**Experimental Stress Analysis**

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**Module No. # 02**

**Lecture No. # 12**

**Ordinary and Extraordinary Rays**

We have started our discussion on transparent photo elasticity in the last class, and I mentioned, light is used as a sensor in photos elasticity.

Then we moved on to find out what is a nature of light. We recall that light is nothing but an electromagnetic disturbance. So you have an electric vector and magnetic vector which are mutually perpendicular and in phase. They are looked at for the purpose of simplified mathematical treatment. We would take the electric vector as the basis to represent the light vector. What we saw is that the natural light is not convenient to be used as a sensor and we need polarization. The equipment used for photo elasticity is polariscope, which are plane and circular.

The first concept is polarization and the second concept is bi refringence. bi refringence is natural to crystals and what we do in photo elasticity is certain polymeric substances become bi refringence. When the loads are removed the phenomenon of bi refringence no longer exits. In order to understand how to get the stress field you need to understand what is bi refringence.

And another important aspect is when you have (( )) patterns you have phenomena of interference. One of the common examples given to understand interference is you go to a point take two stones and then drop. What happens is you have spherical waves form and they interfere when there is a crust. When crust meets you have a constructive interference and when you have the destructive interference where like to the wave is

In photo elasticity, it is slightly different. You have relative retardation which causes the formation of fridges and in the pan when you put two bubbles both the waves are in the same plane. In photo elasticity, you have mutually perpendicular waves with a phase difference between them. What we have to understand is from where these two waves had come.

The difference from other optimal technique is that the vibration requirements are not that stringent in photo elasticity which is the greatest advantage. You will understand it when we look at what happens in a crystal.

Let us first look at what is polarization.

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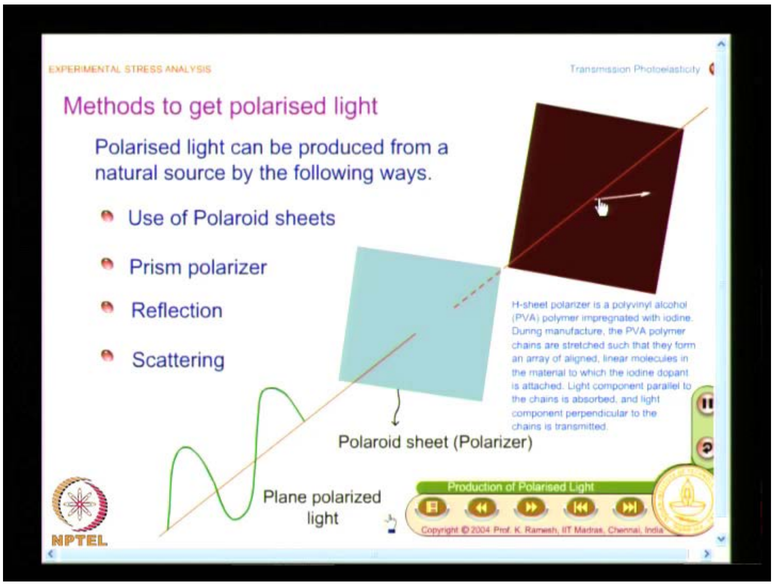


I have a natural light which is so random that the magnitude as well as the direction keeps on changing and a moment you put polaroid sheet you have plane polarized light coming out of it.

Polarization can also be affected by polaroid sheets, prism polarizer, reflection or scattering. In prism polarizer the polarization quality is very high but the field used is very small. On the other hand, in polaroid sheets, you almost get 99.9 percent of polarization and the field used can be as large as even one meter you can do.

And for certain wave lengths reflection or scattering is ideal and at an appropriate angle called (( )) angle you have polarization taking place. This is also another method of getting polarized beam of light and the simplest form of polarization is plane polarized light. Once you have plane polarized light, you could get elliptical and circular polarization.

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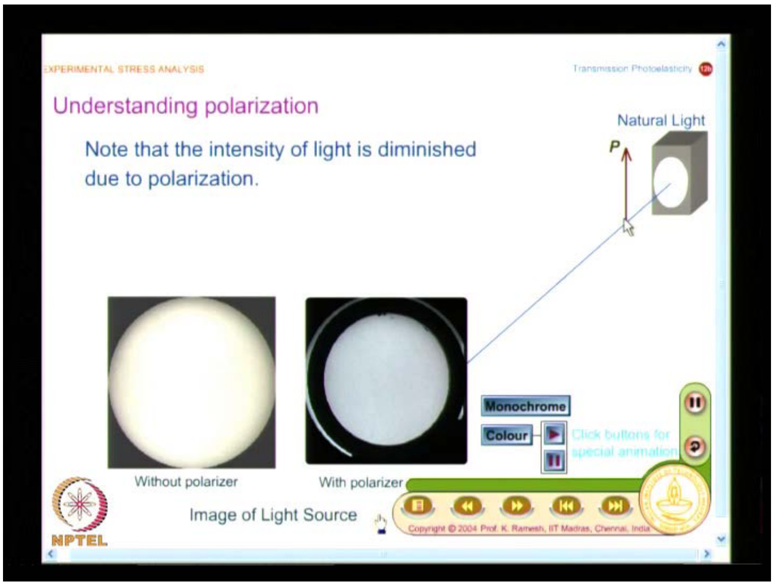


And now, we will look at the justification that the light is polarized. In the last class, we have understood polarization. The moment you send polarized beam of light the amount of light available on the screen is diminished and I show the polarizer simply by a lines sketch and this indicates direction of polarization.

So, what I have here is that by putting this sheet only the vertical component is transmitted. Here I put another element which also allows light only in one direction. When I keep it exactly perpendicular, it should cut, and the second element functions as analyzing the light that is coming out and is impinging on it both physically. As the second one helps in analyzing the light coming on it, it is called as an analyzer.

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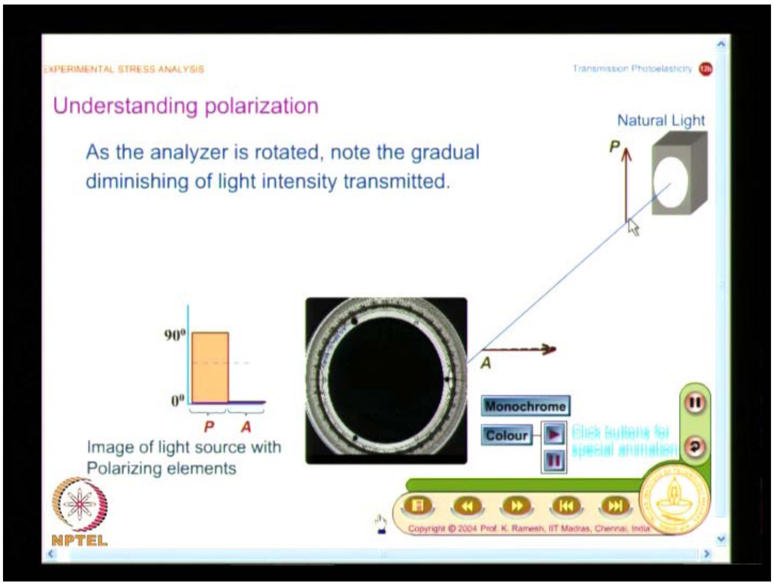


So what I do here is I put an analyzer and I keep rotating it. I find that light gets diminish and when it becomes perpendicular you have light is totally strict and another circle concept was also introduced while discussing this I had an natural light this is the white light source

Here we also looked at monochrome light source. What is the difference between white light source and the monochrome light source? In a white light source you have vibgyor may be with different proportions depending on the light source. So it is a multi-wavelength source and as I said for all numerical development a single wave length is simple and convenient. People also have developed methodologies to find out even in a multi wave length as light source, how to interpret data.

What you saw here was that for the multi wave length also when the polarizer and analyzer cut or crossed the light is estimated. In a mono chrome source also the light is completely extinguish and you see a block.

We will also qualitatively look at how a fringe gets form and what could be a light extension condition. So currently what we are having is only a polarizer and analyzer and we saw that the entire field could be dark or the field could be bright.

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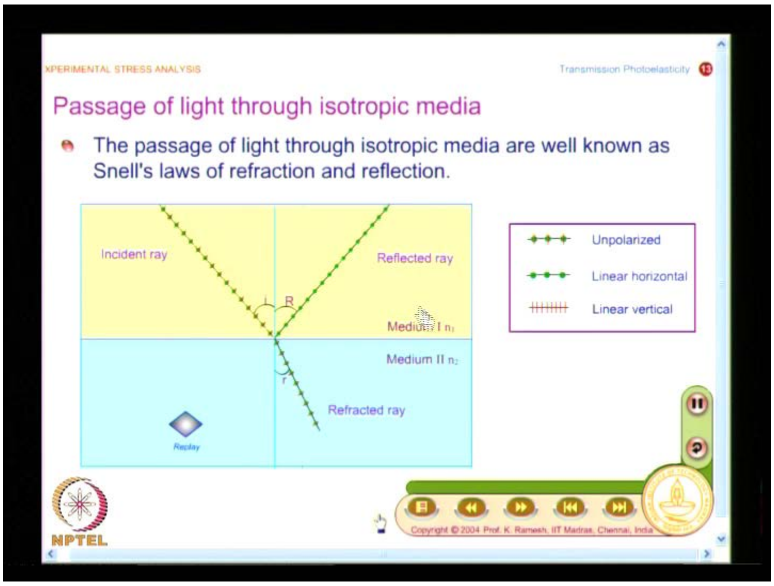


Next we move on to what happens in a isotropic media because when you want to go in to crystal it is better to understand what happens in a isotropic media first.

So when I take an isotropic media I have an incident ray and I have medium one shown by one color and medium two shown by another color. So the light gets refracted and what you have to note here is the representation of un polarized light which is nothing but natural light shown with dot and a straight line.

When you have only dots it is understood that it is linear polarization with horizontal direction and when you have only vertical lines it is linear polarization with vertical direction. Here what you see is that when I send an un polarized beam of light it comes out as un polarized beam of light and also gets reflected as un polarized beam of light.

When you adjust the angle of incident you may get the reflected ray at a particular angle. For glass it is about 50 to 60 degrees and you will get only a polarized beam of lights. So in an isotropic medium when I send a natural light you see light getting transported with in the medium this is one observation and the other observation is we will define refractive index in terms of velocity appropriations which is very crucial from photo elasticity point of view.

` Now we will look at Snell’s laws of refraction and reflection.

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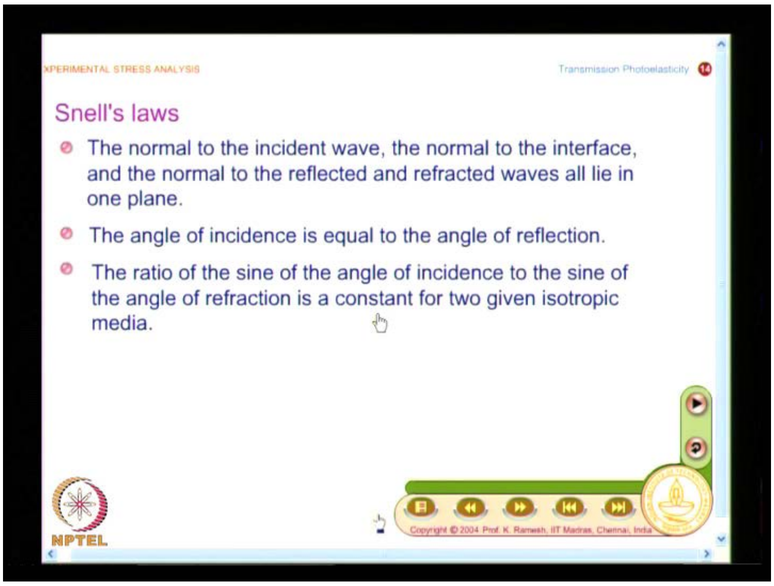


So here the first observation is that the normal to the incident wave, the normal to the interface, and the normal to the reflected and refracted waves all lie in one plane.

The second observation is that the angle of incidence is equal to the angle of reflection. So angle I is equal to angle R, (Refer Slide Time: 09:41)

The third observation is that the ratio of sine of the angle of incidence to the sine of the angle of refraction is constant and depends on the given isotropic media.

So we have that if the medium is air it is known as relative refractive index. If you want go to absolutely refractive index, one medium should be vacuum. You all know that sine I divided by sine R is the refractive index.

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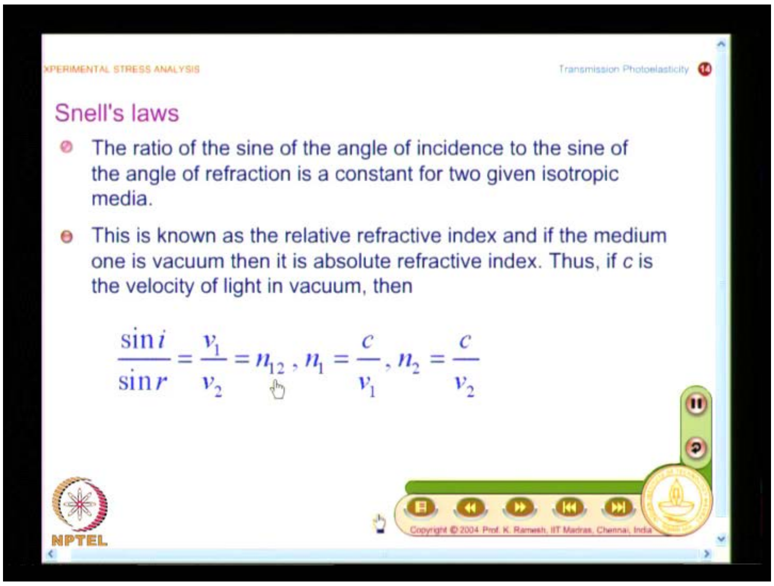


The relative refractive index is denoted by small n and you should not confuse this with direction cosines. Here n 1, n 2, n 3 denotes the refractive index.

Here I see this sin I by sin R as a ratio of velocity v 1 and v 2. We all know that the velocity of the light in vacuum is c. So when you travel in different medium there is a slight change in the velocity and we are going to have some kind of a phase difference which is being initiated into the wave because of the stress information. What you find here is that when the refractive index is different the velocities are different in the media.

So I have a relative refractive index given as v 1 divided by v 2. The absolute refractive index from the medium one is given as c divided by v 1 and for the medium two it is c divided by v 2. For one incident ray I have one refracted ray and it changes as I go to crystals. In photo elasticity you also call this as common path interferometers which are less stringent in vibration isolation requirement.

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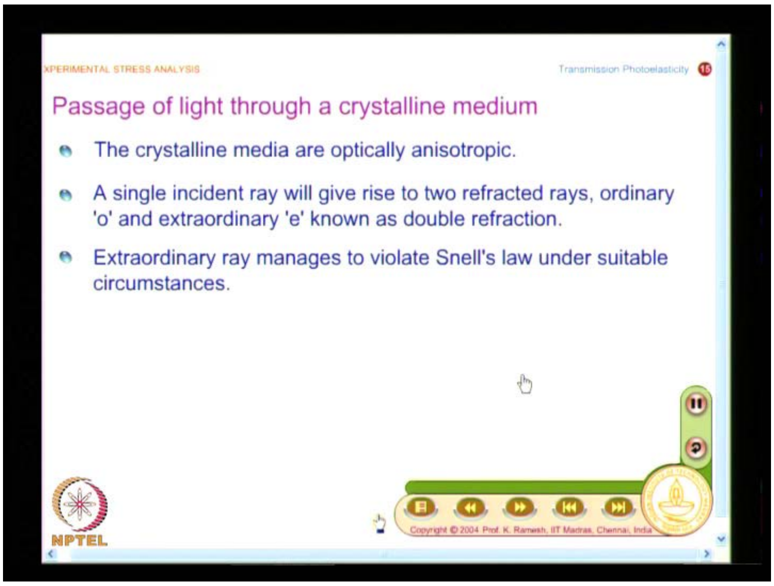


Here let us see what happens when light passes through a crystalline media. The first observation is that the crystalline media are optically anisotropic. So if the property say along all directions then you call that as isotropic and if the property is the function of direction then it is anisotropic.

The second observation is that a single incident ray will give rise to two refracted rays, one called as ordinary ray labeled as o and another called as extraordinary ray labeled as e. This happens because of double refractions.

So what you find here is that the ordinary ray faithfully follows Snell's law and under suitable conditions extraordinary ray violates Snell's law.

So you have for a single incident ray two refracted rays, ordinary and extraordinary. first we will have a look at the ray diagram and then as I mention I will provide you an experimental justifications for all these concepts.

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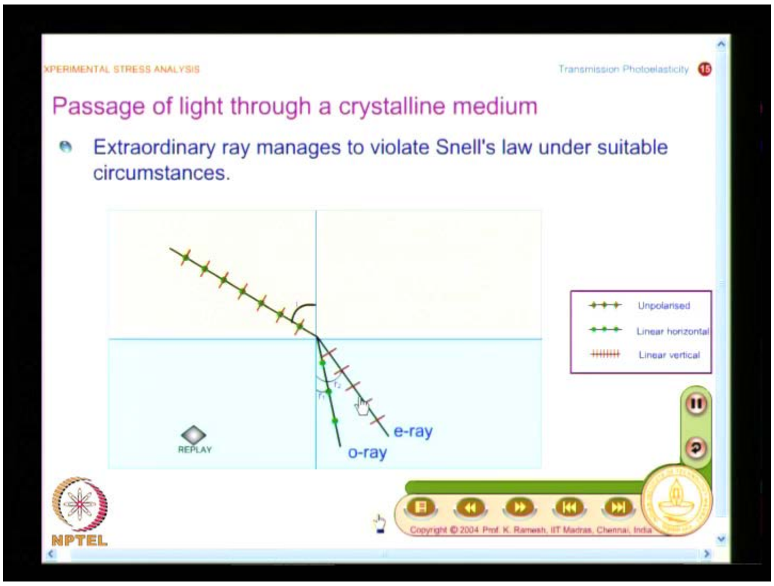


So I have medium one and medium two now is crystal, it is not a isotropic media. So I have a ray which impinges on this carefully look at how the incident and refracted rays are drawn.

First observation is for one incident ray I have two refracted rays, ordinary ray and extraordinary ray. Just look here how these rays are depicted. The incident rays is un polarized but within the medium as the medium does not end there. Within the medium, I find the rays are polarized which is represented by only straight lines on extraordinary ray and only dots in ordinary ray and the planes of polarization are mutually perpendicular.

So I have two simple harmonic motions which are mutually perpendicular. The planes of polarization are mutually perpendicular and in this case you see them as two different rays. In photo elasticity we will develop it and see that they will travel in the same direction and have planes of polarization mutually perpendicular. When I have this angle change I have r 2 and r 1what do you infer is that refractive index is different. But when I go to photo elasticity I look at refractive index as different velocities, so that gives you the information. Say, when you look at holography what happens is light impinges on the model. If you are looking at metrology application, because of the three dimensional shape the depths are different and you have a light which impinges and comes back and hence there is a phase difference. But in photo elasticity it penetrates the model and gets modified with in the model and acquires the phase difference.

So what you find here is that the model behaves like a crystal. The moment you have a crystal a generic understanding is that for one incident ray I have two refracted beams which are travelling at different velocities. Their planes of vibration are mutually perpendicular, but in this case they travel in different directions.

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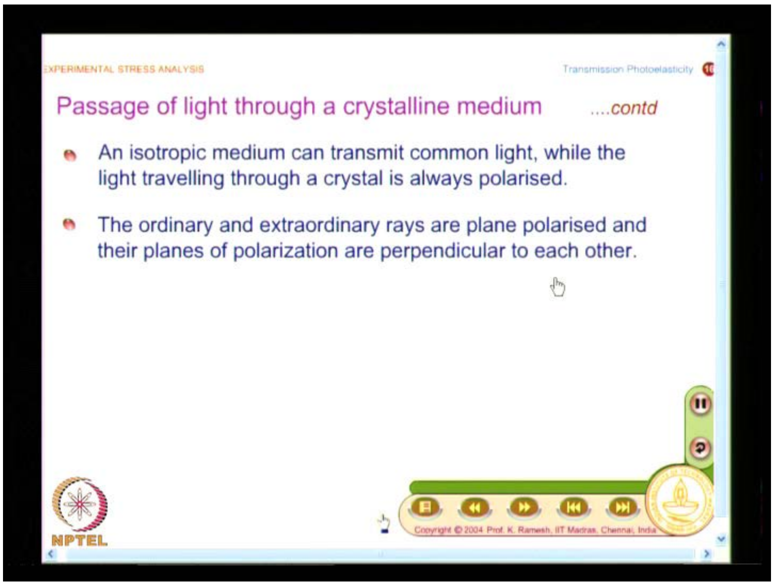


We can summarize this point as an isotropic medium can transmit common light which is nothing but a natural light, while light travelling through a crystal is always polarized.

So with in a crystal whatever the light that travels through is polarized. A crystal cannot sustain un polarized light. What happens in photo elasticity is that the model behaves like a crystal when it is stressed. So that is the reason why we are able to look at modification of the light as a function of stress as a natural crystal will always have bi refringence.

The first and four most observation that we make is that the ordinary and extraordinary rays are plane polarized and their planes of polarization are perpendicular to each other.

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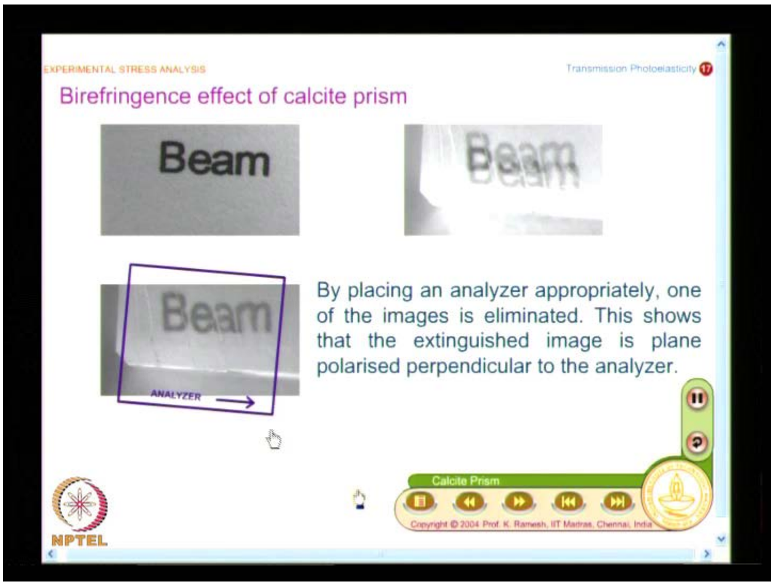


We look at the next slide where I again come back to the later beam. So I have image of the word beam written on a paper and you view it normally. Then what I do I go on put a crystal on top of it and the moment I put it I clearly see two sets of revered beam appearing.

We have seen with in a crystal you have ordinary and extraordinary rays and you see two images. I can do the analysis of polarization of light by taking a polaroid sheet and here we See that the ordinary and extraordinary rays are polarized in perpendicular direction.

Suppose I take a polaroid sheet and place it on the image and rotate it appropriately. What should I anticipate is that at particular orientation one of the images should not be seen only one image should be seen and at another orientation I should see the other image.

So what I do is I am going to a put polaroid sheet on top of it. The intermediate process is not shown only the final process is shown here. So I have a analyzer and I oriented in this direction what do I see is only one of the images that I saw as two in the earlier case.

So this goes to prove two things, you find these rays are polarized that is why I am able to cut off one image and the other thing is that the image vibration is perpendicular to this so that images cut off as it will allow only one component of light.

So in the first case you see when you put a crystal I see two images which indicates for a single incident ray you have two refracted beams and I have shown by putting a polarizer. Because it analyses the light it is named as an analyzer and when I place it appropriately one of the images is eliminated .This shows that the extinguished image is plane polarized perpendicular to the analyzer.

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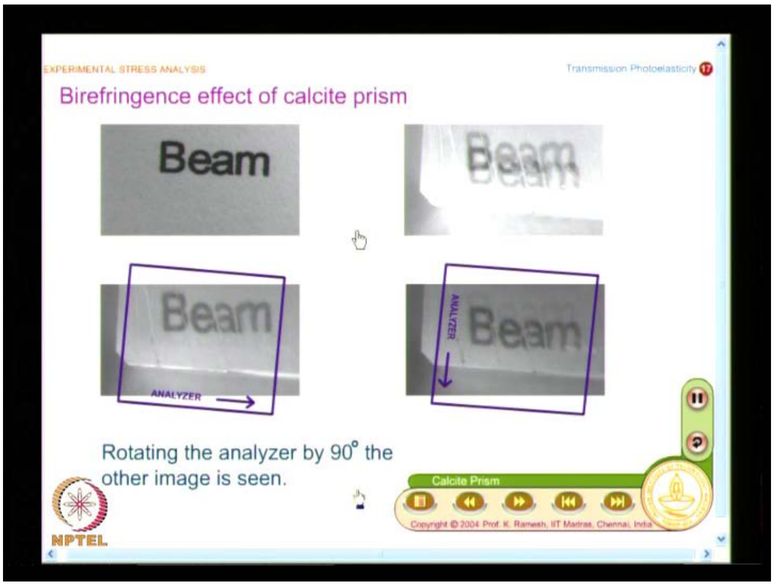


Suppose when I keep this analyzer perpendicular to this then I should see the other image. Let us rotate it and then keep it. Here you can see that I have two images and there is a slight shift the shift.

So I could filter out from this one of the images by employing an analyzer. This shows that the beams are polarized and I get these two images when I keep them at mutually perpendicular positions of the analyzer. We also establish that these two beams are polarized in mutually perpendicular direction.

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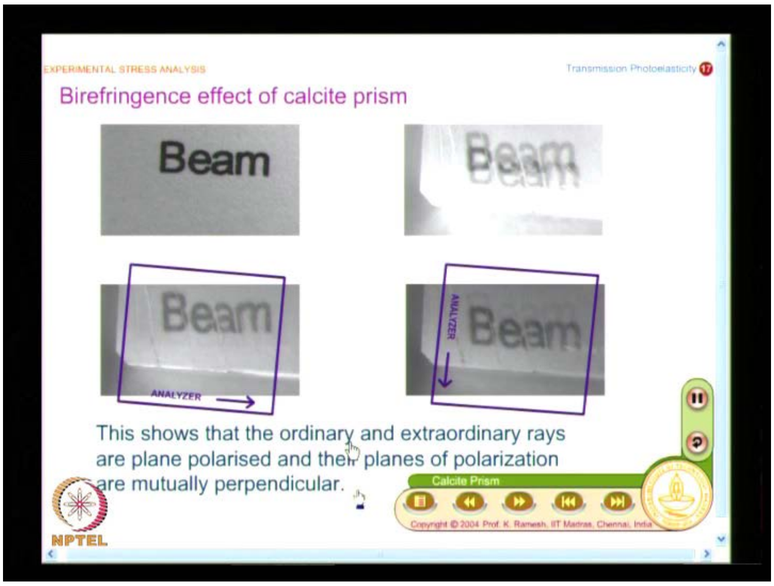


So here we summarize that the ordinary and extraordinary rays are plane polarized which is established by using an analyzer which cut off light perpendicular. So this shows when I extinguish one image the plane of polarization is perpendicular to it and we indirectly show that it is the plane polarized beam of light and the planes of polarization of ordinary and extraordinary rays are mutually perpendicular.

But in photo elasticity we do not want to see two images we want to see in a particularly different fashion. We will also bring in the concept of optic axis and look at the incident ray in relation to the optic axis.

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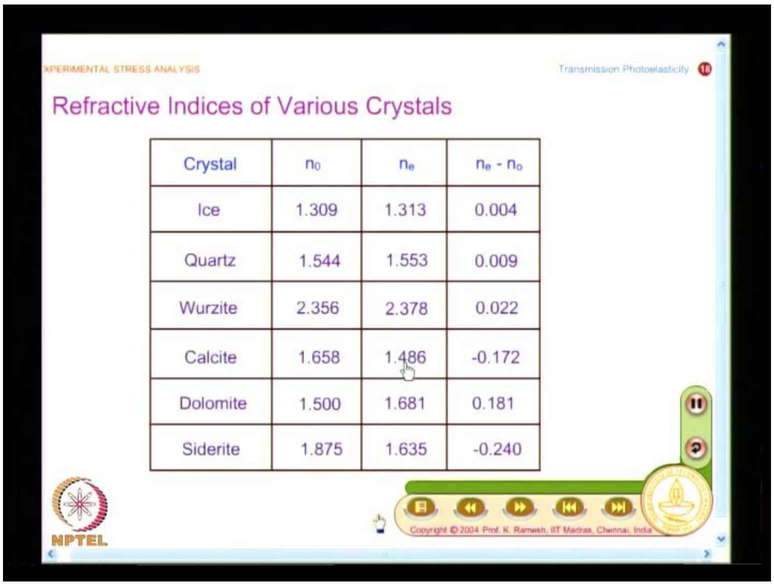


| Crystal | $n_0$ | $n_e$ | $n_e - n_0$ |
|---|---|---|---|
| Ice | 1.309 | 1.313 | 0.004 |
| Quartz | 1.544 | 1.553 | 0.009 |
| Wurzite | 2.356 | 2.378 | 0.022 |
| Calcite | 1.658 | 1.486 | -0.172 |
| Dolomite | 1.500 | 1.681 | 0.181 |
| Siderite | 1.875 | 1.635 | -0.240 |

As I said earlier in natural crystals you have two refractive beams ordinary and extraordinary ray and you also have tables available which give you the ordinary ray refractive index and the extraordinary ray refractive index.

So that gives you an indication that you have two refractive beams with different refractive indices and what is important in photo elasticity is we look at this different refractive indices more from the point of view of velocities.

So in crystal you have this naturally happening and the crystal behavior is introduced because of the stresses. We are in a position to relay stresses to optical behavior. Let us see crux of photo elasticity.

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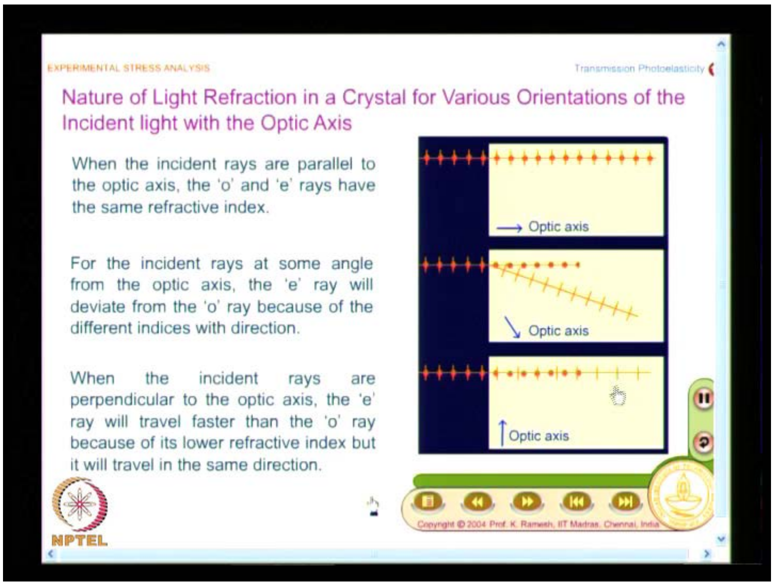


And now what we will look at is the direction of incident of light which is suitable for photo elastic analysis and when you take a crystal you have optical axis. I have a crystal here and I have a ray of light impinging on it. This ray is un polarized beam having a dot and straight line and this direction is parallel to optic axis.

As shown in the diagram I have the ray travel in an isotropic medium where you will have the un polarized beam transmitted as un polarized beam. There is no specialty about crystal behavior here. So what you find here is when the incident rays are parallel to the optic axis the ordinary and extraordinary rays have the same refractive index.

Now we look at another case where we have this axis impinging at a different direction. The optic axis is cut at an arbitrary position where I have an incident ray and I have two refracted beams. The animation very carefully shows that one ray travels faster than the other and you find that this is polarized and the plane of polarization is mutually perpendicular.

So what I find here is for the incident rays at some angle from the optic axis, the extraordinary ray will deviate from the ordinary ray because of the different indices with direction. We have already seen that crystal is optically an isotropic and it is dictated by the incident ray in relation to the optic axis direction. If it is same as optic axis direction then it behaves like an optical isotropic medium and if it is at an angle you see two refracted beams which travel with different velocities and the planes of vibration is mutually perpendicular.

This is not useful in photo elasticity, it is only to understand that within a crystal you have double refraction and you see two images. What you have to look in photo elasticity is that what happens when the incident ray is perpendicular to the optic axis. When it is perpendicular to the optic axis both the ordinary and extraordinary rays travel in the same direction but within the crystal. They will acquire the phase retardation because they have different refractive indices.

So what is summarized here is that when the incident rays are perpendicular to the optic axis the extraordinary ray will travel faster than the ordinary ray because of its lower refractive index but it will travel in the same direction. This is very important and you have already seen from a solid mechanics point of view that photo elasticity provides difference in principle (( )). I also mentioned in one of the earlier classes that refractive index and stresses are a tensor of rank two.

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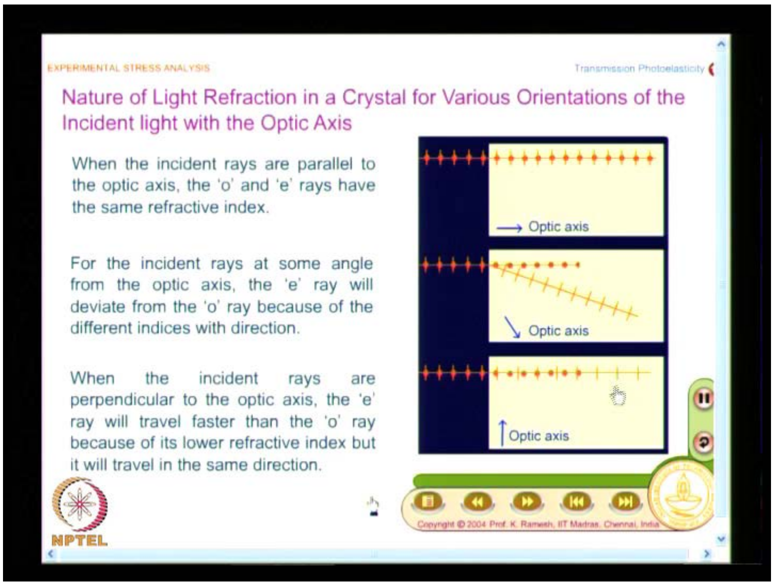


So this is what happens in photo elasticity. The phase retardation is not because of depth change it is because of the way the material behaves. A crystal behavior is perceived as n 1 and n 2 at the point of interest and this n 1 and n 2 could be related to sigma 1 and sigma 2. If I have a crystal at every point, n 1 and n 2 is same and if you have this as a stressed model, n 1 and n 2 change and is dictated by the stresses applied.

Other observation is as I said photo elasticity gives you direction of principles stress which comes from the optics axis and it is perpendicular to it. We are looking at only planer problems.

So now you will understand how optics is related to stresses in a very simplified fashion. Here you are shown two reflected beams but when I rotate the crystals I have only one beam at a particular orientation. This I can show by a demonstration in photo elasticity. One beam travels faster than another beam. At different orientation you have two beams coming that is what you have to anticipate.

I have also said experimental is anticipating the result before performing an experiment. In many cases your anticipation may match with observation and in some cases it does not match with our observation.

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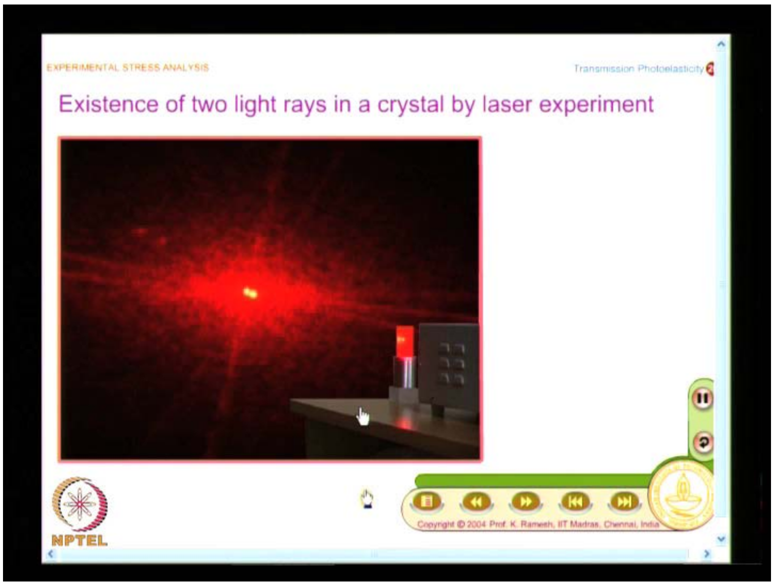


I (( )) laser is experiment. So if a moving crystal is rotated I have laser beam. What you will see is I have a bright spot here and when I rotate the crystal you will see two dots merged into one dot.

See when the light is coming out of the crystal then they will interfere. Suppose I have a light ray which is parallel or perpendicular to optic axis, then both will appear as one dot. Only when I go to photo elasticity I can show fringence. In fact if you recall that in the last class I have shown two plates and when I applied load I saw beautiful patterns. So that shows that light travels with different velocities. In this animation you have to look for one and two dots which will convince that the optic axis has an influence on the behavior of the crystal. The incident light in relation to the optic axis behaves differently.

So I replay it and have a magnified picture. So you could see two dots and then that merges into one. So you see that again when the crystal is rotated I have one dot becoming two dots.

And once you understand this there is joy in doing photo elasticity because you know the in and outs of what happens with in the model. Now we will get into mathematics. What you have here is in normal interferometer both waves travels in the same direction and if there is a phase difference how you will add them.

Suppose I have waves travelling mutually perpendicular and they acquire phase difference, how you will add them.  You have to add it very carefully you cannot do the same addition law like what you have done it when the waves are travelling in the same plane. When they are mutually perpendicular the mathematics is slightly different and we have seen that we have a elliptic polarization. Elliptical polarization can be thought of (( )) polarization in one limiting case and another limiting case are plane polarization.

This comes from your addition of two simple harmonic vibrations which are mutually perpendicular that is what we need in photo elasticity. I have shown you two rays which move with planes of polarization perpendicular to each other and we are already seen that this can be returned as cos omega t or sine omega t. And I also shown that they acquire a phase difference.

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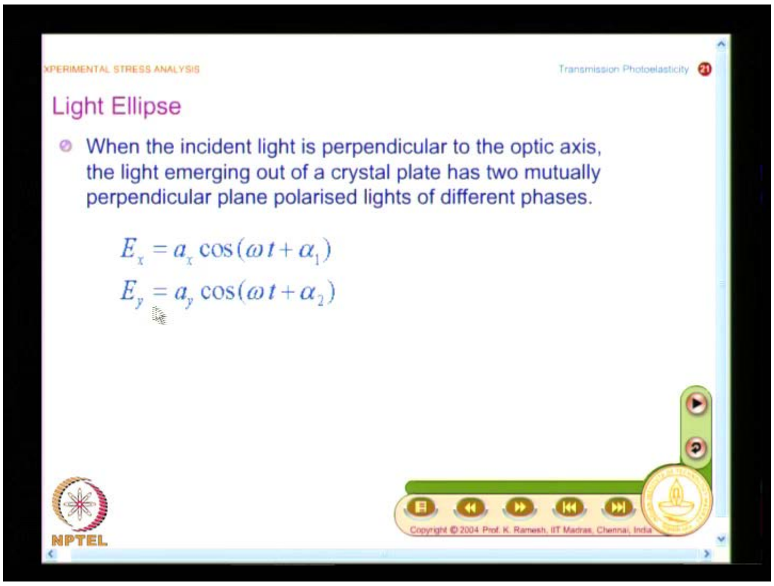


When the incident light is perpendicular to the optic axis the light emerging out of a crystal plate has two mutually perpendicular plane polarized lights of different phases.

So I represent light waves like this. I have two beams of light one is given in the x direction as E x and is equal to a x cos omega t plus alpha 1 and the other wave as E y equal to a y cos omega t plus alpha 2 and they have different amplitudes. Here we are taking generic situation. We keep the amplitudes different and we also have some absolute phase. Alpha 1 can be 0 in a particular case and you represent the second wave as as omega t plus alpha 2.

But what is important in photo elasticity is difference in phase rather than absolute phase. Difference in phase is acquired between the two waves because of this optical behavior induced by stresses.

So it is fundamentally different from other interferometer techniques. Here you have two waves which are mutually perpendicular and acquire phase retardation with in the model which is caused by the stresses introduced. As one ray becomes two, whatever happens to one ray will happen to both the rays so vibration oscillation is not a stringened requirement.

So in this class what we focused was for photo elasticity you need to understand that there are two beams which travel with in the model and they acquire phase retardation and these two beams are plane polarized and the planes of polarization are mutually perpendicular. This comes from an understanding of crystal optics because a crystal by its very natural behaves like this. Whereas in photo elasticity the model temporarily behaves like a crystal when loads are applied and when loads are remove the crystal behavior is different.

So with this background I anticipate that you come out with innovative methods and it helps you to develop your concepts better and may be come and develop a new experimental technique.

Thank You.