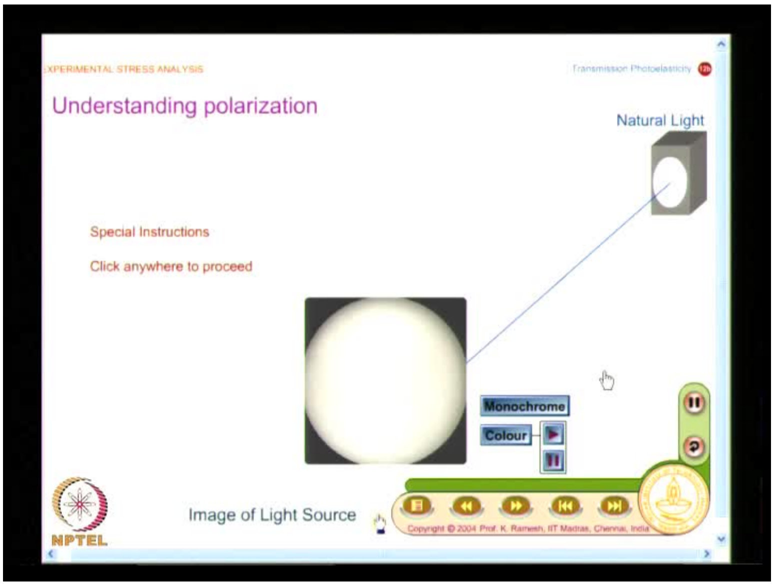
Experimental Stress Analysis

Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module No. # 02
Lecture No. # 15
Plane Polariscope

The last class we had looked at, what is stress optic law and we also saw what is the maximum stress information obtainable from a simple photo elastic experiment. We looked at that we need to find out the fringe order N and theta at the point of interest. Then by in working Moire circle, it is possible for us to find out normal stress difference as well as inplane shear stress. Now, we will have to look at, what is the kind of optical equipment that we need to use to find out the fringe order N and theta at a point of interest. And before we go into that, let us have a look at what we have learnt in understanding polarization.

(Refer Slide Time: 01:03)



XPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity

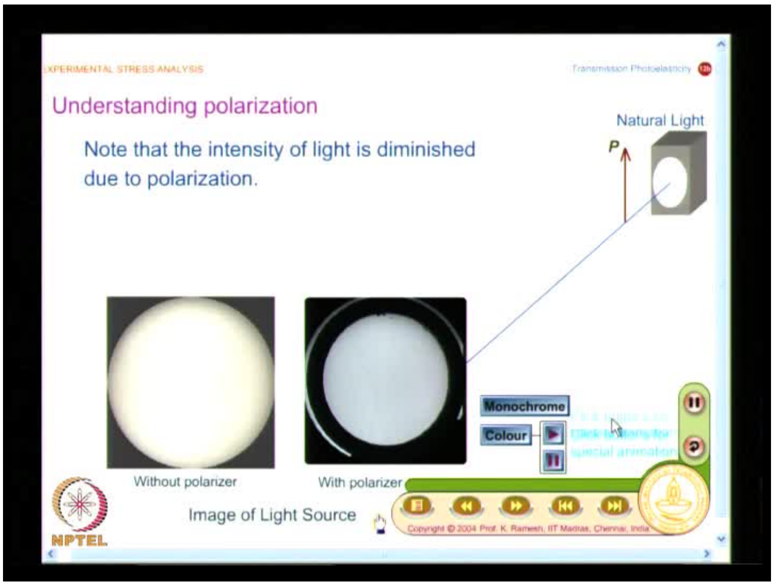
Understanding polarization

Special Instructions

Click anywhere to proceed

Copyright © 2004 Prof. K. Ramesh, IIT Madras, Chennai, India

(Refer Slide Time: 01:15)



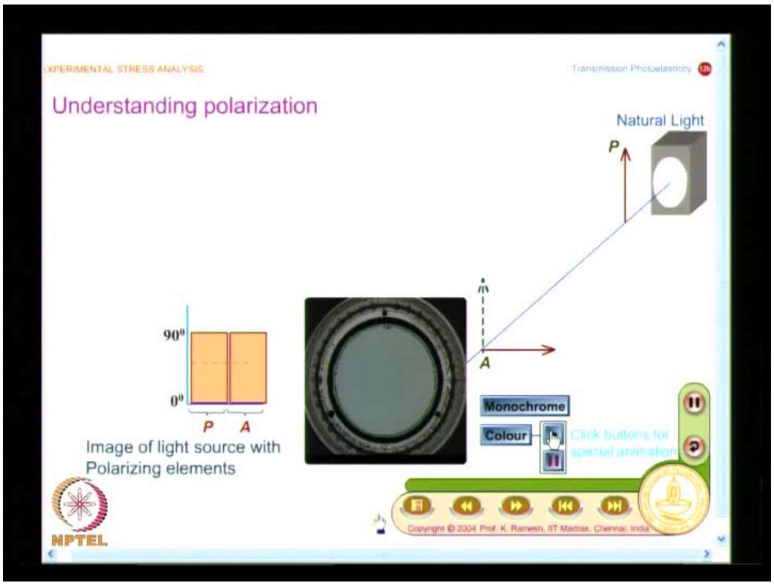
XPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity

Understanding polarization

Note that the intensity of light is diminished due to polarization.

(Refer Slide Time: 01:36)



XPERIMENTAL STRESS ANALYSIS

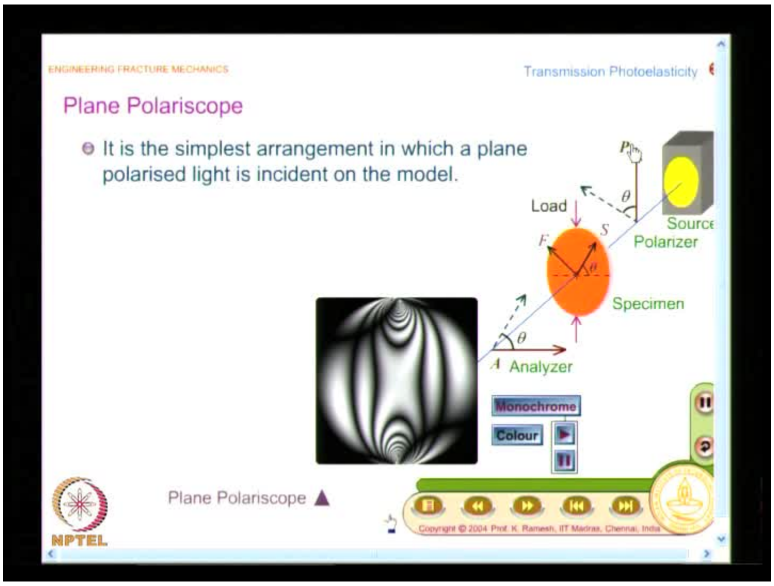
Transmission Photoelasticity

Understanding polarization

So, here you have a natural light, then I put a polarizer, it allows only one component of light. And I want to investigate what is the status of polarization, I introduce another element which is physically the same, and it is kept at different orientations to investigate. And when I do this when, when I rotate this element, you find progressively the intensity diminishes. And when this axis becomes horizontal, that is perpendicular to the polarizing axis, whatever the light that comes out of the polarizer is completely cut off by the analyzer.

So, what you saw was, you saw only a black background, and another subtle point you need to note here is, I am sending a white light. So, this phenomena is independent of wave length. We have looked at, a model behave like full wave plate. When it behaves like a full wave plate, it is a function of wavelength. But on the other hand, when I have a plane polarized light which is impinging on the model; which comes out as plane polarized, and this can be cut off by analyzer which is kept at perpendicular to it, when the light vector is not a function of the phase retardation introduce, there is also a possibility to look at the wave in that fashion.

(Refer Slide Time: 03:23)

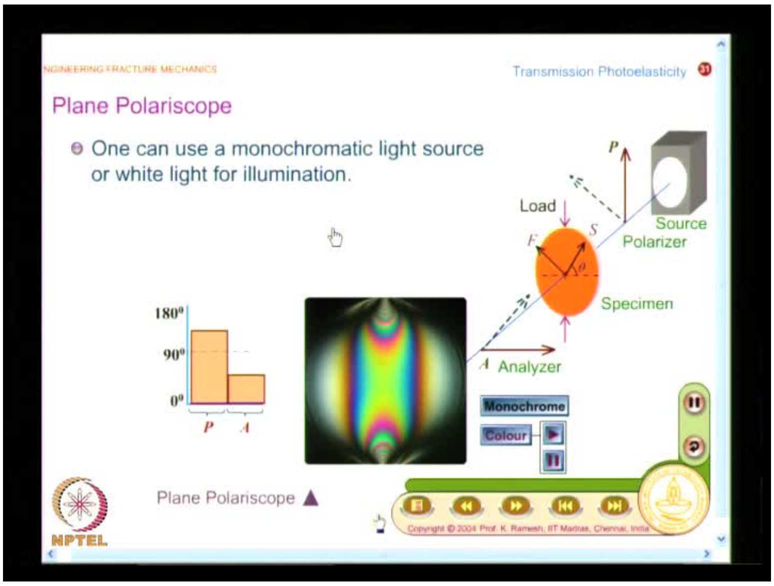

ENGINEERING FRACTURE MECHANICS

Transmission Photoelasticity

Plane Polariscope

- It is the simplest arrangement in which a plane polarised light is incident on the model.

(Refer Slide Time: 04:59)


NGINEERING FRACTURE MECHANICS

Transmission Photoelasticity

Plane Polariscope

- One can use a monochromatic light source or white light for illumination.

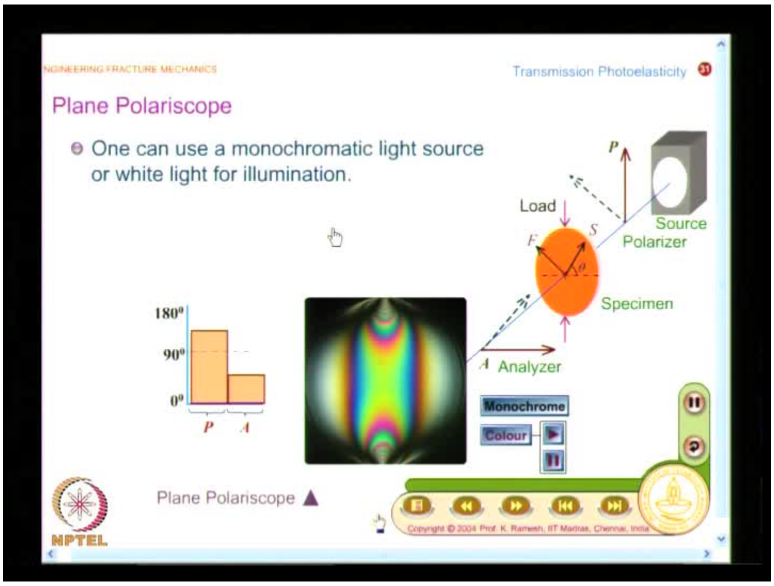
Now, we will go and look at, what happens when I keep a model loaded between the polarizer and analyzer. Because the idea is, we want to find out how the fringes get formed in a birefringence model, that is load. So, what do you find here? I have a polarizer, I have an analyzer and I am sending a monochromatic light source, and I have a model which is birefringence; which is loaded. And what you find here? You have the background is still dark, and within the region of the model you find fringe contours. And you need to make a neat sketch of it. You first worry about light source, put the polarizer, put the model and put the analyzer. And what you find here is, this is the simplest arrangement in which a plane polarized light is incident on the model.

So, what you find here? When the ray hits the model, we label that axis as fast and slow axis. And we have no knowledge whether this is a fast or slow axis, this is only a representation, and I said this matters when I go to digital photoelasticity. For the kind of optical arrangements you have in conventional photo elasticity, whether it is fast or slow, it does not make much of a difference. Then what do you find here? One can use a monochromatic light source or white light for illumination. So, what I can do is, in some monochromatic light source I can also put a white light source. So, what I find here is, when I put a white light, which is nothing but light source of multiple wave lengths, now I find one set of contours are colored, the other two contour that we saw they still remain black. We will spend some time on looking at this again, we will go to monochrome.

Now, what you have to understand. I have a polarizer and I analyze the light that comes out of the model only by using a simple analyzer, which is nothing but a polarizer; but it’s axis is kept differently. So, this will only detect whether the light that comes after the model is plane polarized. If it is plane polarized in the same direction as the input light, then this will completely cut off the light. Because that will be perpendicular to it, it will completely cut off. And when the model can allow plane polarized light as plane polarized light? We looked at in the wave plate. If the wave plate behaves as a full wave plate, then whatever the light that impinges on the model, it goes out unaltered.

So, a plane polarized light incident on the model will come out as plane polarized light, which is a function of wave length. When the moment you say, because it behaves like a wave plate, you will also have to bring in function of wave length, that is the reason why we first look at only monochromatic light source impinging on the model. And you also have a very nice picture here, see this shows that you have stress is continuous, that is the reason why you have a nice continuous contour. So, what I find here is, I could have the regions which are black, are really those regions where the exit light from the model is plane polarized in the same direction as the input polarized light. Because the analyzer is kept horizontal, perpendicular to the polarizer, that light is cut off, so you see this as dark.

(Refer Slide Time: 04:59)


NGINEERING FRACTURE MECHANICS

Transmission Photoelasticity 31

Plane Polariscope

- One can use a monochromatic light source or white light for illumination.

Plane Polariscope ▲

Copyright © 2004 Prof. K. Ramesh, IIT Madras, Chennai, India

And now the question is, I see as two dark bands here, are they same as these other dark bands or not? So, what we will do is, we will now rotate the polarizer and analyzer combination, keeping them crossed and let us see what happen. Now, I rotate them. So, when I rotate them, what I find is one set of contours remains stationary, these contours keep changing and they sweep the entire model. And what you have various this dotted line show, that you have the polarizer analyzer are rotated, they are kept crossed. What can you guess from this? When I was discussing about wave plates, I also mentioned, if the fast and slow axis coincide with the incident plane polarized light, what comes out of the model is simply the same as what is sent inside, there is no modification. And because it is aligning with the fast axis, it is not wave length dependent. So, even when I say, I send a multi wave length light, that will also be cut, we have to verified that statement, but you can conjecture this is what happens.

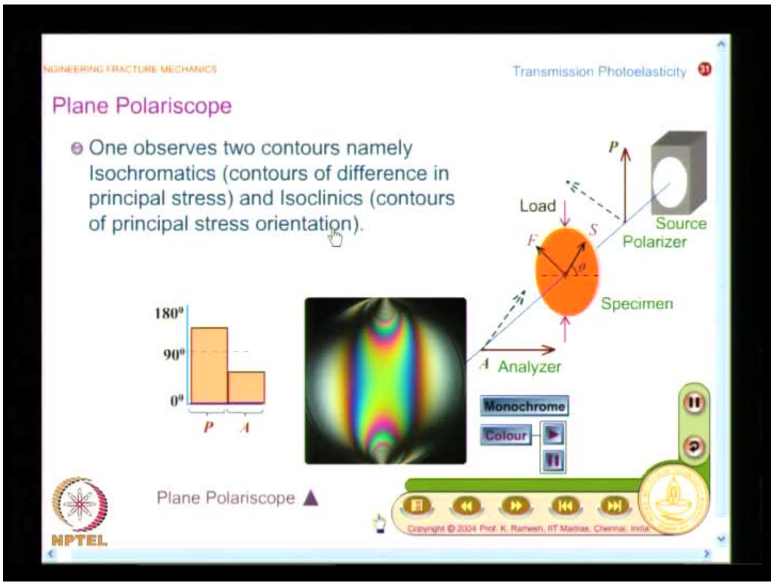
So, what you can now appreciate is, when I keep a model in a plane polariscope, I see two sets of contours. One set of contours, where the model behaves like a wave plate.

Why I see multiple contours? The model cannot distinguish phase difference is 0, phase difference is 2 phi, phase difference is 4 phi, 6 phi, 8 phi, it cannot differentiate that, that is why I get multiple contours.

So, one possibility of fringe formation is the model behaves like the wave plate. So, the input plane polarized light comes out as plane polarized light unaltered. The second possibility is, when I rotate the polarizer analyzer crossed, at different points you have principle stress direction. When it coincides with the principle stress direction at the point of interest, whatever the light that is sent, that is also cut off. I will also provide further explanation, then it will become clear. First make the, look at the animation and find out how they are. Now, what will we do is, we will change the wavelength, we will change from monochromatic light source to polychromatic light source.

So, now I will go for a polychromatic light source. And as I said, these are colored, this I have not provided the reason, but this I already provided, when I rotate the polarizer analyzer, you have this as a black contour. It is independent of wavelength because those contours come because your polarizer analyzer coincides with the principle stress direction at the point of interest. So, whatever the wavelength of light you send, it comes out as plane polarized because the model need not behave like a wave plate. If the polarization change is not because of a model behave like a wave plate, it is because the input polarized light passes through the fast axis there is no component perpendicular to it. So, whatever the wavelength you send, whatever the light that you send, it is cut off by the analyzer because it is perpendicular to the polarizer.

(Refer Slide Time: 12:17)



NGINEERING FRACTURE MECHANICS

Transmission Photoelasticity ㉛

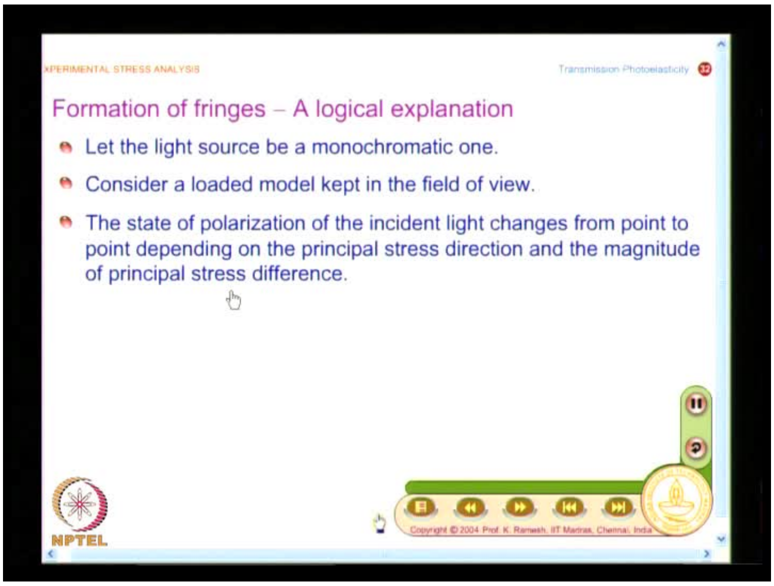
Plane Polariscope

- One observes two contours namely Isochromatics (contours of difference in principal stress) and Isoclinics (contours of principal stress orientation).

So, what you have here is, in a plane polariscope, it is very clear now, one observes two contours, namely isochromatics; which show contours of difference in principal stress and isoclinics; they are nothing but, contours of principal stress direction. And what happens here, when I send, when I want to analyze what are isochormatics, in a white light source I see them as richly colored. So, what happens is, when I keep the polarizer, when I keep the analyzer crossed, when the model behaves like a wave plate, full wave plate, the input polarized light comes out as polarized light; which is the function of wave length. So, what you see when I send a white light is, out of the several colors of white light, one color is cut off, because it will behave differently at different points depending on the stresses developed. It may be a full wave plate for red wave length, it may be a full wave plate, for blue wave length, for difference values of stress. So, one of the colors is cut off, so you see the rest of the colors, that is why in a polychromatic light source, you see the isochromatics as colored. So, you see white minus extension of one of the colors. And isochormatics, the model behaves like a full wave plates. When model behave like a full wave plate, wavelength of the light also is to be looked at.

When the wavelength changes, it will not behave like a full wave plate. What behaves like a full wave plate for red wave length will allow some retardation, more than 2 phi or less than 2 phi, it will not be exactly 2 phi. So, what you find here is, in a plane polariscope, you find beautifully two sets of contours, and now we will also go and develop a logical explanation. Suppose, I want to develop the intense equation, you can anticipate that it should be a function of difference principal stress as well as function of theta.

(Refer Slide Time: 15:15)



Formation of fringes – A logical explanation

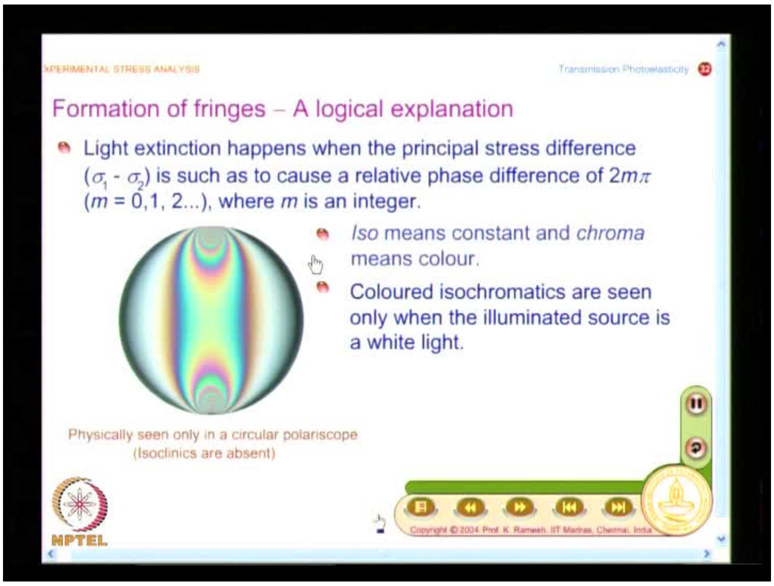
- Let the light source be a monochromatic one.
- Consider a loaded model kept in the field of view.
- The state of polarization of the incident light changes from point to point depending on the principal stress direction and the magnitude of principal stress difference.

So, you will anticipate if I develop the intense equation, it should be a function of delta as well as theta, all that we will look at. Now, we will again develop this argument in the subsequent slides, we will see systematically. Why do fringes get formed? We will try to provide a logical explanation. We have already seen the logical explanation, and this is another verification of whatever we have discussed.

To simply our effort, we will have the light source to be a monochromatic one. And what I do is, I have a model which is loaded; which is kept in the field of view. And what you anticipate? Because the model is loaded, the state of polarization of the incident light changes from point to point depending on the principal stress direction and the magnitude of principal stress difference. Because when you take a disk and then loaded it, near the load application point you will have very high stresses, regions away from it, it will be stress low, and because it is a, stress is a continuum, what you have is, you have an elastic continuum, so when you applied the load, the variations are smooth. That is why you see beautifully smooth fringe contours. Stresses vary gradually from point to point and you also see smooth fringe contours.

And what we do is, by keeping analyzer, we find out when the light is cut. The analyzer will cut the light when the plane of polarization is perpendicular to it. For the plane of polarization to be perpendicular to it, the input polarized light should come out unaltered. And in a plane polariscope, in conventional photo elasticity, we always keep polarizer analyzer crossed. If you go to digital photo elasticity, people also tried keeping them parallel, keeping them at arbitrary orientation, but in conventional photo elasticity, polarizer and analyzer are always crossed.

(Refer Slide Time: 17:37)



Formation of fringes – A logical explanation

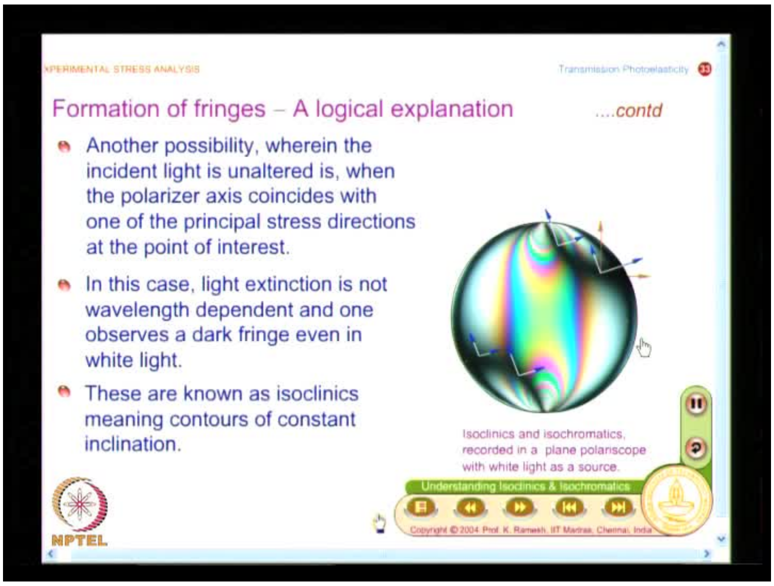
- Light extinction happens when the principal stress difference (σ_1 - σ_2) is such as to cause a relative phase difference of $2m\pi$ (m = 0,1, 2...), where m is an integer.
- *Iso* means constant and *chroma* means colour.
- Coloured isochromatics are seen only when the illuminated source is a white light.

Physically seen only in a circular polariscope
(Isoclinics are absent)

So, what I have here is, the first possibility is light extinction happens when the principal stress difference sigma 1 minus sigma 2 is such as to cause a relative phase difference of 2 m pi, where m is 0, 1, 2, extra. And this I have taken from a different optical arrangement, which is called a circular polariscope for clarity. So, I see one set of fringe contours, and when I send a monochromatic beam of light, I see this as blacken and white contours. Whereas, we have defined isochromatics as, iso means constant and chroma means color. Colored isochromatics are seen only when the illuminated source is a white light.

So, suppose I change the source of light as white, I see them as colored contours. So, you get a fairly good idea that this is how isochromatic contours appear in the case of a disk under diameteral compression. Because stress is varying gradually from point to point and you are having an elastic continuum, I see this as beautiful contours. And the regions where you have shades of light that is coming in, where the retardation is not 2 m pi, it will be either less or more than this. So, it will allow some amount of retardation. You will have an elliptically polarized light, all of that light will not be cut by the analyzer, some light will come and hit the screen. And we have also seen why a fringe appears as a band. Only if those points where sigma 1 minus sigma 2 is 2 m pi exactly, you will have this as plane polarized coming out, and this is cut off completely by the analyzer which is kept perpendicular to it. In other cases, state of polarization will be different, you will also have a component along the horizontal direction, that is why you see fringe as a band, there will be some small amount of light which your eye is not sensitive to distinguish.

(Refer Slide Time: 20:18)



XPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity 33

Formation of fringes – A logical explanation ....contd

- Another possibility, wherein the incident light is unaltered is, when the polarizer axis coincides with one of the principal stress directions at the point of interest.
- In this case, light extinction is not wavelength dependent and one observes a dark fringe even in white light.
- These are known as isoclinics meaning contours of constant inclination.

Isoclinics and isochromatics, recorded in a plane polariscope with white light as a source.

Copyright © 2004 Prof. K. Ramesh, IIT Madras, Chennai, India

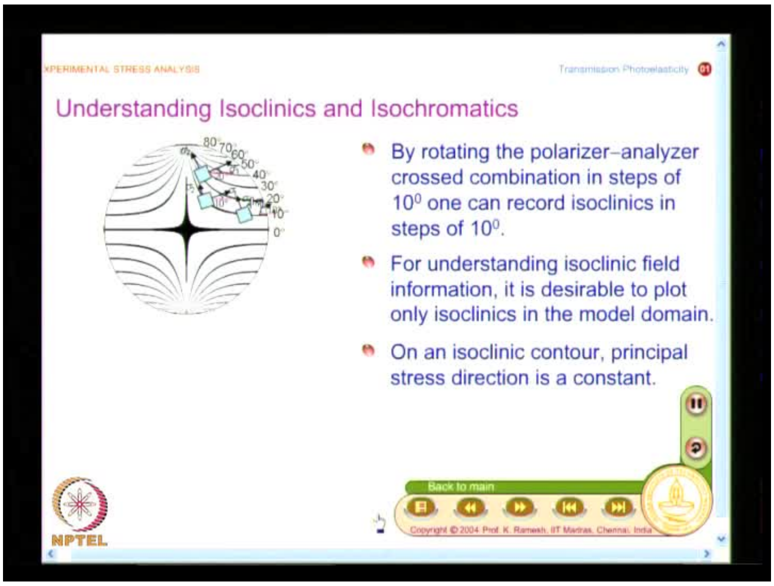
And what is the other possibility? We have seen one possibility, we have to look at other possibility. For clarity what is done is, I have shown one representative isoclinic, and what you need to understand here is I can have multiple branches. I can have one isoclinic here, another isoclinic here, I may also have in some situations a small curve here. So, what is important here is, when the polarizer axis coincides with one of the principal stress directions at the point of interest.

And what is try to be shown here is, I have the reference axis shown in brown, and these are the principal stresses at the point of interest, their magnitude may be different, but if you look at the orientation, the orientation is same. And here again, only isoclinics are shown for clarity, and in fact, there is no unique optical arrangement, we should provide only isoclinics. You have an optical arrangement, which will filter out isoclinics and show isochromatics; which is possible we will also see in a circular polariscope. And I have always been mentioning, in most of the experimental techniques, it is desirable to get fringe information separately. In some cases, you cannot separate them out, in some cases, we can separate them out. In a circular polariscope, I can filter out isoclinics, I can see only isochromatics, but in a plane polariscope, I will always see isochromatics and isoclinics together. For clarity of understanding isoclinics are shown here, but there is no optical arrangement that is available to get you this.

What you could possibly do is, take a photo elastic material which has a very high value of sigma. So, for the given load applied, you may not have significant formation of isochromatics. That is only way which we can see only isoclinics with some level of clarity. And you have a picture of the actual fringe pattern seen in a plane polariscope, and this is the isoclinics, and this is the axis at the point of interest, and the orientation of these axis remain constant. And I have again emphasized, when the polarizer axis coincides with one of the principal stress directions, light extinction is not wavelength dependent. Because I do not have another component, I have only one component passes through, so whatever the light that comes out of the model is completely cut off. And these are known as isoclinics, meaning contours of constant inclination. And it will take sometime for you to appreciate what these contours are. I think the picture is reasonably clear, you want me to enlarge it, I can also replay.

So, what I have initially? I have this as only isoclinics drawn, and you find this is the reference axis and this is the orientation of principal stresses at the point of interest, the length of the vector indirectly indicates it’s magnitude. What is attempted to be shown here is, sigma 1 sigma 2 can have different values, the orientation remains constant.

(Refer Slide Time: 25:20)



Understanding Isoclinics and Isochromatics

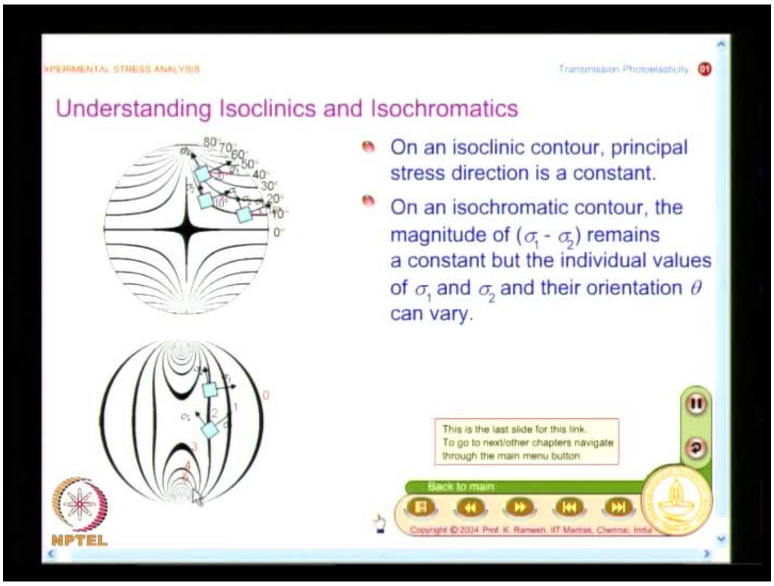
- By rotating the polarizer–analyzer crossed combination in steps of 10^0 one can record isoclinics in steps of 10^0.
- For understanding isoclinic field information, it is desirable to plot only isoclinics in the model domain.
- On an isoclinic contour, principal stress direction is a constant.

And if I see what happens in a plane polariscope with white light as illumination, I see the isochromatics superposed with isoclinic, and this is what you see as isoclinic. And we can also have another way of looking at it, we will also have a look at what way we can represented. What I do is, I rotate the polarizer analyzer crossed combination in steps of 10 degrees, one can record isoclinics in steps of 10 degrees, and plot them. And what you have here is, what is shown here is, I have an isoclinic here which is labeled as 10 degree isoclinic, and what you find here is sigma 1 and sigma 2 can have different magnitudes, but their orientations remain constant in this isoclinic, and if I go to a 20 degree isoclinic, the angle becomes 20 degrees.

And this you have to extract data by several different positions of the polarizer analyzer combination. You do not see this information available in one experimental exposure, this is what you have in conventional photo elasticity. That is why people said, can I plot the isoclinic field for every point in the model domain, and digital photo elasticity has now achieved, you may have to take a few images and process the intensity, and it is possible for you to generate plots of this nature by post processing the data acquired. In a conventional polariscope, I have to keep it at 0 degrees, 10 degrees, 20 degrees, 30 degrees and for each of them go and carefully draw the isoclinic sketch.

And there is also something very interesting, I will not give you the answer. See, this is a circular disk for which you are given the stress field equation and you can find out what is the principal stress direction at every point of interest, and there is something very interesting, this isoclinic meets at the point in the boundary, it will meet only at a specific point and this will also enhance your understanding of solid mechanics. Go back and look at, at what point it can meet, go back and, because this is a free surface and this isoclinic has to come and cut at only at one particular point, which you can look at from solid mechanics, that will give a very interesting result, take that as a home exercise.

(Refer Slide Time: 28:39)



XPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity 01

Understanding Isoclinics and Isochromatics

- On an isoclinic contour, principal stress direction is a constant.
- On an isochromatic contour, the magnitude of ($\sigma_1 - \sigma_2$) remains a constant but the individual values of σ_1 and σ_2 and their orientation θ can vary.

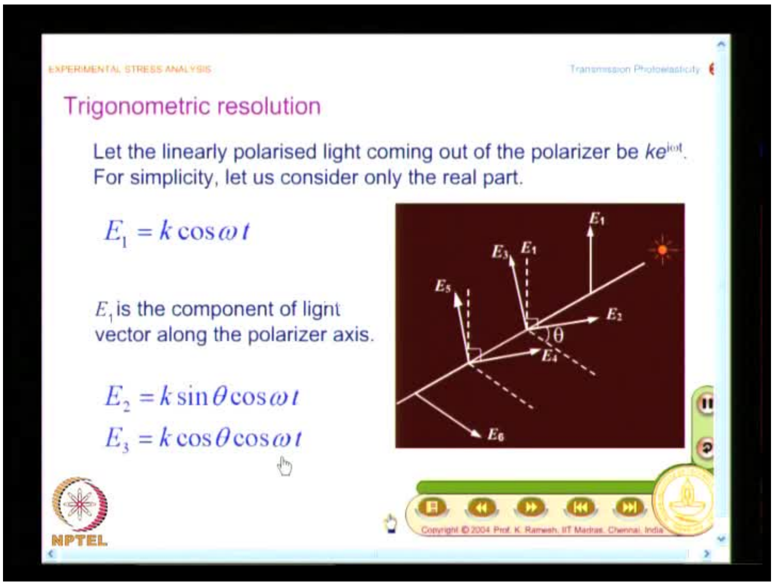
This is the last slide for this link.
To go to next/other chapters navigate through the main menu button.

Copyright © 2004 Prof. K. Ramesh, IIT Madras, Chennai, India

So, what you have as isoclinics is little subtle, and for me to get plots of this nature, I need to conduct a series of experiments, and from that you record them carefully in a conventional photo elastic analysis. And we will look at what are isochromatics. We have looked at what are isoclinics, we will look at what are isochromatics. These are the isochromatics. And what you have here is, the direction may be different, but the difference of sigma 1 minus sigma 2 will remain same on a isochromatic contour. So, that is the only difference. Direction will not remain same here, direction will keep changing from point to point, but the difference of sigma 1 minus sigma 2 will remain same.

And this you can actually put it in one of your earlier contours, you do not have to redraw this. You have one of the isochromatics, on that you can put this information. So, what you find here is, on an isoclinic contour, principal stress direction is a constant, on an isochromatic contour, the magnitude of sigma 1 minus sigma 2 remains a constant, but the individual values of sigma 1 and sigma 2 and their orientation theta can vary.

(Refer Slide Time: 30:08)



EXPERIMENTAL STRESS ANALYSIS

Transmission Photoelasticity

Trigonometric resolution

Let the linearly polarised light coming out of the polarizer be $ke^{i\omega t}$.
For simplicity, let us consider only the real part.

$$E_1 = k\cos\omega t$$

E_1 is the component of light vector along the polarizer axis.

$$E_2 = k\sin\theta\cos\omega t$$
$$E_3 = k\cos\theta\cos\omega t$$

Copyright © 2004 Prof. K. Ramesh, IIT Madras, Chennai, India

So, we have exhaustively looked at, from a qualitative argument, what are isochromatics and isoclinics. And now what we have to do is, we have to go and look at can we establish mathematically, that whatever we have looked at from a logical explanation, and find out what is the equation of intensity of light. For that, we need to go and develop the mathematics. And the simplest way to do is, doing a trigonometry resolution. And what we will do is, let the linearly polarized light coming out of the polarizer, in general is k e power I omega t, because I can have a cos omega t and sine omega t terms. For the purpose of this discussion we will consider only the real part, when we go to Jone's calculus, we will retain as k e power I omega t.

And let us look at how we can label the axis when the light passes through the model, and we will also have a larger picture of it. So, what I have is, I have a light source and the first optical element I have, e is polarizer which is labeled as E 1. Then what happens? You have the model which is shown with enlargement here, that I have a, this is the point where the light enters the model, and this is the point where light exists the model. For clarity, these are shown distance apart. And at the incident plane I will have two axes, we will label them as E 2 and E 3, they are essentially slow and fast axis at the entry point, and these axes will remain because I am looking at a two- dimensional model, essentially it is plane, for convenience we have some finite thickness, for clarity this is shown as enlarged. So, you will have 2 axes, E 4 and E 5.

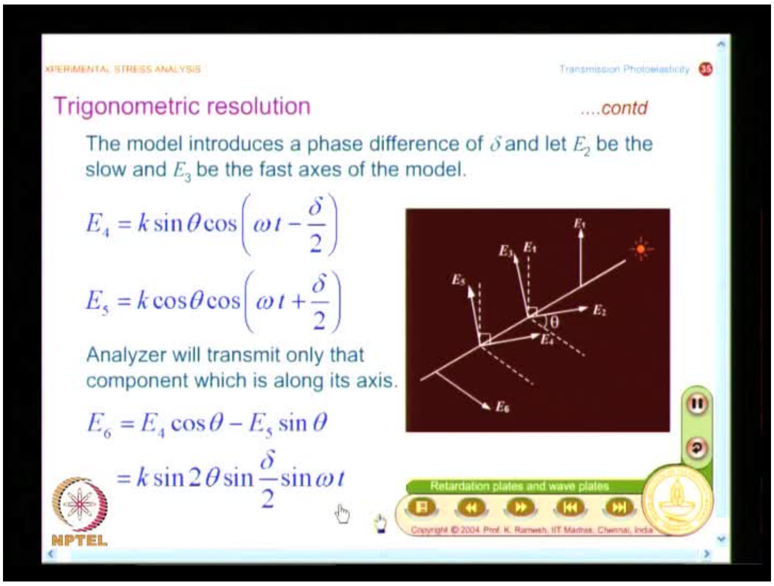
Then, I represent the analyzer which is kept horizontal, the polarizing axis of analyzer is horizontal, and I put this axis as E 6. So, what I need to do is, I need to write the light vector for E 1, find out how do I write the light vector at E 1 and E 3, then I have to find out how to write E 4 and E 5, then finally, I will have to look at only the component E 6 that is filtered by the analyzer. So, if I do all that, I will know what is the exit light that is coming out, which I could simplify and then find out what is the intensity of light transmitted, and we have already anticipated that it should be a function of delta and theta. If we get that, then all our qualitative argument fits nicely with the mathematical development. And because it is a very simple optical arrangement, trigonometric resolution is very comfortable and simple and also provide you a level of understanding. which you will not get otherwise. So, we will go by a trigonometry resolution, later we will also analyze the same problem by using Jones calculus. And you will find Jones calculus is very comfortable when I have several optical elements in the optical bench. Here I have only a polarizer model and analyzer, it is very simple, so trigonometric resolution is quite all right and we will see how do we do the trigonometric resolution.

And what we will do, we will simply say even as cos omega t, k cos omega t. So, I have an amplitude, I have this as cos omega t and you have to recall, when we discussed retardation plates, this is the input polarized light that hits the model, as soon as it hits the model, you have the amplitudes plate along directions E 3 and E 2. So, you can write what is E 2 and also what is E 3. It is fairly simple. You make you are own attempt, then I will show you the, what you have is, when it hits the model, model is loaded, so for one incident light, you will have two refracted beams. Because of this orientation, this is 90 minus theta here. So, I will have a component will pass through E 2 and another component will pass through E 3, essentially the amplitude will change.

So, I have this as k sin theta cos omega t and E 3 as k cos theta cos omega t. It is fairly simple. If you recall the discussion that we had on retardation plates, we spent sufficient time on looking at what happens to the incident light when it hits the model and what happens within the thickness of the retardation plate, a similar thing we will have to look at here.

So, for directions E 2 and E 3, you are able to write the light vector comfortably. And how do you write for E 5 and E 4, E 4 and E 5? These are the axes at the exit of the model. So, within the model, the light has acquired a relative retardation. Suppose, I say retardation is delta, I can say no retardation along direction 4 and full retardation is in direction 5, I can also put 0 and delta, I can also put delta by 2 minus delta by 2 and plus delta by 2. This is for our convenience. Keeping in view that I am going to develop Jones calculus, where the matrices will look nicely, we will take the choice of minus pi by 2, minus delta by 2 in the slow axis and plus delta by 2 in the fast axis, just for convenience, mathematics is not going to change. So, that is what we will do now.

(Refer Slide Time: 37:07)



Trigonometric resolution

....contd

The model introduces a phase difference of δ and let E_2 be the slow and E_3 be the fast axes of the model.

$$E_4 = k\sin\theta\cos\left(\omega t - \frac{\delta}{2}\right)$$

$$E_5 = k\cos\theta\cos\left(\omega t + \frac{\delta}{2}\right)$$

Analyzer will transmit only that component which is along its axis.

$$E_6 = E_4\cos\theta - E_5\sin\theta$$

$$= k\sin 2\theta\sin\frac{\delta}{2}\sin\omega t$$

So, what we say is the model introduces a phase difference delta and let E 2 be the slow axis and E 3 be the fast axis of the model. And E 2, E 3 are, if it is slow and fast, E 4 is a slow and E 5 is fast. And we want do it for E 4 which is considered as a slow axis, so what will I do is, the same amplitude of, that will not change, but the phase I subtract minus delta by 2 and on E 5 I add plus delta by 2. So, what I have here is, E 4 turns out to be k sin theta cos omega t minus delta by 2 and E 5 is k cos theta cos omega t plus delta by 2.

So, do you understand the way that we go about in writing the changes in the light vector as it passes through different elements of the polariscope. We start after polarizer, when it hits the model, we do not look at it as one plane, we look at it hits the model at the front surface, leaves the model at the other surface. Theoretically, when you are looking at a plane model, it is as thin as a plane, but physically we use the finite thickness. So, we will bring in some engineering approximation and we also say that the principal stress direction remain constant within that thickness.

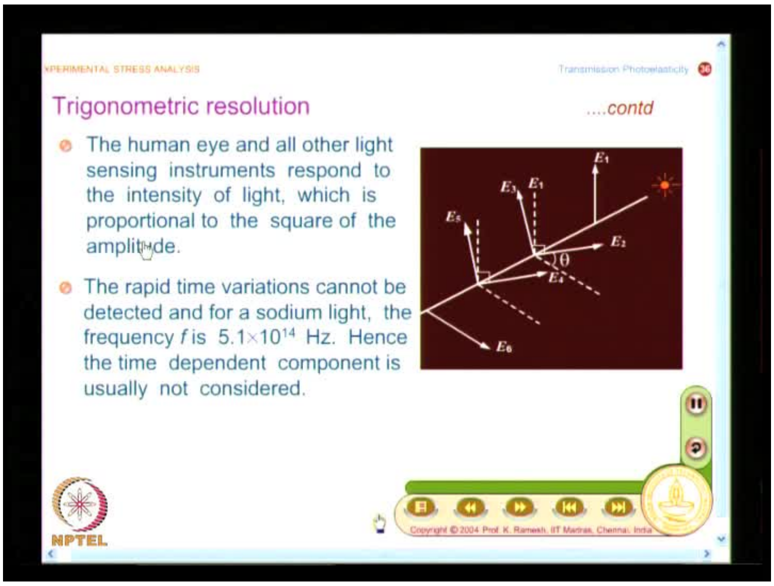
And we do this as two operation. First, there is an intensity amplitude, the amplitude of the light vector changes. The second case, we put appropriately the retardation, then it comes out. So, when it comes out of the model, this E 6 what it will do, this is nothing but your analyzer, this will allow only component of light that is parallel to this. For any other component, if the light vector is perpendicular, it will completely cut off. So, we want to write an expression for what is the light transmitted by E 6. That is very simple, I have E 4, you know this angle, I can find out what is the contribution from E 4 and what is the contribution from E 5. Can you write it, can you write what is E 6?

So, what we do here is, because the optical arrangement is very simple, we are in a position to do trigonometric resolution, and stage by stage we are able to incorporate what changes that happens to the incident light vector. And we know E 6 will pass only the light component parallel to this, we write an expression for E 6 in terms of E 4 and E 5. Fairly simple, E 4 cos theta minus E 5 sin theta and which is also simplified, if we go back, I said for all these you need to have knowledge of trigonometric identities. it is a very simple step, it is not difficult at all.

So, you take this component E 4, multiply by cos theta, multiply by this sin theta and do a simplification, this is the component of light that is transmitted by the analyzer, that turns out to be k sin 2 theta sin delta by 2 into sin omega t. So, what we have anticipated? We have anticipated that this should be a function of delta which is related to sigma 1 minus sigma 2 and theta.

So, I have k sin 2 theta, sin delta by 2, both are appearing. But, what do you record? I do not record the complete light characteristic. All the sensing elements are sensitive only to the intensity, not the complete light vector.

(Refer Slide Time: 42:11)

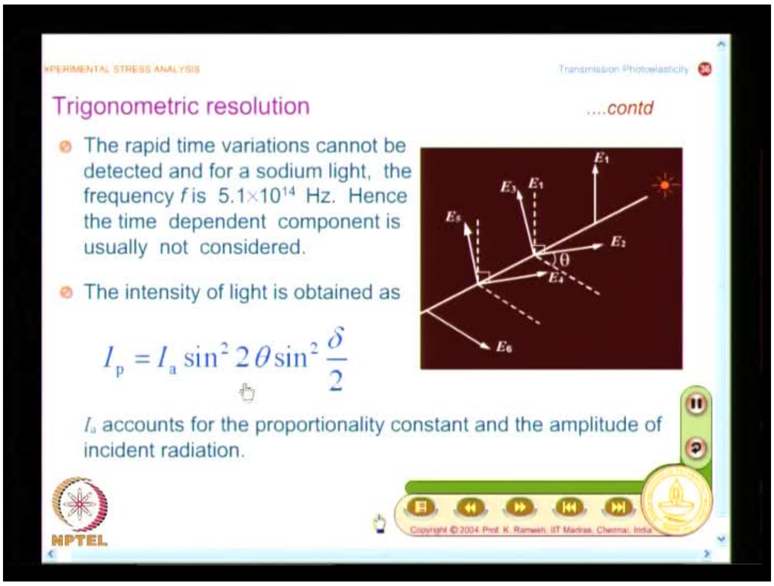


Trigonometric resolution

....contd

- The human eye and all other light sensing instruments respond to the intensity of light, which is proportional to the square of the amplitude.
- The rapid time variations cannot be detected and for a sodium light, the frequency *f* is 5.1×10^{14} Hz. Hence the time dependent component is usually not considered.

(Refer Slide Time: 44:05)



Trigonometric resolution

....contd

- The rapid time variations cannot be detected and for a sodium light, the frequency *f* is 5.1×10^{14} Hz. Hence the time dependent component is usually not considered.
- The intensity of light is obtained as

$$I_p = I_a \sin^2 2\theta \sin^2 \frac{\delta}{2}$$

I_a accounts for the proportionality constant and the amplitude of incident radiation.

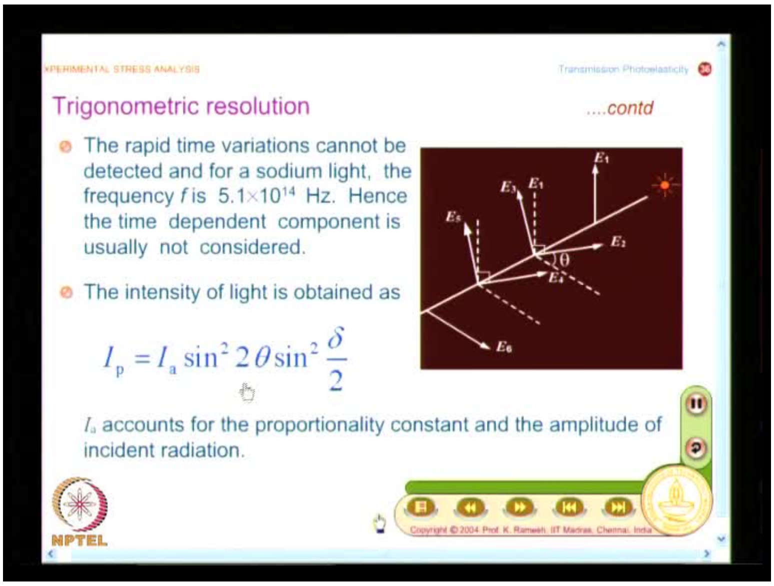
So, that we will have a look at what happens. Human eye is also a light sensing instrument. Human eye and all other light sensing instruments respond to the intensity of light. It is a very key concept. Why we always look at intensity of light is because we respond only to this. And there is a relationship, the intensity is proportional to the square of the amplitude. And this is a point to clarify, you have rapid time variations which cannot be detected and for a sodium light we have the frequency as 5.1 into 10 power 14 hertz. So, you do not perceive the time dependent component and you are sensitive only to the intensity of light transmitted, and intensity of light transmitted is square of the amplitude.

So, what will have to look at is, we have got the E 6, we will look at this E 6 expression and we will square the amplitude and find out what that expression gives. So, that will give you beautifully an explanation what we see as fringes in a plane polariscope. And I am not going to do the complete derivation, I am going to give you the result, which you can have a look at it, and you can actually go back and do this as your homework. And I label this as I p, the p denotes it is a plane polariscope, and I get the final expression as I a sin squared 2 theta sin squared delta by 2.

So we have, with very simple mathematics we are able to find out what is the intensity of light transmitted in a plane polariscope, when a model is kept loaded. And we have said this intensity is proportional to square of the amplitude, what all we have put as I a, it accounts for the proportionality constant and the amplitude of incident radiation.

So, now we have to investigate, when does this intensity expression goes to 0. So, it can happen for two cases. I can have a situation, where sin squared delta by 2 goes to 0. So, those contours correspond to sigma 1 minus sigma 2, they are called as isochromatics. You can have another possibility, where theta goes to 0. When theta goes to 0, essentially means the polarizer axis coincides with the principal stress direction at the point of interest. When it coincides, you have only one light beam travels through the thickness of model, that is completely cut off and this is independent of wavelength. When the incident light coincides with the polarizing axis at the point of interest, which is nothing but the principal stress direction, it gets cut off and it is independent of the wavelength, the light getting cut off is independent of the wavelength. So, that is why you see two contours, and the isoclinics appears black even in white light. The difference is when I rotate the polarizer analyzer combination, these black contours also move. And there could be questions like, suppose you are given a plane polariscope, how will you identify whether it is an isochromatic or isoclinic. What are the ways that you can think of? I have said one which you can easily derive, you keep the polarizer analyzer combine and rotate it, when I rotate it, one contour will move with it, that is isoclinic. Can I make the other contour to move? Why do you say no? You have to say what is it that I have to do. Suppose, I change the load what will happen, when I change the load does the principal stress direction change? Then you have to go back and brush up solid mechanics. See, a linear elasticity, when I increase the load, stress magnitudes change, principle stress direction do not change. So, that is why always say, when you learn experimental stress analysis, your fundamentals of solid mechanics become clearer and clearer. Because certain things you assume, you think you know. When you are confronted with the question, you do not know what is the answer. So, I can also make the other contour move, if I change the load, the contours that are functions of the load are isochromatics, the contours that are not functions of the load are isoclinics. Because isoclinics will not change, should not change, in linear elasticity they will not change. So, that is the way you distinguish between an isochromatic and isoclinic, even in a plane polariscope with simple monochromatic light source. If you are given a white light source, you have a luxury, I see colored contours and then I have black and white contour. But there is one caution, I do not know, this is not very prominently seen in a circular disc, the zeroth fringe order is always black.

(Refer Slide Time: 44:05)



Trigonometric resolution ....contd

- The rapid time variations cannot be detected and for a sodium light, the frequency f is 5.1×10^{14} Hz. Hence the time dependent component is usually not considered.
- The intensity of light is obtained as

$$I_p = I_a \sin^2 2\theta \sin^2 \frac{\delta}{2}$$

I_a accounts for the proportionality constant and the amplitude of incident radiation.

So, you have to distinguish, even in a white light source, what is zeroth fringe order and what is isoclinic, I have to make a distinction. And though in general fringes move, when I apply the load, what will happen to the zeroth fringe order, think about it. See, for every rule there will be an exception, there will be a rule and there will be exception. You should understand both, you should know the rule as well as the exception and why it is so. So, this what you will have to understand.

So, in this class we have started looking at what is the optical arrangement that we need to conduct a photo elastic experiment. And we said that a simple incident of plane polarize light is good enough and I get two contours, one contour corresponds to sigma 1 minus sigma 2, for which I have to find out the fringe order. Right now we have not discussed how to find out N, that we will keep it as a separate discussion. First time we have seen fringes and we have classified one set of fringes as isochromatics and another set of fringes as isoclinics. How to find out N, how to find out theta, we will have separate discussion.

Now, what we will do is, we will go and develop our mathematical skills better, because we want to go for a circular polariscope, where I will have more than one optical element in the path of light, and for me to analyze trigonometric resolution is not convenient, I need to develop a special type of calculus called Jones calculus, that we will take it up in the next class.