Experimental Stress Analysis

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Module No. # 02

Lecture No. # 20

Calibration of Photo Elastic Materials

We have been discussing about transmission photo elasticity and I said one of the key parameters that need to be determined is material stress fringe value. As we use polymers, the material stress fringe value changes from batch to batch as well as over a period of time, there may be small changes.

And as material stress fringe value is the only parameter that relates the experimental measurement for comparison with analytical or numerical methods, you must take sufficient care to determine it with as much accuracy as possible. And I said one of the common models structure widely used is disc under diametral compression.

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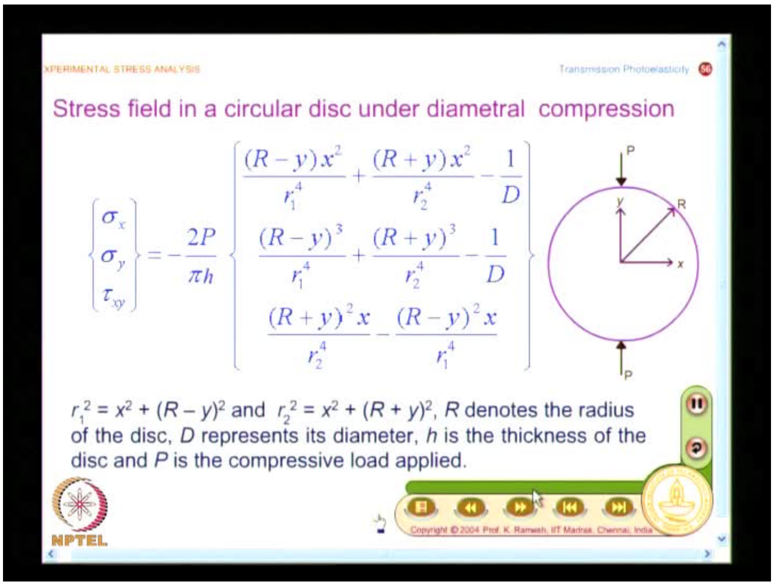

We also seen why we choose a disc under diametral compression; the first aspect is, it is simple to machine, easy to load. Since we have this stress field from theory of elasticity, it is also possible to compare the experimental result with analytical solution. So, in the process, we use this analytical solution to find out the material stress fringe value, because we need a model for which analytical solution is available so that you perform an experiment, find out the fringe order use the stress optic law, instead of finding out the stresses from the analytical computation, plug in the value of the stresses from that you find out the material stress fringe value.

In the last class, we saw the stress field for the disc under diametral compression and I have sigma x, sigma y and tau x y. These are the expression we also noted down earlier and we take the center of the disc as the origin and R is the radius. What I said in the last class was to find out from these expressions, the value of sigma 1 minus sigma 2.

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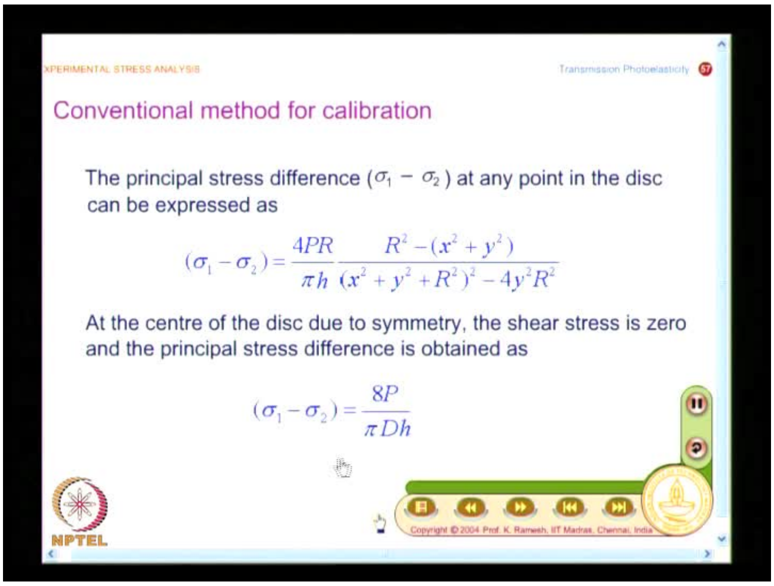

In fact, what we want is, we want to see principal stress difference and since, I had the expression for sigma x, sigma y and tau x y, it is simple for me to find out expression for sigma 1 minus sigma 2. Although for conventional method I need only the values at the center of the disc, I give this as the generic expression x and y with a purpose in mind. The idea is, we will do the conventional method for calibration and later, we will also elaborate on a method which will use as many data points as possible from the field. This is particularly useful with developments in image processing techniques, where acquisition of data becomes lot more simpler and also in 3-dimensional photo elasticity, where they have a stress freezing process, at the end of the process you may get either 1 or 2 disc with stresses locked it.

So, instead of just using the center which will give only 2 data points, you would like to augment the data points, from that point of view also, you need to find out a methodology, which uses several data point from the field. So, keeping that in mind, I am going to have the expression of sigma 1 minus sigma 2 as a function of x comma y and that is given as 4 PR divided by pi h into R square minus x square plus y square divided by x square plus y square plus R square whole square minus 4 y square R square. Once you have this expression, it is very simple to find out what is the principal stress difference at the center of the disc.

You just put x equal to 0 and y equal to 0, you get an expression and I want to simplify and also the expression is popularly written terms of diameter of the disc. So, instead of putting the value as R, the radius you express it as D by 2. So that you have a very popular expression and that is obtained as sigma 1 minus sigma 2 equal to 8 P by pi Dh as simple as that, what we are done here is popularly the diameter is used in expression.

So, I wanted you to replace R as D by 2 and from stress optic law we know, the expression of sigma 1 minus sigma 2 in terms of fringe order and material stress fringe value. Now the focus here is not to find out the stresses at the center of the disc, but to find out the material parameter.

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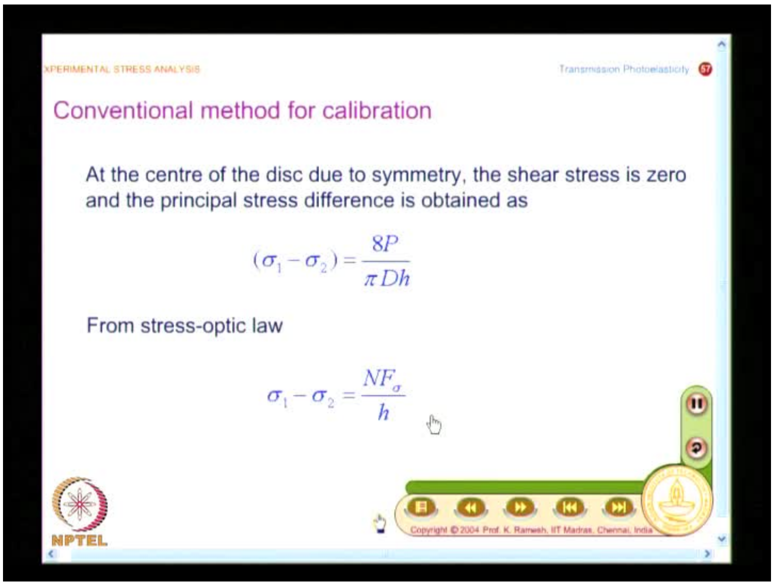

So, that is my focus; that is why I have taken the problem for which I have an analytical solution. The key point here is if you machine the disc perfectly circular, which is easy to do if you have a lathe and if you also loaded properly then, this comparisons can be almost exact so that, the focus is to find out f sigma as accurately as possible. So, you have a good model and you will also have to evaluate N and you have already seen compensation techniques to find out N with at least second decimal place accuracy. So, we can find out n accurately and we can also find out F sigma accurately from the experimental and we will modify this expression in a manner to directly find out what is F sigma and this what (( )) the stress optic gives you NF sigma by h, this we have determined as earlier (Refer Slide Time: 06:37).

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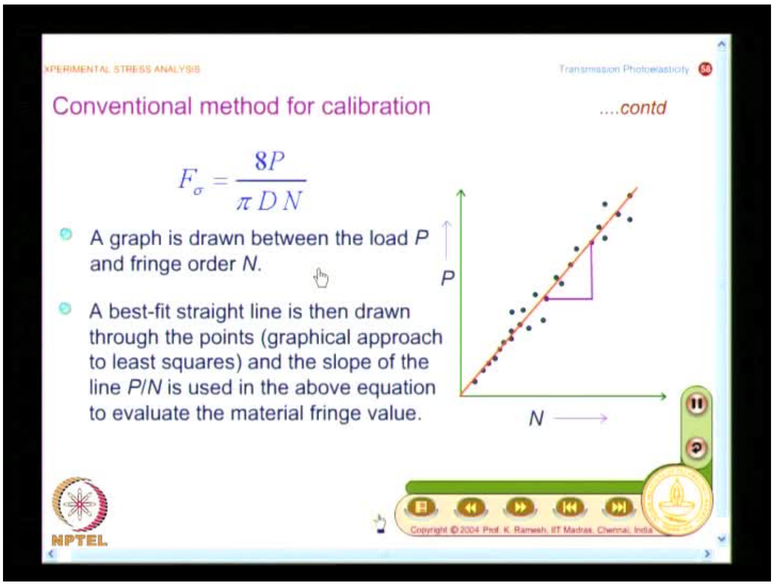

Now, combining these two expressions, I get an expression for F sigma as what is the famous relationship, what is that you have getting it? You work it out yourself, you work it out yourself, it is very simple and what you get here is 8 P by pi D N. What I have here is the very famous expression and the thickness of the model does not come into the final form of the expression; find out the fringe order N and you know the load that is applied, you know the geometric parameter of the disc, so I can find out F sigma (Refer Slide Time: 07:10).

I always mention that as experimentalist, you should not be satisfied with just one measurement; you must make as many measurements as possible. So, that you are able to bring in some kind of a statistical data processing and finally, arrive at the value. And in this case, what you can do is you can keep on varying the load and find out N for all these loads and then you can plot a graph and from the graph, you can determine the value of P by N, then you will have some kind of an averaging process.

In arriving at the value of F sigma and what we will do is we have a now plotted a graph between P and N and I have shown many data points, in practice you may not do so many experiments, you may use 7 or 8 experiments and then you will get 7 or 8 data points and this is shown to illustrate what I can do with this data point. First thing what you find is, there is scattered and scattered is possible in any experimentation; scatter is inbuilt in experimentation that is why this is emphasize, this scatter is emphasize little more you may not have so much scatter, some form of scatter will always exist in any experimentation.

As experimentally what you need to do, you need to make the best value possible one simple approach is I can draw a graph such that the points lie on either side of the graph equally. So, when I do that what happens? I have a graph which is drawn and this is how you have the graph (Refer Slide Time: 09:27) and I have points lying on either side and this process is nothing but, at least square evaluation by a graphical approach.

So, what I have done is I have drawn a graph, it is sensible to draw a graph such that points of scattered lie on either side of the graph evenly and what you have implicitly done is, you have actually done a graphical least square analysis. So, what we have taken the advantage is we have taken advantage of large number of data points; from a graphical approach, we find out what is P by N and once you plus plug in the P by N in this expression, I would get the value of F, F sigma as accurately as possible from a simple experiment and I can find out N accurately by tardy method of compensation, where I have to just rotate the analyzer, for each load I have to rotate the analyzer, find out the fractional fringe order.

I also mentioned earlier, in early days people had a very complex loadings mechanism wherein they will adjust the load so that you have a data point, you have the fringe passes through the center. The data point here is the center of the disc so they have to adjust the load to make the fringe passes through the center, instead if you are in a position to apply a compensation technique and tardy method of compensation is so simple, I just rotate the analyzer I get N accurate. Our focus is to get F sigma accurately, for us to get F sigma accurately, you must measure N accurately and then draw the graph and then from the graph, find out P by N and then use this expression.

This is so for so good, when you are able to do a live load experiment, where I can keep changing the load and find out what is the fringe order at the center (Refer Slide Time: 11:33). Suppose, I do a stress freezing which we would see later, at this point in time you understand, there is a thermal cycling process by which I can lock in the stresses inside the model. And when I lock in the stresses inside the model, I must also place a circular disc under diametral compression within the thermal cycling process, whatever the oven that I use, I must keep this loading mechanism inside, allow also a circular disc to pass to the same thermal cyclic.

Then finally, I will take out the circular disc and in that I have the fringe information, I have the fringe order at the center, even if a full fringe is not passing through the center, I can always find out by tardy method of compensation, but if want to have additional data then I need to have one more disc. For every data you need to have so many discs, on the oven will not have space to keep so many discs under diametral compression.

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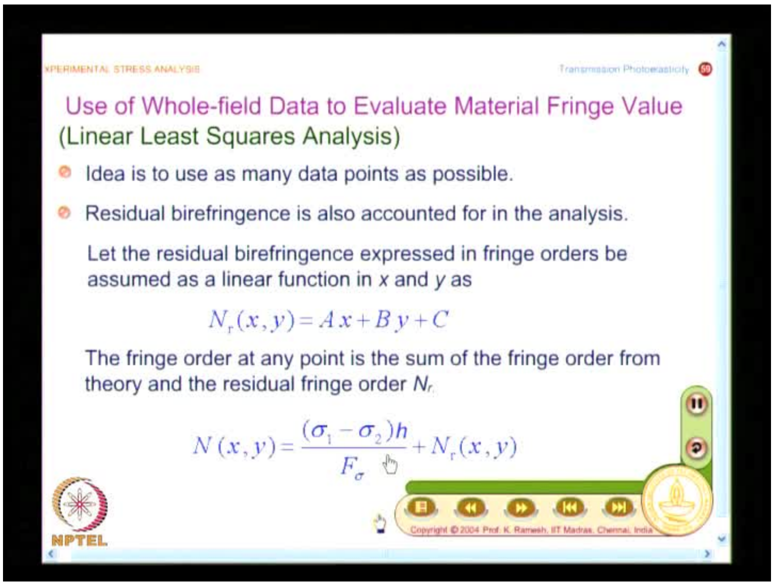

So, you need to think of a different strategy because I record the whole field information, why not I use the whole field information that is the focus. And you have that given as method of linear least square and we want to use whole field data to evaluate material stress fringe value and in this case, the resulting equations are essentially linear and I call this as linear least squares analysis. And the credit to introduce these kinds of methodologies goes to professors Sandford he was the first person to initiate this kind of a thinking in experimental mechanics. This was initially obtained for finding out disc under diametral compression then, it was used for fractional mechanics problems, finding out stress intensity factor and sigma naught x.

We would confine for the time being, how you can find out the material stress fringe value. Now I said that, you can use as many data as possible, what we can go about? Essentially we are going to use a computer to do all this processing, why not we also look at certain additional features to our analytical model. See one of the common problems in any one of this, we also looked at what is the time edge effect as the function of time you have spurious fringes, which we have to avoid for all practical purposes, but you may also have some amount of residual stresses locked in while casting a sheet.

Since I am going to find out the value of F sigma by processing the field information, I can also improve my model by also incorporating the residual by the fringes. Your mathematics may a finalize residual by the fringes is very close to 0 that is welcome, but since we loose the luxury of process in just one data point, we have going to work with the computer and why not we try to have a better model which is logically fine. So, we bring in the residual fringe effect also and that is what we will do and what is that we will do? Engineers are very happy with straight lines.

We will if you want to do anything first, we will first find out whether I can fit a linear graph if I am able to do, that is my result comes I am happy, only the result does not come I go for non-linear. So, one of the very simple aspect what we can do is when I say I will also model the residual birefringence in the analysis, a simplest assumption possible is ideal this as a linear variation, it will be a function of x comma y. My focus is only to find out F sigma, but I bring in one more aspect which I feel which is logically sound so that, I have a better analytical model to handle even situations, where I may have residual birefringence in advertently introduced.

So, what I have is I define a N R as a residual birefringence, which is the function of x comma y varies from point to point, I make this as A x plus B y plus C. So, I am introducing a new function in that case, what are the unknowns? I have F sigma is an unknown, that is the primary unknown that I have to find out and in the process of refining our methodology, we have also introduce the unknowns A, B and C if you have to understand. So, what I have now done is instead of evaluating 1 parameter, we need to find out 4 parameter.

And from your understanding of solving simultaneous equation, if I have 4 unknowns I need how many equations, I need 4. As many number of unknowns as many number of equations I should have, if I have less number of equations it is the problem, if I have more number of equations then also it is problem, I must have equal number of unknowns and equal number of equations.

Suppose, I have more number of equations and less number of unknowns, we also have methodologies to identify only the number of equations matching with the number of unknowns that we do by method of least squares. That is what we will employed here, see one way of approach is from the field you randomly collect lot of data points and simply take an average. The average may not be the right way to do it that is why I emphasize, when you have done a calibration by simple method you have drawn a graph, without your knowledge you have done a graphical least square analysis. So, similar thing we will also do in a situation, where I collect large number of data points that is the way I will develop the methodology and get the equations. Now what I am going to do?

We already have an analytical expression, what is the value of fringe order at a point of interest when x and y is specified. Now, what we say? To that you need to add a residual birefringence N R x comma y. So, that is how we will recast the basic equation. So, if you have fringe order at a position x comma y, it will have two terms; one term contributing it from your analytical expression, which is completely known. If I know the material stress fringe value and if I know the h is known, our focus is to find out F sigma and you can find out an expression from analytical method, what is the expression for sigma 1 minus sigma 2 and to this, we add at a point of interest a residual birefringence N R x comma y.

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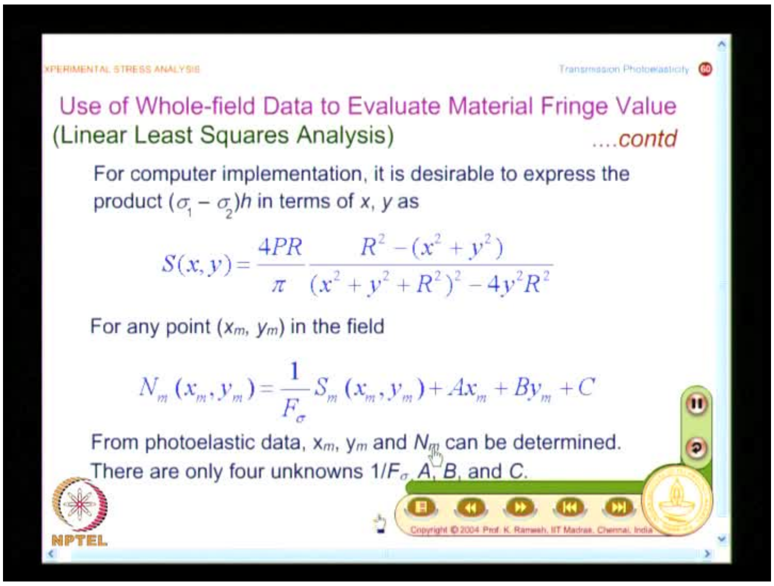

What I am going to do is, we already have an expression for this, we will replace this as the function of x comma y from the generic expression of sigma 1 minus sigma 2 that will be the first step then we will see how to coin equations so that, we do a least square analysis. So, what I have is we already know sigma 1 minus sigma 2 as the long expression and we rewritten as s as the function of x comma y which is nothing but, sigma 1 minus sigma 2 into h.

So, I have this has 4P R divided by pi multiplied by R square minus x square plus y square divided by x square plus y square plus R square whole square minus 4y square R square, instead of writing this complete expression every time, I will simply label it as s as the function of x comma y. So, for the point N I will write this as at the point m, N m is defined as one contribution from analytical expression, other contribution from assumed residual stress field; it is assumed residual stress field.

And from photo elastic data what you can find out? I can find out at every point of interest the value of fringe order N m, that is what is written here. The coordinates x and y and the fringe order at the point M can be determine from the experiment. And if you do a conventional analysis, you have to take a photograph and find out x y accurately and also find out the fringe order, but instead the method becomes advantages only when I go for digital photo elastic analysis.

So, keeping that in mind we will also have a brief discussion on how I can go about an extract these data by using digital photo elasticity. We will first develop the mathematical procedure, for the mathematical procedure to take advantage, we need to collect data conveniently and for collecting data conveniently, digital photo elasticity is a must; otherwise the method is not attractive. So, what I have now is you should recognize the unknowns are 1 by F sigma, coefficients A, B and C and what I have to do is I have to write an error function see in all our least square analysis, we need to write an error function and minimize that error function so that, you get the result which is the best fit for the given data points.

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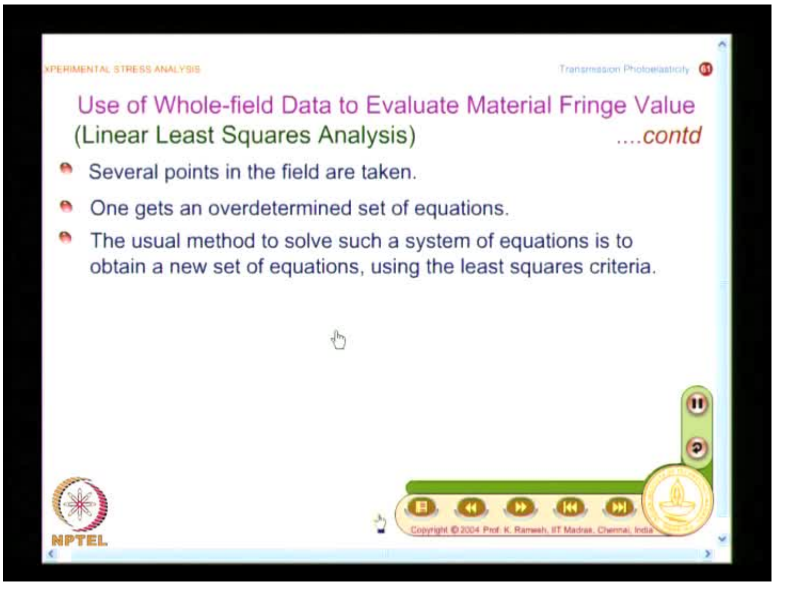

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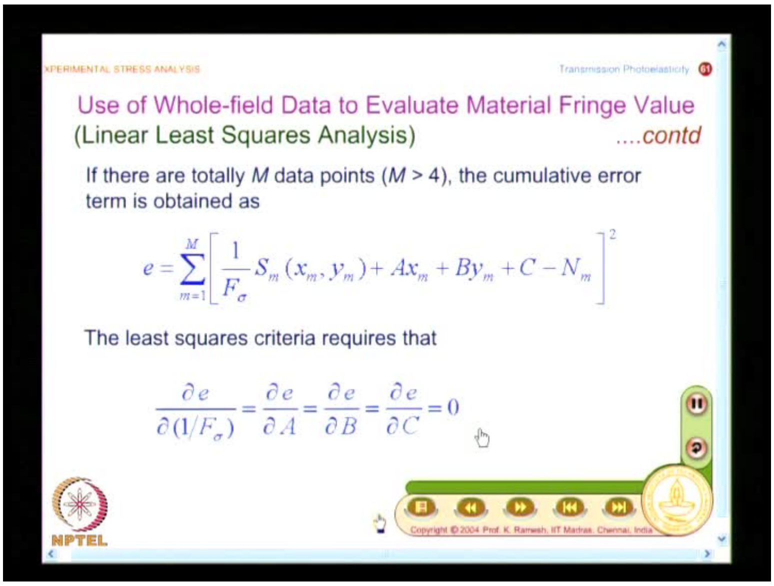

So, first step is we need to find out the error function. So, we define the error now and this is again emphasize we take several points in the field. And you get over determined set of equations and the usual method to solve such a system of equations is to obtain a new set of equations using the least squares criteria and for me get a new set of equations, I must first write the error term.

Because if I have found out all the coefficient correctly and if I take a data point, the error would be zero, because I have evaluated all the parameters correctly and it matches with the data point and I said in any experiment there would be some sort of scatter. So, instead error being zero, we will only minimize the error. So, the error function is very simple and straight forward, you simply subtract the actual fringe order at the point of interest which is experimentally determined.

So, I define a error as we will given the symbol as e and to illustrate the method better, we take M data points and we say M is greater than 4 because I said, you have 4 unknowns, since I have a whole field information available, I can take many data points and I will ensure that I have at least number of equations greater than the number of unknowns. And we will also finalized recommendations for this methodology to work how many data points are recommended. Though our ultimate aim is to find out only 4 unknowns, we would use many data points people will also developed a methodology called sample least square methods because finally, you should not get different values by taking different sets of data. So, when you do a statistical analysis, you must also develop the statistical procedure that, when you adopt the procedure you get one unique value for a given experiment you do not want to have multiple values.

So, you take the statistical methodology to its logical conclusion, so that we will see later. For us to write the error equation, we first ensure that M is greater than 4 if M is equal to 4 there is no need to write this error equation at all and that is not what we want because I may select 4 points, somebody else may select 4 other points and each one will end up with different result.

Here the question is not to get the result immediately, the question is try to get the result as accurately as possible using the whole field information, that is the focus. So, if write the error equation and the error equation is nothing but, I have this analytical expression what I have said is this is from your analytical expression; this is the residual birefringence we have assume and this is the fringe order experimentally measured at the point of interest. And what I do is I have a difference that is why I put a minus sign and take a square of it, suppose I have m data points, I sum all these squares because I am not worried about whether it is a positive error or negative error, I am only interested in the magnitude of the error.

So, I have this as sigma m equal to 1 to M, 1 by F sigma S m which is function of x comma y then, A x m plus B y m plus C minus N m. Suppose I want to construct see this will this is only one equation, when I look at the error is only one equation, suppose I want to employ the least square criteria, what is that I have to do? The standard procedure is you differentiate this expression with respect to the unknowns and make it equal to 0.

So, I will have I have 4 unknowns. So, I differentiate this expression with 4 of these unknowns and make them equal to 0. When you look at mathematically the process may look complicated, but in reality when you look at the final result, it is very easy to implement. See the mathematics may appear complex, but if the final procedure is not simple, people will not use this because the final procedure is very simple and easy to do this has become very popular.

Nowadays people find out material stress fringe value only by processing large volume of data, they do not just go by what you find out at the center alone, there are also reasons for it. See scientist when the level of methodology, they also come out and then say, in which class of problems this methodologies appropriate, why you should adopt this kind of a methodology.

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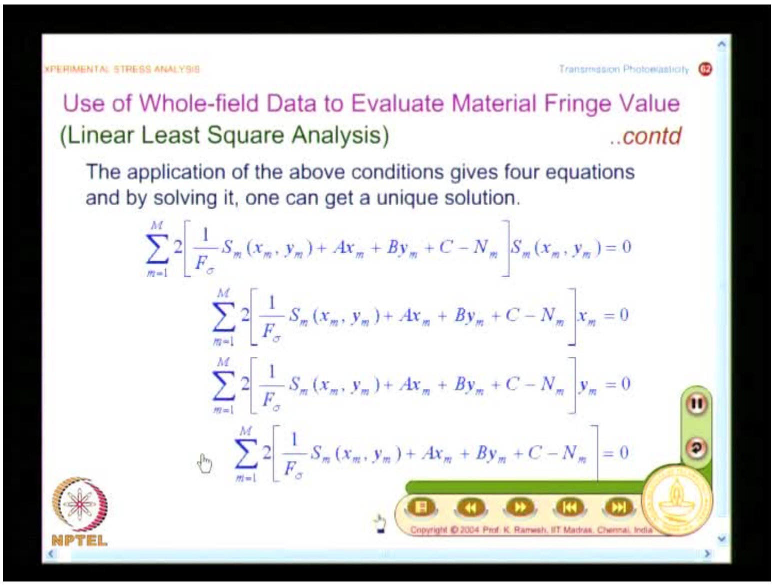

So, what I have here is my focus is to get the value as accurately as possible and I employ the least squares criteria and this criteria is nothing but, dou e by dou of 1 by f sigma equal to 0. Similarly dou e by dou a equal to 0 and I can write this as an expression like this. So, what I have now obtained is, I have taken a large number of data points, I written the error equation I reduce this as just 4 equations; I have 4 unknowns, I have 4 equations and this 4 equations are obtained by employing the least square criteria.

Now it is very simple, once you have this 4 equations you use a simple gauss elimination process, in one shot you got all these 4 parameters. But, how to write the final expression you have to differentiate, I want to do differentiate, I want to do differentiate because once you do for one case you will know how to do this for other cases. How to construct the equation that you need to know, because the idea here is we want to get a unique solution. For us to get a unique solution for multiple data points, we construct only the number of equations equal to the number of unknowns that is what we want to do it and this differentiation is very simple.

See normally you are differentiating with respect to x and with respect to y what do you have to recognize here is the unknowns are A, B, C and 1 by F sigma; that is what you have to do otherwise it is a child's play, it is a very simple expression and I am sure some of you have got it. The expression reduces to as simple as this, you recognized it for every case I will have this (Refer Slide Time: 31:10), this was a square. So, I will have two into this complete expression and differentiation of what is there inside and if I differentiate with respect to 1 by F sigma, this becomes simply S m x comma y, all the other terms goes to 0.

And once you have seen this, writing this for all the other 3 equations is simple and straight forward and what you will have do is, if you have done a course on indical notation by looking at this kind of expressions, you can recast this in a convenient matrix representation; that is what will make the life simple, the equations as such look unwieldy uncomfortable to handle.

The equations are not simple to look at and the method would not become popular, but for very simple implementation procedure. What are the other equations, when you differentiate with respect to a what I have, I will have essentially with this multiply by x m as simple as that. I will have this multiply by x m and if I do it with the b it will become y m, but what I want to think parallelly is when I have this 4 expression, how do I represent this as matrix representation? Parellely think about it, find out whether you are in a position to do it, even if you are not in a position do it, when I show you the solution go back and verify this solution is indeed correct, do not accept it as it is.

So, I have the third equation that is nothing but, I have this as y m and fourth equation will be just this, I will have only one here (Refer Slide Time: 33:16). So, what I found now is from m data points, I have written a error equation and I done the least square criteria by minimizing it to 0 and this results in 4 equations. Right now this looks unwieldy because I have summation of this, I will have so many terms in this series, but on the other hand if you look at as matrix representation, the matrix become very simple because you should understand how matrix are multiplied and if you have done an indical notation, then you would be able to do it.

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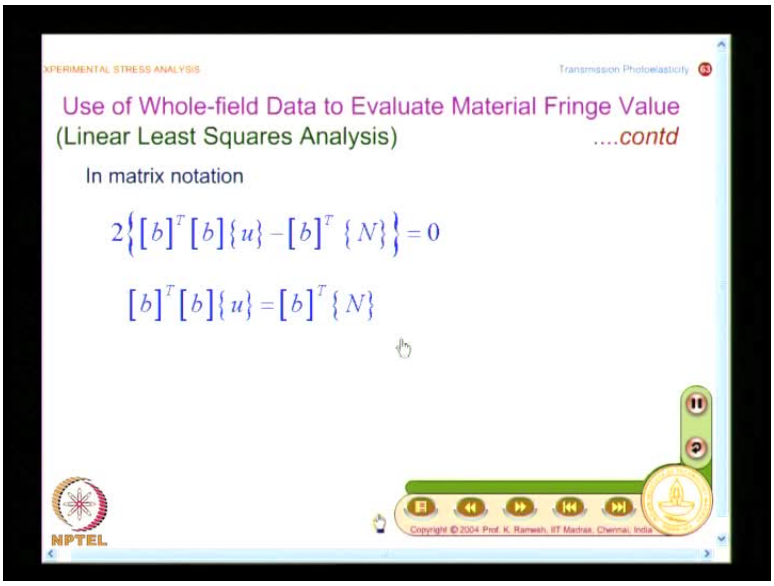

I would show you the answer for the benefit of the class, but I want to verify this solution; I want to verify this in your rooms, how to get this final expression. The final expression is very simple; I have this as b transpose b into u equal to b transpose N. To recognize the set of 4 equations in the matrix notation like this requires some reflection on your indical notation. If I have understanding of indical notation, you can quickly write it if you are not done the indical notation, you write several expressions and then see that this can be re represented in a convenient matrix form. And what is important here is, the matrices what you have as be b transpose u N are very simple.

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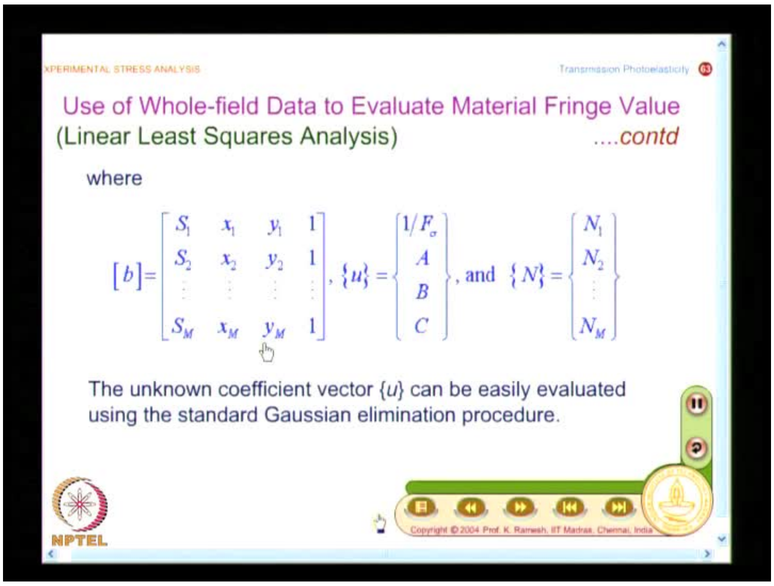

They can be directly written on from your experimental measurement, there is no difficulty at all and you have n number of Gaussian elimination procedures available. Solving this is also very simple, computer time is hardly anything and because there is a linear equation you have, you do not need any iteration just one solution gives you the final answer. But, recognizing the 4 equations into the matrix form is a bit involve not difficult, but I want to verify this and what this matrices b, N etcetera are very simple like this (Refer Slide Time: 35:53), please take down this, b is nothing but, S1 x1 y1 and 1.

So, what I am going to do is, I will find out experimentally for the data point x 1 comma y 1, I will find out N 1; similarly for x 2 y 2, I will find out N 2 and we already have an expression for S. So, for each of these data points, I need to plug in what is S 1, S 2, S 3, S 4 and S m. So, I can find out the matrix b comfortably, very simple and if you look at the vector u I have 1 by F sigma, I have A, B, C these are all the 4 unknowns and the vector N is nothing but, fringe order at several points.

So, experimentally I need to find out the fringe order at several points and it is associated coordinates, that is all have to find out experimentally and from an experimental point of view, I can determine them conveniently if I have digital photo elastic approach, but even manually you can do it. In fact, I have one of the assignment problems, where I have a circular disc under diametral compression with this fringes and I would expect you to extract these data manually.

So that you appreciate that advantage of a digital photo elastic approach, you can also do it manually, you can also do it by the digital photo elastic approach, manual procedure will take time and also can introduced human errors.

So, the idea here is you do not focus on only one data point and you take data points from the field and if you look at the literature, you see a contradiction. In the conventional method, you want to find out the fringe order at the center; in the method where you use whole data point, they recommend because it was tailor made for stress freezing approach, where when you do the stress freezing because the material become reaches it is critical temperature, the load application points will becomes flat and because of that in those applications, the center value does not match well with your analytical solution.

So, you need to avoid the center and take data, this is not the case for live load model; if you are using a live load model, center is also accept. If you are not using the live load model, where using the stress frozen model because the stress freezing process we get the load application points become flat, this method is advantages. When once people develop a method, they must also say under which conditions the method is required, why it is the advantage you have to look at it from the perspective.

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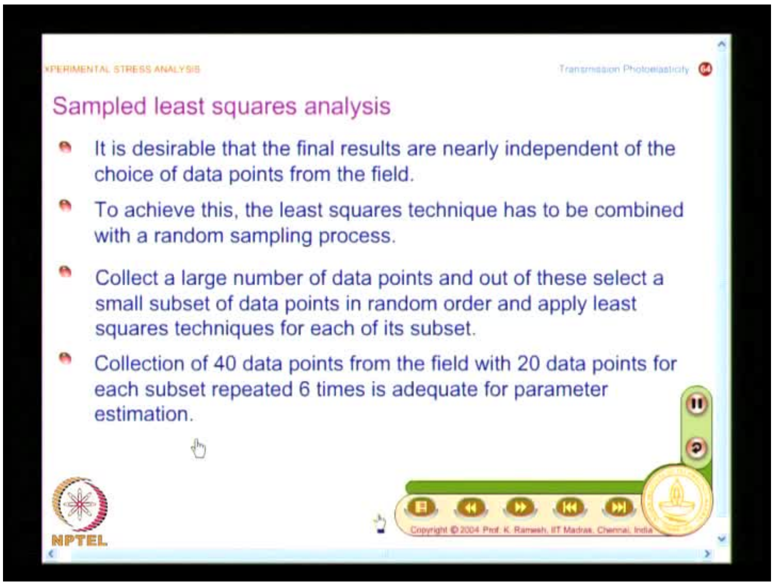

The method has become popular because I can construct matrix b, vector u and N very easily from measurements. And finding out the final result is very simple and straight forward, you can do it by several simple methods and I said when I am developing a statistical method, I must also ensure that I take advantage of the statistical processing completely.

So, that is why I call this as sampled least square analysis, because I want know preferred selection of data points; any set of data points which is select should yield be one unique value. The focus is the final results are nearly independent of the choice of data points from the field, to achieve this, the least squares technique has to be combined with a random sampling process.

This is very simple to implement, there are random number generators available so you can easily do that and what you basically do is, you collect large number of data points and out of this select a small subset of data points, you do this in random order and apply least squares techniques for each of its subset that is all you bring in your randomization.

So, you collect large number of data points, from that you take a small subset and this subset, select from this master data point randomly and there also recommendation how to statistically condition. So, what they recommend is collection of 40 data points from the field with 20 data points for each subset, which is repeated six times is adequate for parameter estimation; because in this case only 4 unknowns have to be determined, for four unknowns they suggest based on experimentation collect 40 data points and from the 40 data points at a time, randomly select 20 of them and repeat this process 6 times and finally, you take the average of this, you will have one unique value for the F sigma and this is called sampled least squares analysis.

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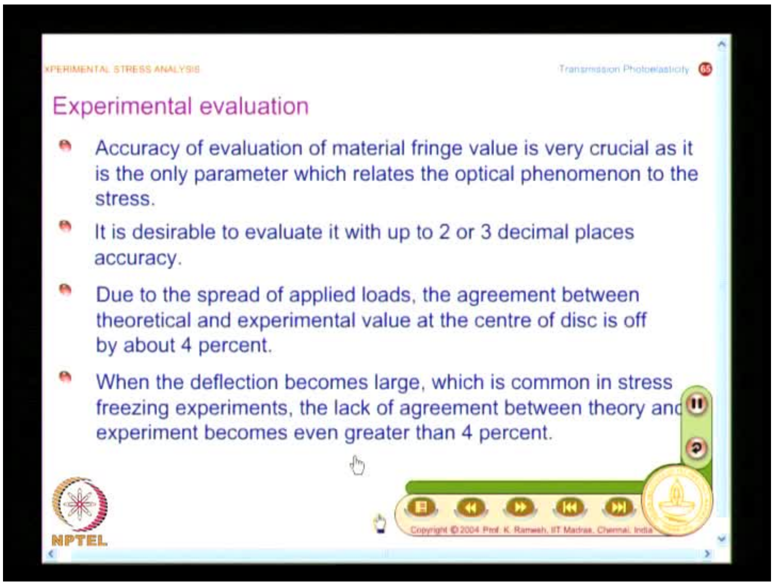

And this I have said earlier, I also emphasize many times F sigma has to be evaluated with desirable accuracy and I want to have 2 to 3 decimal places accuracy and I also mentioned, particularly in the case of stress freezing due to spread of applied loads the agreement between the theoretical and experimental value at the center of disc is off by about 4 percent.

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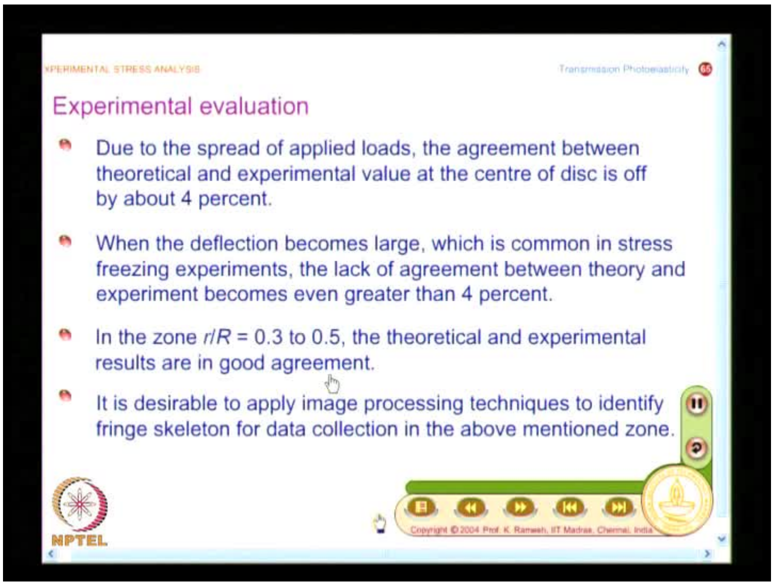

So, in order to improve your agreement, you exclude the data from the center of the disc that is the particular zone people have also identified and when I want to do all these, it is desirable that I go for digital image processing methodologies. So that is what I have here. So, the recommended zone is R by R equal to 0.3 to 0.5 the reason why I do this is particularly in stress freezing experiments, there is lack of agreement between theory and experiment at the center.

So, avoid the center of the disc and I take data in a region R by R equal to 0.3 to 0.5 is an annular region in the circular disc. And as I mentioned earlier, when I have to do all this when I collect large number of data points see manually what we will have do is, you will have to identify the center and then pick out data points, you may have to magnify picture, pick out data points and then do the calculation, it will be very time consuming.

On the other hand, you simply go and click the cursor at selected data points and your computer automatically understand x y positions and also the fringe order, don't you think itself very simple approach? But in order to do that, you need to have some background on what is image processor. So, we need to use image processing techniques to identify fringe skeleton and mind you here, one of the earliest development digital photo elasticity mimic what they did manually, they have not looked at fundamentally what is the requirement and how to go about.

We were finding out the fringe skeleton manually; now, let us find out the fringe skeleton by using a computer that is a way people have looked at it. And those methods are useful in certain applications, though you have phase shifting techniques which give you fringe order at every point in the domain, fringe skeletonization has it role in certain kind of problems. So, what we will now look at is what the basis of this image processing techniques is.

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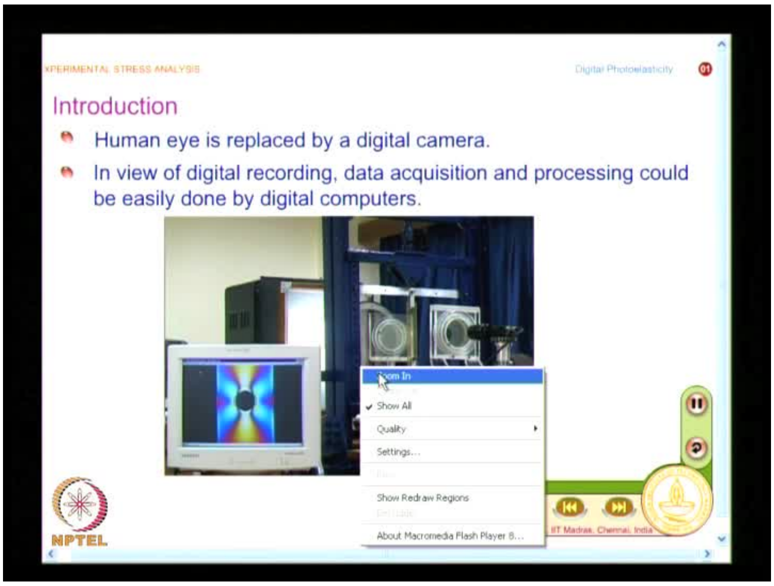

Our focus is to find out the data in this annular zone, but for me do that we need to know certain elements of digital photo elasticity which would be of interest to us. So, what we do in digital image processing? You replace the human eye by a digital camera and this is what I have here (Refer Slide Time: 45:30). What I have here is, I have a basic polariscope and instead of a human being viewing the fringe pattern, I have the CCP camera and I have this model plate with hole and I see beautiful fringe patterns on the computer.

So, what you see here is a human eye is replaced by a digital camera and you do a digital recording and data acquisition and processing could be easily done by digital computers. And you call this whole branch of photo elasticity as digital photo elasticity, but now we have to understand how an image is represented as array of numbers. I have a beautiful animation that animations itself tells you, what is the sequence in digitizing the image. The greatest advantage is the hardware has so developed, you can take this digitization in real time, you have about even a normal camera can give you 30 frames per second and that is what you have here video range is called.

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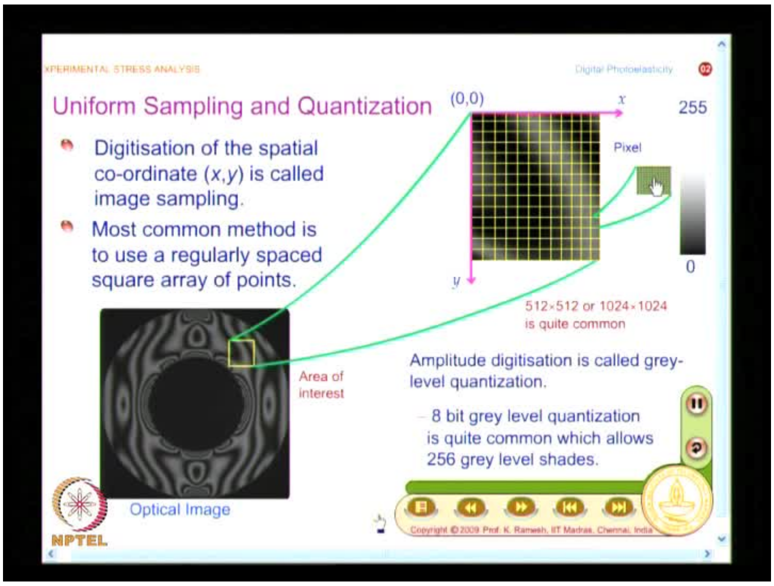

So, what I see here is this is called uniform sampling and quantization (Refer Slide Time: 46:53). So, we do a digitization of the spatial coordinate, when I do a digitization I called that as image sampling. After image sampling, I do a quantization and what do you have here is the most common method for digitization is a regularly spaced square array of points and what I have here is an optical image and what you have here is I have an optical image and for illustration, a small sample area is taken. And what you do for this small sample area? If you look at this I have this, I have this fringe pattern enlarged, this is further divided into smaller area, which are further divided into small areas and this is called a pixel.

So, what I have here is a very small element you find out. So, I have a spatial discretization of the domain as assembly of pixels and the pixel is very small and what you also see here a bar which goes from pitch black to pure white and you label this as 0 for pitch black and for white, you label it as 255. So, what you are going do is for each of these pixels we are going to assign a number between 0 and 255. So, what you get the result? The final result is the whole image will be available as a array of numbers and note down this axis, you have this as origin; all the monitors you have this as origin and x is define like this and y is define like this, y is positive down wards. For all there are graphics applications, this is how you will have the screen origin and when you design your own software to plot, you must take this into account and match it with your suppose I want to plot this circular disc and diametrical compression, where centre is taken as a origin.

So, you should know what is the origin and digital screen, what is origin in your physical problem and use it appropriately while plotting. These are very simple things, but these simple things also you should know otherwise, you get stuck. And what you have here is the spatial discretization could be an array of 512 by 512 or 1024 by 1024, now you have much higher spatial resolution have come, what is quite common is 512 by 512 or 1024 to 1024.

Once you have a pixel element which is abbreviated as pixel, you are providing a number between 0 and 255 and this has come from 8 bit grey level quantization, if I have 16 bit I will have a much more division but, 8 bit is very common. So, the amplitude digitization is called grey level quantization.

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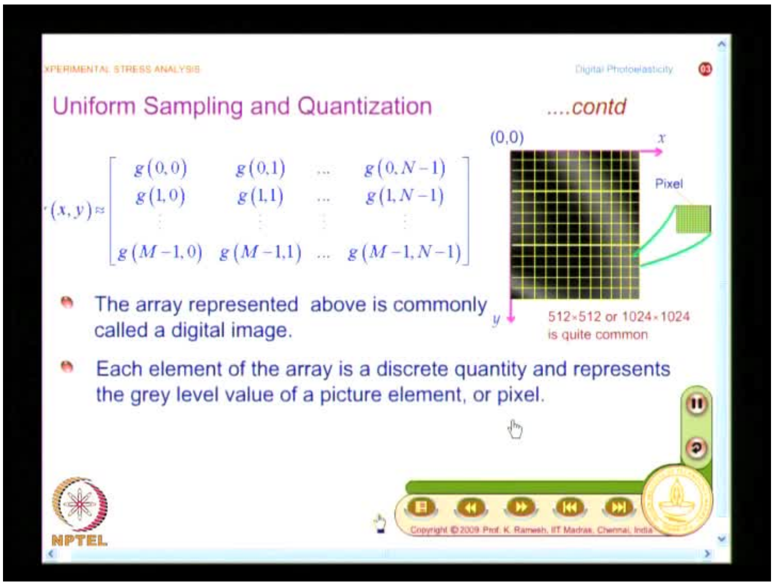

So, I essentially have a number between 0 and 255 representing this image. So, that is what you have in a next slide. So, what I have is when I use a digital camera, I essentially get a matrix consisting of integer. So, what I have is, I have the image available as numbers. So, then I do number crunching, I can extract the features I can see the intensity variation much more closely all that is possible. That is how the digitization is the very key and I am also a position to record the intensively data though in initial development of digital photo elasticity, people only worried about fringe skeleton. Even in fringe skeleton people had binary based methods and as well as intensity based algorithms, and intensity based algorithm perform much better than binary based algorithm.

Later on people had paradigm shift, where they directly recorded intensity data, processes this intensity data to find out fractional retardation and theta every point in the domain. The key to all the development is first digitizing the image and this digitization you can it in real time, that has made the technology very attractive for you to employ in photo elasticity and you have digital photo elasticity that came up.

So, in this lecture, what we had seen was we are looked at conventional method to find out materials stress fringe value. In that we actually found out fringe order at the centre of the disk for various loads and we collected the data in the form of graph and we do a line which passes through the points such that, the points lie on either side evenly and I mention this is the graphical least square approach. Then we said we would not worry only about one data point from a disk, since we record photo elastic fringe which is basically a whole field method, why not I use data from every point in the field? For that we said, we are getting in to a over determined set of equations because the parameter to be determined is only 1 F sigma and if I collect large number of data points, we also felt why not to bring in one more aspect namely the residual birefringence also evaluated as path of your experiment.

So, we brought in 3 more parameters A, B and C. So, finally, we have to find out 4 parameters, but we may end up taking 40 data points that is all the recommendation we saw and we also said that, we will go for a sample least square analysis. So that the final value of F sigma is independent of the choice of data points that I take and I also mention this entire mathematical development looks fine, but from implementation point of view, if you do it manually it looks cumbersome. On the other hand, if you go to digital photo elastic approach, collection of positional coordinates and fringe order becomes lot simpler. And for us to appreciate how digital photo elasticity functions, the basic aspect you need to understand is how the image is digitized, how an image is represented as an array of numbers. So, we have looked at what is sampling and what is quantization.

So, at the end you have the image available as a set of numbers. So, in the next class we will see how to extract the skeleton from such digital images, we will have only a very quick overview of it, we will not get into much of the details, will get in to a quick overview of it then proceed with conventional photo elasticity. So, in between the lectures, I would try to give some aspects of digital photoelasticity and that is how we will also get introduced how conventional photoelasticity could be viewed from a different prospect, thank you.