**Experimental Stress Analysis**

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**Module No. # 02**

**Lecture No. # 23**

**Miscellaneous Topics in Transmission Photoelasticity**

We have been discussing on Transmission photoelasticity and I said one of the very key equations in transmission photoelasticity is a stress optic law, where in you get sigma 1 minus sigma 2 as n f sigma divided by the thickness of the model. And I said f sigma is material stress fringe values, which need to be evaluated with as much accuracy as possible.

We have seen how to evaluate this using a circular disc and diameteral compression and I said one of the most crucial data that you need to collect from an experiment is a fringe order. Fringe order needs to be evaluated correctly from the fringe field, you had seen circular disc which is the very simple circular fringe field, where you have 0th fringe order on the outer boundary and you are able to go and increase set towards the load application point.

We have also seen another example, where ring and diameteral compression, which showed almost all features of a generic fringe field; you had the source, you had the sink, you had the saddle point, you had a singular point, you have isotropic point and that presented a very complex fringe pattern. And when you have a complex fringe pattern, if you restart to colour code, you get very good colours and based on the colour, it is possible for you to identify the 0th fringe order and also indentify the gradient.

And what is important here is, today we are going to discuss certain aspects which use the colour code in a different sense. I said colour code could be used for finding out the gradient and you can also find if the changes are very small, the colour sequence helps you to say, whether the fringe order is increasing at point of interest or decreases at point of interest. So, it is better that we go to the colour code and recapitulate what we have learned, you have a black colour which I label to 0th fringe order and we have the first transmission as tint of passage we have fringe order 1.

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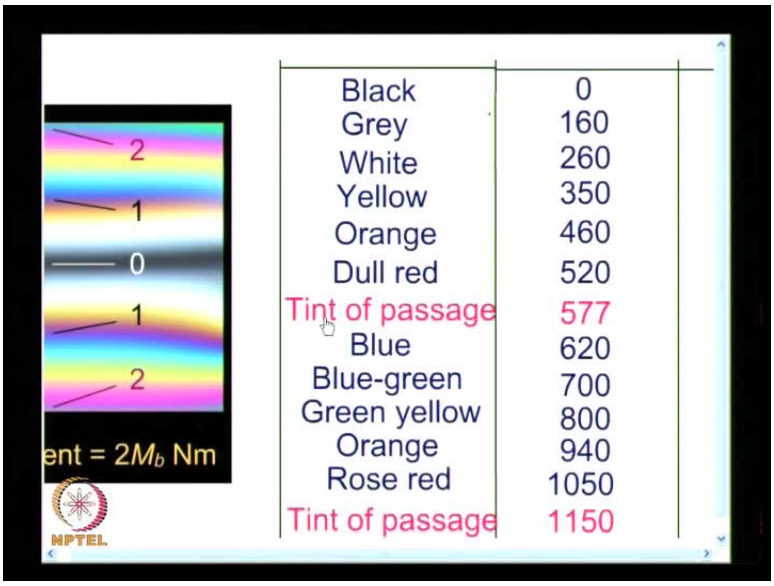


If we look at the colour sequence, it goes from black to grey, white, yellow, orange, dull red and tint of passage. See in actual model situation, if the load changes are small, you know you will not able to see appearance of a new fringe order when we use the monochromatic light source. On the other hand, if we use a white light source, it is possible for you to observe the color change. So, your eye needs to be tuned because the changes may be very small and subtle.

So, the colour change is the only way to identify whether the fringe order as increased or decreased. So, the colour sequence is very important and that is what we see here, I have from 0 to 1, you have a colour sequence like this, from 1 to 2 you have a colour sequence like this, from 1 2 you have blue, green, yellow, orange and rose red. So, if we are focusing on a particular point of interest and when we do some modification on the optical arrangement, the colour sequence will help you whether the fringe order increases at the point of interest or decreases at the point of interest.

And we are going to take our very important topic, which I said earlier also that when we are actually finding out the principal stress direction, photoelasticity gives you sigma 1 minus sigma 2 as well as the principal stress direction at the point of interest.

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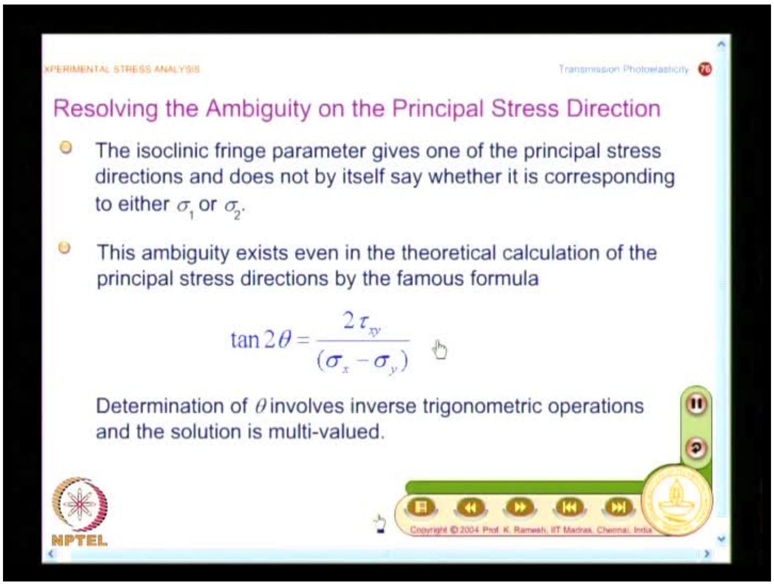


Many of the failure theories you know you do not want the orientation, whether it corresponds sigma 1 direction or sigma 2 directions. I said even in conventional photoelasticity, such a distinction is not generally required but, when you are having an experimental arrangement its all the more desirable that you also find out, whether it represents principal stress 1 direction or principal stress 2 direction. I have mentioned long time back that, you need a calibration of the polariscope to do this, whether this is the problem only in experimental approach, no.

We have also seen the ambiguity exits even in the theoretical calculation of the principal stress directions by the famous formula you all know, tan theta equal to 2 tau x y divided by sigma x minus sigma y. Then you find out theta from this because, theta evaluation uses inverse trigonometric operations, the solution is multi valued you do not know whether it represents theta 1 direction or theta 2 direction, we have already seen principal stresses are arranged algebraically. So, you have a maximum principal stress is given, the value of sigma 1 labeling of sigma 1 and algebraically the next smaller one sigma 2 and smallest you call it as sigma 3.

So, when I say theta 1, I want to know the maximum principal stress direction, how do you associate this? We are seen when the equation is ambiguous in your simple strength of materials, one can always take a ((recourse)) to more circle and resolve whether the theta evaluated is indeed principal stress direction 1 or direction 2. So, you need to have some kind of auxiliary information, suppose you want to evaluate mathematically and we are also seen that, you have this problem posted as Eigen vector and Eigen value problem. In that case for every value of principal stress, you will get the corresponding Eigen vector that will fix theta value to the associated principal stress direction. So, this is what we had seen mathematically, now what we will look at is what the kind of calibration is that we have to do for the polariscope.

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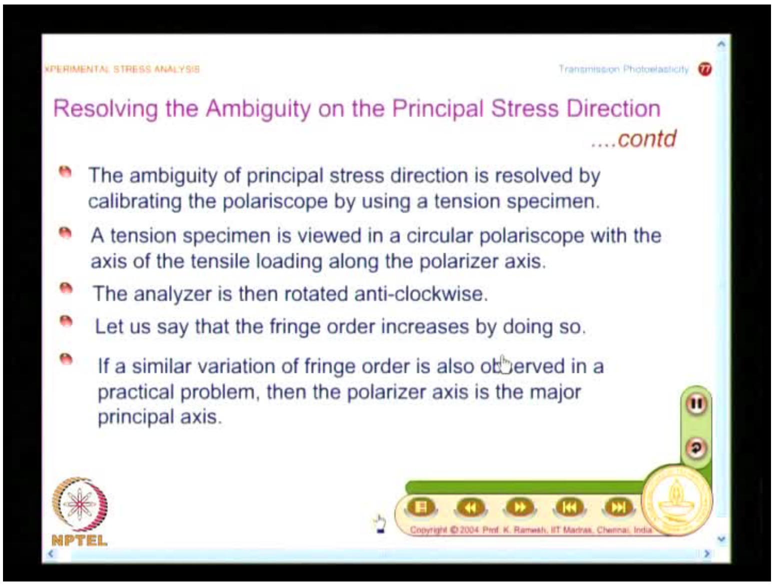


So, what I have here is, I can do this by taking at tension specimen. So, when I take a tension specimen what do I know, when I have tension specimen like this, I know the principal stress direction because that because the principal stress direction, I am pulling it like this perpendicular to the there is no stress. So, the maximum principal stress direction is the direction of the pulling and what I do here is the tension specimen is viewed in a circular polariscope with the axis of the tensile loading along the polarizer axis. I take up a very simple problem and then I align it with the polarizer axis, then I do we have already seen I can use analyzer as a compensated.

So, when I rotate the analyzer anticlockwise or clockwise, the fringe order at the point at the point of interest will change and that precociously what is going to happens and what is going to happens is the retardation introduced would be so small, I would not have full fringe order come and occupy this; if I know the colour code and the colour sequence, it is possible for me to assess whether the fringe order as increased or decreased by a particular rotation of the analyzer. So, the calibration is look at from that point of view.

So, what I do this is I have the analyzer rotated anticlockwise, then I observed how the fringe order changes. Let us say that the fringe order increases by doing so then, what I understand? When the fringe order increases by doing so, I know the original major principal stress direction was along the polarizer axis and when I rotate the analyzer anticlockwise a fringe order as increased.

So, this I keep it as the base information, I go to an actual experimental model and indentify at point of interest and then rotate the analyzer, if the fringe order increases then I say polarizer axis coincide with the major principal stress direction. So, this is the kind of calibration. So, what is summarized here is, if a similar variation of fringe order is also observed in a practical problem, then the polarizer axis is the major principal axis.

So, what we need to do is, we need to have some kind of a calibration to the polariscope. I said this is part and parcel of any experimentation, if you want theta for the major principal stress direction and minor principle stress direction to be determine, that kind of association needs to be done, it is possible to do experiment. In fact, some of the difficulties in digital photoelasticity were how to specify that this corresponds to major principal stress direction or minor principal stress direction. This has cost what is known as inconsistency in an isoclinic phase map and it also cost ambiguity in the determination of fractional fringe order, so though in a conventional photoelasticity, we are not worry about major or minor principal stress direction.

In digital photoelasticity, you need to worry about and they approach problems slightly different but, the fundamental question here is from experimental point of view, is it possible to say the major principal stress direction at the point of interest and minor principal stress direction at the point of interest. You can do that, if you calibrate the polariscope and we are seen what the calibration that you need to do. Then what we move on is simple information.

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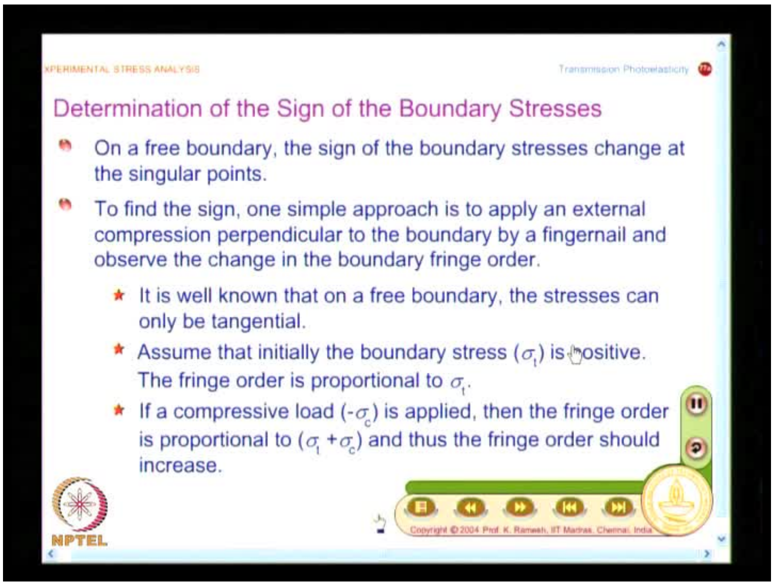


In many of the problems we know, we want know what the sign of the boundary stress is. Determination of the sign of the boundary stresses is also a very important aspect and how to we do that? If I want to find out the sign of the boundary stresses, how to we do that? In the case of beam under four point bending, we knew which fiber is under compression, which fiber is under tension, you do not need experiments to tell you that; even before you go to the experiment, you know the way the deformation of the beam you know, which is the fiber subjective compression which is subjected to tension.

But, nevertheless we need to know how to find out the sign of the boundary stresses and what I can do is to find the sign, a simple approach is to apply an external compression perpendicular to the boundary by a fingernail and observe the change in the boundary fringe order. Here again the boundary fringe order will change by a very small amount and it is advantageous that give you the model in a white light and see the colour sequence when the model is compress by a fingernail. And you know you have to come back and then say, look at the model very carefully and we have to identify depending on the change in the fringe order whether it is positive or negative.

First principle is it is well known that on a free boundary, the stresses can only be tangential. So, this is the first point that you need to keep in mind and for our discussion, let us assume that initially the boundary stress is positive. So, I label it as sigma t, we do not know what is the sign of the boundary stress but, we take up with a simple situation where the boundary stress is positive to start with and whatever the fringe order that you see in the circular polariscope is proportional to sigma t, because the perpendicular direction the stress is 0 because there is the free surface.

So, I see that the sigma t whatever the fringe order is proportional to sigma t; suppose I go on apply a compressive load minus sigma c by a fingernail, then what happens to the fringe order? We are only looking at sigma 1 minus sigma 2 now I play a compression through a fingernail. So, minus of minus becomes positive. So, I get this has sigma t plus sigma c and thus the fringe order should increase. So, what we are looking at is if the boundary stress was positive, a compression by a fingernail on the surface would result in increasing the fringe order at the point of interest. So, what we will do is if the fringe order increasing by a compression, we will say boundary stress was positive, on the other hand if the fringe order decreases, boundary stress is negative.

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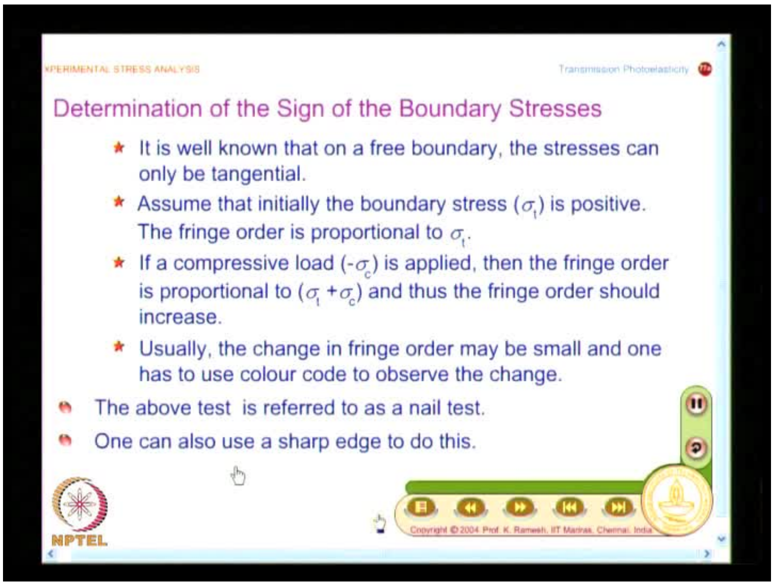


Then the question comes, can I always apply using the fingernail? You know you will also have a short discussion on various photoelasticity materials, we will see for which kind of problem fingernail is good enough, if the fingernail is not sufficient we may have to apply the force by external means, that is the one way of doing it. And that is what is summarized here, one can also use a sharp edge if you are not able to apply the compression by a fingernail, because we use the fingernail to do this test, this is also called as a nail test. And as I mentioned the change in fringe order may be small and one has to use colour code to observe the change, whether the fringe order as increased or decreased, you will get the information by knowing the colour sequence, that is the best way because, these changes are very small and only a colour code can provide you this information.

From this point of view also knowledge of colour code really helps. So, what we are seen is, I can do it by a nail test go and apply the compression and if I am not able to do with the nail use a sharp edge but, we can also think of other ways to do it. Once you understand the principle, we are already seen what is the (( )) soil compensator, we have put a compensation after the model and added compensation or subtracted compensation. So, in a similar fashion I can use an external member to assist you in doing this kind of a test that is what we will see.

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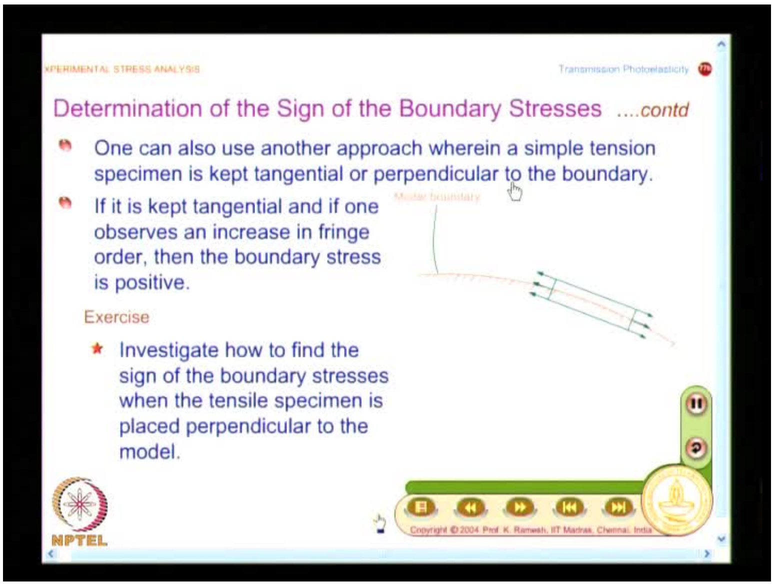


For us to do that, I can even use a simple tension specimen; I can take a specimen subjected to tension, I can do it either keeping at tangential to the boundary or keeping at perpendicular to the boundary. So, what I can also do is, we can adapt another approach wherein a simple tension specimen is kept tangential or perpendicular to the boundary.

So, if it is kept tangential, what will happen? If I keep it tangential to the boundary I am adding the retardation, if the original retardation was positive if I add retardation a fringe order will increase, when the fringe order increases I would say the boundary stress is positive. So, you also can do a similar approach, when I keep it perpendicular do it. So that you take it as a home exercise, because you know here, we have to be very alert in you have logical development of the argument, because if you miss the way you have added the retardation, you may end up with long result; you cannot say if the fringe order increases is always positive, fringe order decreases is always negative that kind of a generic conclusion we cannot arrive at.

We have to look at whether the model was kept tangential to the boundary or perpendicular to the boundary, if it is kept perpendicular to the boundary the reasoning will be different, because you are adding or subtracting the retardation in a direction perpendicular to it. So, what you will have to do is, you have to be very careful when you do this, when it is tangential to the boundary and when it is perpendicular to the boundary.

So, develop the reasoning when the tension specimen is kept perpendicular to the boundary. So, what we have seen, we can find out the sign of the boundary stresses either by using a nail test or by a sharp edge or even by having a simple tension member kept in front of the model and in fact, if you see stress freezing, you can also stress freeze the tension specimen and keep it ready adjust, keep it tangential to the boundary appropriately boundary you do not even have to pull it.

So, that idea you will get after looking at what is the way people employee 3-dimensional photoelasticity analysis, wherein they use stress freezing followed by slicing, similar concept can also be extended to simplify your experimentation. But, the principle comes from your understanding basic understanding of a compensation technique and what happens on the free boundary? On the free boundary, you must always keep in mind that is stress can best be tangential to the boundary and you should never conclude by looking at the circular disc, on a free boundary stresses are 0, its stresses are 0 in the case of the circular disc special case.

We had beam under bending, you had maximum stress on the top and bottom surface they are all again free boundaries and you have to another example, you had ring and diametral compression, you had fringe orders varying on the outer as well as the inner boundary and I caution whenever the fringe order causes a 0th fringe order, the sign of the boundary stress is also changes. And you will also have to recall, if I done a course in theory of elasticity, you know many times you talk about stress concentration, suppose I have a plate with whole and pull it, people know the maximum stress reaches 3 times the nominal stress. What many people do not observe is on the inner boundary of the whole at 90 degrees maximum stress, you have compressive stress developed. So, what you have here is on the boundary of the whole, you need to find out what is the sign of the boundary stress and here again, you have a singular point will help from isoclinic fringe field will help.

I said ring under diametral compression and plate with a whole shares certain commonalities you can identify point of transition because some theory of velocities we know you have maximum stress, suppose I take a tension specimen pull it vertically on the horizontal diameter, you have maximum stress develop and the vertical diameter, you have compressive stresses develop. So, in between there has to be transition. So, on the inner boundary, it is a free boundary sign of the boundary stresses changes and for such applications using an external member is convenient because if hole is very small, I cannot gone put my nail, I can use the tension specimen and then investigate and reconfirm that your understanding of theory elasticity is correct.

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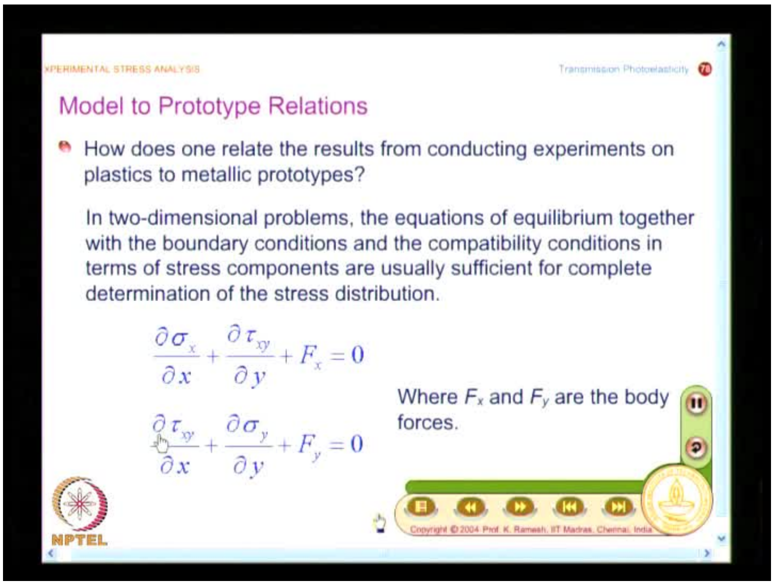

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See now we come back to a very famous problem, you know I have plastic model, this plastic model is poly urethane and I have an aluminum specimen and apply the same diametral compression load and what will happen in the plastic model? Plastic model will deform visible because it has lower young's modulus whereas, aluminum is about 70 gpa and plastic is around 3 gpa in general and this is poly urethane is much smaller than that. Now the question is I use only plastics in the case of photoelasticity analysis, how I am my justified in extrapolating the result from conducting an experimental on plastics to metallic prototypes.

We will go in stages first, we will look at planer problems and when we look at planer problems, we will also simplified it further that we will not look at the (()). So, what I have, the important question that you will have to keep in mind is, how does one relates the results from conducting experiments on plastics to metallic prototypes? You know the confidence comes by looking at the equation of theory of elasticity.

If you look at the theory of elasticity, what do equations say when I am looking at the planer problem, we will have to look at equilibrium equations. And if you are all working on a stress formulation, you will also have to look at compatibility conditions. So, what I have here in 2-dimensional problems the equations of equilibrium together with the boundary conditions and the compatibility conditions in terms of stress components, you must have seen compatibility conditions in terms of strain components, we will also spent 2 minutes on that, these equations are usually sufficient for complete determination of the stress distribution. So, what I need to have is, I need to have a equilibrium conditions, I need to look at the compatibility conditions and satisfies the boundary conditions are the problem.

How does the equilibrium condition looks like? The equilibrium condition is like this (Refer Slide Time: 25:00) you all know, dou sigma x by dou x plus dou tau x y dou y plus f x equal to 0, where this is the body force. And you have dou tau x y divided by dou x plus dou sigma y divided by dou y plus f y equal to 0, I have used the equality tau x y equal to tau y x this expression. Thus this expression has any elastic constants; this expression does not have any elastic constant, when this expression does not have any elastic constants you do not need variation between a plastic and aluminum to behave when you satisfy this condition.

So, the equilibrium condition does not have an elastic constant, fine and what are compatibility conditions, we will go back and then see; if you go back and then see from a theory of elasticity understanding, you have 6 strain components but, you have only 3 displacement components. Suppose I find out the strain from displacements, no problem, from displacements to strain is a simpler exercise.

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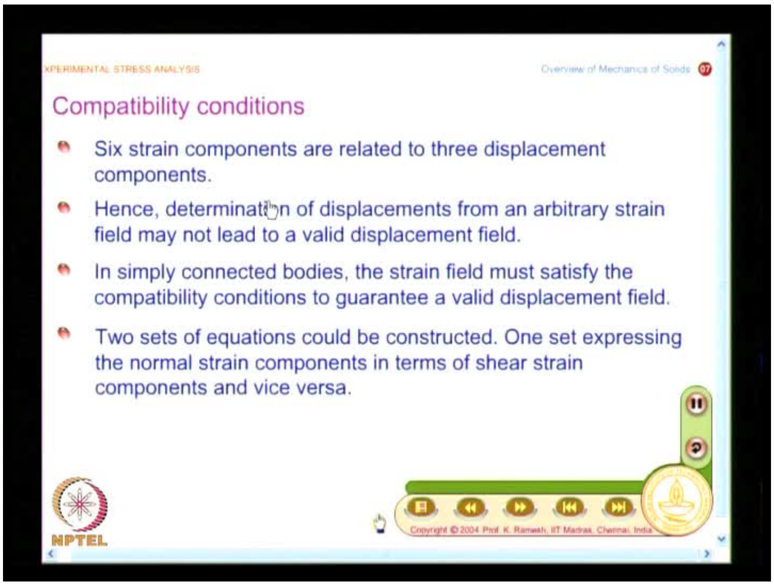


But, knowing the strain components finding out the displacements, you have to bring in the equations of compatibility. If you do not bring in equation of compatibility, you will not have deformation consistent with the loading applied. And what you have here is in simply connected bodies, the strain field must satisfy the compatibility conditions to guarantee a valid displacement field. Though in theory of elasticity you have stress formulation as well as displacement formulation, the very popular various stress function is basically a stress formulation.

In fact, we know stress fashion stress function for a variety of problems. So, you evaluate stresses, from stress evaluates strains, from strain displacement relation evaluate the displacement but, when I go from strain to displacement, I need to invoke compatibility condition; otherwise, my displacement evaluation would not be compatibility. And what you see here (Refer Slide Time: 27:47)? You get two sets of equations could be constructed, one set expressing the normal strain components in terms of shear strain components and vice versa and the equations appear like this.

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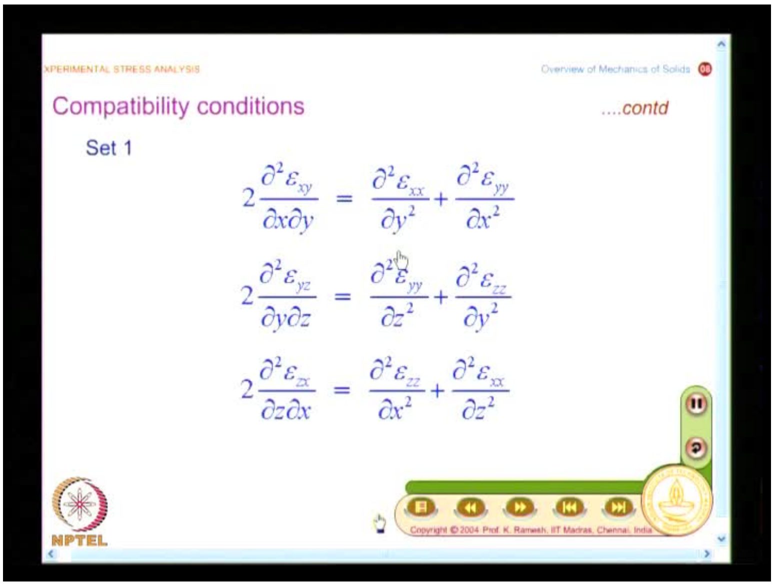


So, when I relate these normal strain components to shear strain components, you have it like this. So, on the left hand side you have only shear strain components, on the right hand side you have the normal strain components, if you go on plug in the expression for a epsilon x x epsilon y y and epsilon x y, this equation will be completely satisfied and since you have confirming a your attention in our discussion only to 2-dimensional problem for the time being, we need only the first equation.

So, this is set 1, I has compatibility condition expressed in terms of strain components; once I have this, if I know the stress strain relations or strain stress relation, I can replace these strain quantities in terms of stress components. In fact, it was one of the home exercise problems, I am only reviewing solid mechanics for the sake of continuity and you have set 1 and you also have another set, which relates your shear strains to normal strain, the left hand side is only normal strain and the right hand side you have only shear strain components.

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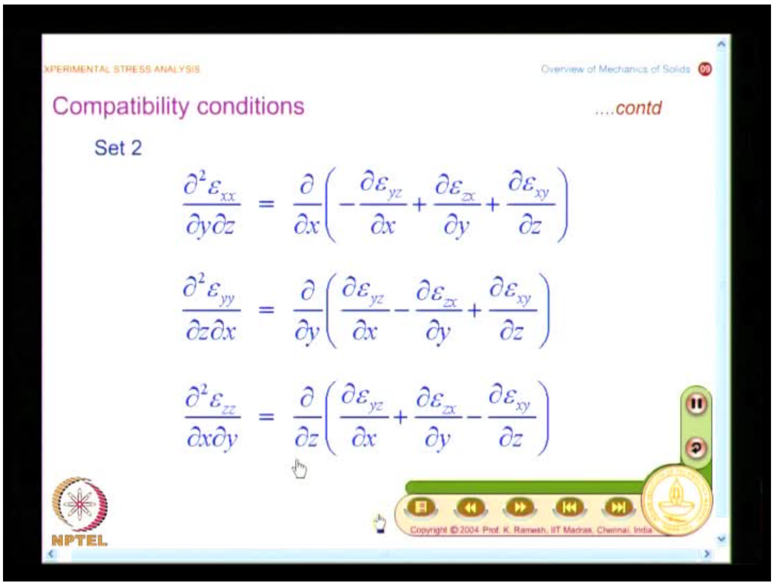


This you must have studied in course in advance mechanics of solids, it is nothing new, it is only looking at this old equations just for continuity and in fact, we were more concerns with set 1 and that is good enough for our discussion.

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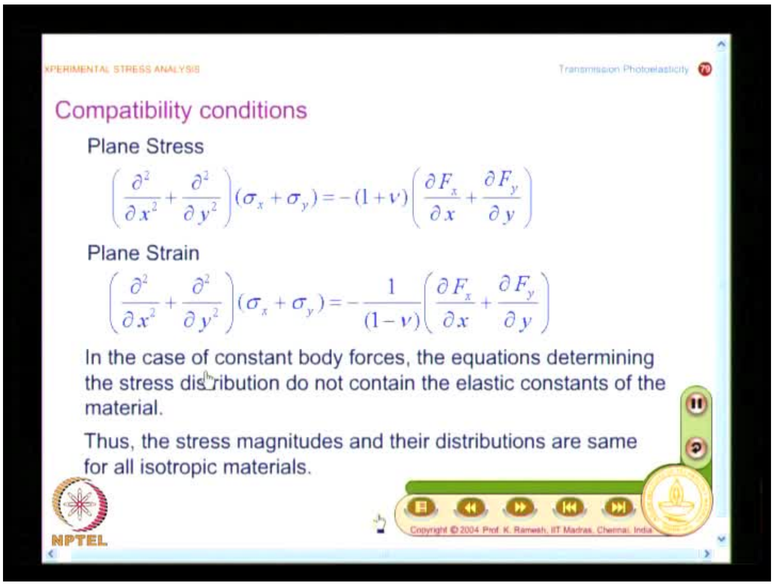


Suppose, I write this expression in terms of the stress components, how do the equations look like and we will look at plane stress condition and plain strain condition separately. The compatibility condition in terms of stress components has the form like this for plane stress. On the right hand side what I have? I have minus 1 plus nu multiplied by dou F x divided by dou x plus dou F y divide by dou y and these are body forces and one of the simplest assumption what we make in most our problems except civil engineering problems, we can consider the body force to be constant or 0.

When it is constant also the right hand side vanishes, suppose the right side does not vanishes the compatibility conditions the function of what? The function of Poisson ratio, which is the material property; here an equally condition was not a function of any of the material property, where as compatibility conditions is the function of Poisson ratio.

Suppose you eliminate the consideration of body forces from the point of simplicity, you find the right hand side becomes completely 0. So, this equation will be independent of elastic constant again and the right hand side will be straightly different when I go to plain strain. When I go to plain strain find out what is the compatibility in terms of stress components, only the first term and right hand side change as minus 1 plus nu its appear as minus 1 by 1 minus nu. So, in the case of constant body forces, the equations determining the stress distribution do not contain the elastic constants of the material.

So, what is the implication? The implication is the stress magnitudes and their distributions are same for all isotropic materials. So, you here the comfort, the theory of elasticity comes your rescue because in most problems, you know body force looking at static problem, it is the dead weight and if it is constant, you are not going to have any problem; only when you have rotating components, where the body force is the function of the radius, then you have to worry the Poisson ratio place its spoil sport.

I said in all the experimental methods, the Poisson ratio will be nuisance in one way of other; you have to live learned live with that. So, what we find here is even though I perform experiment on a plastic, because of the strength of the equation of some the velocity, for a 2-dimensional problem when the body forces are consider constant, elastic constant do not play a role.

The stress distribution is same but, the displacement will be different, because displacement is dictated - displacement or strain they are all - dictated by the elastic constant. So, you have to do this experiment very carefully, you can not apply any low to the plastic model and I said, I correspond whatever the result that I do. You will have to follow the loss of simulative that is we will also see. So, what you find here is working on a plastic is convenience of optics point of you, because it behaves like a crystal, I have birefringence and then I am able to visualize the stresses, all the advantage we have when you work with plastic.

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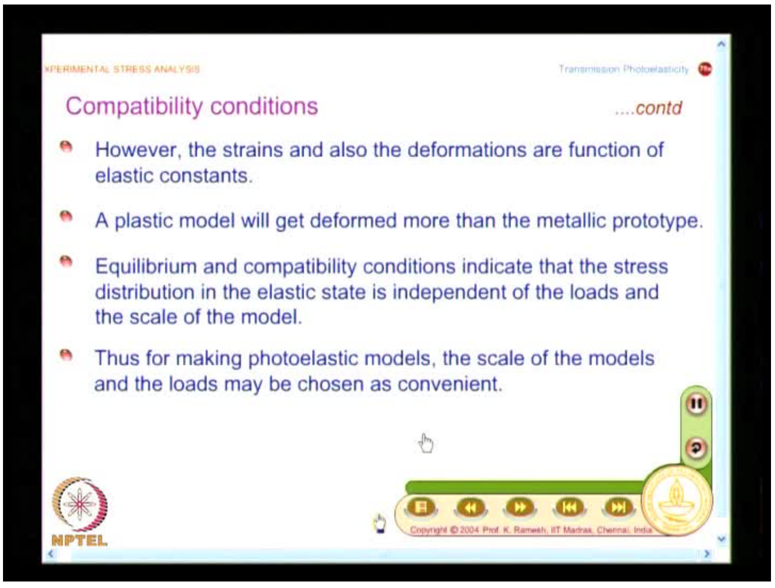

And the results also can be correlated to metallic prototypes from the strength of equation of theory of elasticity and when we go to 3-dimensional what happens? We will see that also, the comfort what you have in 2-dimensional is not same; the 3 dimension is always difficult, you will always have to live with problems. And this is what emphasized again, what you will have to know is, though the stress distribution and its magnitude are same in a 2-dimensional model, when they consider body force as constant, the strains and also the deformations are function of elastic constants. So, in a sense a plastic model will get deformed more than the metallic prototype.

But we will also learn similar to the equations and how do we load the model, what is the level of load that should we applied. So, what we find here is equilibrium and compatibility conditions indicate that the stress distribution in the elastic state - all this are very important, we are not looking at plastic conditions, we are looking at elastic state; we are only having discussion in photoelasticity, we are not looking at photoelasticity. So, with search kind of modeling, the stress distribution is independent of the loads and the scale of the model.

I do not to worry if I do at on the plastic, I do not feel as sorry I could do only in plastic and then compare with metal, you are doing at on plastic and the result are equally valid as for as stress and the distribution is concern; the same is not true for displacement and strain and they are dictated by elastic constant, this subtle different you should understand very clearly.

Even if I have a rotating component, it may be still a 2-dimensional problem, what if it changes from point to point, when your Poisson ratio effect the distribution as per as magnitude, then you will have a model material which as same Poisson ratio as the metallic prototype, which is not possible always. And what you have here? When we make photo elastic models, the scale of the models and the loads may be chosen as convenient and we also keep in mind a plastic model will get deformed more, so I should select the load such that, I do not get into large deformation in plastic.

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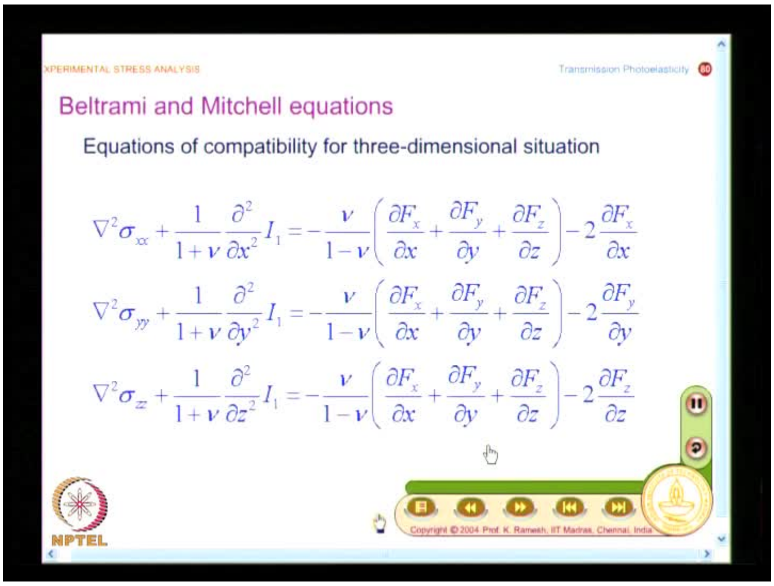


The deformation and strain in the plastic model are matched more or less in metallic prototype that also we will see in the simulative conditions. The first and foremost advantage from a velocities equation is, the stress distribution is independent of the loads and the scale of the models, when we look at in plane problems with body force remaining constant. Now, let as look at how does the compatibility conditions looks, when I go to 3-dimensional situation and these are famously know as Beltrami and Mitchell equations and we not find this several books and experimental of concern about this.

So, I have this as del square sigma x x plus 1 by 1 plus nu, dou square by dou x square I 1 this is first invariance, that is equal to nu by 1 minus nu dou f x by dou x plus dou f y by dou y plus dou f z by dou z minus 2 time dou f x by dou x. What strikes you immediately, when I look at this expression? First it is long; very long expression than what you saw in the 2-dimensional situation, the 2-dimensional situation we could look at when body force is constant and the equation gets simplified.

Suppose body force is constant in 3-dimensional situation, what happens? The right hand side will goes to 0 but, the left hand side has the nuisance Poisson ratio, so the moment you to come 3-dimensional problem, Poisson ratio change will the effect the magnitude of the stress not only the magnitude, even the distribution, this dictation by the Poisson ratio never forget this.

In a practical situation, we may say they influence of Poisson ratio is small, so let us gloss over it, let us permit it because as engineers, we have to live with approximation will make the approximation which is reasonably good and sufficient for your practical application. But, never forget the moment you to come 3 dimensions Poisson ratio is nuisance. Don’t think that only photoelasticity has the sigma, you will see all the coating techniques, the Poisson ratio will place spoils sports; that the grace is whatever the influence it will have it is very small, it is all like second order effects.

But from an analytical understanding, we should know such effects exist; when you are getting the result you take with a fringe of solved integrated dimensional problem that Poisson ratios plays its role. So, that you have to keep in mind and you know I have three such equations, actually you can go back and fill those equation; I have three such equation, 1 is for sigma x x, another its sigma y y and I think I leave that as home exercise, we just have a look at in this class but, go and develop it because this are cyclic repeating.

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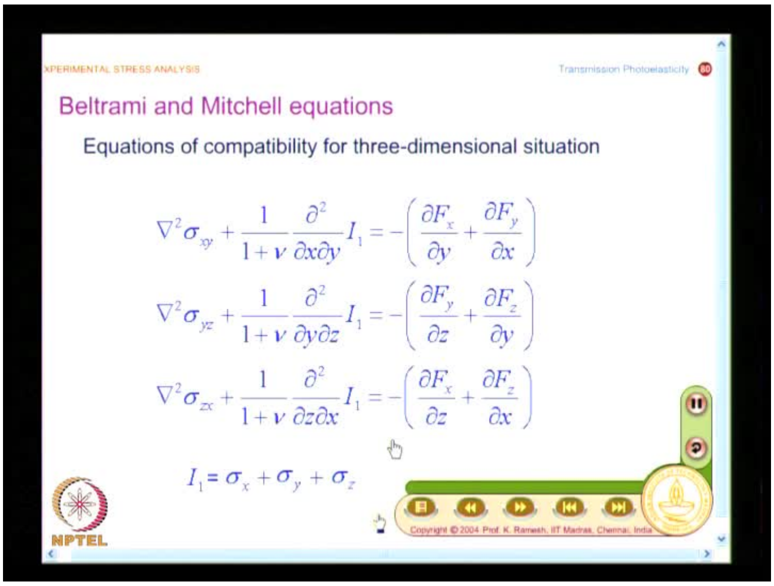

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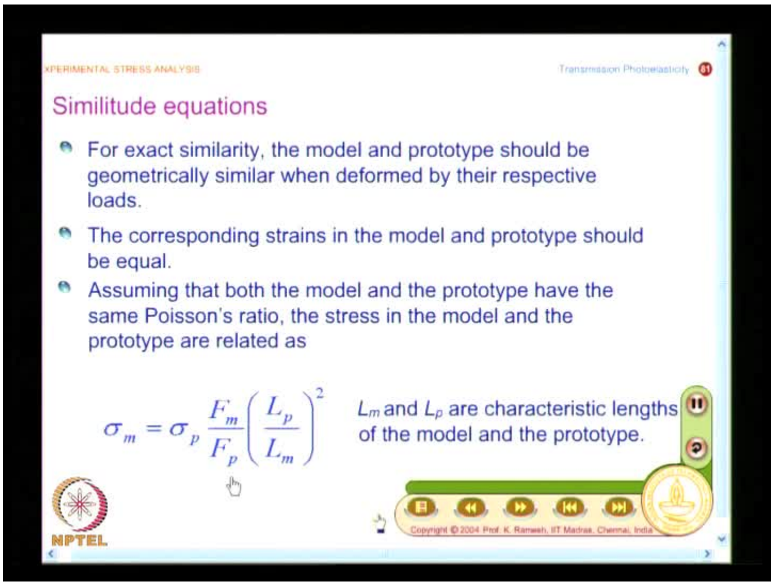


So, I will have another three such sub equations, which are also cyclically repeated and the first equation looks in this form. I have del square sigma x y plus 1 by 1 plus nu dou square by duo x dou y I 1 equal to minus of dou f x by dou y plus dou f y by dou x. So, this also two more equations which you can look at them but, you can easily fill it up from than symmetry of this symbols, by looking at the cyclically changing this will be able to get this. And you have this as I 1 as sigma x plus sigma y plus sigma z see, what is important in a doing a model study is, you will have to follow the similitude equations. For exact similarity, the model and prototype should be geometrically similar when deformed by their respective loads.

So, we have seen when you are working on in plane models for 2-dimensional problems, the stress are not altered, when I go from plastic to metallic prototype. On the other hand, the displacement and strains are dictated by the material property, for the same load a plastic will deformed more and what we look at similitude equation is you do not apply the same load, if I apply the same load as service load in a prototype the model will break completely.

So, you have to apply a load much below the actual service load and you also develop a philosophy of how to apply the load. And what we are looking at here is the model and prototype should be geometrically similar when deformed. So, that is the issue that you take it up to find out what is load by which I can apply on the model. So, what we look at here is, the strains develop in the model and prototype should be equal, this is the desirability. I said in all the experimentation Poisson ratio is an nuisance, assuming that both the model and the prototype have the same Possions ratio, the stress in the model and the prototype are related, I have a symbolism that with suffix denotes whether is a model or a prototype and m denotes the model and this is your actual prototype.

So, I have sigma p into f m by f p in to l p divided by l m whole square. So, what we look at here is, he bring characteristic length from the model as well as the prototype and then we find the stresses in the model, it is related to stress in the prototype like this. Our goal is we want to find out the load, whatever the load that I want to apply on the model, I want to do that in such a fashion when deformed the model and prototype should be having similar strains, this is what you are looking at, that will bring in the elastic constants, how does the equation look like.

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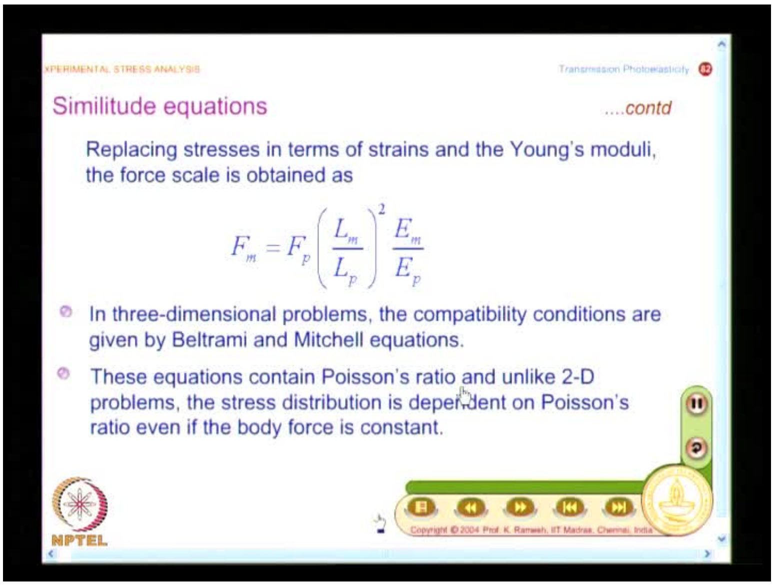


So, what we do is we replace the stresses in terms of strains and the young's moduli, the force scale is obtained as the force is need to apply on the model is related to the force coming on the prototype L m by L p whole square in to E m by E p. So, I bring in the elastic constant and obviously, when a prototype is steel it is about 210 gpa, when model is about 3 in to 4 gpa. So, that mean f m will be much smaller then what is the load that is coming on the prototype with the strength of this equation, what we find, the model as well as prototype when they are loaded they will have similar values of strain as well as deformation.

You do not want do have a large deformation in the model, otherwise the equation are not valid and here again, we will have to bring in a distinction, in 3-dimensional problems the compatibility conditions are given by Beltrami and Mitchell equations, a function of Poisson ratio. Unlike 2 d problems, the stress distribution is dependent on Poisson ratio, even if the body force is constant; that is emphasize, even if the body force is constant 3-dimensional problem is always difficult to handle. What you find here is by using this scale for finding out the force to act on the model, he maintain certain kind of similarity on the strain levels and mind you, this is the obtained with assumption that Poisson ratios are same.

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**Properties of Some Photoelastic Materials**

| Material | Stress Fringe Value $F_\sigma$ (N/mm/fringe) | Young's Modulus $E$ (MPa) | Poisson's Ratio $\nu$ | Figure of Merit (1/mm) |
|---|---|---|---|---|
| Polycarbonate | 8 | 2,600 | 0.28 | 325 |
| Epoxy | 12 | 3,300 | 0.37 | 275 |
| Glass | 324 | 70,000 | 0.25 | 216 |
| Homolite 100 | 26 | 3,900 | 0.35 | 150 |
| Homolite 911 | 17 | 1,700 | 0.40 | 100 |
| SL5180 | 33.62 | 3,275 | 0.36 | 97 |
| Plexiglass | 140 | 2,800 | 0.38 | 20 |
| Polyurethane | 0.2 | 3 | 0.46 | 15 |
| Gelatin | 0.1 | 0.3 | 0.50 | 3 |

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So, it is not going to be same between the model and prototype. So, we need to keep that also in mind, when you are looking at the model ray strain. We have looked at modeled prototype relations, we have also looked at the simulative equations, now let as look at what are the kinds of photoelastic material that you have. I have a quite a variety of them, we also see them physically and also see their properties and what I have here is I have an epoxy disc.

This is the disc made of epoxy and this will have a particular elastic property and I have another model, which is made of polycarbonate (Refer Slide Time: 47:54), this is of polycarbonate even by looking at difference it has a slight yellowish tinge and this properties are very similar in characteristics. On the other extreme, I have the polyurethane model and this also has a tinge of yellow, the difference is I can even compress it with my fingers it has such a low elastic model as I can compress it.

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You can see the deformation, when it is loaded; you can see the deformation, when it is compressed, and you can see my fingers are pressing that is so soft. So, it is very good for models for class illustration and also a good model to do finger nail test. So, when I put the finger nail, it gets compressed very easily. So, I can find out the sin of the boundary stresses by nail test without restarting to any external gadget. And you also have another model this is from a stereo lithographic process. So, this is something like s l 518 resin and I also have on other extreme, this is prospects; if you look at prospects and polycarbonate, they are both transparent.

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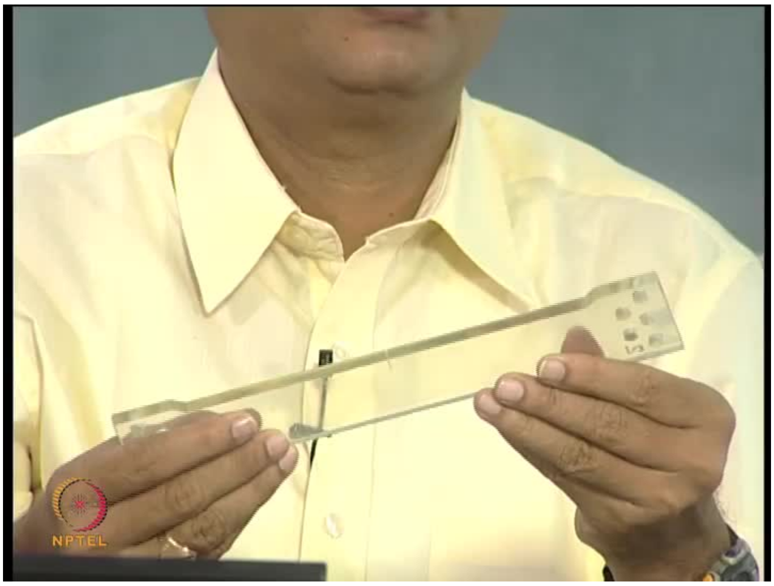

So, this is model of the plate with a crack and we will see the respective properties. You know long time back we have mentioned, when I want to find out isoclinic; it is desirable that is use prosperous model because it has a very high material stress fringe value. So, you will see isoclinic lot more clearly because isochromatic would not develop for smaller loads.

The properties are summarized here, you have some properties are listed, I have a stress fringe value, I have young’s modulus and I have a Poisson ratio and I also have finally, what is known as figure of merit (Refer Slide Time: 49:50). Figure of merit is nothing but, the young’s modulus is divided by the material stress fringe value and the figure of merit is larger, it indicates then is the good model for photo elastic analysis.

So, I have polycarbonate, we are seen polycarbonate and epoxy as similar; they have a polycarbonate has 8 Newton per millimeter per fringe. And this is obtain for a sodium source, we have seen that, it is the function of the wave length and it will also change from time to time and you also keep in mind, you calibrate the material and then use the property. Then I have an epoxy and glass have a very high value of f sigma and Plexiglas is comparable to the glass. So, it is about 140 and polyurethane, which I said that I can even applied load with a finger as a very low as sigma 0.2 and its young’s modulus, is also very small.

It is only 3 mpa whereas, for most plastic is about 3 gpa its only, its only 3 mpa its only 3 gpa for all other plastics and glass is like almost like aluminum in 70 gpa and figure of merit is listed here, it varies from 325 to 3. And if you look at gelatin it is very nice candidate, when you want to analysis the effect of body forces, if you are really looking at civil engineering construction, the use gelatin and make model out of it and bring in the body forces component in the analysis.

And normally you know, many experiment are conducted with polycarbonate and epoxy; polyurethane is good for a teaching purposes and this is also used in civil engineering for modeling layered soil. With recent advancement stereo lithography, you have this s l 5180 that has compare to the common photoelastic material, it has a high f sigma value and it has a figure of merit of 97 and if look at the Poisson ratio, the polycarbonate is much closer to material.

So, the polycarbonate is one of the most prepare modules the material for photoelastic analysis and if can also use epoxy because easy to cost in a laboratory and then prepare models and this gives you idea of what are various photoelastic materials and what we have seen in the class today was, we look at the importance of colour code, we said colour code the sequence is advantages to calibrate the polariscope, to find out the maximum or minimum of principal stress direction and also to find out, the sin of the boundary stresses.

We are looked at nail test; we are also look at how to find out the sin by external member then we moved on to model to prototype relations. So, one of the key learning we learned was in a 2-dimensional problem, when the body forces are constant, the stress and the distribution it is not effected by elastic constants, where as when you have body force varying or when you go to any simple 3-dimensional problem, the Poisson ratio of the model material dictates the stress and its distribution because, there is always different from a metallic prototype, there would be influence of Poisson ratio on the stresses evaluated and finally we will look at simulated equations and then you also had look at what are the various photoelastic model materials that are commonly used and what are the relevant properties.