square plus R plus y square, you have seen R denotes the radius D represents it is diameter, t is thickness, P is the compressive load applied.

And you have the expression are slightly different from sigma y and dou x y and I would like to have a equation written down. And obviously the stress what we now, they are much much complex then what had seed in the case of cantilever beam are the bending problem, or four point bending problem. And naturally, you may not able to calculate sigma one minus sigma two write a way in the class plot the contours, we have depended on the job, the stress field more complex strain field is going to be much complex than this and this is how expression look like.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

The strain field is as follows

$$\begin{cases} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{cases} = \frac{-2P}{\pi Et} \left\{ \begin{aligned} &\alpha \frac{x^2 - \nu\alpha^2}{(x^2 + \alpha^2)^2} + \beta \frac{x^2 - \nu\beta^2}{(x^2 + \beta^2)^2} - \frac{1-\nu}{D} \\ &\alpha \frac{\alpha^2 - \nu x^2}{(x^2 + \alpha^2)^2} + \beta \frac{\beta^2 - \nu x^2}{(x^2 + \beta^2)^2} - \frac{1-\nu}{D} \\ &2(1+\nu) \left[\frac{\beta^2 x}{(x^2 + \beta^2)^2} - \frac{\alpha^2 x}{(x^2 + \alpha^2)^2} \right] \end{aligned} \right\}$$

$\alpha = R - y; \beta = R + y;$
 $r_1^2 = x^2 + (R - y)^2 = x^2 + \alpha^2$
 $r_2^2 = x^2 + (R + y)^2 = x^2 + \beta^2$

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And I have epsilon x epsilon y gamma x y is equal to minus 2 P divided by pi E t and here again I would like a reason sketch of; (()) take down the notes of these expression. I have this is alpha times x square minus nu times of alpha square divided by x square plus a square the hole square and alpha is defined as alpha equal to R minus y beta equal to R plus y. And you have alpha and beta also related to r 1 square and r 2 square as follows, you have r 1 square is equal to x square plus alpha square r 2 square is x square plus beta square. And you have the second term is beta into x square minus nu times of beta square divided by x square plus beta square the hole square minus 1 minus nu divided by D.

So, this is expression for epsilon x, we have long expression for epsilon y and also have very long expression for a gamma x y. And though they are it is better that you have a copy of this your note book for help you for not readily available for in the book that you have you access to have been able to make a copy of this equation. You are seen the stress field and strain field both of them are longish; definitely the displacement field is going to be much longer than what you have seen all long.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

The displacement field is as follows

$$u = -\frac{2P}{\pi t E} \left(\frac{1-\nu}{2} \left[\tan^{-1} \left(\frac{x}{R-y} \right) + \tan^{-1} \left(\frac{x}{R+y} \right) \right] - \frac{(1+\nu)}{2} \left[\frac{(R-y)x}{r_1^2} + \frac{(R+y)x}{r_2^2} \right] - (1-\nu) \frac{x}{d} \right)$$

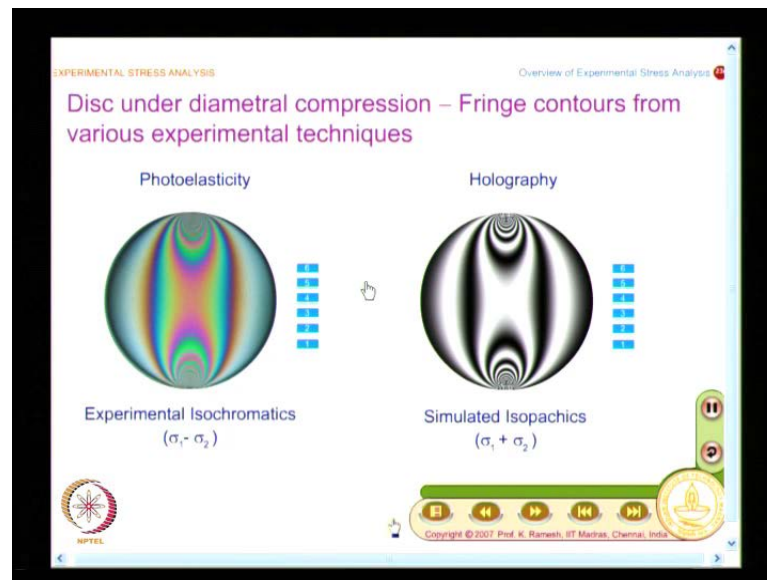
$$v = -\frac{2P}{\pi t E} \left(\frac{1}{2} \ln \left(\frac{x^2 + (R+y)^2}{x^2 + (R-y)^2} \right) - \frac{(1+\nu)}{2} \left(\frac{x^2}{x^2 + (R-y)^2} \right) + \frac{(1+\nu)}{2} \left(\frac{x^2}{x^2 + (R+y)^2} \right) - (1-\nu) \frac{y}{d} \right)$$

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So, it is better so please take a time to write down and you have this is u equal to minus 2 p divided by pi t E into 1 minus nu divided by 2, you have tan inverse x by R minus y plus tan inverse x by R plus y then, you have minus of 1 plus nu times divided by 2 into R minus y into x divided by r 1 square plus R plus y x divided by r 2 square minus 1 minus nu x by d. On similar line we have long expression for the v displacement we have term with a natural logarithm x square plus R plus y hole square divided by x square plus R minus y hole square and the expression goes like this. And an obviously it is very difficult to visualize what could be the nature of the displacement by looking at the equation.

If you have to plot, you have to go to computer software plugging the equation, have the plotting the software callout the numbers collect them and then draw the contours. On the other hand I take the model, I put it appropriate topics, I get stress information, I get displacement information. The moment you want to go for strain write now you do not have a technique whole fill strain information, you can get from strain gauges and plot them. Or from displacement information you can do numerical difference differentiation and plot the strain, but write new you do not have a whole fill experimental technique, you should give you whole fill strain data conveniently. Now, what will look at this, we workout disc and diametric compression it is bench mark problem in photo elasticity.

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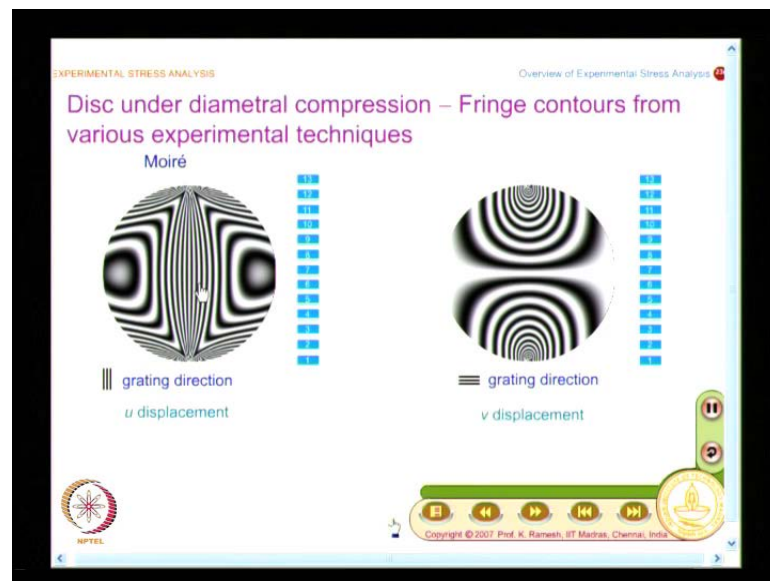
So, we will see the photo elastic contours and we will also see another set of experimental arrangement, where you get contours of $\sigma_1 + \sigma_2$. In photo elastic contours of $\sigma_1 - \sigma_2$ and I said one experimental technique will not give all the information. Suppose I want to information, I want to find out the information individual stress component, one approach could be holographic experiment get simulated; I get simulated isopachics recorded in this case simulated, but get the isopachics recorded. So, when I have photo elastic contours as well as holographic contours, I can process these to find out individual magnitude of σ_1 and σ_2 on the entire field.

And what you not note it down is between photo elasticity and holographic should color contours are photo elasticity, because use white light and get information in color this unit to photo elasticity. Although you also give monochromatic information depend on monochromatic light source and you get black and white information which is processed, holographic you do it only one a single wavelength and the contours are very similar. And I would have a reasonable sketch of this any one of it which gives indication how the fringes are look like. And here you have this as the shape of 8, this as a shape of 8. This you have to note it down, this is use it in our experimental information later.

You can also seen a function of a load applied how the fringes are developed so, what happens the fringes are developed here move out wards at you could see from this

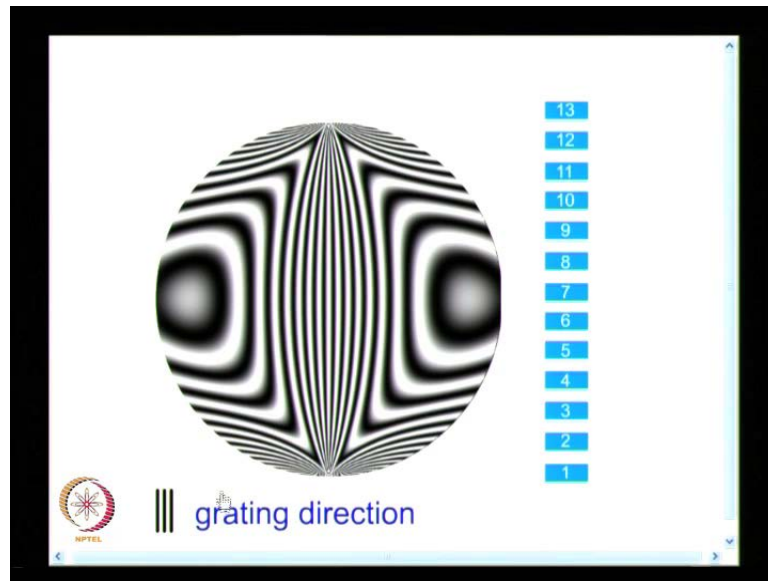
simulation. This also gives indication of doing an experiment write at the laboratory, same thing you can do for photo elasticity fringes developed on move outward this also give a indication how you can go about in labeling fringes, which is complicated exercise you see later. So, in this example we are also shown result from holography and if you want to go for displacement information, what is the technique that will be use we have been seen moiré.

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So, I would see the moiré contours, what I get for u displacement as well as for v displacement how do you decide is I have the grating direction which will be recorded and you could see the contours beautifully.

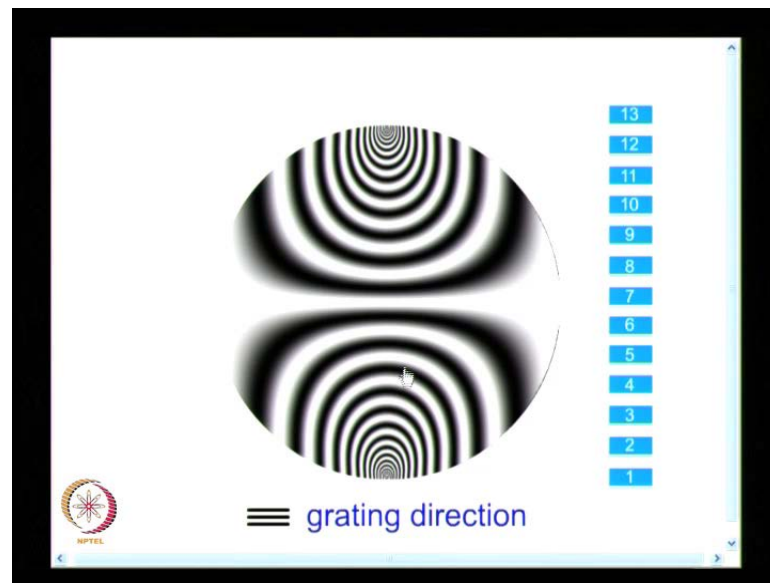
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This we saw this very difficult expression and obviously when you do it will be complicated like this. And I can also change the load 1 2 and gradually increase seen more and more fringes will appear. And probably this sketch you can make, because you do not have many fringes, but it gives you the geometric shape of fringes reasonably get. (()) later point we have occasionally in the experimental result, you could easily say I see patterns earlier then would be for the desk it is only the u displacement. Suppose somebody give photographic without the grating direction on an unknown situation if they want to interpreted you will not be able to given answer immediately.

Unless you know, how the experiment is contacted, how the values are recorded you will not in be a position to interpreted the fringe pattern, interpretation require additional information labeling of fringes are not a task. And for that we will have to know, what are the various techniques available to label the fringes? And some of the later examples have not shown in the fringe ordering elaborately so that you can do this is an exercise label the fringes after we have learn the course.

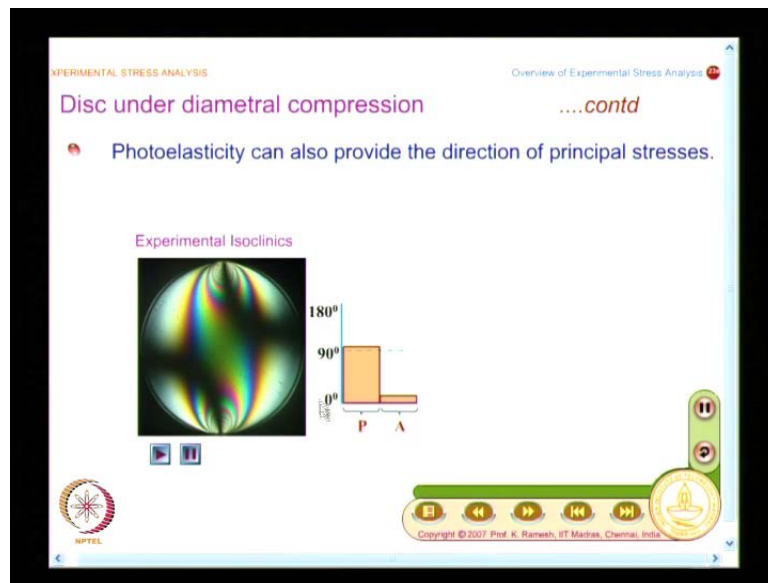
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So, when I increase the load the more and more fringes will appear so that is what I see in the case of the disc under diametric compression and this gives you u displacement. And now, we similarly, we last to see the v displacement and this is what you here grating direction like this. And as I increase the load I go from this gradually, go from this you find this is load application points. And all the fringes come from this and what you find this and you have very broad fringe and as you increase the load do you have more fringes move and occupied different positions.

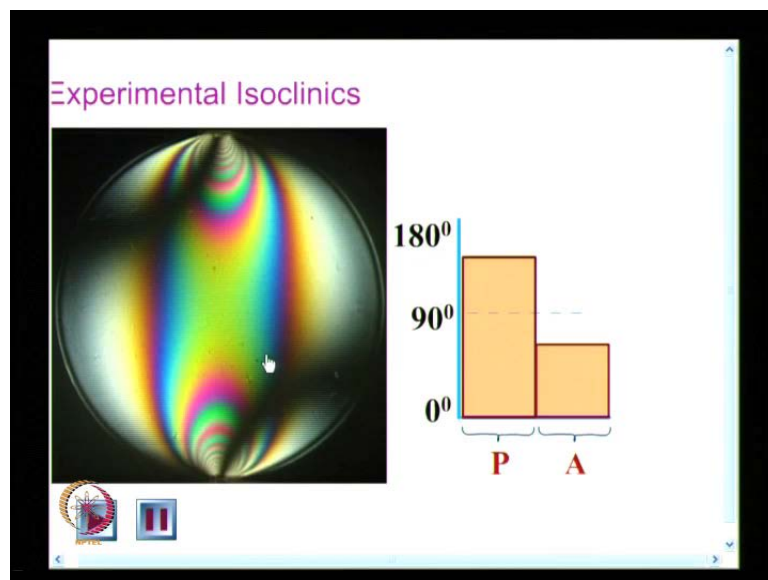
So, it is possible for you to label them appropriately if you have this knowledge, what defined these fringe move at a very high load. So, if you have number attached number also move a long width it so that will see later and here again we have reasonable sketch for a intermediate load note the nature of the displacement field. So, that is gives a certain level familiarity and you feel closer to experimental techniques. And I also get an knowledge how to appreciate visual information.

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And for this example, I also have another set of contours what you can get from photo elasticity, photo elasticity can also give principle stress direction and that want you see here and we will see this closely.

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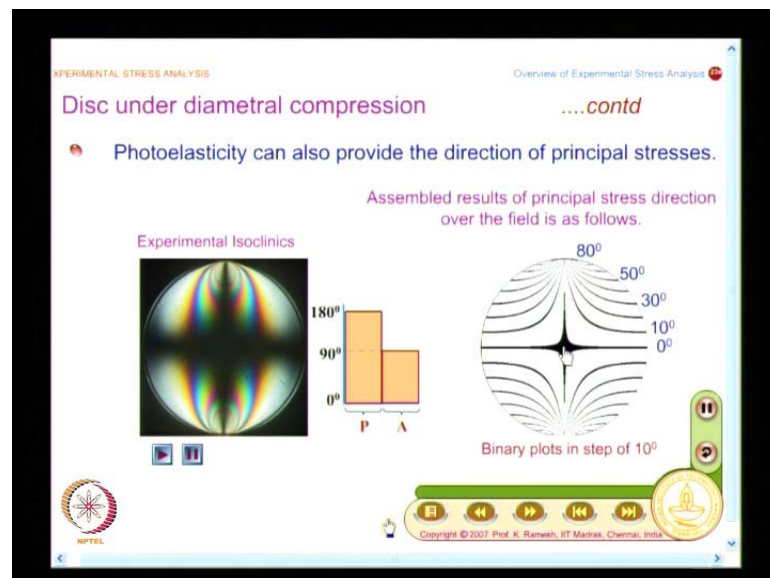
So, what I find here is, I have experimental isoclinic and what you find here is the whole image looks (()) a black contour move over it that is what you see, a black set of contour move over color bands, can you identify the color bands? Because you have seen this

earlier, can you identify the color bands? You can identify the color bands, what the color band show?

Yes contours of $\sigma_1 - \sigma_2$.

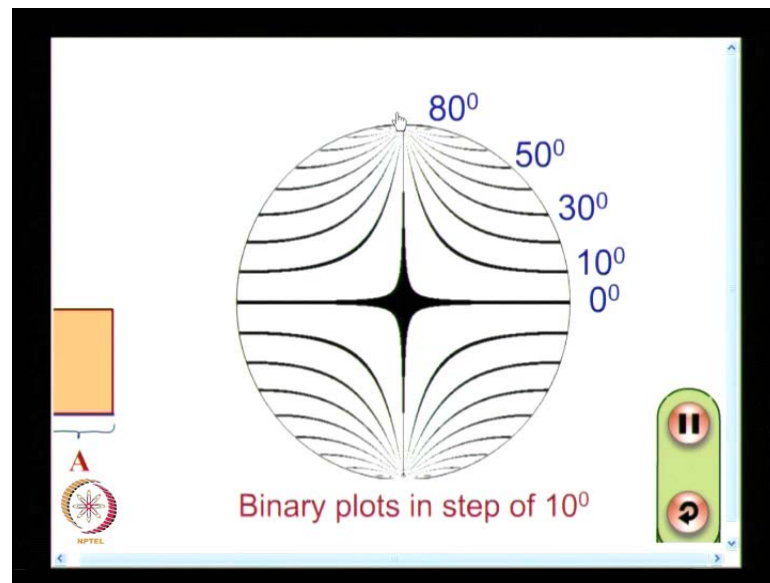
So, what you find here is in this optical arrangement I get two information, one information is $\sigma_1 - \sigma_2$. And information is orientation of the principle stress plane and what you find if you have two information super imposed clarity is lost so, you always like to have independent information then processing data collection everything becomes much simpler. And what you seen in this example, I have one set of contours that move this may not would be understand this stage. What it is indicate a direction of polarizer analyzer is the two optical elements they are kept at mutually perpendicular decrease, for these angles for these contours move?

(Refer Slide Time: 49:05)



So, what you are do is form data collection point of you, I have to set it at fixed angle and try to make a reasonable angle sketch of, what this contours are? And what shown here? So, what you have here is I have the set of contours.

(Refer Slide Time: 49:15)



Which is a binary plot in step of 10 degrees this you make it sketch of it so, I have to process this information and extract this in this form, I do not have an optical arrangement which would give me this set of contours directly. So, I have a 0 degree isoclinic, 10 degree isoclinic, 20 degree isoclinic, 30 degree isoclinic and so on and I; we call this is isoclinic's. In fact, when you do it chapter on photo elasticity know in detail what these isoclinic's are iso means constant, clinic means inclination, isoclinic means are contours of constant inclination, constant inclination of what? Constant inclination of principle stress direction. So, photo elastic experiment it possible for to get sigma one minus sigma two contours.

And you can also get by an appropriate optical arrangement contours of principle stress direction. And what I find is here I have sigma one minus sigma two contours super impose over isoclinic contours, I do not seen all isoclinic in one shot, I have to scene the image and pick out this information, let this as the separate image of isoclinic`s. So, without getting into the experiment details, I have try to project what is store for you and some experimental technique you get the information separately. In the case of moiré I get u displacement, I get v displacement separately and getting them separately is better. Through I have to do one more experiment getting them separately an optical arrangement is for better from processing point of you. Then getting the image of sigma one minus sigma two super impose on the isoclinic pattern, that you what seen here. And

the next problem take on his, we would take on the problem of a clamped circular plate under a central load.

(Refer Slide Time: 51:42)

EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Clamped circular plate under a central load – Analytical solution

$$w = \frac{Pr^2}{8\pi D} \log \frac{r}{a} + \frac{P}{16\pi D} (a^2 - r^2)$$

$$\frac{\partial w}{\partial x} = \frac{4xw_{\max}}{a^2} \log \left(\frac{\sqrt{x^2 + y^2}}{a} \right)$$

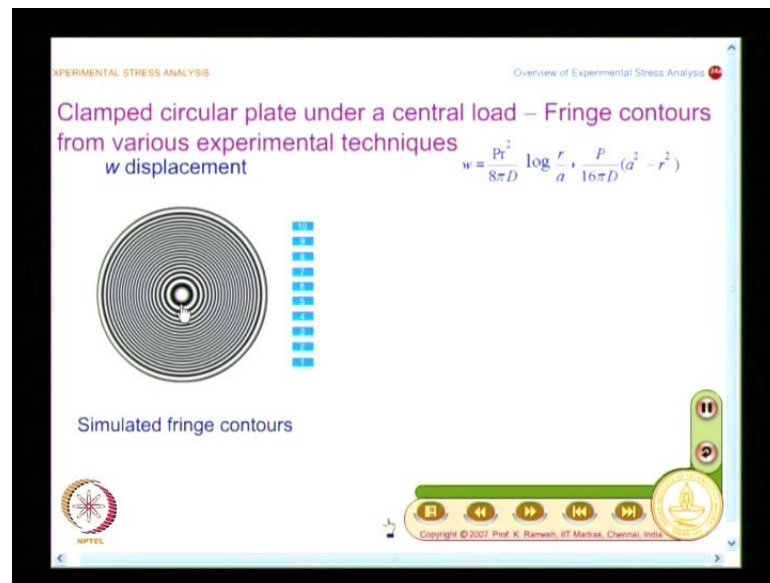
$$\frac{\partial^2 w}{\partial x^2} = \frac{4w_{\max}}{a^2} \log \left(\frac{\sqrt{x^2 + y^2}}{a} \right) + \frac{4x^2 w_{\max}}{a^2 (x^2 + y^2)}$$

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And what I would in appreciate is I have this clamp circular plate this is clamp at all around the beriberi and subjected to a central load. In fact, in the kind of problem where they do out of displacement, they take this as the bench marking example, this as the bench marking example and using this only they justify the method. And what you have here is have expression for w out of place this placement, you have the expression for dou w by dou x, you also have the expression for the curvature dou square w by dou x square. In all this cases you have logarithmic terms appearing in this and the expression are definitely complex and you would able to find out this from a study on theory of plates.

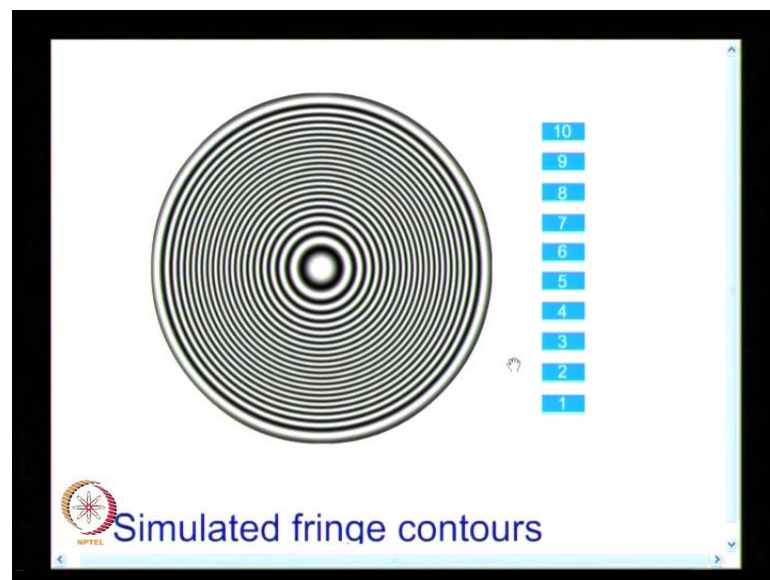
Where you have sufficient theory is develop to find out how to get this expressions. And essentially when I want to go and find out whether I have the circle interferometry working alright. I would not taken disc under diametral compression, but I would rather take a clamp circular plate with a central load to test even by experimental setup. So, by doing that, I would be able to get that information. And similarly, what I have here is, I have shown in the slope, you also have slope in direction x slope in direction y. And when I go find out, I get this pattern like this.

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So, what I have here is I said nature is so good that it gives you. So, I have simulated fringe contour, I have concentric circles nature is so good, in the case of beam and bending you saw horizontal line in the case of clamp circular plate.

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You see beautiful circle and this is again the function of loader plate. So, you have the concentrate circles that they come and this is what you have? And as the load is increase you have more and more circles.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Clamped circular plate under a central load – Fringe contours from various experimental techniques
w displacement

$$w = \frac{Pr^2}{8\pi D} \log \frac{r}{a} + \frac{P}{16\pi D} (a^2 - r^2)$$

Simulated fringe contours

Experimental fringe contours (Speckle Interferometry)

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And what you find is if I want to go and look at an experiment, I can get this some speckle interferometry. And I will magnify this and show this here you do not find the fringes as clearly mark dark and white.

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Experimental fringe contours

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You have speckle pattern and these are called correlation fringes. And here again is the function of loader plate, I can apply the load 1, load 2, load 3 and so on and you do not have the level of contrast that you have in the case of photo elasticity. What is the advantage here? You able to get out of displacement by hole fill technique, but they do

not have I contrast so, if you look at any of this speckle interferometry methods they expand lot of time on filtering. On the other hand photo elastic fringes are very high contrast and you do; need to do less pose processing. Then, what you can do in the case of speckle interferometry and this fringes you have and again the function of load.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Clamped circular plate under a central load – Fringe contours from various experimental techniques

Slope Fringes

$$\frac{\partial w}{\partial x} = \frac{4xw_{\max}}{a^2} \log \left(\frac{\sqrt{x^2 + y^2}}{a} \right)$$

$$\frac{\partial w}{\partial y} = \frac{4yw_{\max}}{a^2} \log \left(\frac{\sqrt{x^2 + y^2}}{a} \right)$$

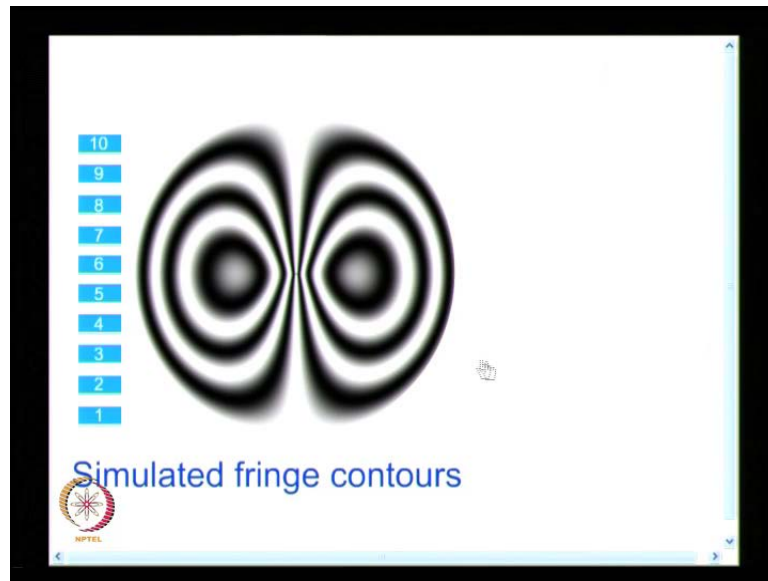
Simulated fringe contours

Simulated fringe contours

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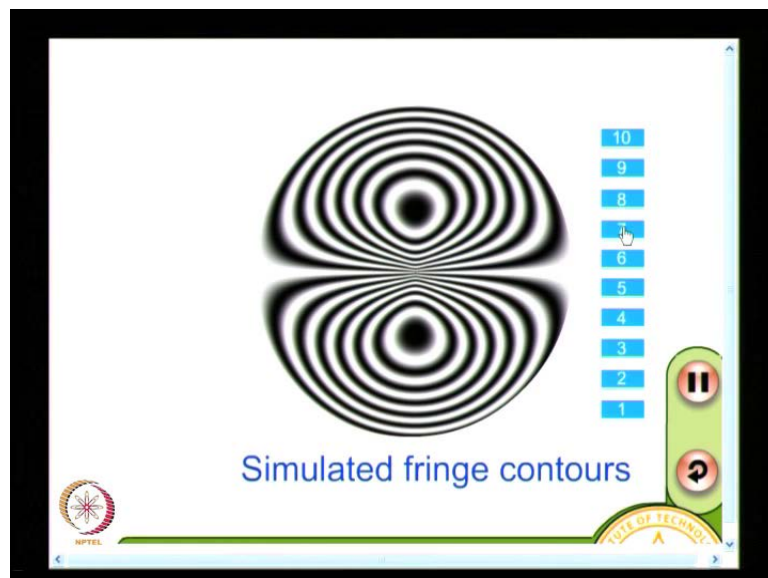
Also make a need reasonable sketch of this. Suppose I want to see and go on how do the slope pattern look like this and the slope pattern look like this, slope fringe are like this. You have the expression this and you have the slope in the direction x, slope in the direction y and this is the fringe pattern you have. And what you have here is I have increase the load so, you could say the people also say these as butterfly fringes, they look like wings of butterfly and they call as butterfly fringes.

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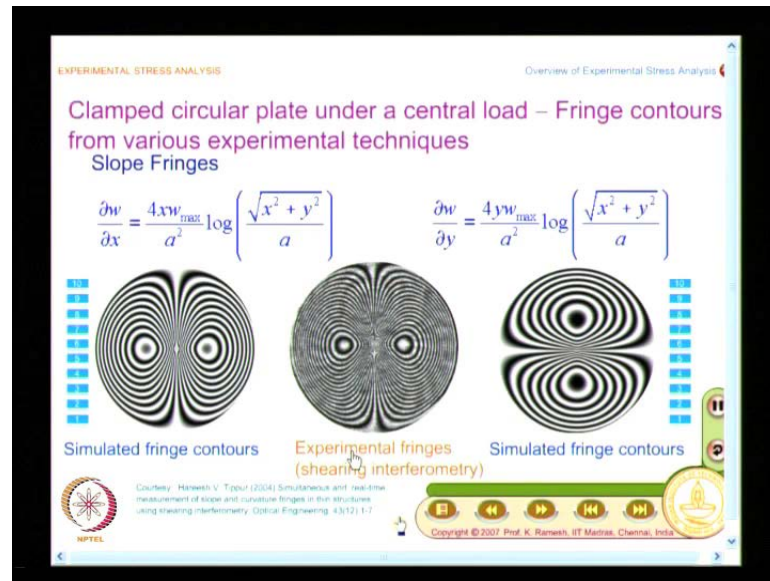
Particularly in the case of non destructive testing and I can do the honey come testing any delimitation you could be find out easily. If you see butterfly, should not feel happy should feel disturb there is the delimitation. The butterfly pattern why they called it is in the case of commencers they you do not want to say that dou w by dou x contours a simply say that the butterfly fringes. And you have the similar situation when have you have the slope in the y direction also, this are called as butterfly fringes pattern.

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And this you have look at it and oriented different direction and that is what you have. And now, what you at look at it, suppose is an any experiment which can revile this are also simulated contours, these are expression here, I have simulated when I increase load I get that. And what you find here is a very resent experiment on speckle interferometry has given very nice set of contours on thin vapors.

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And you know, in fact it matches very well the; what you have as the simulated pattern, this is how see in the experiment. And these is from the wake of professor Harish V Tippur is a best friend of mine and these has come in the journal of optical engineering will get more details of these fringe pattern from this. And courtesy goes to optical engineering and these society for optical engineer spie. And you can note down these reference good looks for more details of these fringe pattern. And these; was the very recent work these is on a silicon wafer so, it is simultaneous and real time measurement of slope and curvature, you see only the slope fringes here and what you would be see.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Clamped circular plate under a central load – Fringe contours from various experimental techniques

Curvature fringes

$$\frac{\partial^2 w}{\partial x^2} = \frac{4w_{max}}{a^2} \log\left(\frac{\sqrt{x^2 + y^2}}{a}\right) + \frac{4x^2 w_{max}}{a^2(x^2 + y^2)}$$

Simulated fringe contours

Reflection Moiré

Courtesy: Prof. G. Subramanian, IIT Madras

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Now, is you also see this curvature and curvature information is looks like this. And simulated fringe contours are like thus if you go to experimental technique, you have nice set of curvature fringes obtain and you may thing from the experimental fringes be an seen an earlier these borders are not smooth.

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Reflection Moiré

Shearing Interferometry

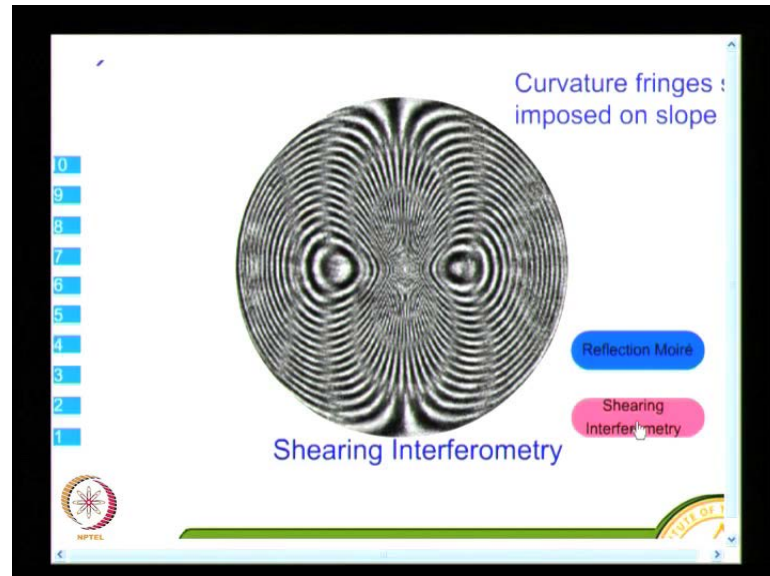
Reflection Moiré

Courtesy: Prof. G. Subramanian, IIT Madras.

They are the jacket, but you see the shape as footers in here these a result is steam of a G Subramaniyan, he was an exponents on moiré interferometry in the country and the is from reflection moiré. And what you load know is even to identify the optical technique

to get these is a challenge and you will know the difference only when you want to find the curvature in shearing interferometry.

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Shearing interferometry gives information that like this and what you see here, do you see the curvature I will magnify the further, what you see here is I have on the background slope fringes, on the slope fringes you see faintly the curvature fringes like. When I shown sigma one minus sigma two contours super impose with isoclinic, you feel uncomfortable. Now, you will say that was much better it has very good contrast. Here I get curvature information, but the advantage here is though it looks little dull, the advantage is the optical arrangement used gives both slope and curvature information in one shot, that is what the optical this is again the wake from process Tippu and this is the optical from engineering.

So, you can get more information on this from reading the paper like this. So, I can say comfortably now you have a fairly good idea on how do the optical patterns look like. Optical patterns essentially good certain physically contour and these physical contours dictated by the physics expect experimental technique. You have seen individually experimental contours, you have also sample of super impose contours in an some cases you get in super impose information, only the certain cases you will be able to separated. And from experimental point of you both, suppose I have a time vary phenomena I

would like record both, I would record both slope and curvature by a very different optical arrangement.

Though the quality of information slightly of a poor quality I would record both them information together. And I am sure at end of this lecture I would fairly reasonable idea how do the hole fill representation of the stress field σ_1 minus σ_2 contours and displacement fields by an large look like, that give certain limit of familiarity certain level of affinity. And as we go further you would able to find out how to get this contours by yourself, what is the principle that use and also how to interpret not just we happy with the shapes of the contours, but also get the actual magnitude with essence of confidence, thank you.

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