

**Experimental Stress Analysis**  
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**Lecture No. # 27**

**Correction Factors for Photoelastic Coatings**

We have started our discussion on photoelastic coatings in the last class. And I mention photoelastic coatings can be called as birefringent coating taking to account that we paste a birefringent material on the specimen surface and we also saw the material become birefringent, when the model is loaded.

So, whatever the polarization optics as well as the crystal optics basics that we developed in transmission photoelasticity or directly applicable to reflection photoelasticity will certain minor changes; I also mention, because we analysis reflected light - this is also called as reflection photo elasticity. And one of the main challenges here is how to interfere the specimen stresses from observing fringe patterns that is seen on the coating.

So, from the optics first we find out what are the coating stresses? And from the coating stresses using the principles of mechanics of solids, we go and find out what are the specimen stresses.

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**Coating stresses**

Stress strain relations

$$\epsilon_1^s = \frac{1}{E_s} (\sigma_1^s - \nu_s \sigma_2^s) \qquad \epsilon_1^c = \frac{1}{E_c} (\sigma_1^c - \nu_c \sigma_2^c)$$

$$\epsilon_2^s = \frac{1}{E_s} (\sigma_2^s - \nu_s \sigma_1^s) \qquad \epsilon_2^c = \frac{1}{E_c} (\sigma_2^c - \nu_c \sigma_1^c)$$

Coating stress

$$\sigma_1^c = \frac{E_c}{E_s(1-\nu_c^2)} \left[ (1-\nu_c\nu_s)\sigma_1^s + (\nu_c - \nu_s)\sigma_2^s \right]$$

$$\sigma_2^c = \frac{E_c}{E_s(1-\nu_c^2)} \left[ (1-\nu_c\nu_s)\sigma_2^s + (\nu_c - \nu_s)\sigma_1^s \right]$$

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And what we had seen in the last class was whatever the strain develop on the specimen is faithfully transmitted to the coating, and individually you can write the stress strain relations for the specimen as well as the coating. And idea was using this set of relations; it is possible for you to estimate the coating stress  $\sigma_1^c$  and  $\sigma_2^c$ . And this side asked to work it outing your room and I things some of your done it and the expression looks like this. And what we have here is  $\sigma_1^c$  is given as  $E_c$  by  $E_s$  into  $1 - \nu_c^2$  into  $1 - \nu_c \nu_s$  into  $\sigma_1^s$  plus  $\nu_c - \nu_s$  into  $\sigma_2^s$ . And  $\sigma_2^c$  is given as  $E_c$  by  $E_s$   $1 - \nu_c^2$   $1 - \nu_c \nu_s$  into  $\sigma_2^s$  plus  $\nu_c - \nu_s$  into  $\sigma_1^s$ , and I said when I go and look at what is brittle coating there again the expression for coating stress remained same.

There we will find out individuals stress component by analyzing iso (( )) as for as photoelastic coating is concern though we develop individually the expression for  $\sigma_1^c$  and  $\sigma_2^c$ , essentially you get only difference in principle stresses or principle strains. So, that what we are going to look at, we will look at the difference in the principle stresses. First we will find out how can I express specimen stress difference, as the function of the coating stress difference, then coating stress difference can be replaced by your principles of optics, you will have a strain optical relation and then plug in it and do it.

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### Specimen stresses

- The inter relationship between the difference in principal stresses of the specimen and the coating is obtained as

$$\sigma_1^s - \sigma_2^s = \frac{E_s}{E_c} \frac{1 + \nu_c}{1 + \nu_s} (\sigma_1^c - \sigma_2^c)$$

- The principal stress difference in the specimen can be found as

$$\sigma_1^s - \sigma_2^s = \frac{E_s}{1 + \nu_s} \frac{N \lambda}{2h_c K}$$

- It is interesting to note that the elastic properties of the coating material do not appear in the above equation.

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So, the specimen stresses are expressed as follows, you have the young's modules of the specimen divided by the young's module of the coating multiplied by  $1 + \nu_c$  divided by  $1 + \nu_s$  into  $\sigma_{1c} - \sigma_{2c}$ . Whatever the expression that we are got individually for  $\sigma_{1c}$  and  $\sigma_{2c}$  can be recast in a form, which relates the specimens stress difference to coating stress difference. And now what we will do? We will replace the coating stress difference in terms of the fringe orders.

If it is the transmission photoelasticity, we will simply have this is  $N_s \sigma$  by  $h$ ; in reflection photoelasticity I said, you have to have the light with is entering the model and coming out the off the model. So, you have twice the distance of the thickness the light travels the path. So, one modification you will have two times in a thickness coming in the denominator, other one is instead of expressing it as material stress fringe value, this lot more convenient to express in terms of material strain fringe value.

So, we lose all those interrelationships, we also looked at what is interrelationship between  $f_\sigma$  and  $f_\epsilon$  and your also seen  $f_\epsilon$  is  $\lambda$  by  $K$ . So, what I am wanted to do is, I am go to write the final expression, which involve the fringe order and  $K$ , because  $K$  is supplied by the manufacturer for whatever the photoelastic coating material, they normally supply the strain coefficient  $K$  as part of the supply of the material, but it is also desirable that you calibrated, because all polymers slowly changes the properties or period of time. So, it is desirable that you also evaluated.

So, now, what we are going look at is, we will look at the principle stress difference of the specimen in terms of the fringe order and the strain of the coefficient  $K$ . So, what I have here is, I have the elastic constants of the specimen come into the picture, which was not the case in the case of transmission photoelasticity. I had only  $N f_\sigma$  by  $h$  here it is  $N f_\epsilon$  divided by  $2 h c$   $f_\epsilon$  is replaced by  $\lambda$  by  $k$ , but in addition you also have young's modules of the specimen divided by  $1 + \nu_s$ .

So, what I haven photoelastic coatings is our focus is to find out the stresses develop on the specimen. The difference in principle stresses on the specimen is related to the fringe order through this expression, but even this expression is not complete, because I have said while developing the theory, we are merge certain assumptions and all this assumptions you will not be able to fully satisfied in your experimentation. So, whenever you violate this we bring in a correction factor to improve our result.

So, we will see that also in the case of transmission photoelasticity the famous stress optic law was  $\sigma_1 - \sigma_2 = N f \sigma / h$  that was the golden rule. Still we are not come to look at what is the final expression that is off value in reflection photoelasticity. I need to have one more parameter; it is only an intermediate step. I cannot stop my photoelastic coating development of mathematics with this set of expressions.

I need to have one more factor that is needed and we will also see how that factor varies from different type of loading situations and another point of interest here is, if you look at this expression. The elastic properties of the coating do not appearing the above equation that is one general observation and I also mention in our student see here, I said one of the important issues in photoelastic coating is what is the maximum fringe order obtainable?

Which question we never raised in transmission photo elasticity. It was not critical, because the loading is different. It is basically the strains that develop on this specimen surface or transmitted to the coating the level of loading on this actual coating is very small and if you look at this expression you can actually replace this by the yield stress and find out what is the maximum fringe order and you will find that number is small and that number is going to be small that also we will take at a plated.

Why I am reminding this is? There are certain peculiarity sense reflection photoelasticity and if you understand those peculiarities, whatever you have learned in transmission photoelasticity the same comfort you have in interpreting what are isoclinic? What are isochromatics? All that knowledge is same only the interpretation is slightly different here and you should know the peculiarities here, what are the peculiarities it will have?

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EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

### Correction Factors for Photoelastic Coatings

- The following points are to be considered for a finite thickness coating.
  - ★ The coating reinforces the specimen.
  - ★ A strain variation exists over the thickness of the coating.
  - ★ Mismatch of Poisson's ratio between the coating and the specimen needs to be accounted for.

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Then we move on to what are the correction factors? Why they are needed? And I said the moment time talk out the photoelasticity coatings. The discussion is complete only when we talk about the correction factors and first consideration is the coating reinforces the specimen and why we have to look at this? We are having a finite thickness coating.

Suppose, I have a magic material, I simply going spray it on the actual specimen there which was done in the case of image correlation. You have a white paint or the thermo elasticity you put a black paint, where the coating thickness is negligible. In terms of micron it is will be, but in the case of photo elastic coating have already said coatings of the order of 3 millimeters or not and common. So, you are handling a coating, which as a finite thickness. So, when I look at finite thickness you should also look at whether the coating reinforces the specimen significantly and I also said there is the comfort.

People started developing photoelastic coating with glass as the coating material and glass was having a young's modules of 70 g p Hs comparable to the waste material that definitely reinforces. Whereas the coating that are employed on metals have young's modules of order of 3 GPa, which is very negligible when you compare to aluminum or steel or braze it is very, very small and on the similar note. If you go for low strength alloys as well as rubber, then you need to develop the coating material, which has much less young's modules than the base material.

So, the issue here is you are having a finite thickness coating and definitely you have to appreciate that coating can reinforce specimen. You must take corrective measures for either you bring a correction factor or minimize the reinforcement of the specimen by the coating and another issue is a strain variation exists over the thickness of the coating. This is very important in which class of problem do you think that strain variation can exist. See if the coating is very, very thin you do not have to really worry, but if I have a thick coating, if I apply a tension absolutely no problem thickness does not really signify you will have an average strain approximately over the thickness.

The moment, I go for bending or torsion where the distance from the neutral fiber dictates the strain developed. So, definitely in problems, where you have to do tackle or evaluate the loads due to bending or torsion a strain variation exists over the thickness of the coating and this needs to be addressed completely.

See, when we looked at transmission photoelasticity the famous specimen, we used for calibration was a circular disk and diametric compression. We also saw what are the reasons for it? And on the other hand when I come to reflection photoelasticity, I would use a cantilever beam as the favorable calibration specimen for evaluating the strain coefficient  $K$ . So, that a beam is subjected to bending and if I do not use the correction factor my evaluation  $K$  would be unknown.

So, even for a simple calibration test I have to accommodate a correction factor. This is what is the peculiarity in reflection photoelasticity, and finally, I always been saying in all the experimental technique position's ratio has a role to play and you have a mismatch of Poisson ratio between the coating and the specimen and that needs to be accounted for and what we would do cleverly is, We will develop correction factors for many of the simple loading situations assuming that there is no Poisson ratio mismatch. So, you will have some kind of corrections and we separately address for certain class of problem, you could also accommodate Poisson ratio mismatch only through class of problems we will bring in how to accommodate mismatch of Poisson ratio? And we will also see how you find out stress concentration factor? When I use a photoelastic coating test, there is a slight modification that we need to do it is not as simple as you saw in the case of transmission photoelasticity mainly, because of mismatch of Poisson ratio, which becomes significant in those class of problems

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EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings 10

Correction Factors for Photoelastic Coatings ....contd

So the principal stress difference equation to be rewritten as

$$\sigma_1^s - \sigma_2^s = R_f \frac{E_s}{1 + \nu_s} \frac{N_f \lambda}{2h_c K}$$

Where  $R_f$  is the correction factor.

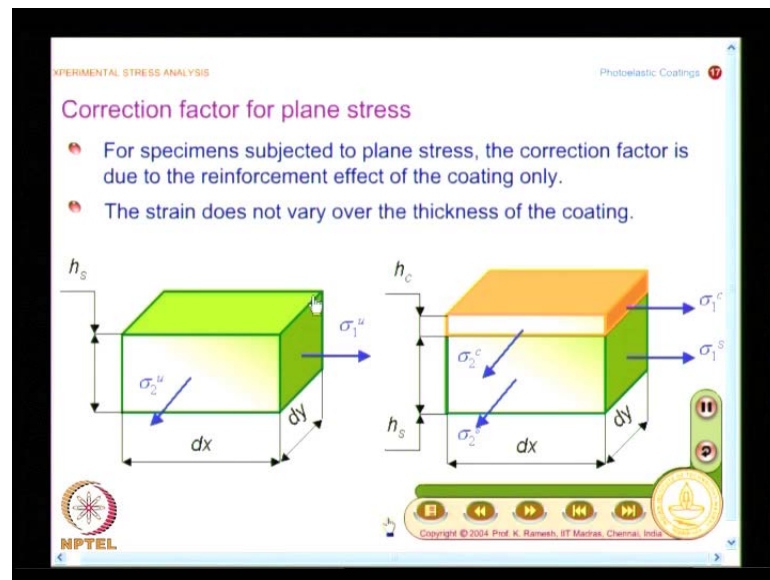
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And we will take a one of the very simple problems to start with and before, we are going to that as I said the final expression that you need to know in reflection photoelasticity is as follows. A difference in principle stresses of the specimen namely  $\sigma_1^s - \sigma_2^s$  is given as  $R_f$  into young's modulus of the specimen divided by  $1 + \nu_s$  into  $N_f \lambda$  by  $2h_c K$  and whatever the discussion we had on color code is equally applicable in reflection photoelasticity.

So, I will have to find out what is the fringe order accurately at a point of interest. Once I know fringe order  $N_f$ , I know the specimen material properties and also the coating properties and if you know what is a wavelength of light that have used and also the specimen we ((C)) the coating property namely the strain coefficient. I can find out  $\sigma_1^s - \sigma_2^s$  and what we will have is? We will have this correction factor evaluated for in plane stress, bending, torsion and internal pressure and so, on and so forth - that is all we will have.

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We will have different expressions for R f. And R f is use, because it is essentially a reinforcement factor and will first take up what are the correction factors for plane stress. This will really establish, what is the basic procedure to evaluate the correction factoring; the credit goes to (( )) and its co-workers, he really established the advancements of photoelastic coatings. And for specimens subjected to plane stress, the correction factor is due to the reinforcement effect of the coating only.

The strain does not vary over the thickness of the coating and what I am going to do is, I am going to take an uncoated specimen subjected to planes stress and I will consider a specimen with a coating. So, what you let look at is the moment type coating, the coating can also participate in load sharing. Now, the question is what is the share of load the coating takes in?

So, in the process the specimen is loaded less. So, will essentially compare what are the strains develop for a given loading on the uncoated specimen and compare to the coated specimen from which I will get the correction factor. Because my requirement is by looking at the reading from a coated specimen, I should evaluate the strains that would develop in an uncoated specimen, but coated specimen only I can do experiment, uncoated specimen I cannot do experiment.

So, I bring in a correction factor to do this and this shows the load sharing what you have, I have a coating for the discussion, it is put very thick. So, that you see the coating



is sitting very prominently. So, you have the specimen thickness as  $h_s$ , coating thickness as  $h_c$  and in a uncoated specimen I will have the specimen taking the load of  $\sigma_1^u$ , the super script the u denotes the these are the stresses develop in a uncoated specimen.

In a coated specimen, what you say, I will have a stress developed as  $\sigma_1^s$  on the specimen and  $\sigma_1^c$  on the coating. So, now, I will have to essential do a force balance, I will essential do a force balance and find out what is the share of  $\sigma_1^c$ , and what is the share of  $\sigma_1^s$ . And this is how we will develop the methodology to find out the correction factor? See the problem is simple enough why we take a simple problem is? You understand the physics behind how one goes about in evaluating the correction factor and you can easily write.

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EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

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Correction factor for plane stress

Force balance

$$\sigma_1^u h_c dy = \sigma_1^s h_s dy + \sigma_1^c h_c dy$$

$$\sigma_2^u h_s dx = \sigma_2^s h_s dx + \sigma_2^c h_c dx$$

$$\varepsilon_1^c = \varepsilon_1^s$$

$$\varepsilon_2^c = \varepsilon_2^s$$

The strains in the uncoated and coated specimen are related by

$$\varepsilon_1^u - \varepsilon_2^u = \left[ 1 + \frac{h_c E_c (1 + \nu_s)}{h_s E_s (1 + \nu_c)} \right] (\varepsilon_1^c - \varepsilon_2^c)$$

$$R_f^a = 1 + \frac{h_c E_c (1 + \nu_s)}{h_s E_s (1 + \nu_c)}$$

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When I go to the force balance, I can write  $\sigma_1^u H_s$  into  $dy$ , because this is the area and this could be a... there could be a force component from the coating, there could be a force component to specimen and you will have the expression written down like this  $\sigma_1^s$  into  $h_s$  into  $dy$  plus  $\sigma_1^c$  into  $h_c$  into  $dy$  known similar fashion I can also find out what is the force on the other direction and I get this as.

So, what we have here is, I have the force balance written down and we have assume to start with the coating is well bonded to the specimen. So, that the strains on the coating, on the specimen are identical and this is possible only when poisson ratio as same fully. So, that the assumption that we make and when I do this and also find out what way, the

uncoated specimen strain difference is related to the coated strain difference I have an expression like this. This is easily obtainable from these set of force balance equation and making this strains equal, I can write this expression and what I call this factor 1 plus  $h_c$  by  $h_s E_c$  by  $E_s$  into 1 plus  $\nu_s$  by 1 plus  $\nu_c$ , I call this as a reinforcement factor and you will see a super script coming here. I have said in generally it is  $R_f$ , because it is actual loading I have the symbol as  $B$ . So,  $R_f$  denotes the correction factor that is needed when I have to interpret the results from a simple axial test simple axial test how do interpret, I have to bringing a correction factor, there is no way.

Then we move on to more complex problem of if the specimen is subjected to bending how do you go about and evaluate the correction factor. So, what we will do is for illustration because you all know bending reasonably well we will find out how much of it you remember now. So, that is what I always been saying when you are learning a course of strength of material you **you** should not forget it when your learn advanced courses they form as the fundamental for you to build up the theory in the higher level courses. The moment type put a coating what happens? it is behaves like a composite beam. So, that is what we are going to look at. So, our interest is we are looked at what is the kind of correction factor that I need for in plane stress.

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**Correction factor for bending**

Strain variations through the coating thickness exists.

**Actual strain**

**Maximum strain**

Labels:  $h_s$ ,  $h_c$ , NA, CL,  $A$ ,  $\rho$ ,  $M$

$$\epsilon_1^s = \frac{y}{\rho} \text{ for } -(h_s \cdot A) \leq y \leq A$$

$$\epsilon_2^s = -\nu_s \epsilon_1^s$$

$$\epsilon_1^c = \frac{y}{\rho} \text{ for } A \leq y \leq (A + h_c)$$

$$\epsilon_2^c = -\nu_c \epsilon_1^c$$

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Now will move and find out how do I go about for a correction factor when the specimen is subjected to bending. So, what **what** you have here is if I have a specimen which is

uncoated for a given bending moment, it could have its neutral axis like this and strain very linearly over the depth of the beam. Now what I have here is on top of this specimen I put a coating material then this system behaves like a composite beam. So, what is the first knowledge in the case of composite beam, what will happen to the neutral axis? Neutral axis will get shifted.

So, we will find out what is the shifting the neutral axis, and we will also have to look at and very prominently because you are looking at bending when I have a finite thickness of the coating strain is not going to be constant over the thickness - it varies, it varies linearly. So, also bring in an average strain over the thickness of the coating. So, this is what I have to calculate, so I have to find out go back and find out from your conventional knowledge of how do you analyze bending of the beam, we will take up a very simple case of pure bending. And we will look at how do we find out a shifting the neutral axis - that is what to shown here there is shifting the neutral axes. And you can also have a clear picture of the beam and to label various quantities.

So, you look at I have a thickness of the specimen, I have the thickness of the coating, and the neutral axis no longer coincide with the centralized axis of the specimen cross section by a gets shifted and we have label the distance from one of the ends of the specimen top surface to the neutral axis as  $A$ . And you have a radius curvature  $\rho$ , and I could see very clearly within the coating there is strain variation. It is not a constant, but you know principle stress direction remains a same. So, I could also replace this for the purpose of mathematical analysis by an average strain which is good enough to find out what kind of fringe pattern that you to get.

So, from the mathematics, I will let find out what is this shifting the neutral axis, and we have what **what** we have done we have done force balance in the case of in plane stress loading, the force balance was very simple and straight forward; its mathematics is very simple. And other hand mathematics slightly involved in you have to go back to your fundamental understanding of, how you analyze the beam and bending. Here again **again** write the force balance, I can also write the moment balance.

So, these are all the equation that are needed for you to find out the shifting the neutral axis and also estimate the average strain difference and related to uncoated specimen mathematics is slightly involved I am not going to derive every step, but I am going to

give you key intermediate stresses - that will give you an idea how we are gone about. And what I have here is how do I write the expression for the specimen strain and coating strain. If you go back to the way that you develop the bending of beams the distance from the neutral axis and the curvature if you know, I can write an expression for strain. I will to put the limits appropriately and that is what the expression looks like.

So, I have  $y$  by  $\rho$  this is very famous expression. And this specimen strain is from this fiber to this fiber. So, that is given as minus  $h_s$  minus  $A$ . The  $y$  lies between minus of  $h_s$  minus  $A$  to capital  $A$ . And from  $A$  to  $A$  plus  $h_c$  you have the straining in the coating. And we also know the straining the transverse direction on the specimen is related to the strain in the longitudinal direction by minus  $\nu_s$  times  $\epsilon$  along the coating is related to minus  $\nu_c$  times  $\epsilon$ . This expression is simple that is why it said we should not effort to forget whatever that you are learned in strength of materials. We **we we** build on fundamentals and if you want to understand advanced courses you should know what you are done in your earlier courses. This memory is very important that go a long way and while writing this expression we do not know what is value of  $A$ ,  $A$  has to be calculated from mathematics.

**Editing Completed.**

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EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

Correction factor for bending

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Equilibrium of forces acting along the axis of the beam (width  $b$ ) requires

$$\frac{b}{\rho} \left( E_s \int_{A-h_s}^A y dy + E_c \int_A^{A+h_c} y dy \right) = 0$$

The position of the neutral axis is found to be

$$A = \frac{h_s}{2} \frac{1 - \left( \frac{E_c}{E_s} \right) \left( \frac{h_c}{h_s} \right)^2}{1 + \left( \frac{E_c}{E_s} \right) \left( \frac{h_c}{h_s} \right)^2}$$

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And first let me write the force balance. So, the force balance you could write it in this fashion straight forward on a beam and pure bending the net force is zero for I have 1

written down for this specimen I have another expression written down for the coating and this read like b by roan to young's modules are a specimen integral with the lima its goes from a minus Hs to a y d y plus young's modules of the coating and the integral goes from lima its a to a plus Hc y d y is equal to zero and this expression when you saw I finally, get what is the expression for a that terms out to be Hs by 2into 1 minus E c by E into Hc by Hs whole square later we will replace this by convince symbol replace this e and probably I replace this by g and. So, on and this is divided by 1 plus E c by E into Hc by Hs whole square.

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EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

Correction factor for bending

....contd

Equilibrium of moments about the origin gives

$$\frac{b}{\rho} \left( E_s \int_{A-H_s}^A y^2 dy + E_c \int_A^{A+H_c} y^2 dy \right) = M$$

The radius of curvature is obtained as

$$\frac{1}{\rho} = \left( \frac{12M}{bE_s h_s^3} \right) \left( \frac{1}{H} \right)$$

Where

$$H = 4(1+eg^3) - 3(1-eg^2)^2 / (1+eg)$$

$$e = \frac{E_c}{E_s} \quad ; \quad g = \frac{h_c}{h_s}$$

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So, by looking at the equilibrium of forces I am able to find out what is the expression for this gives the neutral axis position it is measure form the top fiber of the specimen now I return the equilibrium forces then I have to write the equilibrium of the moments that will give me the curvature the Idea is to find out what is the curvature and so on and so forth. I also mentioned that I will replace the ratio of young's modulus has e and ratio of thickness has g that makes my life simple in a expressing the expressions and the equilibrium of moments again has to 2 terms 1 corresponding to the specimen and 1 corresponds to the coating and if we have physically Identified what were the lima its of the specimen it goes term a minus Hs to a and coating goes term a to a plus Hc rest of the expression is very simple and straight forward and this is the bending moment applied mind you we are considering a problem where the specimen is subjected to pure bending and when I solve this equation I get the radius of curvature is obtained as 1 by rho equal

to twelve m divided by bingo E into Hs cube into 1 by Hand His the very complex expression His given as four into 1 plus e G cube minus three into 1 minus e G square whole square divided by 1 plus e g you know this is for your understanding you knowing my examination I do not test your skill of memory you know, but, you also have provision for you to bringing a four size sheet an either side you can write Important final expression of formulae and you filter out what formulae from the course to fill it in there that itself is a learning because once you have learn the subject thoroughly you know what is Important and what is not Important because once you function as a engineers you should know the methodology very well for details engineers look at hand books and take out this expressions. So, that comfort you will also have when you write the final examination for final examination I allow a **a** four size sheet on 2 sides you know if you have a skill to write the entire set of expressions that is your left your choice no Xeroxing of the sheets is allowed hand return sheets are permitted. So, you do not have to worry that I have such complex expressions do you have to remember this you do not have the remember them.

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**Correction factor for bending** ....contd

The average value of the principal strain difference in the coating is

$$(\epsilon_1^c - \epsilon_2^c)_{ave} = \frac{1 + \nu_c}{\rho h_c} \int_A^{A+h_c} y dy$$

$$(\epsilon_1^c - \epsilon_2^c)_{ave} = \frac{6M}{H} \left[ \frac{(1 + \nu_c)}{bE_s h_s^2} \right] \left[ \frac{(1 + g)}{(1 + eg)} \right]$$

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,But you should know what you write-in your final expression that will come only if you know how you are solve it now what we do is we also define the average strain difference in the coating and that is given as 1 plus nu c divided by rho H c the integral goes from the limit a to a plus H c and I have y d y that is to be integrated and final expression for this takes the form like given here. So, the average strain difference

developing the coating as follows  $6M$  by  $1 + \nu_c$  divided by  $b E_s h_s^2$  into  $1 + g$  by  $1 + e g$ . So, in all these expressions you know  $e$  denotes  $E_c$  by  $E_s$   $H_c$  denotes  $H_c$  by  $H_s$  that kind of symbolism we have to develop and that makes your writing of expression symbol. So, now, you have methodology how to go about and write the final expression for correction factoring bending I have know have a expression..

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EXPERIMENTAL STRESS ANALYSIS Photoelastic Coatings

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However,

$$(\varepsilon_1^u - \varepsilon_2^u) = \frac{6M(1 + \nu_c)}{bE_s h_s^2}$$

$$(\varepsilon_1^u - \varepsilon_2^u) = R_f^b (\varepsilon_1^c - \varepsilon_2^c)_{average}$$

$$R_f^b = \frac{(1 + eg)}{(1 + g)} \left[ 4(1 + eg^3) - \frac{3(1 - eg^2)^2}{(1 + eg)} \right] \times \left( \frac{1 + \nu_s}{1 + \nu_c} \right)$$

This correction factor accounts for both reinforcing and strain variations through the coating.

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For average strain difference and I will also related to the uncoated the strain difference then I job is done and what we have a uncoated strain difference this behaves like a simple beam where as when I have a coated specimen it behaves like a composite beam. So, this is what I have here finally, I have uncoated strain difference is given as  $6M$  in to  $1 + \nu_c$  divided by  $b$  in to  $E_s$  in to  $H_s^2$  and this is related to the average strain difference by a correction factor  $R_f^b$ . So, here we have replace the super script from  $a$  to  $b$   $a$  denotes axial loading and  $b$  denotes the bending loading and  $R_f^b$  is finally, obtained as  $1 + e g$  divided by  $1 + g$  in to  $4$  in to  $1 + e G^3$  minus therein to  $1 - e G^2$  whole square by  $1 + e g$  the whole of it multiply by  $1 + \nu_s$  divided by  $1 + \nu_c$  and what does this correction factor doing the development we have same there is a strain variation over the thickness of the coating. So, whatever the correction factor that I got accounts for both the strain variation in the coating as well as the reinforcement effect and you know this kind of expression is needed even when you want to calibrate the photo elastic coating material because there you're going to

cantilever beam under the bending and its subjected to bending loads. So, you have to use correction factor right from calibration of the coating material.

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EXPERIMENTAL STRESS ANALYSIS Photoelastic Coatings

**Correction factor for bending of plates**

- For thin or medium thick plates, in regions of uniform stress, the correction factor is

$$R_i^{bp} = \frac{(1 + emg)}{(1 + g)} \left[ 4(1 + emg^3) - \frac{3(1 - emg^2)^2}{(1 + emg)} \right]$$

$$e = \frac{E_c}{E_s}, g = \frac{h_c}{h_s}, \text{ and } m = \frac{1 - \nu_s^2}{1 - \nu_c^2}$$

- For the restricted case of pure-bending loads, the correction factor may be applied for thick plates too.

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So, it is very Important and what are all the waste that I can use this from whatever the expression that I am got I could also gone extrapolated for bending of plates will see a little while later for thin or medium thick plates in regions of uniform stress the correction factor is given as b p denotes bending of plates and you have a very long expression for this and we bringing 1 more Identity m **m** denotes 1 minus nu s square divided by 1 minus nu c square we already seen what is e what is g and now we are introduce another parameter these are all for convenience write-in the expression convenience fashion.

So, for bending of plates I have r s b p is given as 1 plus e m g divided by 1 plus gin to four in to 1 plus e m G cube minus three times 1 minus e m G square whole square divided by 1 plus e m g and this is for a generic equation for the restricted case of pure bending loads the correction factor may **may** be applied for thick plates too you knowing the initial development of a photo elastic coating people have analyst all this they brought the correction factor and also comparison what is the.



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EXPERIMENTAL STRESS ANALYSIS

Photoelastic Coatings

### Combined in-plane and bending loads

- Frequently, a structural part is subjected to a combination of plane and flexural loads.
- For thick coatings, proper weighing of the correction factors is important.
- One approach could be to find a coating thickness such that the factors  $R^p$ ,  $R^{pp}$  are equal.
- If done, one will observe that the thickness of the coating is larger than the thickness of the structure.

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Correction factor if I use the simple bending equation and a plate equation and that Identity will see we will see know that a when I have  $\nu_s$  equal to  $\nu_c$  I do not separately go and find out correction factor for bending of plates I can use simply the correction factor for bending what we are develop earlier now these are all the developments in the whole days where they had only slight role to work with now you have computer software available you have to just potent your thickness of the specimen and thickness of the coating and coating and material properties you can even selected from database you instantly get a correction factors even if it is not available you can write a own code. So, that days are different now though the expressions look very long and unwell the it is necessary even to do simple calculations in reflection photo elasticity you cannot Ignore it and I had already mention the whatever the correction factor that we do they are not for common geometrical distribution such as a hole or a series of holes it is also not applicable when the plate has an abrupt change in thickness and what is the what I have to do this put the thick coating and find out which are all the stress concentration zones remove the coating put the thin coating in those regions where you want to find out the stress concentration area. So, you do not need a correction factor for analyzing those region where you have stress concentration this is the way they solve the problem from a engineering point of you because the correction factor determination becomes difficult..

In fact, I am going to show you correction factor for a torsion I am going to give you only final result if you look at the development of the necessary equations for correction very complex when you go for it is gone be much complex than what you seen earlier we have only develop a systematic procedure for in plane load and also for pure bending for all other cases we right only the final expression we will take the result from the literature and use it and this is also discussed these are all these are all to show what kind of contradiction that you may how to face you will haven general combined loads I have combined in plane and bending loads for thick coatings proper weighing of the correction factors is Important and 1 of the approaches could be to find a coating thickness such that the axial load correction factor and bending load correction factor are equal suppose I do this thickness of the coating is larger than the thickness of the structure..

So, such contraction exit. So, the whole Idea here is we look at what is the correction factor like I said the finding out the maximum fringe order obtainable in a photo elastic coating testing is an issue finding out the suitable thickness for a given photo elastic coating test is also a discussed issue people use different philosophy when I have used a correction factor 1 philosophy is why not I use the thickness such that correction factor equal to 1. So, were I do not have to apply a correction factor, but, I will modify the thickness of the coating..

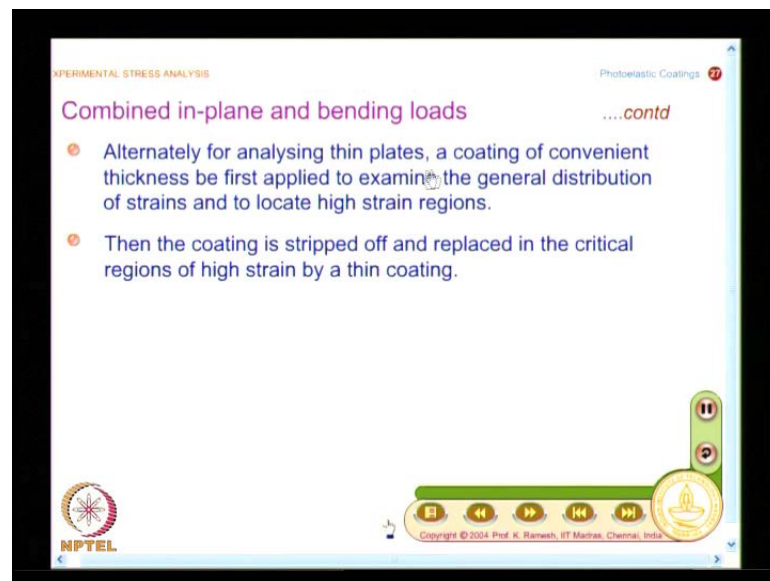
So, you have such a philosophy is also. So, some of these modification and thickness is you could do when you when you're using a plastic where you are going to caste and you can adjust the thickness to a requirement and then paste it over the actual specimen on the other hand if you Import or buy from your supplier even that contour plastic in the gel state you do not have a choice on multiple thickness available from the supplier we will have certain discrete values. So, you have to leave with discrete values and another issue is when you having a coating when you are doing a plastic if you go and pressed with your hand there is a possibility of small thickness variation.

So, another philosophy of thickness determination could be small variation thickness should not affect the correction factor drastically. So, there are multiple approaches in finding out the thickness of the coating Ideal thickness of the coating, but, finally, it is all dictated by the what is the coating readily available and you go paste it and done do the

analysis, but, from an academic point of view you should also look at from a holistic view that what are all the variations possible.

So, when you have  $R_f a$  equal to  $R_f b_p$  from make it I will have a thickness of the coating larger than the thickness of the structure; obviously, not desirable that is what is Implied here.

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What is Implied here is when you do this it is not desirable and this is was I am saying off and on for analyzing thin plates a coating of convenient thickness be first applied to examine the general distribution and locate high strain regions then the coating is stripped off and replaced in the critical regions of high **high** strain by a thin coating. So, this is the generic procedure for any problem if you use a very thin coating you do not worry about the correction factor correction factor becomes very Important when the thickness is significant why you need a sufficient thickness if I do not of sufficient thickness I do not see fringe pattern on the other hand if I have a very precise equipment then I can go and use smaller thicknesses. So, that is the advantage when I go for digital photo elasticity whatever the digital photo elastic principles which we are learn and also equally applicable to reflection photo elasticity. So, when I go for digital reflection photo elasticity I can afford to use a thin coating and do my experiment.

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EXPERIMENTAL STRESS ANALYSIS Photoelastic Coatings 20

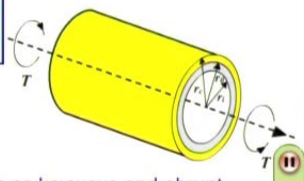
### Correction factor for torsion

- Coating reinforces the member.
- Strain gradient exists through the thickness of the coating.

$$R_f^t = \frac{2}{(1+c)} \left[ 1 + \frac{G_c(c^4 - 1)}{G_s(1 - a^4)} \right]$$

$G$  = shear modulus  
 $a = r_i/r_o$   
 $c = r_c/r_o$

- Not applicable for the regions such as keyways and abrupt changes in diameter.



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So, my new sense from correction factor effort becomes minimal now we take a what are the correction factoring the case of a torsion and I am going to give you the expression directly here against strain gradient exists that is what you have to note down both and bending and torsion there is a strain variation and we have this denoted as  $r_f t$  denotes it is torsion and this takes a form like this  $\frac{2}{1+c}$  and  $c$  you defined as ratio of the coating radius divided by the outer radius of the specimen and  $a$  as inner radius of the specimen divided by outer radius of the specimen the expression is lot more simpler here I have  $\frac{2}{1+c}$  in to  $1 + \frac{G_c(c^4 - 1)}{G_s(1 - a^4)}$  and what you get finally, you get finally, not applicable for the regions such as keyways and abrupt changes in diameter. In fact, when you take any practical shop it will have keyways it will have changes in diameter. So, are all those cases which correction factor is not applicable. So, this is the contradiction correction factor your are able to find out for simple problems from complex problems it is not applicable, but, the saving grace here is because it is the regions of stress concentration even a thin coating can provide you enough information. So, that is how its solve form a engineering point of you. So, when you initial do the test Identify shown of high strength concentration then use a thin coating if you look simple geometry you know everything where is there is stress concentration where there is a reasonable value of strain all this heuristicalinformation you know the problem comes only when you have a very complex situation wherein your experiment should rescue like a said in the case of concord failure

photo elastic coating Identified very precisely the zone of stress concentration. So, Identifying shown of stress concentration in a generic complex problem is challenging. So, from that point of if a coating test can really help for you have the specimen here you have the coating and when I apply torsion you have a strain variation in the coating and all this is accommodated in this correction factor and another very Important engineering problem is pressure vessels you look at you're the how sold the cylinder gas cylinder its pressure vessel your cooker is a pressure vessel your air craft is a pressure vessel.

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EXPERIMENTAL STRESS ANALYSIS Photoelastic Coatings

### Correction factor for pressure vessels

- Coating restrains radial displacements of the vessel.
- Coating restrains longitudinal deformation of the member.

The restraints are not equal for the transverse and longitudinal directions and the evaluation of correction factor is more complicated.

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So, pressure vessels are very Important from engineering point of view and what you have here I have inner cylinder which is put with the coating. So, it behaves the composite cylinder and again have this specimen subjected to internal pressure as well as pressure from the coating and the coating is subject to some amount of internal pressure the evaluation of correction factor correction factor is more complicated in these class of problems. So, you are going to have a very long expression it is not got to be simple we are not going to look at the full derivation as such nevertheless you should haven your notes because pressure vessels are. So, Important you will need to have the expression for correction factor. So, what you find is coating restrains radial displacements of the vessel that is why you have a pressure acting on the specimen coating also restrains longitudinal deformation of the member, but, you always have a saving escape route, but, the coating is made of lowing smart less.

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EXPERIMENTAL STRESS ANALYSIS Photoelastic Coatings

**Correction factor for pressure vessels** ....contd

- In the case of a long cylindrical pressure vessel with closed ends, the correction factor ( $R_p^c$ ) is given by

$$\frac{1}{R_p^c} = \left[ \frac{2(1-2\nu+c)(1-\nu)}{(1-2\nu+c^2)+P(1-2\nu+a^2)} - \frac{(1-2\nu)}{(1+P)} \right]$$
$$P = \frac{em(c^2-1)}{(1-a^2)}$$

- Valid only for regions in continuous vessels remote from closures, discontinuities, nozzles, etc.

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So, some of these effects are small and not very significant and you have to be a little bit more careful. This is slightly retardant on a different fashion for convenience you get only 1 by r f p and I have this as  $\frac{2(1-2\nu+c)(1-\nu)}{(1-2\nu+c^2)+P(1-2\nu+a^2)} - \frac{(1-2\nu)}{(1+P)}$  where p is given as  $\frac{em(c^2-1)}{(1-a^2)}$  you have already seen how do defines c and a and case of a similar definition applies here to and like any other correction factor here again it is valid only for regions continuous vessels remote from closures discontinuities nozzles etcetera. So, that is what you have to keeping mind correction factors are very Important and correction factors are develop for simpler loading situations and we are seen we have a stress concentration with a finite geometry even theory we cannot solve unless I evaluated analytically and then compare it there is no way I can find out the correction factor with modern developments you could possibly develop the correction factor by a final analysis any complex problem can be solve and then you could find out a correction factors, but, the people have at trusted from engineering point of view what they have said is use the photo elastic coating to Identifies zones of high strain regions and in those regions use a thin coating and do away with the correction factors that is the very good strategy, but, why correction factors are needed correction factors are needed even for the you that is the basic experiment and you use for finding out the strain coefficient K and unless you use the correction factor K will not determined accurately we will see the true words end of the lecture on from photo elastic coatings.

So, what we have seen in today's lecture was I emphasized in photo elastic coatings correction factors are very Important when you focus is to find out specimen stress is different you also have correction factor coming on the right hand side and the expression in terms of the fringe order and strain coefficient and you also saw the elastic constants of the specimen appearing on the expression and we will seeing a later class what is the maximum fringe order obtainable from a photo elastic coating test you can use the expression and find out and this is an issue you cannot neglect the maximum fringe order obtainable in a photo elastic coating test because that is needed for planning your experiment you may not see very high fringes even when the material yielded. So, you will see only very less fringes in photo elastic coatings that is the reason why people prepare white line as a elimination for photo elastic coating **thank you**.