Experimental Stress Analysis

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Module No. # 02

Lecture No. # 14

Retardation Plates, Stress-Optic Law

The main focus of discussion in the last class was, when the relative retardation is changed, you are able to get light of different ellipticities, azimuth and also handedness. So, that gives you a hope by measuring the characteristics of the light, it is possible to find out what is delta. And in fact, there is a whole body of optics literature what you call as ellipsometry, which tries to find out the azimuth, which finds out the handedness and also the ellipticity. Fortunate in photoelasticity, particularly two-dimensional photoelasticity, we do not have to go to that much detail, we can simply use a plane polariscope or a circular polariscope and analysis the light, exit light characteristics. So, it is lot more simpler, only when we go in for a three dimensional photo elastic analysis, we invoke certain aspects of ellipsometry in more detail.

And what we looked at next was, for all our photo elastic analysis, it is desirable that we understand what is a light impinges on the model. And we said that, the simplest light that you can impinge on the model is plane polarized light. And for getting a plane polarized light, what we did, we said that we are using a sheet polarizer. And a sheet polarizer is like this, that is what we saw in the last few classes earlier. You have a sheet polarizer, which is very convenient for you to rotate, and it is also easy to have a larger field, and it is desirable how this acts like a filter, because I said when you have a natural light, this acts like a filter and you get only plane polarized light that comes after this.

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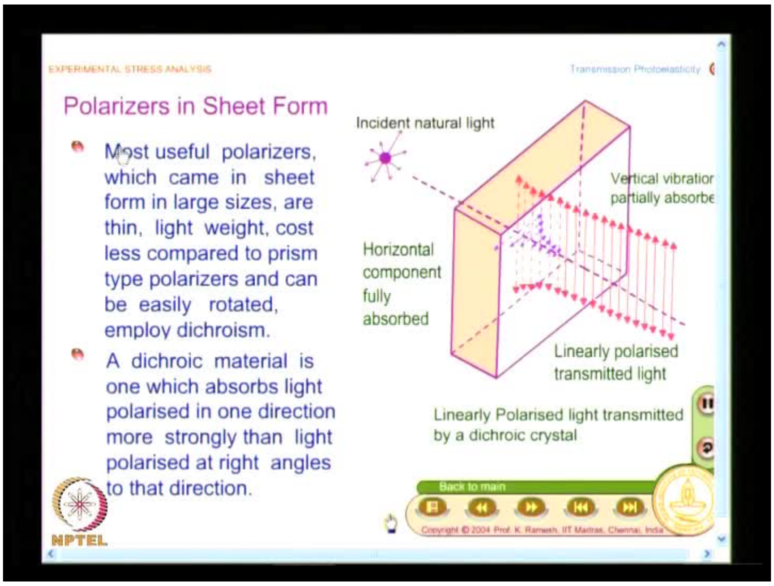

And for you to understand this, you should know some aspects of crystal optics, only then you will be able to appreciate even the physics behind a sheet polarizer. And that is what we have looked at in the last class, polarizers in sheet form. And what we learnt was, within the polarizers sheet, the horizontal component is absorbed by the polarizer. And for illustration, this is shown, which a large thickness, in reality it is very thin, and this phenomena is called dichroism. So, a dichroic material is one which absorbs light polarized in one direction more strongly than light polarized at right angles to that direction.

So, what you find is, from the natural light, it allows a vertical component, vertical vibration are only partially absorbs, and you have this horizontal component is fully absorbed. So, you have the net result is, for the natural light source, you get only a plane polarized light.

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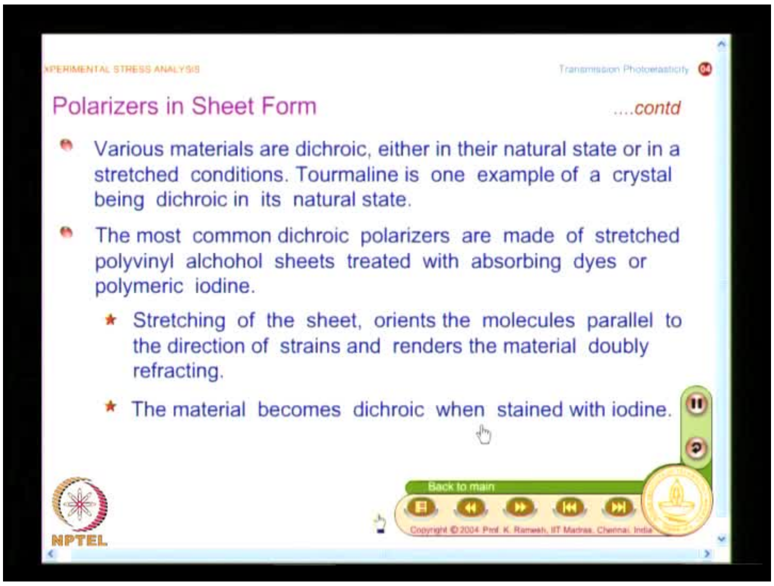

So, a linear polarized light is transmitted by a dichroic crystal. And if you look at the nature, you also have certain materials which are dichroic. So, what you find is, you have a tourmaline is one example of a crystal being dichroic in it's natural state. Like we have seen natural crystals which is birefringent, and you also have tourmaline, which is one example of a crystal being dichroic in it's natural state. And what we have seen while making the polarized sheet was, you had polyvinyl alcohol stretched. So, the other possibility is the most common dichroic polarizers are made of stretched polyvinyl alcohol. In general, they are stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine. So, what you have is, various materials are dichroic either in their natural state or in a stretched condition. Tourmaline is one example of a crystal being dichroic in it's natural state, and when I come to the common polarizers sheets, they are made of stretched polyvinyl alcohol sheets treated with absorbing dyes or polymeric iodine.

And let us look at what each of these steps really influence. So, what you find is, stretching of the sheet, orients the molecules parallel to the direction of strains and renders the material doubly refracting. So, the first step is you stretch it and because of the stretching process, the sheet becomes doubly refracting. So, it behaves like a crystal. You have two refractive indices, you have ordinary and extraordinary travel through it. And what happens is, when I, the material becomes dichroic when stained with iodine.

So, dichroic means it absorbs one component of light vector, it allows the other component. So, essentially you get plane polarized light.

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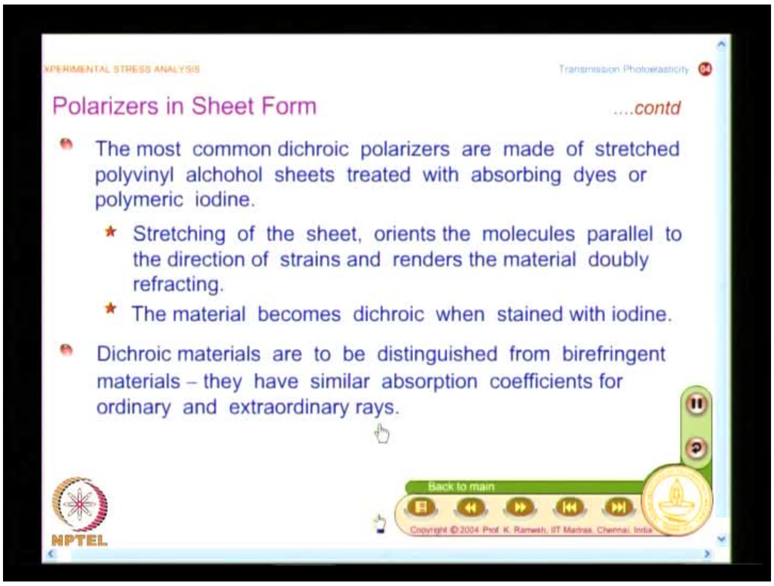

And you should also make a distinction, and this is very important. The dichroic materials are to be distinguished from birefringent materials. Birefringent materials have similar absorption coefficient for ordinary and extraordinary rays. See, for you to do photo elasticity, you need birefringent materials, and birefringent material the absorption coefficient is same for ordinary and extraordinary ray. But for polarized sheet, you have dichroic material, it absorbs one of the rays completely and it allows the other ray unhindered, and you are able to see. And you should also see a very subtle point here, see I said engineering is approximation, and when you are looking at polarization optics, I said after the polarizer and till the analyzer, we do not assume any absorption of light intensity. So, in reality there may be a small absorption of ordinary and extraordinary rays, which could be neglected. So, we made that kind of an approximation from practical stand point.

So, what is important is, we need to know what is the characteristic behind sheet polarizers, because that are really advanced photoelasticity analysis, because if we have to work only with Nicol prisms, then you had only very small area for you to analyze the amount of polarized light, availability is small, the region of interest could be small, once you have sheet polarizers, I can have a large sheet and large models can be looked at comfortably.

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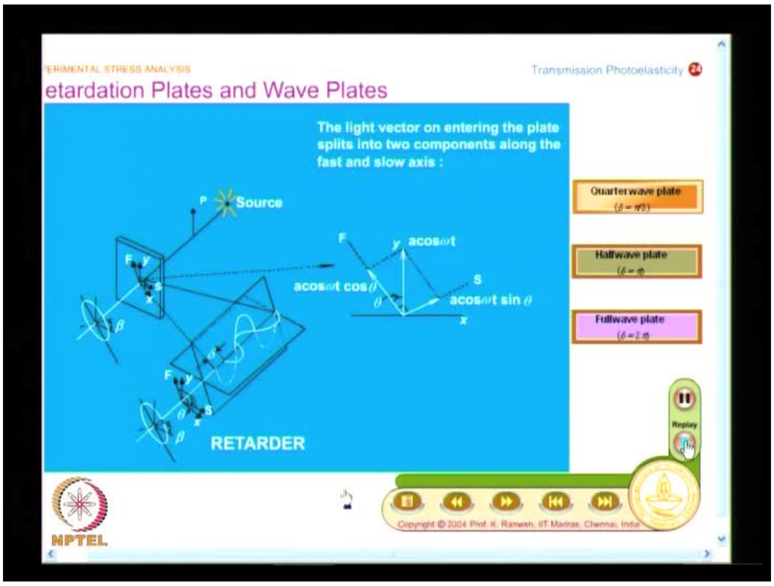

And what is fundamental to all of this photo elastic analysis is, understanding what are retardation plates and wave plates. And this animation you will be looking at it again and again in this course. This is the cuts of photo elasticity, we will again have a look at it. I have a natural light source, becomes polarized when it hits the front surface of the model. You have this split into two light components. They travel within the model, acquire a retardation and in general, you get an elliptically polarized light.

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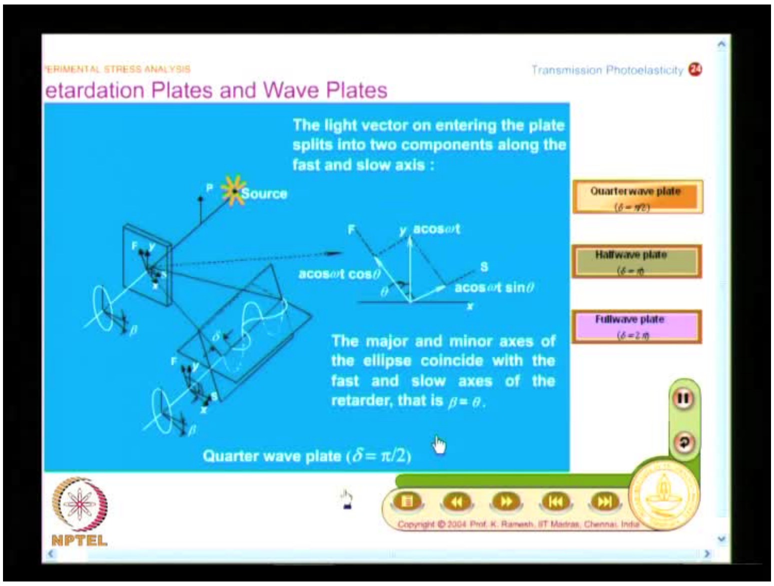

And we have seen, by changing delta, the characteristic of the ellipse can change. And there are few important cases, which are of relevant to photo elastic analysis. And we keep using a optical element called a quarter wave plates, the name signifies it introduces a retardation of delta equal to pi by 2. And this is essentially a crystal plate, the property is same at every point on this body of the crystal plate. And what we have learnt as the process of this animation was, you find one of the rays travels faster and you have this on this plane, the ray travels faster and you call that axis as f axis. And you have another plane which is mutually perpendicular to this, the ray trails behind it, you called this as a slow axis. So, once you go to a crystal plate, you will always look for a fast and slow axis. And what is the advantage is, when delta equal to pi by 2, we have already seen from various states of polarization, the major and minor axis of the ellipse coincide with the reference axis, here it is labeled as fast and slow axis. And essentially, the azimuth of the ellipse is 0, if I have this as horizontal. If I have this axis as horizontal and vertical axis, the azimuth will coincide with the major and minor axis of the, the major and minor axis of the ellipse will coincide with the reference axis, fast and slow axis.

So, that is an advantage. So, this understanding is very much important, when we look at an elaborate optical setup, where how do these optical elements contribute to formation of different types of fringe pattern. Because we are also going to look at a plane polariscope, we are also going to look at circular polariscope, in conventional photoelasticity. If you go to digital photoelasticity, people have no restriction on what kind of input light you should send, people had experimented on various combinations, and there you will know a physical appreciation of how these element contribute to the complete experiment, you will be able to understand it better.

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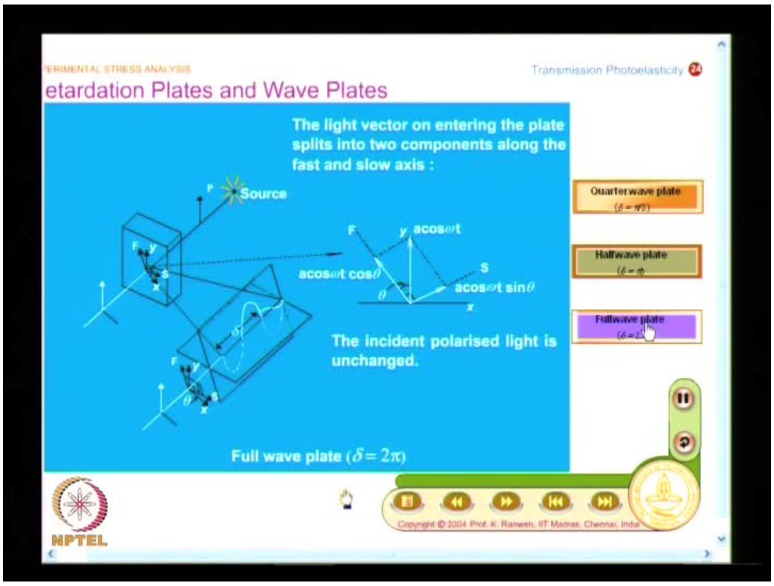

On one hand, we have quarter wave plate, on the other extreme, I have a full wave plate. What is the definition of a full wave plate? It introduces the retardation of one complete wave length. So, it is as good as, the crystal plate is not there, when I have a full wave plate, it is as good as the crystal wave plate is not there. So, this could happen at 2 pi, 4 pi, 6 pi, the essential process is same. So, whatever the input light I send, the same light will come out at the exit point. And this is a very important aspect, and this is what we will use it for investigating what happens in a plane polariscope. Though we developed various states of polarization by looking at elliptically polarized light, that knowledge is essential for appreciation. For understanding plane polariscope, you will have to just analysis whether the light coming out of the model is plane polarized or not.

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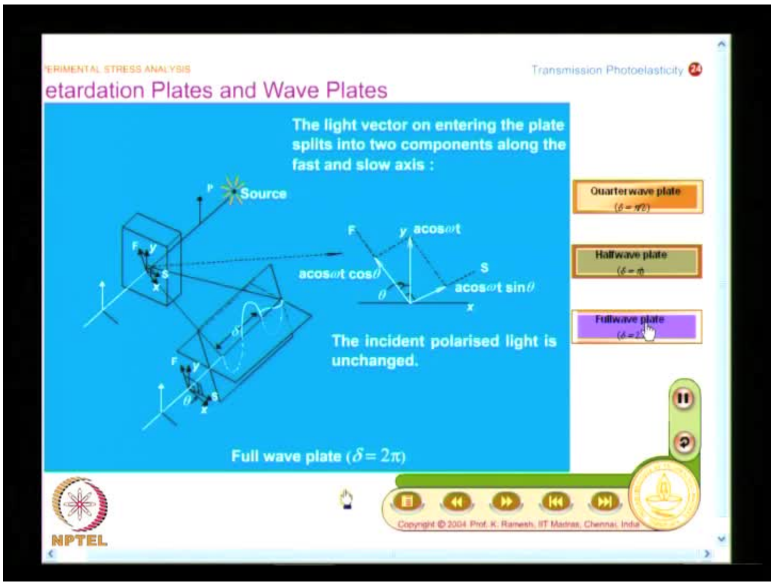

And what you find here. I have also shown semantically that the thickness of the plate is increased, to provide you large value of retardation. And what happens when I go to a half wave plate? When I go to a half wave plate, a simple argument is, I will have instead of a cos omega t, suppose I give only in this there is a retardation, this will become a cos omega t plus pi, so that is nothing but, minus a cos omega t. So, you will have a component here. So, you will have, the resulting will be in this direction. So, a plane polarized light which is instant on the model, remains plane polarized but rotated by angle 2 theta. It is very interesting.

So, what you find here is, when you go to three dimensional photoelasticity, we also learnt what is a rotator that could be reasonably understood, when I look at how a half wave plate behave, it is for a plane polarized light. So, similarly people found there are ways that you can rotate the light ellipse, the elliptic characteristic will remain the same, only the azimuth will change it’s direction. So, that is what a concept of a rotator.

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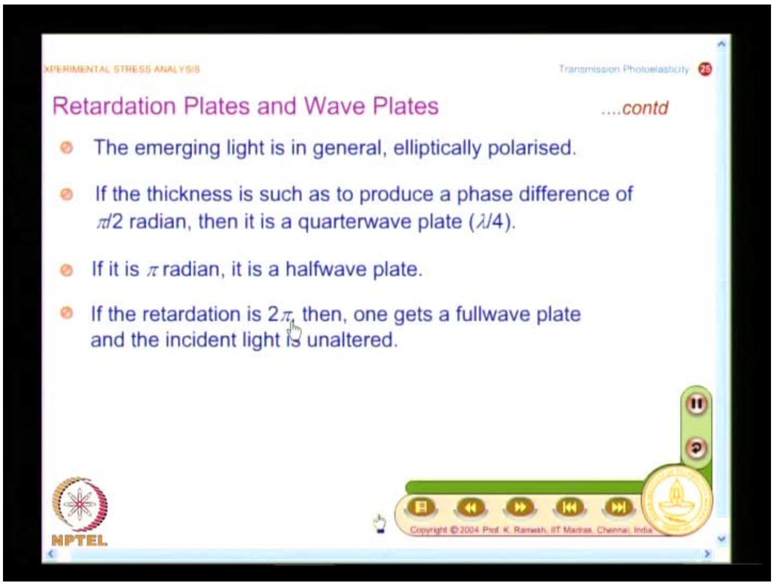

So, what you have here is, in a half wave plate, I have delta equal to pi, in a quarter wave plate, delta is equal to pi by 2, and a full wave plate, delta equal to 2 pi. Let us summarize this is concept of also summaries in this slide. And so, what you find the first observation is, when I have a crystal plate, even when I send a linear polarized light impinging on the crystal plate, the emerging light is in general, elliptically polarized. So, that is the general observation number one. And if I so adjust the thickness of the plate to produce a phase difference of pi by 2 radian, then I call that as a quarter wave plate, it is also labeled as lambda by 4 plates. If the retardation is pi radian, then it is a half wave plate, if the retardation is 2 pi radians, one gets a full wave plate and the incident light is unaltered. And this understanding is very important. Even before we go and find out the expression of delta, a clue is given and the sketch was also drawn, that thickness is one parameter which I could play with to get different values of retardation. That is the simplest, when you look at the expression, when you look at, it acquires retardation within the model.

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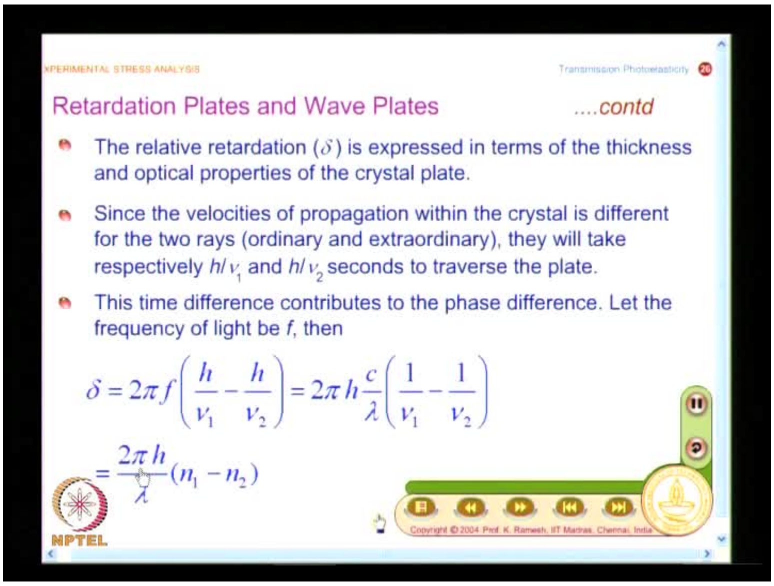

So, it is easy to anticipate by changing the thickness. So, what we will have to look at is, when I say a crystal plate, I should look for two reference axis, one is the fast axis and slow axis, and I also have to know what is it's refractive indices, n 1 and n 2, I should know what is it's thickness. So, what I will now try to do is, I will get an expression for delta, which is a function of the optical properties of the crystal plate. That should be our next goal. Your first thing is, we said delta is very crucial, any changes in delta is reflected in the nature of exit light ellipse, and now we go back and find out whether delta could be evaluated from the parameters of the crystal, whatever the crystal plate that we think of. So, that we will do that. So, what we want to do is, we want the express the relative retardation in terms of the thickness and optical properties of the crystal plate.

So, what you want to do is, when I say optical properties, what are the properties. I have refractive index n 1 and refractive index n 2, or you can also classify it as ordinary ray refractive index and extraordinary ray refractive index, that is fixed for a crystal. The difference between a crystal and a model is, every point in the crystal has same behavior, every point in the model in general, will have different behavior, depending on the stresses introduced. Suppose, I take a tension specimen, apply uniform tension, then it will behave like a crystal plate only, because you are applying a uniform state of stress, so every point will become identical. Leaving that apart, in a generic situation, the local stress state dictates what would be the properties of the crystal at that point of interest.

Now, what we want to do is, we want to get an expression for delta as a function of the thickness and properties of the crystal plate.

So, how do you go about, what is the clue? Say, one ray travels faster; suppose I fix the thickness of the, because in practice, we will have a model of a particular thickness being analyzed. So, thickness of the plate is fixed. So, one ray will travel the say had a the thickness faster than the other. So, looking at, in other words, the time taken to traverse the thickness by these two rays will be different. And that is why we looked at, when we learnt Snell's law, I said you have learnt in your Physics course at the school level, there you worried only about sin a by sin r, you never bothered to look at as ratio of velocities, and I said in photoelasticity, we have a purpose, we want to look at it ratio of velocities, now we use that knowledge and identify that the rays will take different time interval to traverse the thickness.

Suppose, I have v 1 as the velocity, v 2 as the velocity, I will have h by v 1 and h by v 2 is the time taken to traverse. And then, I have omega t as the phase and omega is nothing but 2 pi f. So, using this input, it is possible to write an expression for delta. That is what we are going to do. So, refractive index, whenever we want, we will look at as ratio of velocities, and we look at as a sensor, when I want to related it to stress. So, I use it in a way that help my theoretical development. That is what I am going to do.

And so, what you find here is, the velocities of propagation within the crystal is different for the two rays, they will take respectively h by v 1 and h by v 2 seconds to traverse the plate. And we take h as the thickness of the plate. t, if I use, it indicates time. So, we want to have a different symbol for the thickness. And we take advantage of our understanding on refractive indices. So, this time difference contributes to the phase difference. Suppose, we have the frequency of light be f, then I can write the expression for delta. Can you try out? How you will write delta, make an attempt, even if it is wrong, it is fine, that is how you learn things.

So, what I have here is, I know that ordinary and extraordinary ray travel with different velocities and I understand, because it has to traverse the same thickness, these will take different time intervals. Now, the question is, can I write an expression of delta in terms of the parameters that I know. You can go in stages, first you take the time difference, then look at how it can be converted into phase difference, then bring in certain identities, f can be written in different ways, and finally, write down that expression in terms of difference in refractive indices. That is how we will go, we will look at difference in velocities first, difference in time taken, then finally, write it as n 1 minus n 2, that is my requirement, that is how I want the results to be reported.

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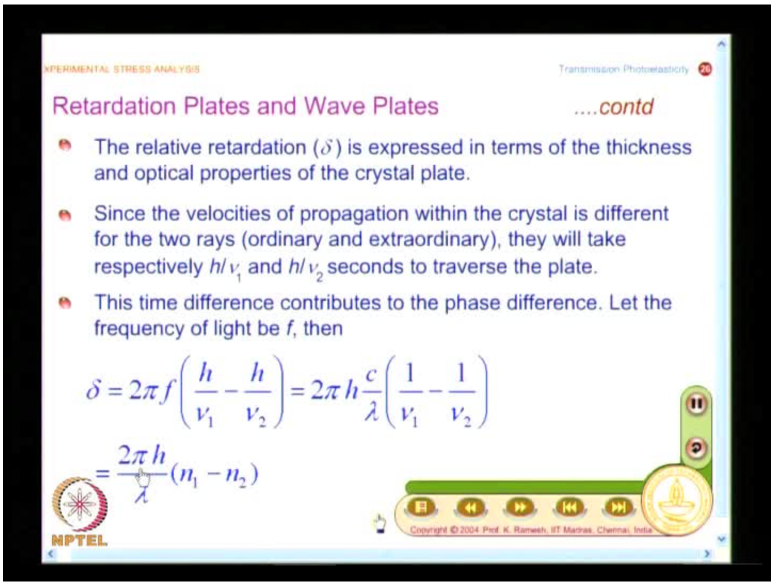

I can do that, it is very simple, I think some of you might have got it, and that is what you have here. So, I have this as 2 pi f h by v 1 minus h by v 2, add again, rewrite f as c by lambda, where c is the velocity of light. And we have already seen, if I write c by v 1 c by v 2, I can write it as n 1 and n 2, these are absolute refractive indices. And you have to note a very key important observation on this expression, I have the expression for delta, this is given as 2 pi h by lambda into n 1 minus n 2.

So, what you find? Suppose I say, I want to have a delta equal to pi by 2, I want to have a quarter wave plate, I can find out what is the thickness corresponding to that. And what is hidden here? h will become a function of wavelength, it is very important. See, I said from mathematical development of photo elastic analysis, the mathematics becomes lot more simpler, if I confine our attention to monochromatic light source. Where does this come? Till now, we have not looked at, we wanted to see colors, so we used white light, we enjoyed seeing those bright colors. But when I come to mathematical analysis, we find a crystal plate behaves like a quarter wave plate for a given wave length. The same is applicable for wave plates as well as the model behavior.

So, we would confine our attention, our mathematics will become lot more simpler, if I use monochromatic light source and do my photo elastic analysis. Now, you have also achromatic quarter wave plate; where the plate gives a phase difference of pi by 2 for different wave lengths, for a range of wave lengths, you have such plates available. When there is a problem, you have an opportunity for research, and research has developed a way to overcome this. So, that goes on parallely. So, what you have to understand is, in photoelasticity, why the wave length is important, monochromatic wave length comes hidden in the expression. Now, what we will do is, we will go back and then see what great scientists have contributed, how this could be related to sigma 1 minus sigma 2, more by induction rather than clear mathematical development.

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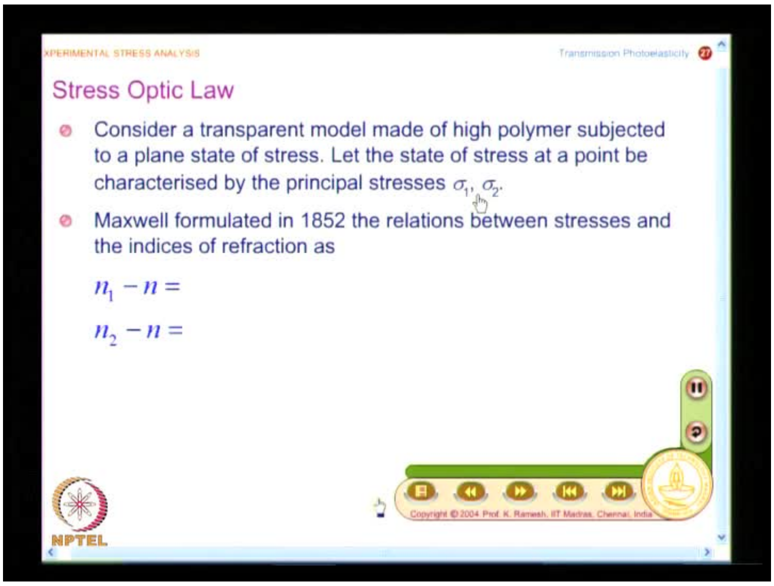

So, what you have here is, we called this, this law relates stress and optics, and I call this as stress optic law, it relates stress and optic, so I call this as stress optic law. And when I do this, I consider a transparent model material and this is made of a high polymer, and we also take for simplicity, subjected to a plane state of stress. And what you will have to note is, each statement you have to qualify. Let the state of stress at a point be characterized by the principle stresses, sigma 1 sigma 2. I am looking at a two-dimensional state of stress, I can have a matrix involving sigma x, sigma y, dou x y and so on, but I can also represent the same stress sensor in terms its principle stress values. So, that is what is indicated here. Let the state of stress at a point be characterized by the principle stresses, sigma 1 sigma 2.

And Maxwell, in 1852, formulated relations between stresses and the indices of refraction as. He conducted a series of tests, and then found out that n 1 minus n is related to sigma 1 minus sigma 2, it is a function of the material constant. So, what he found was, he found a direct stress optic coefficient C 1, and there is a transverse stress optics coefficient. And what he found was, that n 1 minus n, that is n 1 and n 2 are the refractive indices of the ordinary and extraordinary ray, and n is a refractive index in the unstressed state.

So, based on a series of experiments, he was able to establish a relationship, n 1 minus n is equal to C 1 sigma 1 minus C 2 sigma 2, and n 2 minus n equal to c 1 sigma 2 minus C 2 sigma 1. A similar exercise could be extension for three dimensions, which I am not paying attention now. And what you find here? We are interested in n 1 minus n 2. So, I subtract these two equation, then I can group the terms, and mind you, that C 1 and C 2 depends on what is the transparent model material that I going to use. So, there is the material parameters that comes in the formulation. Whatever I see, is also a function of the material that I use. The arithmetic is very simple. If I want to relate it to sigma 1 minus sigma 2, the arithmetic is very simple, there is no great deal about it. But to understand the physics, we have to look at how a crystal behaves, reinforce ourselves that for a one single incident ray there will be two refracted rays, the two refracted rays are plane polarized in mutually perpendicular direction, in general, they will be elliptically polarized, when it comes out the crystal. All that knowledge is required to appreciate the link, but if you look at the mathematics it is very simple.

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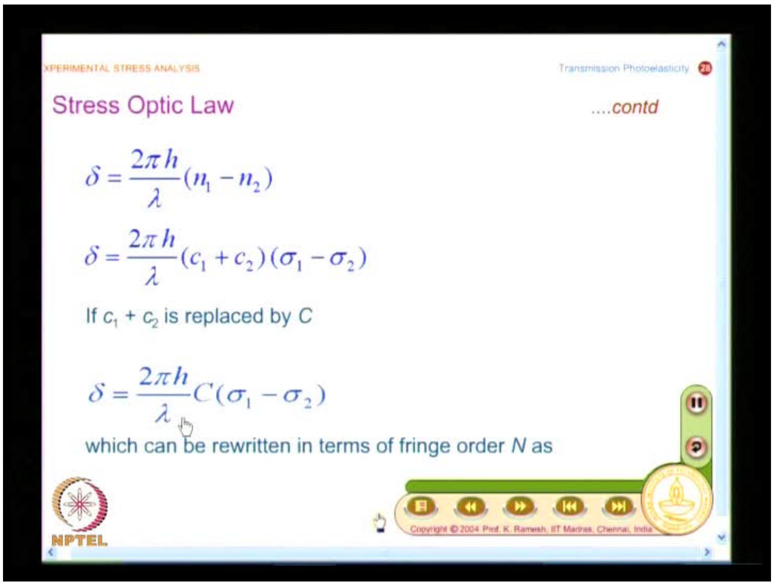

Now, I am going to have rewrite this delta in a form convenient for us to use, and those steps are fairly straight forward, there is no great mathematics involved here. So, I have this as 2 pi h by lambda n 1 minus n 2. Now, we know n 1 minus n 2 in a different form, so I put this as C 1 plus C 2 into sigma 1 minus sigma 2. And what I have is, this is the material parameters and for convenience we replace it by another symbol in order to differentiate it from the velocity of light, we use a capital C, I can recast this equation.

So, I have this as 2 pi h by lambda into capital C into sigma 1 minus sigma 2. So, it is a function of the material that I am going to use, and it is also a function of the wave length, that is very important. And if you go to any of the optical techniques, you would not tell the retardation in terms of radians. It is lot more convenient, if I label it as fringe orders, and I do not know how many of you have really looked at what is a fringe order. If you look at fringe order, it is defined as delta by 2 pi. So, if I say fringe order one, the relative retardation is. What is related retardation, delta by 2 pi, I say. So, the retardation is 2 pi. If I say fringe order and one, and fringe order one, fringe order two, fringe order three, you can go, and I can also have partial fringe orders. So, what we will do is, we will recast it, we will segregate the term delta by 2 pi.

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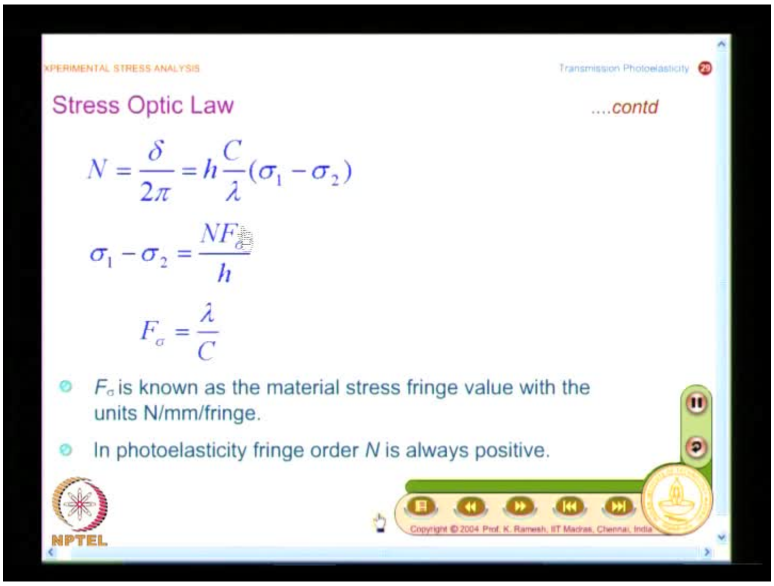

Then, the expression becomes lot more simple to look. So, I will write this N, N as delta by 2 pi. So, I can recast this equation as h into C by lambda sigma 1 minus sigma 2, and I can write sigma 1 minus sigma 2 finally, as in this fashion, NF sigma by h. And we have introduced a new symbol for the term lambda by C in this fashion. And mind you, this is a very famous relation in photoelasticity, sigma 1 minus sigma 2 equal to NF sigma by h, a very famous relation. And if you know only this expression, it is not sufficient, it is misleading, that is why I put immediately F sigma equal to lambda by C. Because if you look at the basic equation, because you bundled some of those quantities by a new symbol, there is a chance that you may misinterpret where is the wave length comes in the expression, there could be mistake like this, and when we ask questions in the examination, then we understand that you have not understood it, until then it looks as if it is crystal clear, only one questions are asked, you find that your understanding is not complete.

So, do not remember only this final expression, always think that F sigma is a function of wave length. Why we say function of wave length? Why we emphasize this? If you go to photoelasticity benches, some of the earliest benches, they had a mercury arc lamp, that was one of the easily available monochromatic light source, then people had sodium vapor lamp. So, these two wave lengths you come across, some of the old polariscope they may have only a mercury arc lamp, some of the recent polariscope, they may have white light as well as the sodium vapor lamp, and some of the material property stabled may have been obtain for a particular wave length, and you may have to use that property for your mathematical analysis, then I have to convert from one wave length to another wave length. So, that is, all that you can get by looking at F sigma is a function of wave length. And this is considered as independent of wave length for the most part of analysis and this also gives that this is a linear expression within limits. Suppose, I apply load, which are very close to plastic region, and I have very high stress gradient, this relationship is no longer linear, and you have to use it with caution. See, sometimes you look at an expression, whether the expression tells you or not, you assume many things.

So, one of the first wrong assumption that is possible is, it is independent of lambda, is a wrong conclusion you can arrive at, also, because people have introduced F sigma for convenience. And F sigma, how it is defined, this also has units newton per milli meter per fringe, an F sigma is known as the material stress fringe value, and this has a units like this, and when a plug in here, I will get stress as empea, that is the purpose here.

And many things you can understand from this expression. See, when I will introduced fringes from photoelasticity, even when you looked at the famous problem of four point bending, where you had tension and compression side, the fringes where always labeled positive integers. On the other hand, when we went to Moire, you found that fringes are numbered both positive and negative, whereas in photoelasticity you always number it as positive integers. Why is it so? That comes from this expression. Because when I say sigma 1 minus sigma 2, I always arrange the principle stresses in algebraically decreasing order, that is sigma 1 is the algebraically is the greatest, sigma 2 is middle and sigma 3 is the least. So, this will always be positive.

Let me go back to my other question that I raised about three classes back I took. An aluminum disc and also a polyurethane disc. I said, polyurethane is one of the photoelasticity model material. And then, imagine that I apply a same load, I said what is the nature of the stresses developed. Because polyurethane is a plastic, which has a low Young’s modulus, you can visually see the deformation. Aluminum is so strong, it has about70 gpa, and this is about 0.3 gpa, it is very small value. So, that deformation is definitely different, there is no two opinion about that. The question I asked was, for the same load and for the same size, how the stresses would be? It is a plain problem. Have you brushed your solid mechanics and found out? What do you anticipate? Stress will be same. I am happy to hear that. That is the very key point, without which there cannot be any photoelasticity. Suppose, I have three dimensional model, story is different. This we will see towards the end of the discussion on photoelasticity, we will look at the relevant mathematical equation and then show, I said in all the experimental techniques the Poisson ratio is a nuisance value, the Poisson ratio will do the spoiled sport, when I go to a three dimensional problem.

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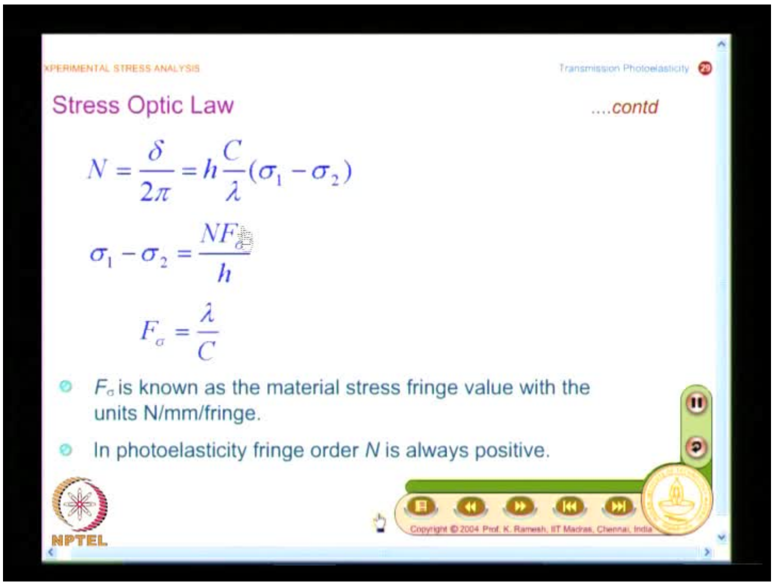

In planar problems, the stresses are same, it is very advantageous. Now, let us look at the expression, now what I have. See, if I have to use this expression, my interest is to find out sigma 1 minus sigma 2, that is very clear. From the experiment, I will have to find out what is the fringe order. And depending on the material that I use, if you look at photoelasticity, for class demonstration we bring in polyurethane, and then you have polycarbonate, you have epoxy, you have Perspex, you have even glass, they are all photoelastic material, and even the recently introduced stereolithography resin, they are all photoelastic sensitive material. And we use this for certain purposes, in class it is easy for me to apply the load and then show the generation of fringes very conveniently,

And when I do an experiment, I want certain amount of satiability, I do not want model to deform and introduce large deformation, when I introduce large deformation, the whole analysis becomes different. So, I want to minimize deformation. So, that is one reason why I choose different material, that is also another reason availability, then you have what is called time edge effect, we will see all those issues later. So, the essence here is, there will be chances for you to use models of different material. So, each of this material, I need to find out the material stress fringe value. Now, the question is by looking at this expression for a given problem, I change the material, what would happen to the fringes. We have just now seen, I take aluminum disc or a polyurethane disc, for the same load applied stresses do not vary. Instead of aluminum disc, I am going to have arielle disc or poly carbonate disc or perspex disc and so on. So, in such a scenario, what happens? Sigma 1 minus sigma 2 will not change at a point of interest. So, this product will change appropriately. So, if I have F sigma is small, I will have more fringes, if I have F sigma is high, I will have less fringes, this product will remain a constant. And if we look at the kind of problems that can be coined, the arithmetic is very simple, if you understand the physics behind it, if you anticipate that this is how it has to, the left hand side, here it is the left hand side, sigma 1 minus sigma 2 does not change, and only the right hand side changes. So, they will adjust. So, you will see more fringes, less fringes. More fringes, less fringes is not the indication of the values, you need to know the material parameter, only when you know that, I can evaluate the stresses. And also this is very important, if I find out this parameter in accurately, then all my match between experiment and, if I want do the comparison, whatever I do from the experiment and analytical methods, they will not match if I measure this quantity carelessly. I have to do sufficient care in finding out.

So, what we will do is, we will have a detailed discussion on how to find out F sigma as accurately as possible. And from photoelasticity point of view, we will have to find out how to get the fringe order N. I cautioned you even several classes back that in all optical techniques finding out the fringe order is tricky. You do not get it in the first go, you have to use auxiliary method, you will have to develop engineering equipment, you have to verify from varies methods of finding out, and then another question is, you find only fringe order and the fringes, you do not find out in between fringes, so you have to use compensation techniques.

So, finding out fringe order is an issue. So, if I have to find out the stresses, I have to know the fringe order N and material parameter. So, what we will proceed is, we will first go and see how to get the fringe order, then we will have discussion on how to find out F sigma. But even before we discuss on these issues, let us have a look at what is it that photoelasticity can give. See, I said photoelasticity can give you directly only fringe order and the principle stress direction. How it is going to give, how we have to do, we have to look at the optical arrangement, understand, and then go about it. But even before get into that details, we can now find out from our strength of materials knowledge, can I extract what information if I know these two quantities that would be of interest.

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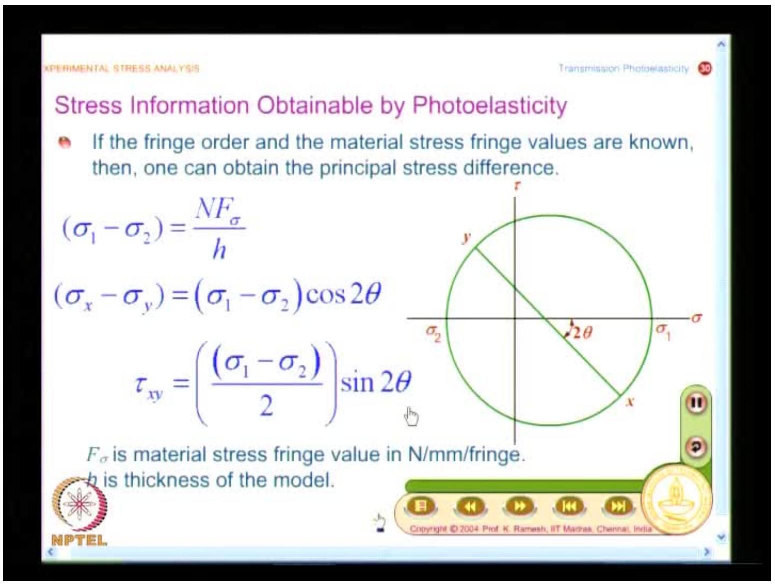

Because even before we want to do an experiment on photoelasticity, you can be assured what I can get as information from stress analysis point of view to the extent possible. So, that is what we will see in the next slide. So, what is the stress information obtainable by photoelasticity. So, here for this discussion, we assume at a point of interest you know the fringe order, we have not yet looked at the fringes, how to find out the fringe order, what is the optical arrangement, all that we will take it up later. Suppose, I know the fringe order, I know the material stress fringe value, and I also find out theta, what I can do? So, I will go to the Moire circle and look at what it is. And take an advantage from your knowledge of most, it is not new, it is all we build on your understanding of solid mechanics, strength of materials, the foundations have to be strong, that is why we had a review on solid mechanics, you should know about Moire circle, you should know that stress is a sensor and Moire circle represents this beautifully. And what I have here, I have sigma 1 minus sigma 2 is given as NF sigma by h, and if I know the Moire circle, I can easily write, sigma x minus sigma y equal to sigma 1 minus sigma 2 into cos 2 theta.

You all know, Moire circle you draw in the sigma and tau plane, I draw a circle and each point denotes the plane, and here it is x plane and this is y plane. And in Moire circle, all these angles are twice angles, that is why they are at 90 degrees it is shown at 180 degrees. And when I have so many points on the boundary, it shows all the possible infinite planes, you can find out what is the normal and shear stress absolutely, no problem. So, that is why it is a beautiful representation. I do not know whether you looked at Moire circle from this point of view. When I said all the possible state of stress in all the infinite planes, when I have a point of interest, that is what you understand as stress sensor and a beautiful graphical representation is Moire circle.

So, on the Moire circle, every point on the circle denotes a particular plane, and using this you can also find out what is the principle stress plane, what is the magnitude of sigma 1, and what is the magnitude of sigma 2, and simple geometry will help you to find out what is difference in normal stresses and also the shear stress. What is the value of shear stress? You all know it. It is simply sigma 1 minus sigma 2 divided by 2 into sine 2 theta.

So, what you find is, from photoelastic analysis a simple normal incidence can give you fringe order N and theta at point of interest. And if I know the material stress fringe value of the model material, then I can go use Moire circle, find out normal stress difference as well as in plane shear stress.

So, I can find out in plane shear stress very comfortably. And this is what I said, if you remember and recall, one of the very important problems in engineering is, I have a three point bends specimen, I want to find out what is the variance shear over the depth, I said when I go closer to the point of loading, though you have read in your simple strength material course that shear varies parabolically over the depth, this is no longer so when I go very close to the load application point. And I also mentioned, doing this analytically is possible; however, you have to represent this concentrated load by several Fourier harmonics is a circus. On the other hand, photoelasticity can give this information directly.

So, what do you need to find out. Closer to the point of loading, I need to find out tau x y, tau x y means, I have to find out fringe order and theta. And in fact, you will do this as part of one of your laboratory experiment, you will find that so nice, so elegant. It is a very key point in strength of materials, though you learn it varies parabolically near the load application point, near the surface, shear is maximum. And even if you want do it by numerical analysis, you have to discretize the model very carefully, and also model the concentrated load as accurately as possible.

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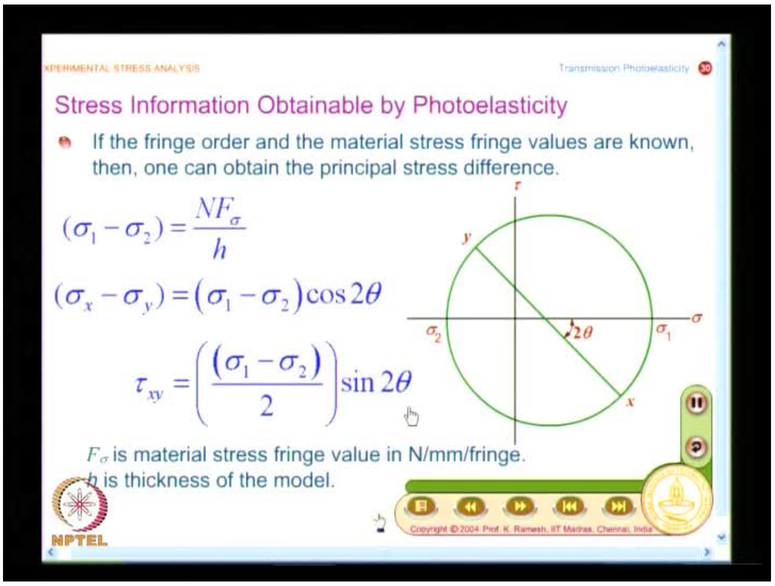

And is there anything like a concentrated load? It is an abstraction. Is there anything like a rigid body? It is again an abstraction. The concentrated load and rigid body goes together. In reality, all bodies are deformable. So, you live on approximations. And same concepts are required when we go and understand and interpret what is the result from photoelasticity analysis.

So, what we have looked at here is, stress information obtainable by photoelasticity. Suppose, I know N, theta and F sigma, I do not have to say I get only difference in principle stress, I can also find out difference in normal stresses and in plane shear stress. So, in this class what we have look at was, we started looking at how to understand the physics behind the simple polaroid sheet, we found that they display the behavior of dichroism, very useful, we have taken advantage of that. Then we moved on to find out an expression for relative retardation and we found it is a function of the thickness of the crystal plate and also depends on the wave length. And we also reasoned out, why mathematics becomes simpler when we use monochromatic light source in photoelasticity analysis.

Then we moved on to establish what is stress optic law. Maxwell has conducted a series of experiments, he found out there is a material parameter also comes in the equation, and I cautioned this material parameter is also a function of the wave length, you should not forget that. Because if you look at the expression, sigma 1 minus sigma 2 equal to NF sigma by h, it does not given an impression that is a function of lambda, but you have to keep in mind it is a function of lambda.

So, the problems could be F sigma is determined in one wave length, I do it in experiment in another wave length, so you may have to do the modifications, then I use different materials, then I have to do. If you look at arithmetic in photoelasticity analysis, they are very simple, but the physics behind it, little involved, that is what we are looking at it. And once you know the physics, you can easily solve the problem and the most challenging and crucial aspect is finding out the fringe order value. That requires some type of training, understanding, and that is where digital photoelasticity aids in minimizing your effort. Thank you.