Experimental Stress Analysis

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Lecture No. # 24

Three-dimensional Photoelasticity

Till now we have looked at aspects of two-dimensional photoelasticity; we developed the stress optic law; we also saw that for any photoelasticity experiment, the first step one has to do, is to evaluate the material stress fringe value, and material stress fringe value is a very key parameter. That is a only parameter with relates the experimental information to compare with analytical and numerical solution, and we looked at the circular disk and diameter compression is the suitable model to find out f sigma as accurately as possible.

The next challenge was how to label the fringes. In fact, we had a long discussion on how to find out the fringe order look at the isoclinic fringe field, I so clinic fringe field futures. Also aspects of two-dimensional elasticity, what advantage you get from an understanding, what happens in a free out work corners, how that knowledge would be a effectively utilized in ordering the fringes. And we also found whatever the fringes we observe could be easily related to physical parameters.

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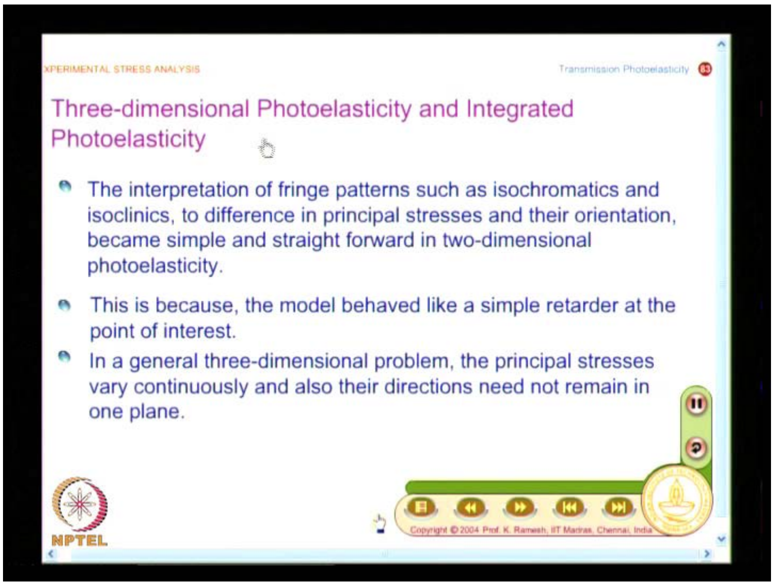

Now let us look at how we can go and analyze three-dimensional problems. So, we have three-dimensional photoelasticity, and it is important to look at the interpretation of fringe patterns, such as isochromatics and isoclinics, to difference in principal stresses and their orientation, became simple and straight forward in two-dimensional photoelasicity. We have no difficultly at all... Why is it is so? How we have been able to find out. That is chromatic represent sigma 1 minus sigma 2 contours, and how isoclinics represent the principal stress direction contour.

The basic idea was the model behaved like a simple retarder. And what you had was the thickness was considered as as as small as possible, we were looking at plane problems. So, what you had was the principal stress direction remains same, even though the thickness is a about 3 to 6 millimeters. So, because the model behaved like a simple retarder, you are able to interpret isoclinic as contours of principal stress direction, and isochromatics as contours of principal stress difference.

On the other hand if you look at a general three-dimensional problem, the principal stresses vary continuously, not only this the orientations also vary, and their need not remain in one plane. This is the very crucial aspect you know just, because of this the analyses of three-dimensional problem becomes extremely complex, and once you face a problem you know engineers are smart enough to device methodologies to overcome whatever the difficulties. So, we have two methods essentially one intelligently uses aspects of two-dimensional photoelasticity, for interpreting three-dimensional photoelasticity.

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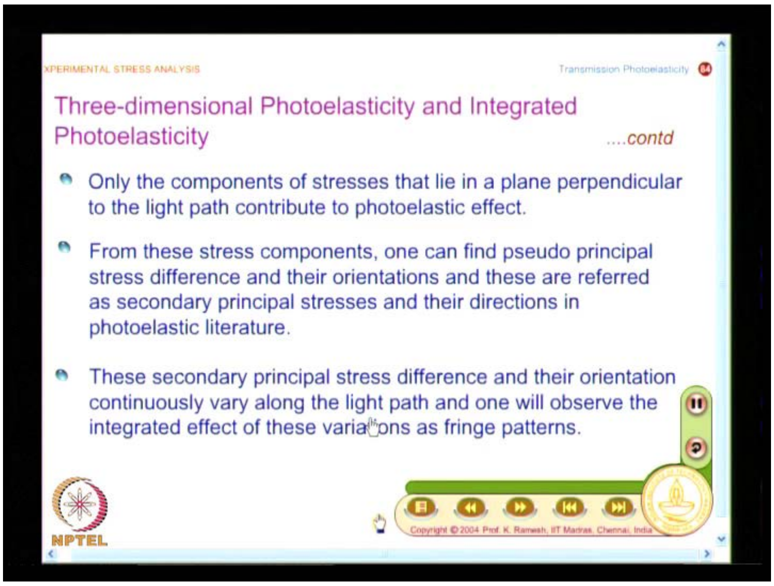

How do we do that we look at? We will also look at another aspect where we record the fringe pattern as such and go for a integrated photoelasticity where you develop appropriate mathematics which is quite complex we will see those details, but before we get into it we need to know another important aspect, only the components of stresses that lie in a plane perpendicular to the light path contribute to photoelastic effect, this is very, very important, because what you have here is the moment you want to graduate to analyze three-dimensional problems.

We also need to develop certain additional terminologies, one of it what we will try to develop is what are known as secondary principal stresses. So, in order to develop that we have to understand what causes photoelastic effect. Only those stress components that lie in a plane perpendicular to the light path contributes it. So, in a three-dimensional problem what i will try to do is I will try to find out, what are known as pseudo principal stress difference, because you will have a stress tensor in general 3 by 3 matrixes, and you will have to evaluate depending on the direction of the light.

So, the light path is also very important and we should find out stress components perpendicular to the light path, and from a solid mechanics point of view you will have at the point of interest principle stresses and their orientation, those definition change when way want to apply photoelasticity. So, I will essentially get only pseudo principal stresses in a plane perpendicular to the light path, and in order to distinguish this from conventional understanding of principal stresses.

These are referred as secondary principal stresses and their directions. This is well understood and establish in photoelastic literature, and what we will also have is as I had mention, the stresses vary from point to point in a three-dimensional model. The secondary principle stress also varies from point to point, and that is what we will have to account for when we want to interpret what the fringes that we see. So, the secondary principal stress difference, and their orientation continuously vary along the light path and what you normally get on the screen is the integrated effect of these variations as fringe patterns, because I do not get the information of one plane.

I have a serious of plane as a light passes through a model. So, what I get as a output is a integrated effect of all these planes, that is why the interpreted becomes much more involve in a three-dimensional photoelastic analysis, but what is greatest advantage photoelasticity offers a methodology to penetrate into the model and find out the stresses interior to the model. This is the specialty, suppose I have an inclusion and I want to analysis the stresses around it photoelasticity is an ideal choice, and when people are developing a contrast stress problem; the maximum stress occurs in the surface.

It is not on the surface, if you look at any of the coating techniques, if I take a bridle coating or if I take a photoelasticity coating or if I take strain cages or even digital image correlation I essentially get only the surface information. If I have to find out the information interior in certain optical technique it is possible, in strain cage also is possible, but you can do it one or two selected points with lot of effort from your point of you. On the other hand photoelasticity offers you how to find out the stressor interior to the model, this is greatest advantage.

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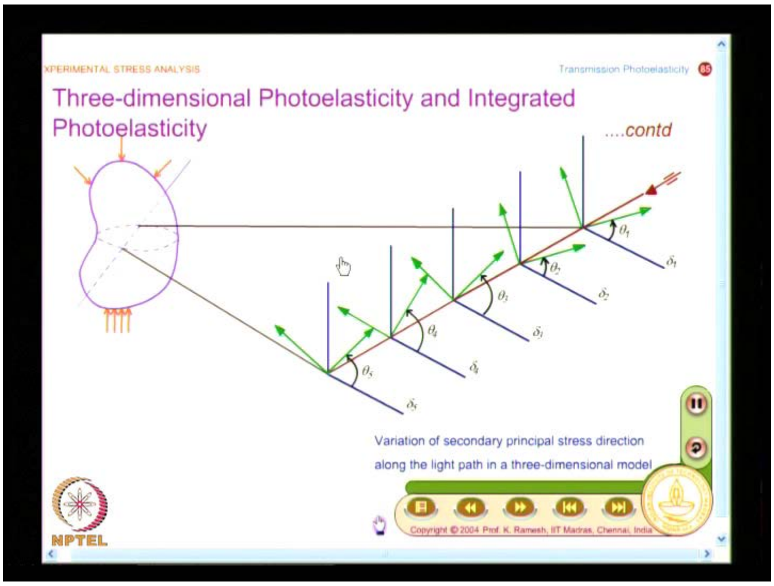

Now let us look at what really complicates our analysis. So, what you have here is a three-dimensional mode is shown, and I have a light path that passes through the model; and this passes through several planes are the model, and this is expended and a representation is given. And what you have here is suppose I take the plane one, because that behaves like retarder. So, now I have a serious of retarders along the light path. And we have always seen each retarder can be represented by the retardation delta, and it is orientation theta.

In a two-dimensional problem you had only one plane. So, whatever the light that you sent, whatever the fringe pattern you observes, you could related to theta as well as delta without any difficulty, but what complicates in a general three-dimensional model is that this orientation change as as the function of the plane. Not only the orientation, but also the retardation changes from delta 1 to delta 2 delta 3 delta 4 delta 5 and so on... I have only shown five representative planes for this... Light path incident.

You will have many planes, and you can consider all are them as individual retarders, from a mathematical point of view you will have several Jones matrixes for these retarders, and you can multiply them. And get the integrated effect and in order to write this you have already discussed, but what are represented is actually secondary principal stress direction, and the difference we have already define, what is the secondary principal stress difference.

So, the moment you come to three-dimensional photoelasticity, you will have to know how to handle variation of the principal stress direction along the light path as well as the variation of the magnitude. And whenever, you face the problem like this you know mathematician first try to solve the problem by looking at simpler cases, then one case I may have a thick model, but the stress distribution is such, the principal stress direction does not change over the thickness only the magnitudes vary.

This is one type of problem, there could be another class of problem where the magnitude remains same, but there is a rotation of the principal stress direction along the length path, light path, and problems of this nature people have found at even analytical solution, how to go about and evaluate the stress distribution. So, the moment you come to three-dimensional photoelasticity, you cannot confine your knowledge development only to the plane of interest, but you have to bring in the stress distribution along the light path.

So, that makes the problem complicate, and have a reasonable sketch of this diagram. So, I have a three-dimensional model and the essence here is the orientation as well as the retardation changes along the light path, and obviously one single light path will not give you all the parameters of interest I may have to have multiple light paths, for me to find out the stress distribution. So, you understand what kind of complexities that comes in a three-dimensional photoelastic analysis.

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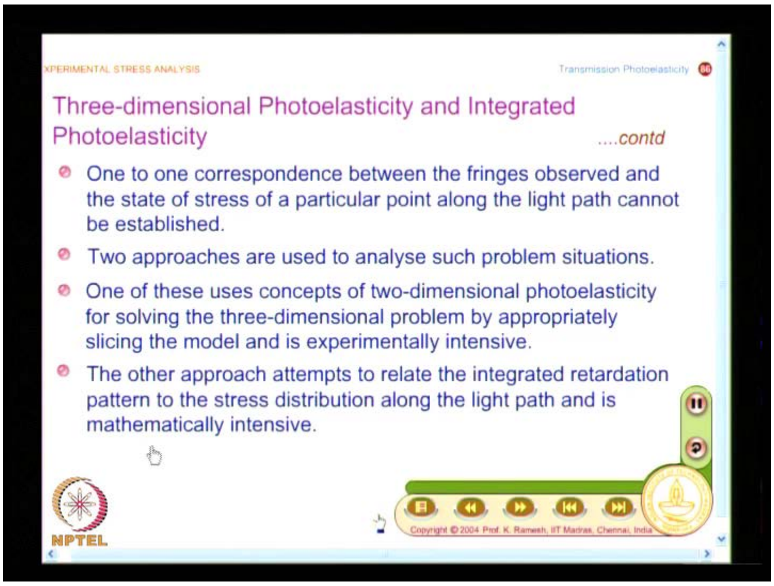

So, what you find here is one to one correspondence between the fringes observed and the state of stress of a particular point along the light path cannot be established state away, which I can find out if I solve the problem I will able to get this answer, where as in the case of two-dimensional photoelastic analysis we were able to get it directly. We have no difficulty in associating the isoclinics to principal stress direction, and isochromatics to principal stress difference. So, now you have two approaches that are used to analyze such situations.

That is what I said when engineers look at a problem that try to find out solution. And one of these uses concepts of two-dimensional photoelasticity for solving the three-dimensional problem by appropriately slicing the model this was possible, because of a very unique phenomena called stress freezing, the method is experimentally intensive, see if you have a problem, when you want to approach you have to pay a price for it. The price may come from either experimentally intensive, if you want to simplify the experimentation then it, because mathematically demand it.

If the problem is complex you have to pay a price for it, there is no no escape from it. You will have to do it you can circum and take, only in a manner which is possible from the methodology (( )). So, because of stress freezing people where able to extend two-dimensional photoelastic analysis for analyze in three-dimensional problem intelligently, but it is experimentally intensive; that means, I have do stress freezing then I have do slicing all that we will see now.

The other approach is directly record the fringe pattern and you have that as integrated retardation pattern and that needs to be interpreted, because I need to assume a stress distribution, and obtain the parameters based on mathematically solving the problem. So, the other approach becomes mathematically intensive, either it has to be experimentally intensive or mathematically intensive. Mind you anything interior is always difficult, what will have to look at is? If I want to find out what are the stresses interior.

I have a methodology you have to look at from a positive point of you. In fact, many of the interstitial problems in the 1930's and 50's have been solve by exacts to three-dimensional photoelastic analysis, that is how many of the fundamental designs were perfected. Even now even the a3 80 a landing here was done by three-dimensional photoelastic analysis and photoelastic coating.

In fact, both booing and alas use extensively photoelasticity and photoelasticity coating for many of those a aero structural components, and also in a space industry people use this, because in those are industries where any human error in design could be disasters. So, they have to be very clear about the performance of the structure, and all these experimental methods really play a role in such situations.

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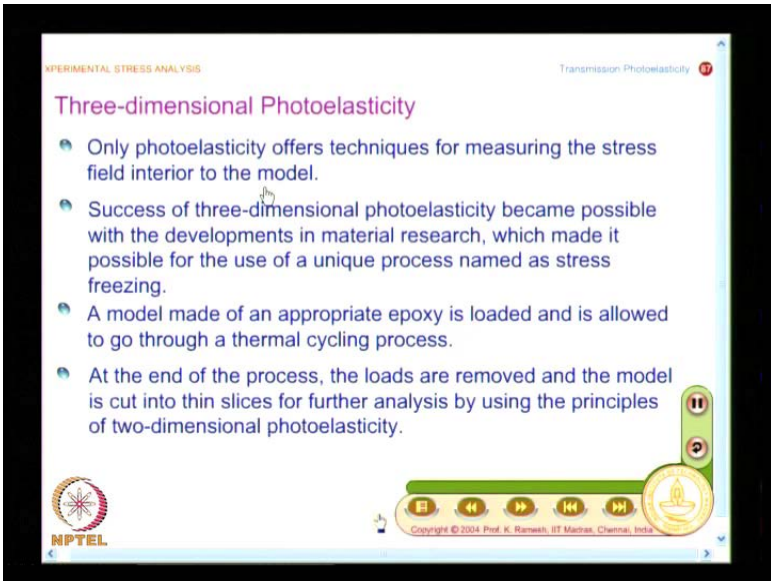

So, now, we will look at what is the method of stress freezing and slicing. So, we have emphasis earlier also, that only photoelasticity offers techniques for measuring the stress field interior to the model. That is one of the disadvantages and this was possible, because of a unique process named as stress freezing. So, I have also mentioned many developments in engineering is, because of advancements and material research. So, you had developed epoxys and they exhibited a nice property, when I go over a thermal cycling the stresses get locked in to it.

So, it is a material research which as contributed to the development of stress freezing. So, what you do in this a model made of an appropriate epoxy is loaded and is allowed to go through a thermal cycling process. It is actually going through a thermal cycling process, because at the end of the process the stresses get locked you call this as stress freezing. We will also see an analogy, why we call it as stress stress freezing. So, what you find is the model goes through a thermal cycling process with the loads apply, and at the end of the process, the loads are removed and the model is carefully cut into thin slices, that is very important.

If you do not slice it properly the processes of slicing can introduce machining stresses. That is should be avoided, you have to be very carefully and doing it. That is why said, when you want to solve the three-dimensional problem slicing it what I am really looking at when I make it as a thin slice, I can invoke the model behaves like a two-dimensional model, whatever the stresses locked in or from a three-dimensional situation, from mathematics point of you I analyze it has a two-dimensional model.

So, I take the advantage of two-dimensional photoelasticity, but in order to do that I have to slice it very preciously as thin as possible. So, that the principal stress directions remain constant within the thickness, and the stress field is actually from a three-dimensional situation. So, the advantage here is analysis can the done by using the principles of two-dimensional photoelasticity, that is the greatest advantage.

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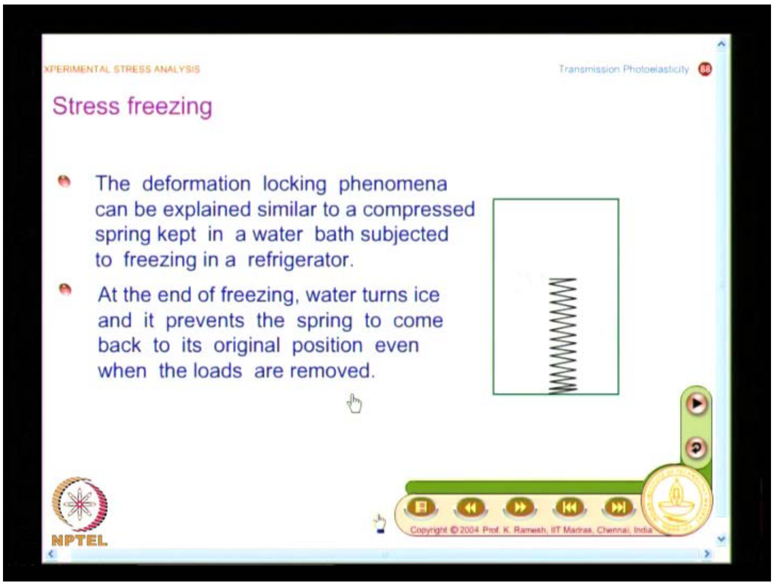

And what I do here, this is what you have a stress freezing, and I said the I will give an analogy. So, what I am doing is I am taking a spring I compress it and put it in water and keep this in a fridge, what will happen? I keep the spring compress and I apply the load and I can keep it in the fridge, and what will happen is the water will and became ice and it till prevent this spring to come back. The compressed spring will remain compressed, and something similar to that has happen in the epoxy also, and because I do a freezing to keep the spring compressed.

The process of thermal cycling in fact, I heat the model I do not cool the model. In this case I cool water and it becomes ice in actual stress freezing I only heat the model, but because the phenomena is similar, and it is also nice to call stress it into the model. So, I can the stress are locked in now I can take out slices carefully, and in analyze only those slices. So, that is what you see here, the water turns to ice and it prevents the spring to come back its original position even when the loads are removed. I do not have to keep the load here.

Now I have the animation also shows the load is remove when this was water load was necessary keep it compressed. The moment it becomes ice you do not need to keep the load to keep thus spring compressed. And something similar to that happens in the epoxy, and we will also not just talk about explaining the process I will also show by making a cut in the model, the essential stress distribution is not disturb. If you cut it very carefully if you cut it very carefully, the stress lock will not get disturb.

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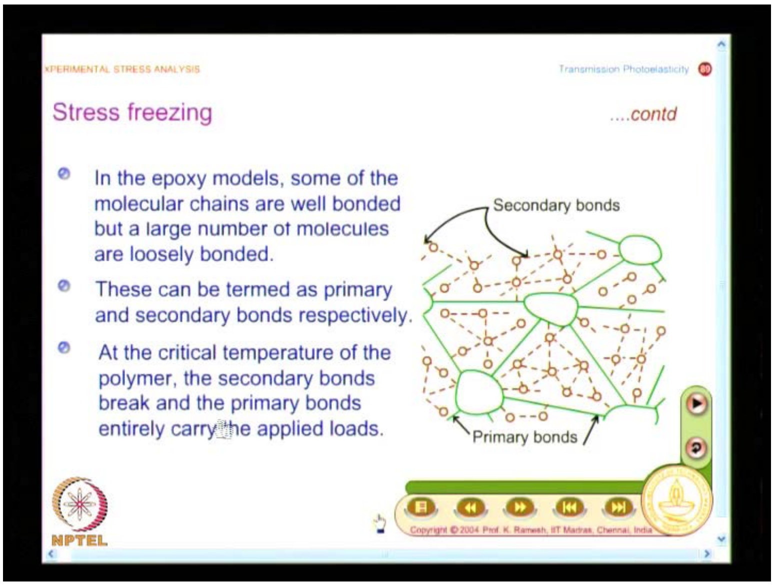

And what you have here, I have this epoxy can be thought of as assembly of secondary bonds as well as primary bonds. When I have an epoxy schematically this is shown, in epoxy models some of the chains are well bonded, but a large number of molecules are loosely bonded. So, the well bonded bonds you call it as primary bonds and the chains that are loosely bonded you call them as secondary bonds.

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And what happens is when I apply the load, and I will also raise the temperature. I keep it as what is known as critical temperature of the polymer; the critical temperature is something like about 110 to 120 degrees, and people also now operate at a lower temperature for various reasons. You do not even have to go to 100 degree centigrade you can also operate at 90 degree, there are requirements for certain problems people also operate at slightly lower temperature than this, but what happens of the critical temperature though it is not visually seen, the secondary bonds break at the critical temperature, and the primary bonds entirely carry the applied loads.

This happens at the molecular scale, you will not see that at the critical temperature it is gone be liquid plastic know. Whatever the secondary bond break you similar to the flue it and the primary bonds gets compressed are depending upon whatever the load that you have apply, and even the loading design for stress freezing as to be carefully chosen, because the material properties change at the critical temperature. So, if I have to maintain the load essentially you will find, the loading is obtain through a dead weight load, because the load will remain constant, load should not change.

And if I have to all internal pressure, they may have a mercury column to do that and people also have done very complex objects like turbines planes. When it is rotating at high speed people have forces and stress field, because of centrifugal forces by a stress freezing processes. So, complex objects have been analyzed by three-dimensional photoelastic analysis and this as very successful process. Though it is experimental intensive, there are technicians and skilled workers who can do this comfortably. So, the critical temperature of the polymer, the secondary bonds break and the primary bonds entirely carry the applied loads.

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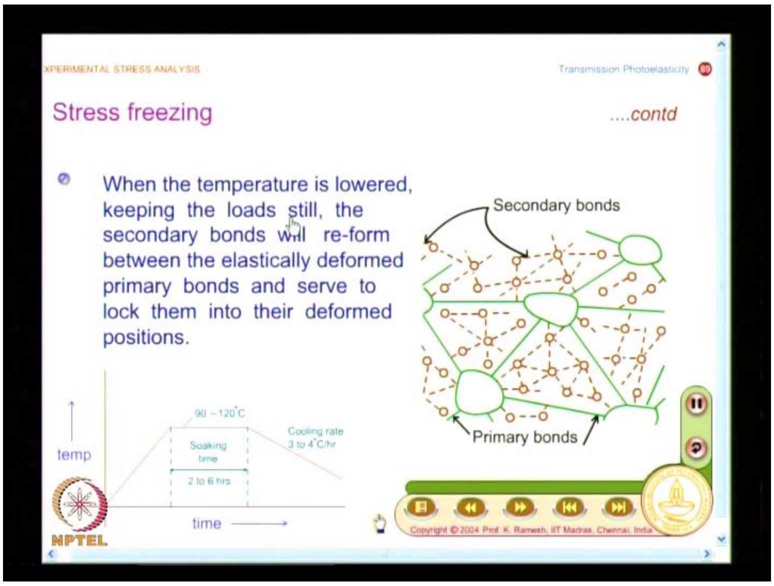

And this is what you have thermal cycling process, very similar to what I do in the case of keeping a string compressed, when the temperature is lowered, keeping the loads still, and the secondary bonds will re-form and prevent the elastically deformed primary bonds to come back it is original position. And hence, the stresses are lock, and at the end of the process I remove the load.

Like what I have in the case of spring put in water, and frozen in to an ice at the end of the process I remove the load still, the spring remains compressed. So, similarly in a stress freezing process I essentially go through a thermal cycling. At the end of the process I remove the load, and let us see, what is the thermal cycling process?

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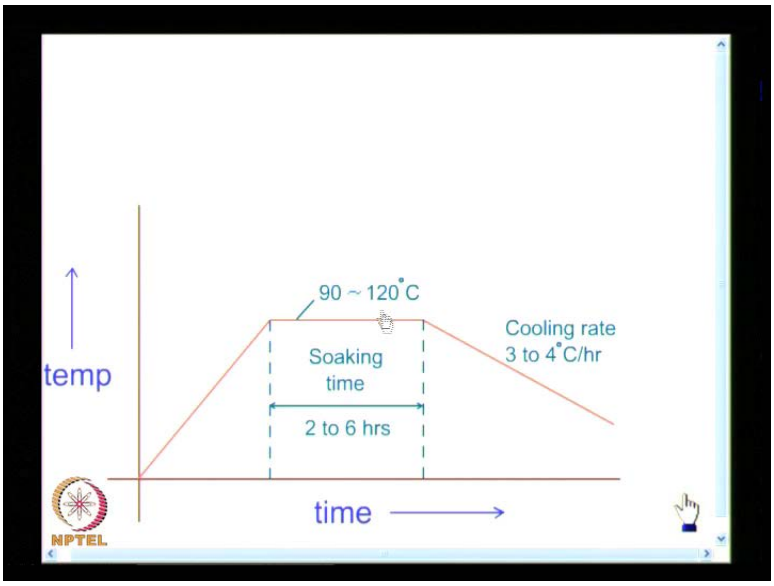

And this is the thermal cycling process, I have a gradual heating I reach the critical temperature, then I so get at that temperature for several hours depending on the complexity of the model. Until the entire model reaches the particular temperature, then cool it at a rate of 3 to 4 degree centigrade per hour. So, gradually equal it and this is a temperature as a function of time. So, the stresses are locked and in the deformed position the primary bonds remain. So, when I remove the load the stresses indeed get locked.

We will see that by an example, we will take the circular disk under diametric compression we will look at how the fringes develop when the model as gone through a thermal cycling processes, then I put a slot and show the original stress distribution remains un altered. So, that is the proof of the pending that stress you can do, and you can also do slicing and slicing has to be, and very, very carefully that is very important, and in order to do the cooling what you normally do is you just switch of the Furness. Even the normal Furness cooling is good enough to limit this, and this is the very, very powerful processes people have adapted this.

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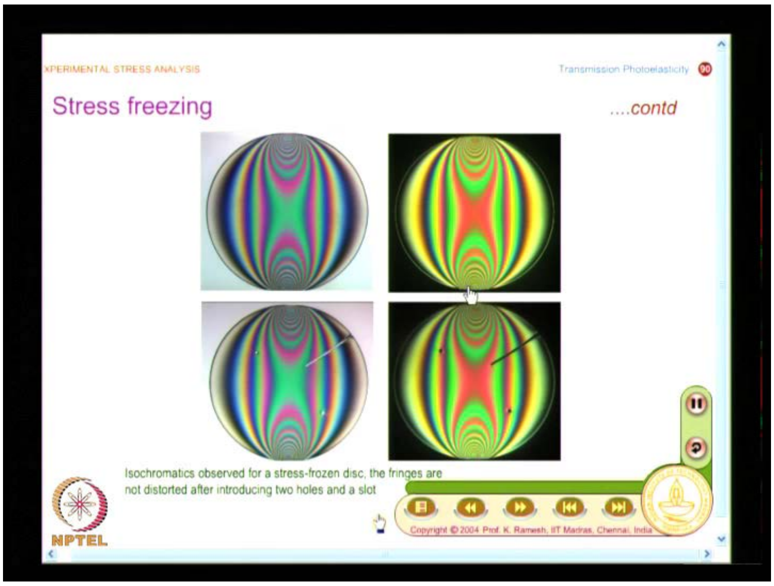

And here you have the fringe patterns, I have a bright field as well as dark field and this is the fringe pattern that you get after the thermal cycling process and what you see here I will also enlarge it, what you see here is after the thermal cycling processes. A flat is made which is very clearly seen in a bright field, and you could see very clearly. There is no discontinuity in this stress distribution it only that this portion is removed.

And similarly, if I do the slicing properly suppose I make another cut here I will have a slice like this... so, that slice will retain the original stresses that have been locked. So, when I do a machining process I must take sufficient care that I do not introduce any kind of machining stresses, and you have to also have a cool end. So, that to take away the heat, and you use a single point cutting tool. There are many restrictions on how you do it.

And you need a mechanic to be train to handle this kind of problems, they will use a high speed tool, single point tool and then minimize generation of machining stresses, and if you take that kind of a care. What this fringe pattern show is when I have a slid made the fringes are not disturbed, and you have a slide difference here this is the hole is made you know you finds small, though the hole is made reasonably well, there is some small disturbance it is not as good as what you had access lot.

So, the person was made the hole need to improve is processes, because you have some small disturbance. So, even this could be eliminated by careful machine. So, what this slide show is there is possibility to lock the stresses by a thermal cycling process, and there is also a possibility to remove the portion of this model as sliceses, and sliceses would retain that stresses that have been locked from a three-dimensional loading here I have taken a two-dimensional model.

I will also show your another example, of the three-dimensional model which is very complex I will show you that, but the point is well made that a thermal cycling processes can freeze the stresses. That is very important the second aspects is by careful machining it is possible to cut sliceses, which retain the stress distribution, and this is shown in color and you have also the patterns recorded and black and white, because the black, and white even if take this whole you know you would not see that circle difference.

I said that hole measuring could have been improved slightly better, and when you see a black and white fringe pattern, you will not notice that the hole has some small minor issues. Then you see in a color you will see the color change, in a black and white will not see the change. You will see that this is as good as the slap that you have made, and that is the reason why I showed the fringe pattern in color and color variation, we have also seen that has been used for calibrating the polar scope used for finding out the sign of the boundary stresses.

Where ever you come across small variations in fringe orders, color sequence is the best way to look at and I am need you to make a sketch of this that fringe patterns are not disturb. So, you need to have only this sketch, but when I make a slit the fringe patterns are not affected if the machining is done carefully. If you go to a cut it you will definitely generate large amount of stresses, you have to be very careful in doing the machining processes.

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And also show you another example, which is a much complex problem. This is just to give a flavor, what kind of complex geometric shapes people have made, and this is a model that is used in a rocket to transfer that thrust from a stress motors to the main body, and you have a ball and socket joined here. So, I have a spherical ball here, and you have a threaded connection, you have a bush and the relative size is shown with the help of a figure of jump clip here.

And what you do is the parts are fabricated, and then assemble, loaded, and stress frozen and for stress freezing I said you have to apply a dead weight, and that dead weight is sudden, after stress freezing the slices are cut, and the typical slices are shown. This is just to give a flavor how a complex geometry has been analyzed and see a beautiful set of fringe pattern.

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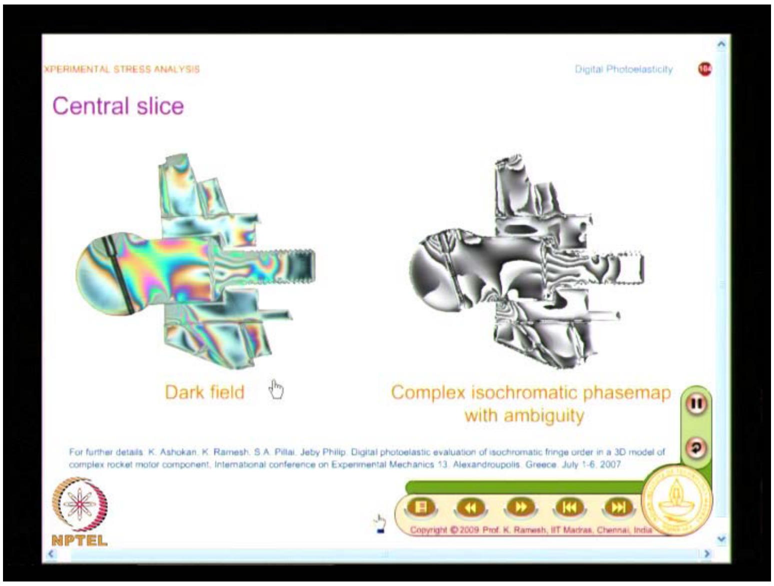

This is the center slice and what you find here is this is the ball, and these are all this the push and these are the outer plate, and you see the threaded connection, and you know this also illustrates you are seen three-dimensional model. From a three-dimensional model a two-dimensional slice is taken. This is just to give you see I want analyze the three-dimensional problem. So, I need to take a three-dimensional model, and then I apply the loads by following simulated equations, and also model to proto type relation I find out what is the actual load that should apply do the stress freezing.

Once you are done the stress freezing stresses are locked, from that whichever is a portion of important you make a slice, and then apply two-dimensional photoelastic analysis to interpret the data. And mind you in a problem like this, what is the slice that I should take is the one of the important question that you should answer, and what is the direction of light path that I should take, all these are issues you should decide, what is the light path that I should investigate; what is the slice that I should investigate, how many slices I require and there has to be a very careful planning of the slice.

Stress freezing is a very elaborate procedure you know people also you know the f sigma value that the material stress fringe value rustically comes down. It may be around for a epoxy which is around 11 or 12 at room temperature as stress freeze temperature, it will reduce to 0.3 also... And one of the challenges in three-dimensional photoelasticity analysis is what is the load, that I should apply. So, people calculate based on how many fringes are also you want to see, and you do not what introduce large deformation, because the material becomes very plastic at critical temperature.

So, that is why in similitude equations we have seen, we must try to maintain parity between the strains develop in the actual model, actual prototype and the model. So, many times you will find you make a complex model, the model break in the stress. So, you ways one or two models if you do not do the calculations correctly or with the problem is too complex and your estimation of loads have gone well beyond the capacity of models to with stands.

So, in all the stress freezing processes model making is very difficult and finding out the actual loading to do that is also equally challenging, and there are you know now we have also shown by using a three-dimensional finite element analysis, how you can get guidance to select the load, guidance to see which slices have to be get all these people have looked at. And you have a great bone with rapid prototyping which has come in a big way in manufacturing.

So, you have stereo photography is processes which is fortunate that whatever the resins that are used in stereo photography or photoelastically sensitive, and if I look at rapid prototyping the model is made from a cad drawing. So, you have a cad data you translate that into a model, you grow the model in fact. So, that makes your model making simple in photoelastic analysis. The same model can be utilize for finding out stresses by stress freezing and slicing, and the same cad model could be used for doing a finite element analysis also. So, now you can do a 3 D finite element analysis as well as three-dimensional photoelastic analysis, starting from the same cad model.

So, that way you have a combination of experimental and numerical approach to solve very complex problem, people have solves such complex problems and that is why this technique is very popular, and I am show some of you have taken down this fringe pattern, and this shows what you have as a phase map we will have occasion to see when we discuss on digital photoelasticity. What is the difference between fringe pattern and phase map, and a focus is to find out fringe order at every point at the model domain. And this gives you, how the complex three-dimensional model would be stress frozen and slice for you to the analysis.

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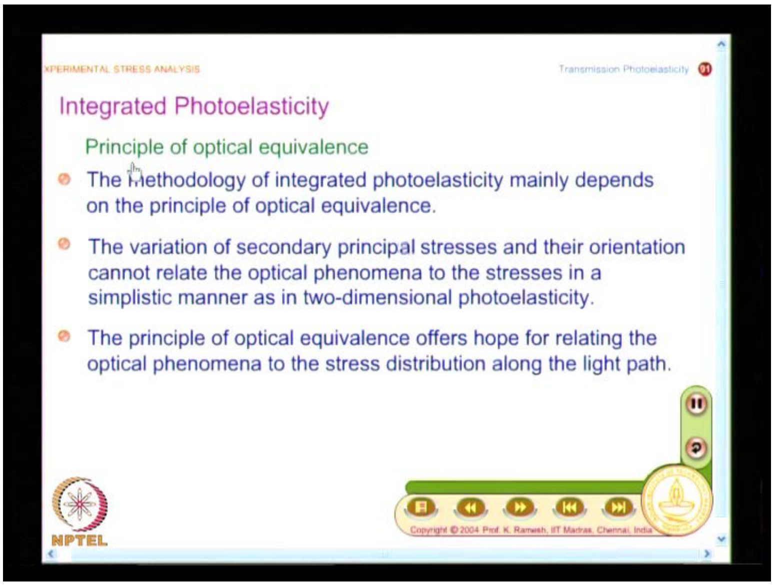

How we take out we are seen that the conventional three-dimensional photoelastic approach. Involves stress freezing and slicing, and you you must have really seen that it is mathematically this is experimentally intensive. I am sorry it is not mathematically intensive is experimentally intensive. Now we are go to look at the process called integrated photoelasticity, and this fringes on a very important principle called principle of optical equivalence.

This is the very intelligent conception, and we look at it more from optics point of you and entire methodology of integrated photoelasticity depends on the principle of optical equivalence. And let us see what is the principle of optical equivalence and we have already seen that there is a variation of secondary principal stresses, and also their orientation and that is the reason why you are not able to interpret by using simple concepts of two-dimensional photoelasticity unless you slice them.

Suppose I want to see the model as a whole the principle of optical equivalence offers a hope. So, here I need to determine the stress distribution along the light path. You are not looking at stress components on a particular plane, but stress components variations along the light path. So, essentially it is the tonsorial tomography, say in medical imaging they have a cads scanner.

The cad's scanner essentially finds out in your hex cal if there is a tumor it can go, and find out locate where the tumor is what is it size. So, that surgeon can go and operate upon or take some corrective measures. So, the focus is only to find out the location and the size. So, there are calls k r tomographies. The moment I want to find out stresses interior to the model at every point need six components, and normal tomography itself is difficult, scalar tomography itself is difficult.

And if you are going to find out six components per point is called tonsorial tomography, and you can image what is the level of mathematics that you require to unravel this, and in this case even to approach a mathematical methodology. You need principle of optical equivalence without this principle you cannot even formulate the mathematics behind it.

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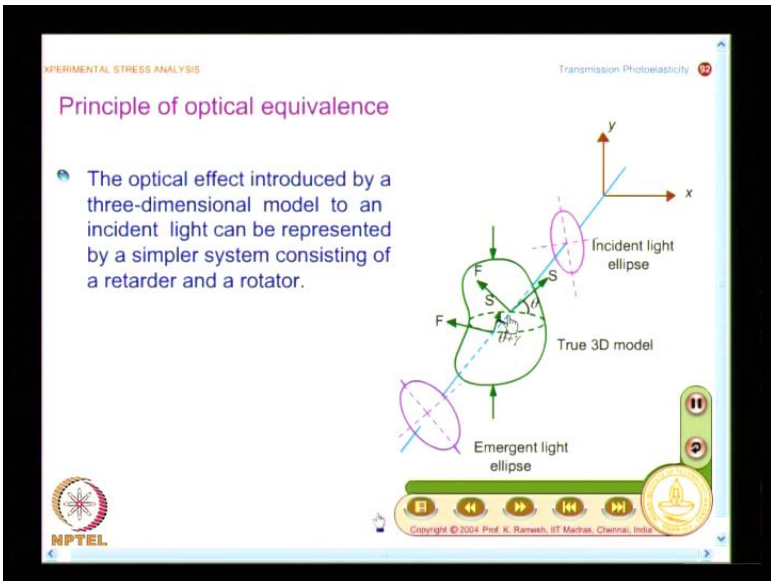

And let us see what is the principle of optical equivalence, here also you need to make a neat sketch and let us hypothesis what we call it as optically equivalent, and we also need to bring in new terminologies. See what I have here is I have a three-dimensional model and we are seen in general you will have a light ellipse impinging on the model, and because of the stress distribution along the light path. Whatever the incident light would get alter, and essentially this will come out as some other light ellipses with some assume it and orientation.

So, this is well understood. So, what you have here is when you send a light ellipse it will come out as some other light ellipse. In a two-dimensional model, whatever the changes could be related to one single plane, in a three-dimensional model it could be related to the stress distribution, and in order for me to develop the mathematics I need to introduce new mathematical entities. I need to recognize a difference what happens at the inlet point.

I have a slow and fast axis, and I call this as characteristic parameters. I bring in another terminology called characteristic parameters. So, when the model then the incident light enters the model, you have an orientation theta and you also have at the exit planed another orientation theta plus gamma. These are not coinciding with principal stress direction at the inlet plane and exit plane, these are optically determined and this is called the primary characteristic direction and this called the secondary characteristic direction, and you call gamma as a characteristic rotation, and you will also have two delta you call that as characteristic retardation.

So, you attach a new terminology call characteristic parameters. Instead of two parameters, you will have three parameters now to define the three-dimensional model. In a two-dimensional model, you define retarder you had a retardation and orientation here you have a retardation labeled as two delta you call it to distinguish from two-dimensional analysis you call it as characteristic retardation, because this is the characteristic retardation often optically equivalent model, and you call theta as primary characteristic direction and theta plus comma as secondary characteristic direction.

And what I say as optically equivalent model this is what I have in a actual model. Now I will replace the actual model by a set of optical elements which would essentially modify the incident light ellipse to the same characteristic as the model modified, and what we will see here is I will have a retarder as well as a rotator required. I need to have two optical elements, the model has to be replaced by a retarder as well as a rotator, in a two-dimensional problem, you have only retarder in a three-dimensional problem I replace the actual stress distribution along the light path of interest by a retarder and a rotator which gives the same exit light ellipse for the given incident light ellipse. So, this gives a via media, instead of looking at stress distribution.

Now, we are simplified the problem to find out the parameters of the retarder, and the rotator. That is how the problem is refreezes. So, we will have look at the points whatever I have said is now summarized. So, it can be replaced by a simpler system consisting of a retarder and the rotator; a retarder will give retardation and a rotator will rotate the light ellipse. We have already seen when we are discussing a half wave plate.

In fact, rotates the incident plane polarized light by and by an angle. So, it is possible for you to conceive a generic concept of a rotator, which will rotate ant light ellipse to any angle of your choice. So, the idea here is I want to find out an equivalence of what happens inside the three-dimensional model optically, because we have only providing an optical equivalence. We have only looking at the incident and exit light ellipse is we match the incident and exit light ellipse by replacing the three-dimensional model by a retarder. And rotator that is why this called a principal of optical equivalence.

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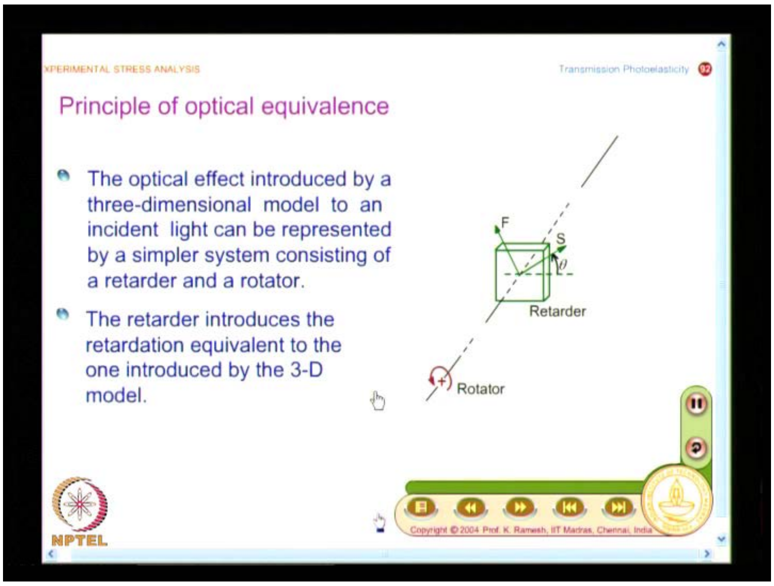

And you see what is that we are doing I am putting a retarder, and I am putting a rotator, and let us see what happens you observe the animation, now let us see what happens when the incident light hits on the retarder. And after the retarder what happens? What it happens and after the rotator how it behaves, just observe the animation know.

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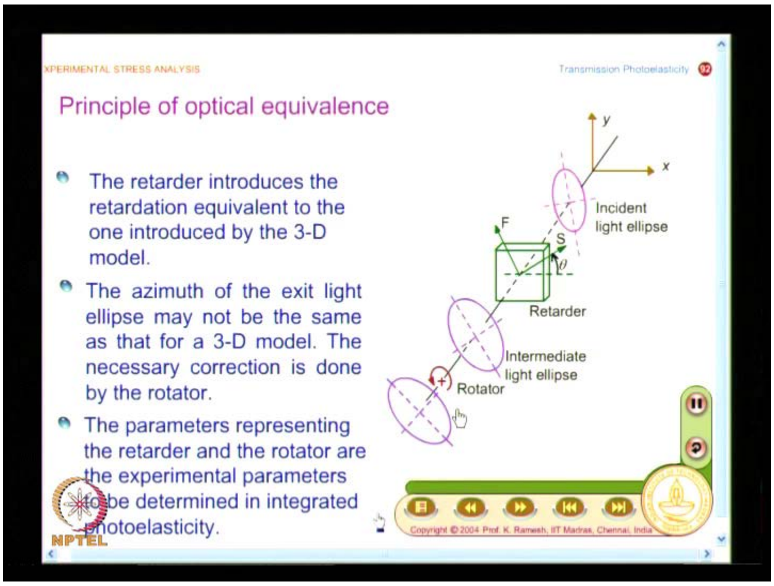

I have an incident light ellipse. So, it gets transformed whatever the transformation by the retarder may not guarantee the same azimuth, and rotator rotates it is... So, that the azimuth matches with the actual three-dimensional model. So, whatever the three-dimensional model you had and we are only confining to the particular light path. Whatever at happens that light path is replaced by a rotator a rotator here and a retarder.

What it does optically is whatever the incident light, the same exit light characteristic as that of a three-dimensional model is maintained by this. So, experimentally what I am going to find out is I will try to find out the parameters of the retarder and the rotator, and theta refers to primary characteristic direction, theta plus gamma is the secondary characteristic direction. So, what you find here is the retarder introduces the retardation equivalent to the one introduced by the 3-D model.

The azimuth of the exit light ellipse may not be the same as that for a 3-D model. So, the retarder does the job of modifying the incident light ellipse to exit light ellipse, but it may not guarantee the azimuth being same. Whatever the correction that is necessary is to be done is done by the rotator. So, experimentally you would determine the parameters representing the retarder and the rotator. So, I will have to determine three quantities when I go to three-dimensional photoelastic analysis, not one quantity and its interpretation as physical variable is very challenging it is not simple, and what you can think of is I can have a series of retarders.

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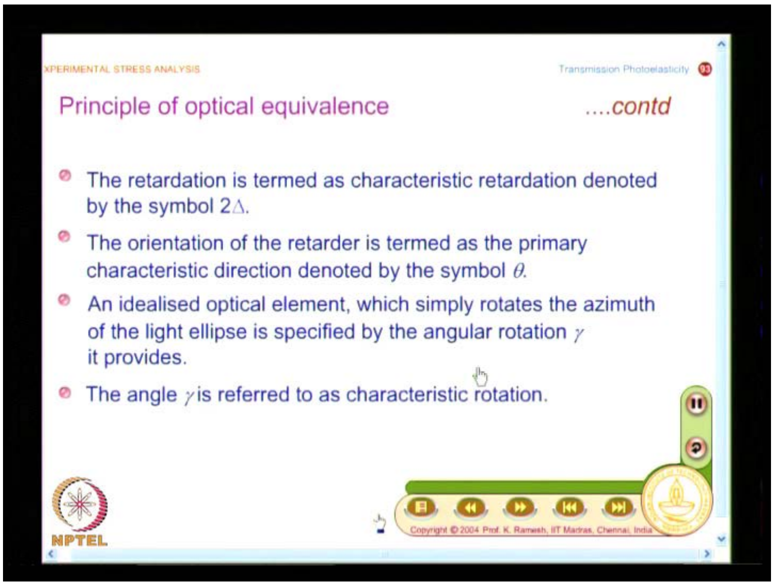

And analyze it by Jones calculus and whatever I have said, the retardation is termed as characteristic retardation labeled as 2 delta. The orientation of the retarder is termed as the primary characteristic direction denoted by the symbol theta. The rotation is given by gamma; the gamma is given as labeled as characteristic rotation. So, these are all the three quantities determined experimentally for a three-dimensional photoelastic analysis.

Suppose you look at how do, I go about my intension is not to solve a problem in integrated photoelasticity. My interest is to give a flavor, what kind of mathematical complexities that come in an integrated photoelastic analysis. Once I know the retarder and the rotator at a point of interest, I can construct Jones matrixes for it suppose I take up a problem, because whenever you take up a problem or new methodology we solve a known problem.

Suppose I take a three-dimensional problem and I know the stress distribution along the light path, then I can analytically write the Jones matrix for each all the planes multiply all of them. I will get one final Jones matrix compare these two, and find out the parameters related as stress distribution. So, I need azimuth stress distribution field. When I azimuth stress distribution field reasonably I will have several coefficients if I have so many co-efficient, I need so many equations.

So, I need to have multiple optical path, and multiple collection of the characteristic parameters. So, you get a flavor that it is mathematically highly demanding, it is not going to be simple. On the other hand even though stress freezing is experimentally demanding, it is much simpler mathematically. You can see what is happening physically. In fact, if we look at glass manufacture they introduce deliberately residual stresses, and because the process is well understood and also the class thickness is very small.

People have successfully employed integrated photoelastic analysis for such class of problems, for a generic class of problems it is becomes extremely difficult. So, what we have seen today was an over view of three-dimensional photoelastic analysis, we have essentially looked at what is the basic difficulty, when we want to apply photoelasticity to a three-dimensional problem. The principal stresses vary along the light path also their orientations vary along the light path.

Then we introduced concept of secondary principle stresses then we said if you have nice processes of stress freezing I can lock the stresses, and take out the slices of my interest and extrapolate. Whatever you have understood in two-dimensional photoelastic analysis to analyze the slice, the other approach was replace whatever that happens along the light path optically by an optically equivalent model.

That gives you a via media, because that gives you what to measure experimentally experimentally. I have to measure two delta theta and gamma, and also gives you an indication, how complex the mathematical analysis could be in an integrated photoelastic analysis. So, the idea is to give a flavor and this almost brings to a completion of transmission photoelastic analysis, thank you.