Experimental Stress Analysis

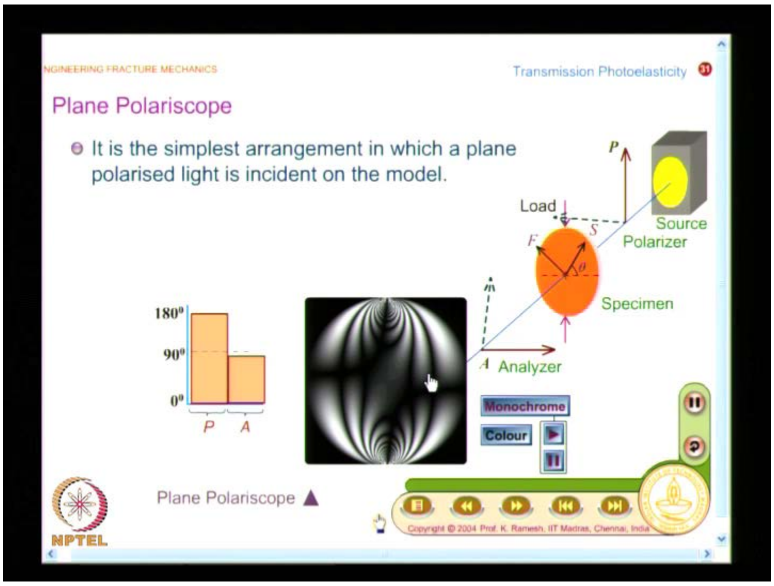
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module No. # 02
Lecture No. # 16

Jones Calculus

Let us continue our discussion on transmission photoelasticity. We have looked at from crystal optics point of view, we need to find out how photoelasticity as a technique developed. And we looked at that the model is temporarily birefringent when the loads are applied. So, you were able to relate aspects of crystal optics, and we combined stress and optics, and understood what stress optic law is. The stress optic law famously gives sigma 1 minus sigma 2, and if we want to find out that, I need to find out the fringe order and the materials stress fringe value f sigma. So, we have looked at in the last class, sigma 1 minus sigma 2 is N f sigma by h and I said, you need to find out N, for finding out N, you need to go for optical arrangement, and then you will also have to find out for the given model material what is the material stress fringe value. If I know N and if I know f sigma and if I know the thickness of the plane model, it is possible for me to find out using mechanics of solids, inplane shear stress as well as normal stress difference, if I also know the principle stress directions at the point of interest. So, this we had seen.

(Refer Slide Time: 02:01)



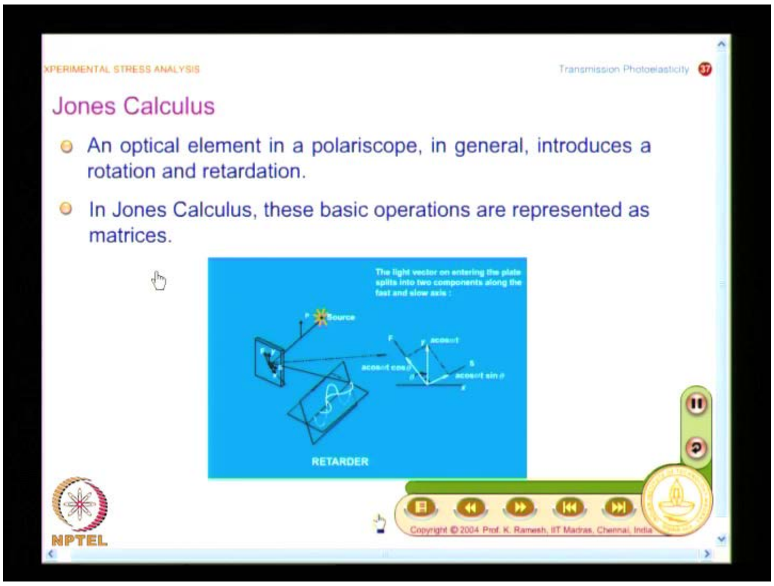
Now, the next aspect was that we need to go and find out the fringe order and we need to have an optical arrangement. We started with a plane polariscope. In a plane polariscope, what is that we saw? We had a light source, then we put a plane polarizer, then you had a model which is loaded, and you analysis whatever the light that comes out of the model by an element similar to polarizer, but the axis is different. Because you do the analysis of exist light ellipse, you call this as analyzer.

And what we saw? In general in a plane polariscope, you would see two fringe contours. One fringe contours moves as the polarizer and analyzer crossed positions are moved, and you have another contour which is stationary. And we have also looked at, suppose I increase the load, the contours that are stationary will move, because they are effect of the load applied. And we have also looked at, from principles of solid mechanics in linear elasticity, the principle stress direction do not change, only the magnitudes, you have tan 2 theta equal to 2 tau x divided by sigma x minus sigma y, when the loads are increased, the individual magnitudes of shear stress and normal stress vary, but theta as such will remain constant. And this, we also understood this could be used advantageously even in a plane polarizer scope with monochromatic light source. How to distinguish a contour being isochromatic or isoclinic. You can rotate the polarizer analyzer crossed and if the contour moves, then you call it as isoclinic. Suppose I increase the load, if the contours move, then I find those are isochromatics.

Subtle difference comes when you are dealing with zeroth fringe order. And what we did in the last class? We had analyzed what the light as it passes through the model, how it comes out after the analyzer. And we said intuitively, because I see two fringe contours, the intense equation should be a function of two parameters, one is the relative retardation delta, another is principles stress orientation theta. And what we did was, we did a simple trigonometric analysis. And what is it that we did? We had this polarizer, and after the polarizer you get plane polarized light which hits on the model, when it hits the model we look at these entry plane and exist plane. On the exit plane, the model introduces a retardation which is indicative of the stresses developed at the point of interest, then whatever the light that comes out you analyze it using the analyzer. And we found out what is the light transmitted by the analyzer, which was a function of delta as well as theta.

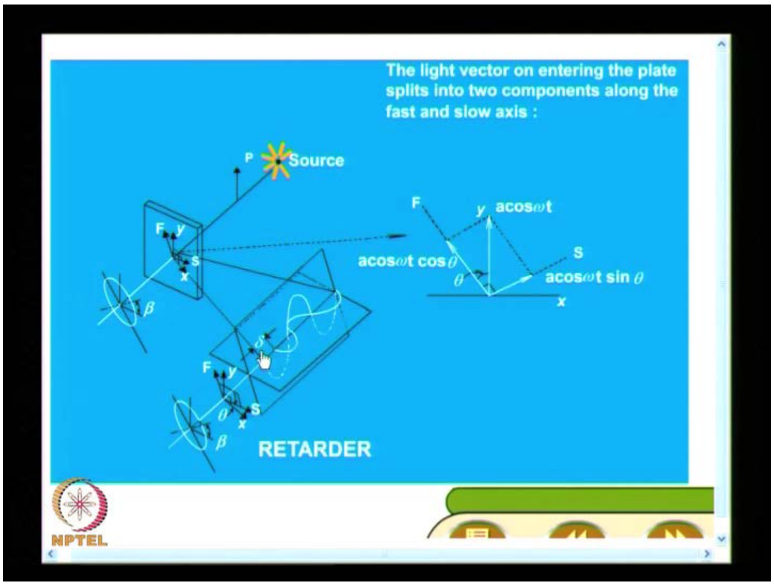
And in this case, you had only simple optical element, a polarizer, model and an analyzer. So, trigonometric resolution was sufficient and convenient for you do it. And we also saw that we are going to look at circular polariscope. In a circular polariscope, I need to have two more optical elements. I need to have a polarizer, a quarter wave plate, model, another quarter wave plate and another analyzer. So, I have more number of optical elements. When I have more number of optical elements, employing trigonometric resolution could be cumbersome. You will still get the result, that is not an issue, you can always get the result by trigonometric resolution, but it is better to develop an appropriate mathematics, which helps you to analysis when I have more optical elements in my experimental setup. And this is known as Jones calculus.

(Refer Slide Time: 06:19)



And what is Jones calculus? We will have to understand first of all that an optical element in a polariscope in general introduces a rotation and a retardation. And our interest is, you want to have this as matrix operators, with that the analysis becomes lot more simpler.

(Refer Slide Time: 07:03)



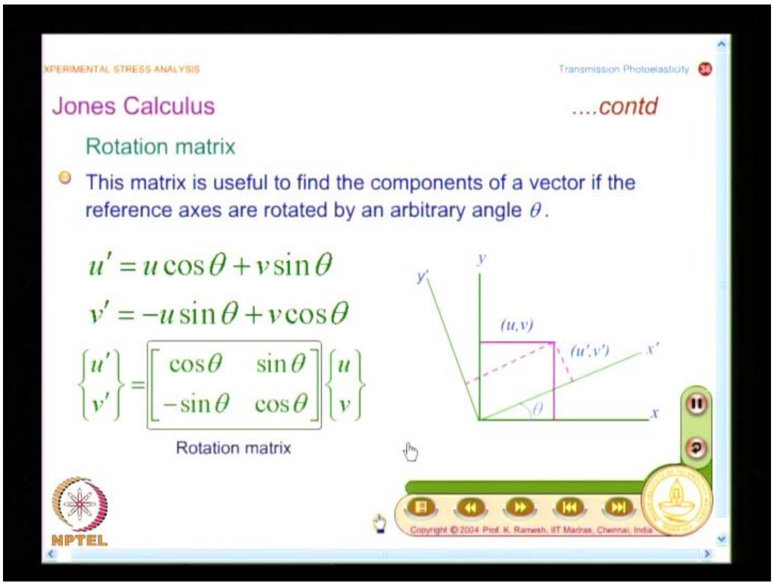
So, this is what we want to look at. The basic operations are represented as matrices. And I said in one of the earlier classes when we introduced this, we are going to see this animation again and again in the course, and we had actually analyzed a crystal plate, and what we want to look at is, when light impinges on this, what happens within the crystal, we have schematically shown with a larger sketch, what happens on the plane on which the light hits the front surface of the crystal plate, and what happens within the crystal is shown here. And if you look at what happens on the front surface, I have the reference axis as x and y, and the crystal we have reference axis as slow and fast, which is rotated at an angle theta.

So, what you have is, whatever the light that is represented with respect to xy axis, they have to be represented with respect to the slow and fast axis of the crystal plate. When you are looking at the model, it is the slow and fast axis at the point of interest which coincides with the principle stress direction, and whatever the retardation introduce within the model is shown here schematically. And this retardation is a function of the stresses developed in an actual model. In a retardation plate, it will be a function of the thickness of the plate and also the refractive indices of the ordinary and extraordinary rays, that is what we had seen.

So, what we need to do is, if I want to represent this crystal plate or the model which is loaded behaves like a crystal, we want to replace this by one matrix, which represents a rotation; another matrix, which represents the retardation introduced. So, if I understand these two in matrix notation, it is possible for me to give a mathematical representation of how the retarder can be represented. So, the moment I do this, if I have several optical elements, it just to put those matrices in the order in which they are arranged, multiply them and you will be able to comment on what is the intensity of light transmitted.

So, what we are looking now is, we have to look at what the retarder has done the function. See, we have looked at by impinge a plane polarized light, that is a very specific situation. In a generic scenario, what you are going to look at is, you are going to have light vector represented with respect to x and y axis, and the model as such has reference axis as slow and fast, which is oriented in general at an angle theta. So, I do a rotation when it enters the model, within the model it acquires a retardation. So, these two operations I would like to first get the matrix representation. If I do that, it will make my life a lot more simpler. So, that is the goal, we will look at.

(Refer Slide Time: 10:45)

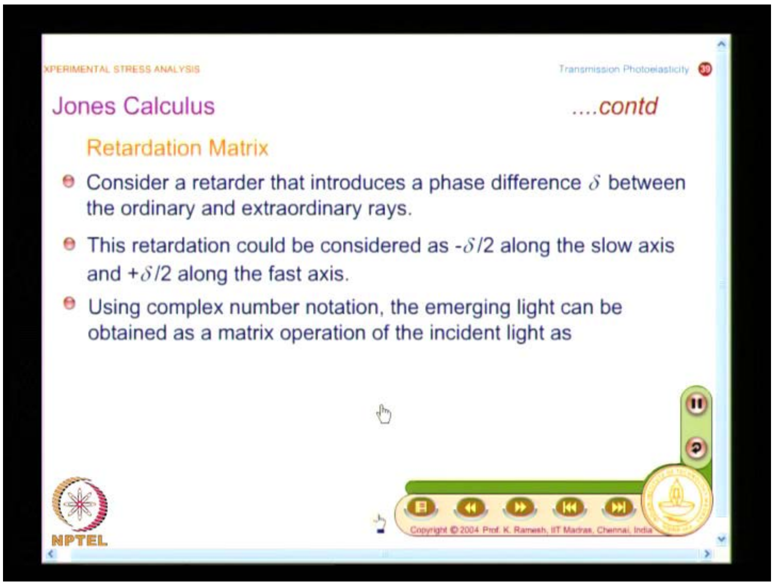


So, what I have is the rotation matrix, and rotation matrix you would have done in many of your other engineering courses. Whenever you come across transformation, one of the simplest transformation is rotation. To find the components of a vector, if the reference axis are rotated by an angle theta. And this is fairly simple and straight forward, it is only remembering what you have done in some of your earlier courses. So, what I have here is, I have the reference axis x and y, I have a point labeled as u comma v. So, I can find out u, I can find out v. And what I want is, I have another coordinate system at angle theta, I have this as x prime axis and I have the y prime axis just perpendicular to this.

So, the transformation is simply obtained by finding out the expression for u prime and v prime from simple geometry. That is all you need. This you would have done in your earlier course, it is not something new, I think you can brush up your old memories and write on your own, whether you can write the rotation matrix, or in other words find out what is u prime, what is v prime in terms of u, v and theta. And you need to simply apply geometry here and then get the relevant components. It is not difficult. If you start writing all these in the class, then revising your notes become lot more simpler. It is only remembering what is it that you have done in your earlier courses and it is fairly straight forward. So, I am going to have u prime as u cos theta plus v sin theta and I have v prime as minus u sin theta plus v cos theta. And this you would have done in your earlier courses, it is only recalling what you have learnt. And this could be represented as a matrix operator, what we are going to do is, I know u comma v, I want to find out u prime v prime, and this can be obtained by pre multiplying u v by a matrix cos theta sin theta minus sin theta cos theta. So, I have this rotation matrix.

So, what I have is, when the light enters the retarder, I would put this rotation matrix to find out the components of light that passes through the slow and fast axis. So, I will have a rotation matrix. And you have to be very careful about what is the notation that you are using, theta, how it is defined. See, in our development we define theta as the orientation of the slow axis with respect to the horizontal. One can also develop with respect to fast axis, one can also develop with respect to slow axis. We will consistently find out theta as the orientation of the slow axis with the horizontal. We need to have a sort of a convention, we use this type of convention for all our mathematical development.

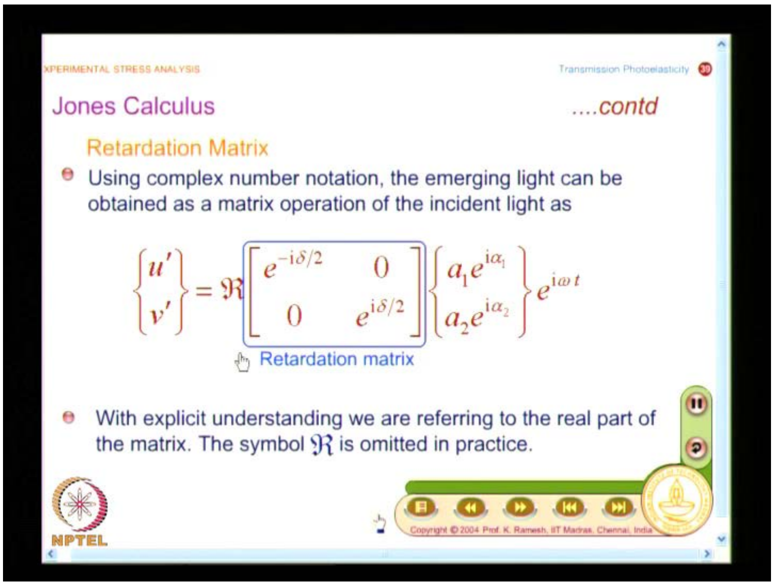
(Refer Slide Time: 15:09)



So, now you have the rotation matrix. We will also try to find out how to represent the retardation introduced within the model by a retardation matrix. And what we want to do? We will consider a retarder that introduces a phase difference delta between the ordinary and extraordinary rays. And how we introduce this delta? This can be written down in many different ways, and I said that retardation could be considered as minus delta by 2 along the slow axis and plus delta by 2 along the fast axis.

So the idea here is, we would like to have a nice matrix representation and we will follow some convention to facilitate how we analysis different optical arrangement. If we develop some equations to start with, the same set of equations could be used if we follow the same convention, that is the idea behind it. And we would like to have slow axis as orientated at angle theta. And what we will do is, we will use complex number notation and we will represent the emerging light, this makes our life lot more simpler and that is what we will do. And when I do this, I can represent the retardation introduced within the model comfortably as like this.

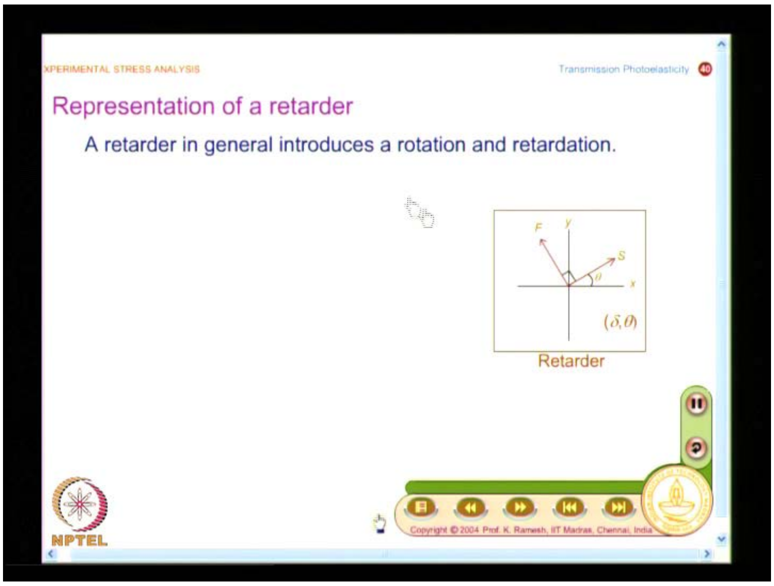
(Refer Slide Time: 16:49)



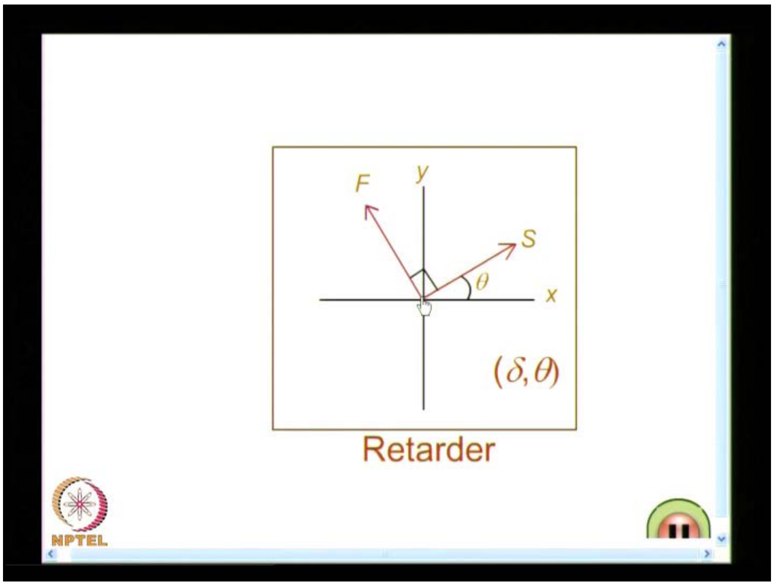
So, what I have here is, I have u prime v prime which is equal to, we take the real part of e power minus i delta by 2 0 0 and e power i delta by 2. And why I have e power minus i delta by 2 here? Because I understand that this is the slow axis and this is the fast axis. So, because I write by retardation matrix with the first element as e power minus i delta by 2, I should always represent theta referring to slow axis with respect to the horizontal. And this is the general light that is impinging on the retarder, this what happens inside the retarder. Within the model or within the crystal plate, you have ordinary and extraordinary beams, acquire a phase difference of delta, which is represented as one half split to the slow axis, another half split to the fast axis. And what we do in all our mathematical development is, we will remove this R, we will not write that explicitly for convenience. We understand that we deal only with the real part and we will simply have the retardation matrix written as such.

So, what we have now seen is, we understood a retarder in general introduces a rotation and a retardation. And rotation matrix, you have seen in many of your earlier courses, it is nothing new. When you are having a vector, if you want to represent the vector with respect to another axis which is rotated, you use the rotation matrix, the same rotation matrix comes here also. And within the retarder, because of the, if it is a model, it is because of the loads that is applied, the waves acquire a retardation, and if it is a crystal plate, you may have a quarter wave plate, you may have a half wave plate or you may have a full wave plate to fill in your optical arrangement. So, it will give a particular value of retardation for a given wave length. You understand that we do all this development for monochromatic light source. We have already seen retardation is a function of wave length. And our mathematics will become simpler if I confine my attention first to monochromatic light source, later on you can develop, if time permits we will also develop, see what happens when I have a multiple wave length.

(Refer Slide Time: 20:02)



(Refer Slide Time: 20:12)

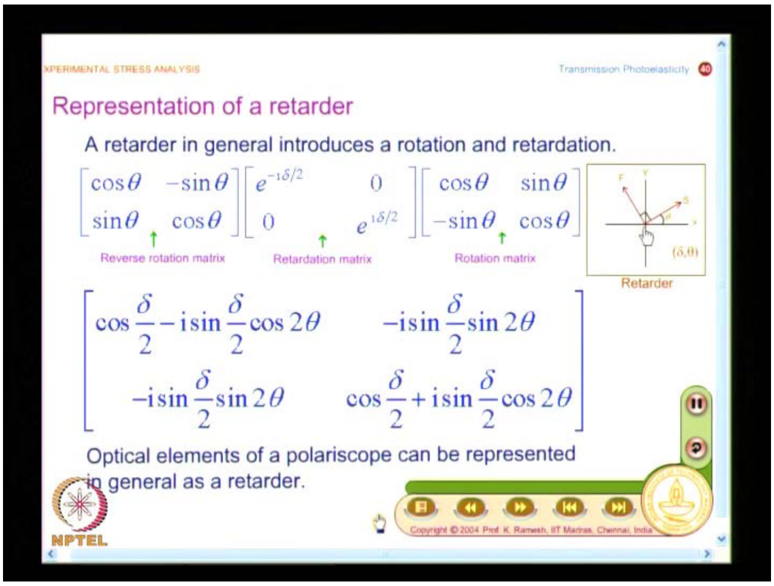


Now, what we will look at is, these two are elements only, now we will have to understand what happens in a retarder, how do I mathematically represent a retarder, that is what we will see now. So, what we are going to do is, we are going to have representation of a retarder. And here I have given for clarity, when I take a retarder, I will recognize a retarder having two axis, I will label one of them as slow and one of them as fast. And I will find out the angle theta of the slow axis with respect to the horizontal, my reference axis is horizontal x and y. And I will always represent the retarder by two parameters, what is the retardation introduced and theta.

So, delta is the retardation introduced by the retarder and theta is the orientation of the slow axis with respect to the horizontal, this you should never forget. Because when we develop mathematics, we need to follow some convention, we are only representing a convention here and this convention we will consistently follow for all our analysis of other optical arrangements as well. And what you will have to know? I have this as a reference axis, the model has slow and fast as the reference axis, but I would finally like to have reference axis as only x and y, and within the model I have a retardation introduced, but this retardation also I will introduce only along the slow and fast axis. These are all subtle points, see if you understand this now, when we go and develop compensation techniques, why we do the steps involved in a compensation technique, you will immediately understand, carefully listen to what I say. When you take a retarder, I have a reference axis, slow and fast, we introduced minus delta by 2 along the slow axis, plus delta by 2 along the fast axis and it comes out as a light ellipse. When it comes out as a light ellipse, I will still want to have this represented with respect to the x and y direction. So, retardation within the model is introduced only along the slow and fast axis, only then I can do that, that is what the retardation matrix explicitly gives you.

So, whatever the matrix operation I do, I must rotate it back to reference axis. So, if I want to mathematically represent a retarder, I need to have three matrices to represent a retarder with x y as the reference. Is the idea clear? I will repeat again and you will also do the mathematics behind it, then it will become crystal clear that this is what will fully represent the action of a retarder if you have x y axis as the reference. I do not want to keep changing my reference axis from one element to another element. Element has a reference axis, that reference axis we said, one you can label it as slow axis, one you can label it as fast axis. Though you do it arbitrarily, when you are actually making a polariscope, you can match your mathematical development with the polariscope that you have. That is called calibration, that is required in digital photoelasticity, that may not be required in conventional photoelasticity because fast and slow axis do not play a major role in conventional photoelasticity. But when you learn the photoelasticity subject in 2010, you will have to know aspects of digital photoelasticity as well. So, it is better that you learn the knowledge which will fit into the digital photoelasticity comfortably.

(Refer Slide Time: 24:17)

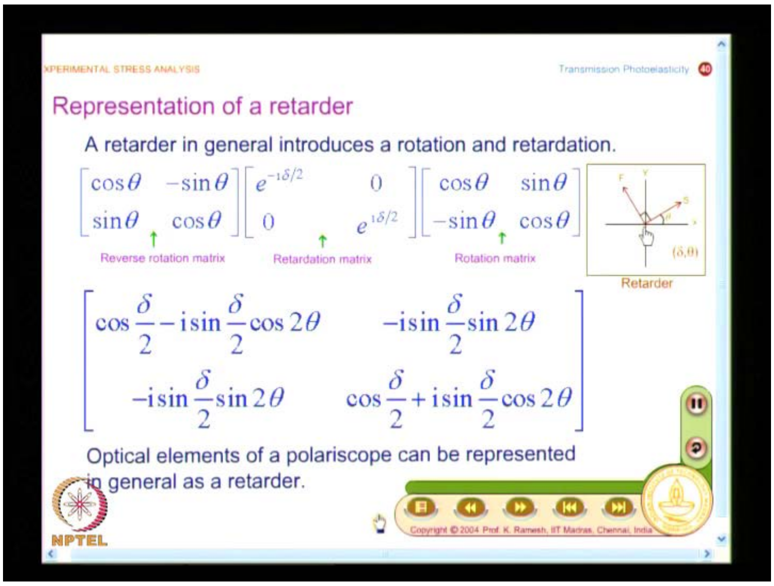


So, what we are now looking at is, I have a retarder, I have a reference axis x comma y, the moment light hits the retarder I will have a rotation matrix, then I will have a retardation matrix, then a reverse rotation matrix so that whatever the exit light ellipse is referred back to x and y reference. So, this is what I am going to do. So, what I am going to have is, I am going to have a rotation matrix. So, we will right one after the other, also look at the animation, you will have this appearing in a particular sequence, that is how light gets modified in a retarder. When it hits the model, you have this rotation matrix cos theta sin theta minus sin theta cos theta, then within the model it acquires a retardation, and this retardation matrix and rotation matrix are interrelated. Because I say theta refers to slow axis, because it refers to slow axis, I put retardation matrix in this fashion, I put e power minus i delta by 2. If I had referred theta with respect to fast axis, this would change to what? This would become e power i delta by 2. Because I refer theta with respect to slow axis, I put this as e power minus i delta by 2, then fast axis will give e power i delta by 2, and this would be referred with respect to what, I have already rotated with respect to the reference axis of the retarder, I introduced retardation only along those reference axis, so this definition will have only the crystal plate or model reference axis.

But we want to have, if I have multiple optical elements, it is better each optical element output is referred back to the x y axis, then I can repeat the same thing for every optical element, then you have a simple matrix manipulation that will tell you what is the exit light. In trigonometric resolution, you have to find out it enters the model, it introduces a retardation within the model, and then rotate back, all those steps you have do it individually, now you do not have do that. If I have once, I multiply all these matrices and keep it as a ready reckoner, you just plug in the value of delta and theta into this, provided you define delta and theta correctly, theta refers to slow axis with respect to the horizontal is the convention we follow, you can follow any convention, but the convention we follow is that.

So, now I will also have a reverse rotation matrix. So, I have a rotation matrix as light hits the retarder, within the retarder it introduces a retardation, and whatever the modification that has been done by the model I want to refer it back to the original reference axis as x comma y. So, I put a reverse rotation matrix. But what I want is, I want you to do the matrix multiplication right in the class, and when you do this, you expand this e power minus i delta by 2 as cos delta by 2 minus i sin delta by 2 and do the simplification. So, that is what I have been saying, in photoelasticity you need to have matrix manipulation, you also need to know trigonometric identities. If you know these two, development of mathematical aspect of photoelasticity is lot more simpler. And once you do this matrix multiplication and have this result as aready reckoner, then finding out what happens in a quarter wave plate or what happens in a half wave plate or what happens in an actual model, it becomes very simple for you to write.

(Refer Slide Time: 24:17)



In fact, I want you to develop it right in the class, so that the understanding becomes complete, and when you refer back to the notes, you do not have to feel that this is something different, you have already derived it in the class and it is fairly simple and straight forward. All of you know how to do matrix multiplication and how to simplify complex quantities, take a little bit of your time and then do this.

I think some of you are half way through. And in matrix multiplication even the sequence matters, if I have put the matrices in a particular order, multiply them in the same fashion. And when I do the final simplification, the final matrix takes a form like this, and if some of you can verify. So, I have this as cos delta by 2 minus i sin delta by 2 cos 2 theta, and I have this as cos delta by 2 plus i sin delta by 2 cos 2 theta, and if you look at closely there is only one small sin change between the diagonally elements, and I have this as minus i sin delta by 2 sin 2 theta and this is minus i sin delta by 2 sin 2 theta.

So, this represents completely what the retarder influences the light that impinges on it. It accommodates what is the orientation theta, it also accommodates what is the retardation introduced by the retarder.

So, the representation of a retarder clearly shows that I can represent it as a matrix like this. If I know this matrix and it is also very easy to remember, they are not very complicated terms, I have cos delta by 2 cos delta by 2 here, and there is only a sign change, i sin delta by 2 cos 2 theta instead of cos 2 theta, I have this has i sin delta by 2 sin 2 theta. So, it is also very easy to remember, and even if you forget, you can always go back and write this basic multiplication sequence and get the result even at the examination, that is not difficult. But what you will have to keep in mind here is, we have taken theta as the slow axis with respect to the horizontal.

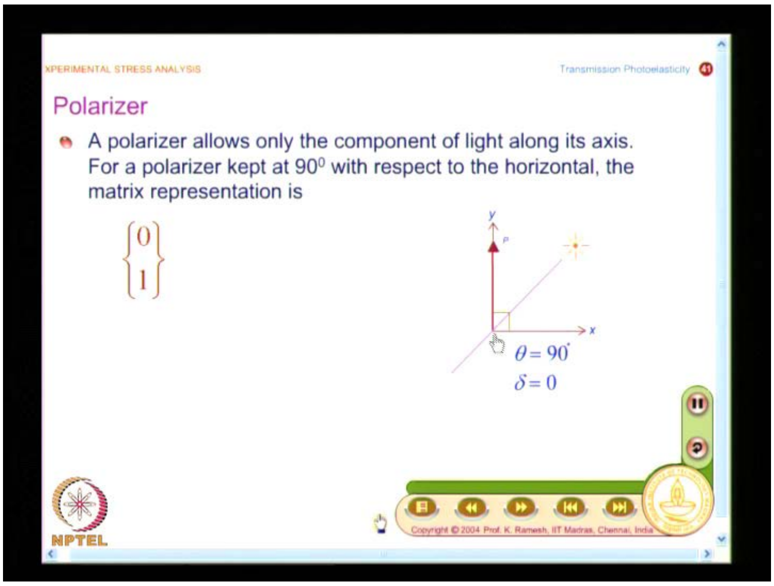
So, now what we will do is, we will look at what are all the optical elements that we may think of in a polariscope, we may have a polarizer, we will have a quarter wave plate. So, I need to know, how do I represent the polarizer, how do I represent a quarter wave plate. And a model will be represented as a retarder, because model I do not know what is the delta it introduces and what is the theta it introduces, we do not know, we want to find out as part of the experiment the value of theta as well as value of delta. Delta, you indirectly find out by finding out the fringe order. And theta, you find out from the isoclinic angle.

So, model can always be represented by a complex matrix like this, and this is for a retarder. When I say a retarder, you have to keep in mind, I have this axis in an actual model corresponds to principle stress direction, and I implicitly look at only a single plane. In practice, you have a finite thickness because there is two- dimensional model, surface free, you also make an assumption that principle stress direction does not change within the model thickness. Suppose, I take a three- dimensional model and take a slice out of it, in general the principle stress direction will change from plane to plane, you should never forget that. So, that is why two- dimensional photoelasticity is fairly simple, I could correlate whatever the result I get from optics to physical parameters comfortably. The moment I go to three- dimensional photoelasticity, what do I do, mathematics becomes very complex. In order to simplify the mathematics, I said there is a stress freezing and slicing process, I have a three- dimensional model, I lock in all the stresses, I take out the slice. When I take out the slice, I will say that, I will use a very thin slice, bring in engineering approximation, principle stress direction remains constant. So, this is how the approximations come.

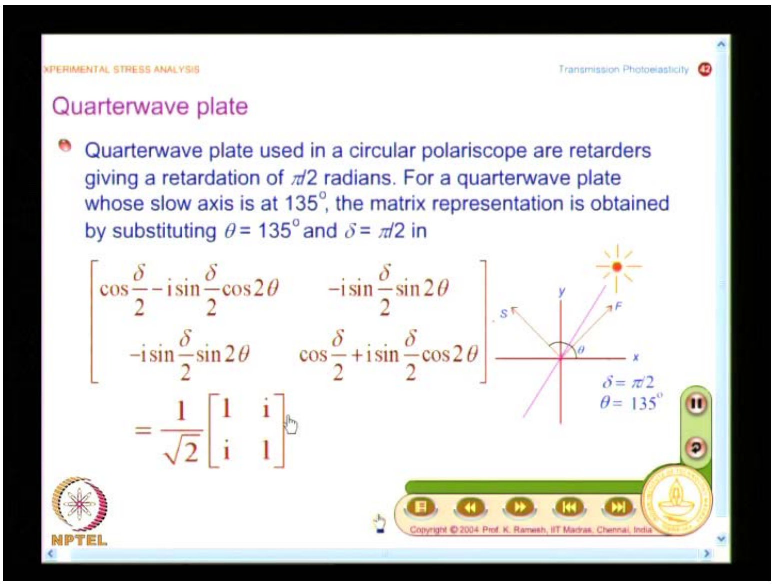
See, you should know the procedure, you should also know what are the approximations involved. You should not conclude, you have taken a model of 6 millimeter thickness and put a diametral disk compression, you have analyzed it. You have analyzed it because it is a two- dimensional model and thickness is also sufficiently small, and because of the way I apply the stresses, principle stress direction remain constant is a reasonable approximation over the thickness, only then interpretation is possible. Because what is the interpretation that we finally come to? We essentially find out what is the slow and fast axis and these axes should remain constant over the thickness of the specimen. So, thinner the model it is better. But if you have a thin model, when I apply compression it will buckle. So, you need to have thickness, so that buckling is avoided. So, these are all practical considerations. So, you have to understand the practical consideration and also appreciate the approximations involved.

And what we have seen in a retarder is, retarder introduces a rotation, and within the retarder you are adding or subtracting retardation only along the fast and slow axis, that is only along the principle stress direction, remember this, then we do a reverse rotation.

(Refer Slide Time: 36:19)



(Refer Slide Time: 37:19)



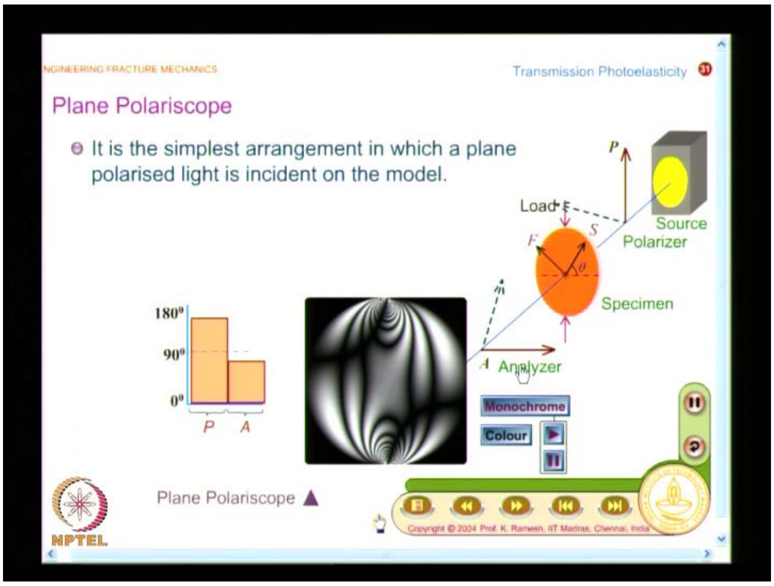
First, we will take a very simple optical element like a polarizer wherein there is no great mathematics involved, it is only a representation for convenience. And what I have here is, I have a simple polarizer, I can say that theta equal to 90 degrees and delta equal to 0. I do not even have to go for a matrix representation, I can simply represent this as a vector. I send only a component of light along the y axis is what my vector should say, and that is easily represented by 0 comma 1. A simple vector like this is sufficient to represent polarizer, nothing more is required. Only when I come to a quarter wave plate, I need to have a look at how it is orientated and it is a retarder, but the retardation is known, a quarter wave plate gives a retardation of pi by 2 radians. So, that is what we will have a look at it, and then we have this as quarter wave plate, and quarter wave plate used in a circular polariscope are retarders giving a retardation of pi by 2 radians. So, we know the retardation. And we keep the quarter wave plate at appropriate angles, one such angle is 135 degrees, the other angle is you can also keep it at 45 degrees, and look at how the retarder is represented. So, what I have here is, I have the reference axis as x and y, I have a quarter wave plate with slow axis, and when I show theta I show the reference from the horizontal to the slow axis, theta is the angle of the slow axis, and fast axis is represented here.

So, when I have a quarter wave plate orientated at 135 degrees, I write delta equal to pi by 2, theta equal to 135 degrees. On the other hand, if I put theta equal to 45, I would get a different answer. Unless I keep the quarter wave plate slow axis at 45 degrees, I cannot put theta equal to 45. So, you have to represent theta very carefully. Theta is always, in our representation, orientation of the slow axis with respect to the horizontal. Once I know delta and theta, it is child's play. You have an expression what a retarder does, plug in the values of theta and delta, and because these values are very nice, even your sin cosine terms reduce to known simple quantities, it is very simple, make an attempt and try to find out what the values are.

The retarder is cos delta by 2 minus i sin delta by 2 cos 2 theta, then I have minus i sin delta by 2 sin 2 theta minus i sin delta by 2 sin 2 theta, similar terms here, this is symmetric actually. Cos delta by 2 plus i sin delta by 2 cos 2 theta, this is the representation of a retarder. Now, you want to find out for a quarter wave plate which is orientated at 135 degrees, the angle is also given. Why do we have it as 135 degrees? We want to have a circularly polarized light. Because we want to have a circularly polarized light, we keep it at 135 and 45 in conventional photoelasticity. In conventional photoelasticity, these are the two orientation of the quarter wave plate popularly used.

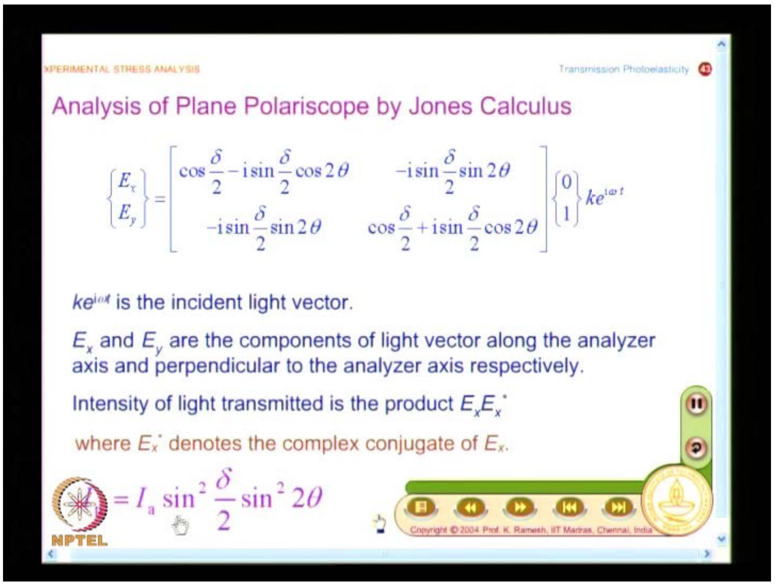
So, when I substitute delta equal to pi by 2, theta equal to 135, you will get a very simple expression, and it really makes your life simple, when you have to go and analysis a circular polariscope. And I want to right now plug in the values of theta and delta and simplify, and it will definitely make your life simple when you want to analyze circular polariscope. In fact, I would recommend all of you to try out trigonometric resolution for a circular polariscope, then analyze the same problem by Jones calculus, then you will find out how Jones calculus is very simple. Until then you will not find out because you have to do matrix multiplication, you may find, I have to do matrix multiplication which have studied long time back, I am not able to do comfortably now. But if you do that, it becomes lot more simpler to analyze any type of optical elements kept in a polariscope, at any orientation you could comfortably analysis. And in fact, Jones calculus paved way for the developments in digital photoelasticity also, you could analysis any given optical arrangement very quickly by employing Jones calculus. I think now you must have done the simplification, please verify the result, it is simply 1 by root 2 1 i i and 1, and if I change to theta equal to 45, you will have a very small change in this representation, as simple as that.

(Refer Slide Time: 42:21)



So now what we will do is, we will go an analysis what happens in a plane polariscope. So, what we are going to do is, we have looked at a plane polariscope, where I have a polarizer, I have a model and I have an analyzer. So, what I need to do? I need to put polarizer in it's mathematical representation, then just replace only the model by the matrix that you have solved, that is all you have to do, I do not even have to worry about the analyzer. If I look at only the horizontal component of that, I know what is the light transmitted by analyzer, it is as simple as that, it is too simple. For a plane polariscope, trigonometric resolution is also needed because that gives you understanding what happens in each optical element, that is needed. We did a logical explanation, we did a trigonometric resolution, we will also analyze the plane polariscope by Jones calculus. The reason is, you understand all the three methodologies. A logical explanation may not be possible when I have many optical elements, many optical elements will complicate your logical explanation.

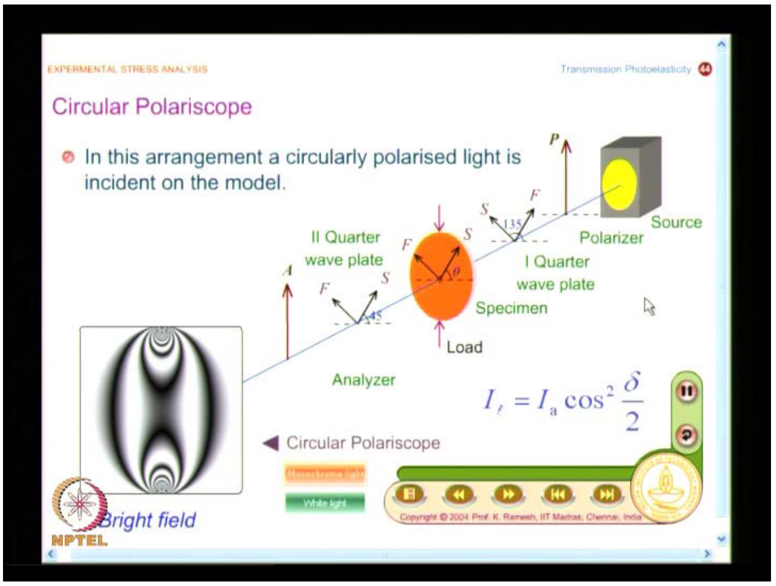
(Refer Slide Time: 43:37)



So, we will see what happens in a plane polariscope. So, we are going to say analysis of plane polariscope by Jones calculus and I will repeat the animation, you will see here my interest is to find out the light vector E x and E y. And I would also write this starting from the right, first I will say this is the light impinging on the model. So, I will put k e power i omega t first, then I will look at what is the first optical element, represent the optical element. First optical element here is polarizer and you have already seen how to put polarizer, I will simply put 0 and 1. And what is the next element? You have only the model and you have already written down how to represent a retarder. In fact, you can put the rotation matrix, retardation matrix and reverse rotation matrix, and then finally arrive at this. But instead if you directly write, it saves your time and very simple to remember, it is not difficult, which you will be in a position to do.

And this would give me what is the component of light along the x axis y axis. And what I need to find out? I need to find out the amplitude, because I said all the sensing elements only record amplitudes. So, that is what we will see now. So, what I have here is, k e power i omega t is the incident light vector, E x and E y are the components of light vector along the analyzer axis and perpendicular to the analyzer axis respectively. And intensity of light transmitted is simply E x into E x star, where E x star is the complex conjugate. And once I know E x, I can easily find out E x star, and this matrix multiplication is very simple, and I get the expression for light simply as I a sin square delta by 2 sin square 2 theta, which is same as what you have got by trigonometric resolution, absolutely no change. Only the mathematical procedure is different and here both trigonometric resolution and Jones calculus require the similar effect, there is no change in the effect involved, effect is almost similar. You really do not see the advantage of using Jones calculus for a plane polariscope, but definitely improves your understanding how to apply Jones calculus in analyzing optical elements of polariscope.

(Refer Slide Time: 46:41)

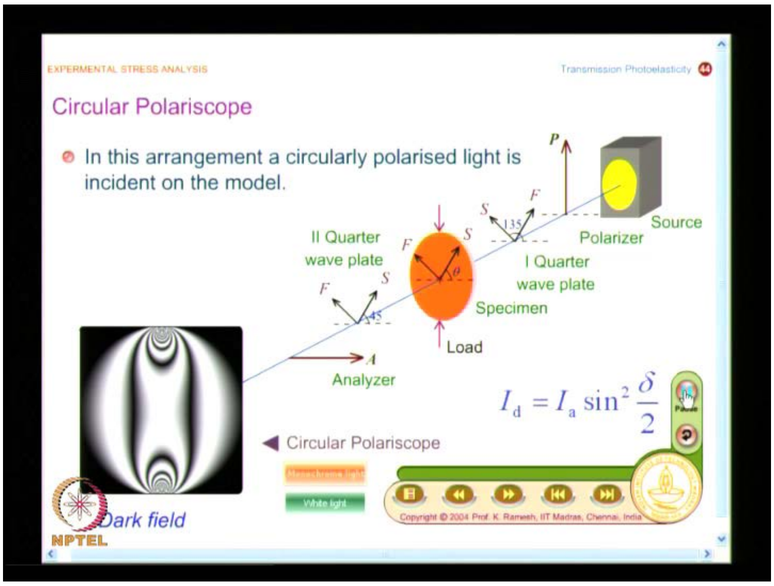


Now, what we will do is, we will look at what is a circular polariscope. I have the optical diagram here and let us see what are all the elements that you have. I have the light source, after the light source I put a polarizer, then I put a quarter wave plate which is orientated at 135 degrees, then I put a model which is loaded, after the model I put a second quarter wave plate and this quarter wave plate is kept at 45 degrees, then I put an analyzer.

So in a circular polariscope, I have two elements before the model and two elements after the model. And if you watch it very carefully, I have the fast and slow axis and the slow and fast axis of the second quarter wave plate are crossed. What I have in first quarter wave plate and what I have in second quarter wave plate are aligned such slow axis of second quarter wave plate is perpendicular to the slow axis of the first quarter wave plate. And I have the polarizer and what I see here is, I keep the analyzer parallel to polarizer, then I have the background as bright.

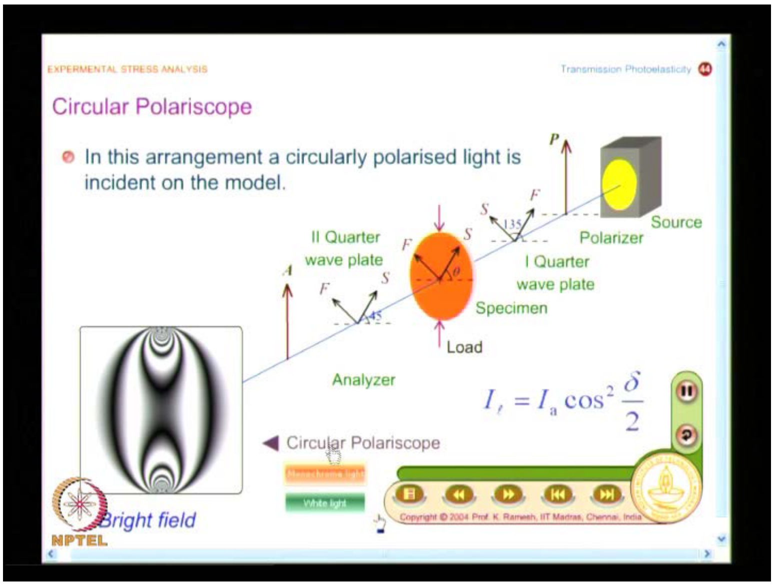
I will repeat the animation, you will see that when the analyzer is horizontal, in addition to quarter wave plates being crossed, polarizer and analyzer are also crossed. You will see the background as dark. All that we will develop equations later, first make the observation. And I also bring in the expression which you can derive, we will also derived, and what you find here is the intensity of light transmitted is a function of delta alone, it is no longer a function of theta. So, what I seen in the screen? I see only one set of fringe contours. So, first knowledge in using a circular polariscope is, instead of two fringe contours I see only one set of fringe contours. And I repeat the animation, you just have a look at it.

(Refer Slide Time: 49:25)



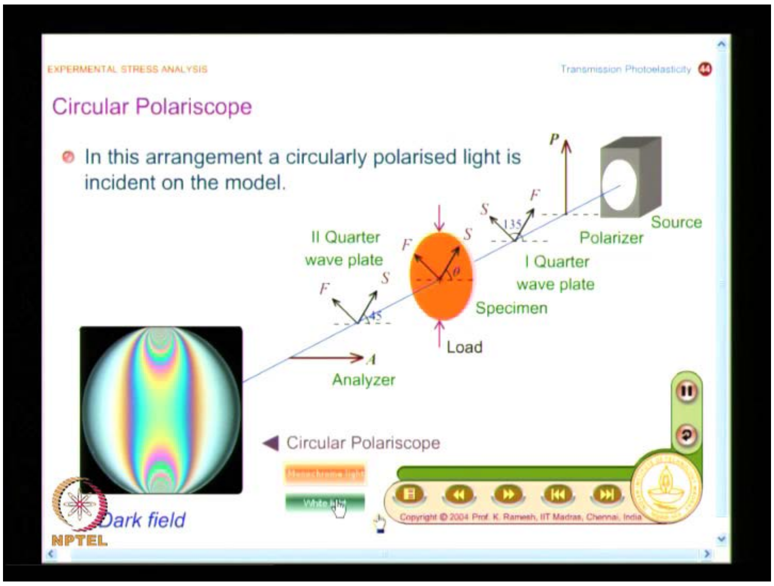
So, what I have here is, I have the polarizer, first quarter wave, plate, model second quarter wave plate, no change is done. Instead of analyzer vertical, I have kept analyzer horizontal, I see this as background as dark, and I see fringes also in a particular fashion. See, if you recall, in a plane polariscope, we had only the dark field, we never look at the bright field, bright field was not giving any information. And here I have the light intensity transmitted as I a sin square delta by 2. And when I keep it vertical, what I have? I have this as bright field and this equation also has changed.

(Refer Slide Time: 50:18)

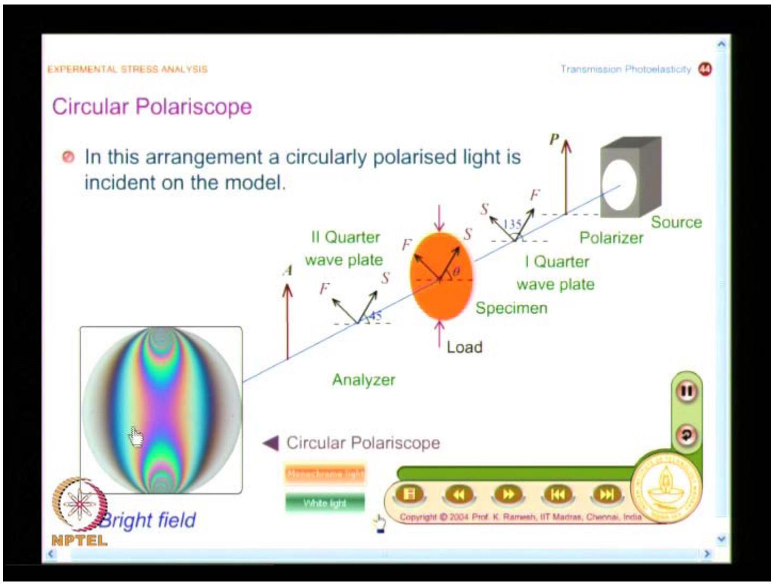


Now, you know the intensity light transmitted. So what I would suggest is, do the analysis of light that passes through the circular polariscope purely by trigonometric resolution. Look at what happens in the polarizer, what happens in the first quarter wave plate, what happens in the model, what happens in second quarter wave plate, then how do you look at in the analyzer and then which component you are looking at. Both horizontal and vertical component give you physical information. So, take this as an exercise, complete this exercise and come for the next class. Do it only by trigonometric resolution, it will run into several pages, and we will do the same thing with Jones calculus, you will find how elegant and simple it is. So, from now onwards, we will use only Jones calculus for all our development, but please do the analysis by trigonometric resolution for a circular polariscope. I have a polarizer, I have the first quarter wave plate, I have the model, I have the second quarter wave plate and I have the analyzer.

(Refer Slide Time: 51:51)



(Refer Slide Time: 51:54)



And you can also have a look at this, what happens in a white light, when I put it in white light I see colored fringes beautifully. And this flip between a dark field and bright field and watch what happens. Whatever these two are complementary fringe patterns, look at that, whatever the gaps that you see in one field it will fill by the other field. Just look at the animation. If you watch from that perspective, you will find that there is a 90 degrees shift. So, that is what we will see.

In this class what we have looked at was, initially we looked at, what we do in a plane polariscope, what kind of fringe patterns you get, and we recalled that we did a trigonometric resolution to find out what happens to the light that passes through the model and how you perceive it at the analyzer, we found that it is a function of delta as well as theta. And I mentioned, when you have multiple optical elements doing trigonometric resolution is lot more cumbersome and it is better that we develop a new mathematics called Jones calculus, which will simplify your analysis when you have multiple optical elements.

So, in Jones calculus you try to represent the modification introduced by each of the optical elements as a set of matrix operators. So, we found that a separate matrix for rotation, a separate matrix for retardation, then we represented how to mathematically identify the role of a retarder. We had a mathematical representation of a retarder and I said that, in this the angle theta refers to slow axis of the retarder, that is how the final matrix was obtained. And once you know the retardation matrix, any set of optical elements in a polariscope can be comfortably analyzed, if you know delta and theta.

Finally, we looked at what are the elements in a circular polariscope and I suggested that you try to do a trigonometric resolution, and you will find that it becomes quite cumbersome for the number of optical elements that you have. On the other hand, when we do by Jones calculus, mathematics becomes lot more simpler, which we will see in the next class.