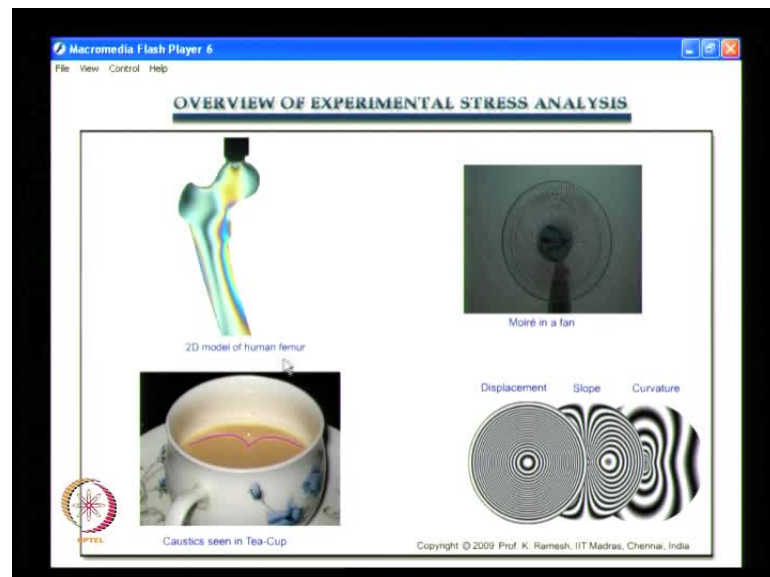


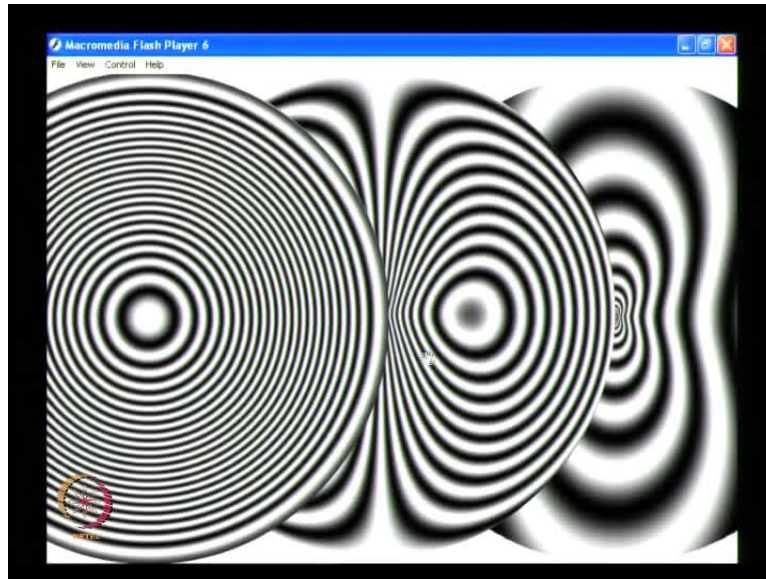
Experimental Stress Analysis
Prof. K. Ramesh
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Module No # 01
Lecture No # 10
Selection of an Experimental Technique

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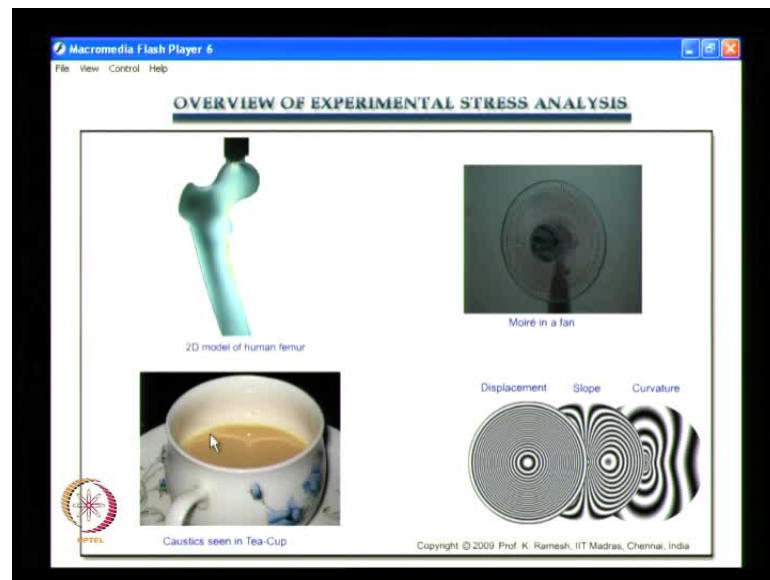
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We come to the concluding part of our discussion on overview of experimental stress analysis. This slide was shown at the beginning of this chapter and now you must be familiar in identifying various techniques that are depicted in this slide. Here you see the usage of photo elasticity for finding out the stress patterns and you find the use of techniques like moiré holography or shearography to reveal various fringe patterns. You find this as the out of plane displacements. Now you know that this is the slope, this is the curvature information and each of this requires separate optical arrangements. You may get this displacement by the technique of holography or speckle interferometry or shadow moire. What you see is the simulated fringe patterns and when you really make an experiment the fringe patterns will follow this closely but the quality of the fringes depends on the technique that is being used.

The moment you come here you see this as butterfly fringes which you get from shearography. You can also get it from a variation of moire and this curvature fringes from shearing interferometry. So now you have a familiarity how the whole field information looks like.

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So this knowledge will go a long way in appreciating what patterns you have and how you have to interpret them when doing the experiments. These two corner slides bring out the physics for each of the techniques. You see this as a method of caustics which is also seen in the case of moiré. You also have beautiful moiré patterns coming out and the term moiré has an origin from fringe.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Selection of an Experimental Technique

- One of the crucial steps in experimental analysis is to identify a suitable technique(s) for a given problem on hand.
- The selection of a technique depends on several factors such as
 - ★ Time available for analysis
 - ★ Level of accuracy required
 - ★ The range of strain/stress to be measured
 - ★ Influence of extreme conditions like high temperature, high strain rate etc.
 - ★ Thoroughness of the study required
 - ★ The cost permissible for the study

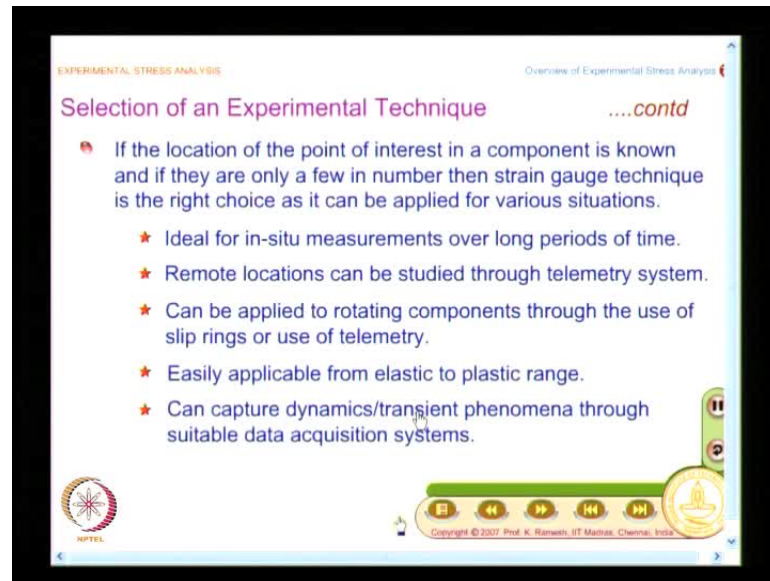
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We have been discussing on selection of an appropriate experimental technique and we will go back and review what we have seen till now. One of the crucial steps in

experimental analysis is to identify a suitable technique for a given problem. We have seen that it depends on several factors like time available for analysis, level of accuracy required, range of strain or stress to be measured, influence of extreme conditions like high temperature, high strain rate etc. You also have to look at what thorough study is required and finally the cost permissible for the study.

(Refer Slide Time: 03:59)



EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Selection of an Experimental Techniquecontd

- If the location of the point of interest in a component is known and if they are only a few in number then strain gauge technique is the right choice as it can be applied for various situations.
 - ★ Ideal for in-situ measurements over long periods of time.
 - ★ Remote locations can be studied through telemetry system.
 - ★ Can be applied to rotating components through the use of slip rings or use of telemetry.
 - ★ Easily applicable from elastic to plastic range.
 - ★ Can capture dynamics/transient phenomena through suitable data acquisition systems.

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Then what we looked at was if the location of the point of interest in a component is known and if it is few in number then strain gauge technique is a right choice as it can be applied to various situations. One of the greatest advantages of strain gauge is that it is ideal for in situ measurements over long period of time as it is necessary for monitoring structural health which you need to do that for bridges and dams. This comes as an ideal technique for remote locations, rotating components and also applicable from elastic to plastic range and also dynamic or transient phenomena

(Refer Slide Time: 05:19)

EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Selection of an Experimental Techniquecontd

- If one wants to get a whole field appreciation of the stress field then photoelasticity is the right choice.
 - ★ Useful for quick comparison of different designs.
 - ★ Can be applied to a range of problems from static to dynamic analysis.
 - ★ Through its many variants easy to measure residual stresses, assembly stress and stresses interior to the body.
- In fact, an intelligent use of photoelasticity with strain gauge technique can solve a variety of problems that commonly occur in normal design scenarios.

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So what we find here is that it is more of a general purpose technique which is advantageous when you go in for strain. We have also looked at in detail about certain issues in using photo elasticity. Suppose I want whole field appreciation of the stress field, then photo elasticity is the right choice. We have seen for a stress concentration problem plate with a hole, plate with an elliptical hole for crack problem and spanner with different designs.

So when I want to know what happens on the entire field, photo elasticity would do it quickly and this can be applied to a range of problems from static to dynamic analysis. You have many variants of photo elasticity which can be selected for measuring residual stresses, assembly stresses and most importantly stresses interior to the body.

Similarly when I want to go and look at what is interior to the body the problems are always complex and photo elasticity provides the revenue. Recently you have seen in the paper that Honda has asked certain components to be called back in the recent **cost** that they have fabricated and released in the market. So when you are really looking at the design you have such flaws and people have to call back and then replace the component. In such applications a detailed analysis involving both numerical and experimental analysis is a must. If you go back in history, in many engineering applications people have done in detail three dimensional analysis using photo elasticity. So this is one of the strong points as far as photo elasticity is concerned and what you

have is that an intelligent use of photo elasticity with strain gauge can solve a variety of problems that commonly occur in normal design scenarios and also in special situation. In fact you have concorde aircraft though it is grounded after the accident in France, was one of the aircraft which had very least amount of failures in the field. There were no accidents till it was grounded. It had a tail rudder which was failing repeatedly.

You know in those days about in seventies, finite element analysis was done to find out the cost for the rudder failure. From finite element point of view, the rudder is found to be reasonably strong to withstand the service loads which never failed. So what they did they took a strain gauge and put it on the rudder and made measurements. When they compared their strain gauge values with design calculations it showed that it is within the permissible limits. Something what they could not identify from a numerical technique or a point by point experimental technique was the cost as the component was failing in the field. So they had to go for a whole field experimental measurement. They applied a photo elastic coating and found that the maximum stress point was slightly shifted and the strain gauge was reading only two third of its maximum value. The moment they applied photo elastic coating, they identified the point of stress concentration and put the strain gauges which made the design perfect and the problem was solved. So what you need to do is in many practical problems, identifying what to do which is the critical area itself is a challenge.

(Refer Slide Time: 09:58)

The slide is titled "SELECTION OF AN EXPERIMENTAL TECHNIQUE" and is part of an "Overview of Experimental Stress Analysis". It discusses special techniques for addressing special problem situations. The techniques listed are:

- ★ Moiré interferometry has found wide acceptance in electronic packaging industry to measure thermally induced stresses on tiny components.
- ★ Speckle and holographic methods are found to be suitable for the analysis of micro electromechanical systems (MEMS).
- ★ Thermoelastic stress analysis is useful to measure stresses developed due to fatigue or random loadings.

A note at the bottom states: "The methodology seems to be promising for unconventional applications such as stresses introduced in slamming of automobile doors!"

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So a combination of whole field and a point by point technique help you to solve day to day design problem. In special situations you can always fall back on either photo elasticity or strain gauges. You need special techniques to address special problem situations and the moment you think of moire interferometry you have to think of electronic packaging industry which has found wide acceptance.

In electronic packaging industry, to measure thermally induced stresses on tiny components moire is ideally suitable as the components are very small where I cannot go and paste the strain gauge. Moire gives the whole field information and researches convincingly show that moire is applicable for these classes of problems. For MEMS application speckle and holographic methods are ideally found to be suitable. In an MEMS sensor we have seen how a speckle method could be employed and how they have used holographic method to study a micro gear rotating at 360000 rpm. So in MEMS application if you have the need think of either speckle or holographic methods.

Then you have the method of thermo elastic stress analysis which is useful to measure stresses developed due to fatigue or random loadings. It is a very special application people have applied using thermo elastic stress analysis. The methodology seems to be promising for unconventional applications such as stresses introduced in slamming of automobile doors. In fact you know in automobile industry people are very particular about the shape, so they keep changing the shape of the automobile. They all have streamline contours that travel at very high speeds and there is a pressure on them to bring out new variance. All this is a sheet metal work and you need to find out for the given life whether the doors will withstand. So slamming of the door is a random loading as it is a three dimensional problem. You can apply thermo elastic stress analysis to it as you have a contoured shape where you need to slam it and make the measurement.

So as technology advances you get newer problems. In fact if you compare, in seventies and eighties the cars did not have smooth contour they had box type. Now what you have is streamline contours, new manufacturing technologies and more and more increase in speed. All these bring in new class of problems which also require newer techniques to solve.

(Refer Slide Time: 13:56)

EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Selection of an Experimental Technique *....contd*

- ★ Holography is ideal for recording mode shapes of a vibrating component.
- ★ Speckle methods in its variation of shearography is very useful as an NDT tool in aerospace industries.
- Like photoelasticity and strain gauge techniques, the methodology of digital image correlation is attractive as a general purpose stress analysis tool.
- ★ The technique is still developing and its accuracy level needs to be improved further for low strain measurements.

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The moment you think of vibration, holography is an ideal choice. If you want to find out the mode shape you would normally sprinkle sand or lycopodium powder on a flat object. As long as I am working with the flat plate of different shapes I can do the analysis with sand or lycopodium powder. If I want to do it for a turbine blade which has a twisted complex aero foil shape then holography can be used because it is essentially out of plane measurement technique. More composite and honeycomb panels are used in space technology. So they all have the delamination problem.

Speckle methods in its variation as shearography is very useful as an NDT tool in aerospace industries. As I have been saying photo elasticity and strain gauges are general purpose tools, similarly methodology of digital image correlation is also attractive as a general purpose tool. This technique is still developing and its accuracy level needs to be improved further for low strain measurements. Sometime back the low strain was about 160 micro strain and now people say that you can go up to 50 micro strain. So as the technology is improving you are also improving the capability of the technique to measure smaller values of strain.

(Refer Slide Time: 16:41)

The slide is titled "SELECTION OF AN EXPERIMENTAL TECHNIQUE" and is part of an "Overview of Experimental Stress Analysis". It contains the following text:

- The above guidelines are not complete and the experimentalist's acumen in deciding an appropriate technique or an intelligent use of an existing technique to unconventional situations is always welcome!
- In experiments, there is no upper limit for accuracy!
 - ★ As one improves the experimental setup, the accuracy can be improved.
 - ★ The deciding factors are the cost and time available for analysis.
- Based on the stress-strain history, a broad classification of suitability of different techniques for different ranges of strain is indicated.

The slide also features a navigation bar at the bottom with icons for back, forward, and search, and a copyright notice: "Copyright © 2007 Prof. K. Ramiah, IIT Madras, Chennai, India".

What you have to keep in mind is that whatever the guidelines we have discuss so far are not complete and the experimentalist is acumen in deciding an appropriate technique and an intelligent use of an existing technique in unconventional situations is always welcome. You should never forget that your engineering acumen is required to intelligently attack complex problems with simpler combination of existing methods. Doctors have the acumen in detecting what is the disease you people have and he is only able to identify the disease correctly and solve your problem.

So similarly engineers also have to understand what is the requirement of the given design scenario, select an appropriate technique and solve the problem. Another issue is what level of accuracy you can expect in experiments and you have to understand that there is no upper limit for accuracy. You can keep improving the accuracy in an experimental technique, you may start with a simpler rate and make some measurements and once you understand the scenario you may want to improve it as much as possible.

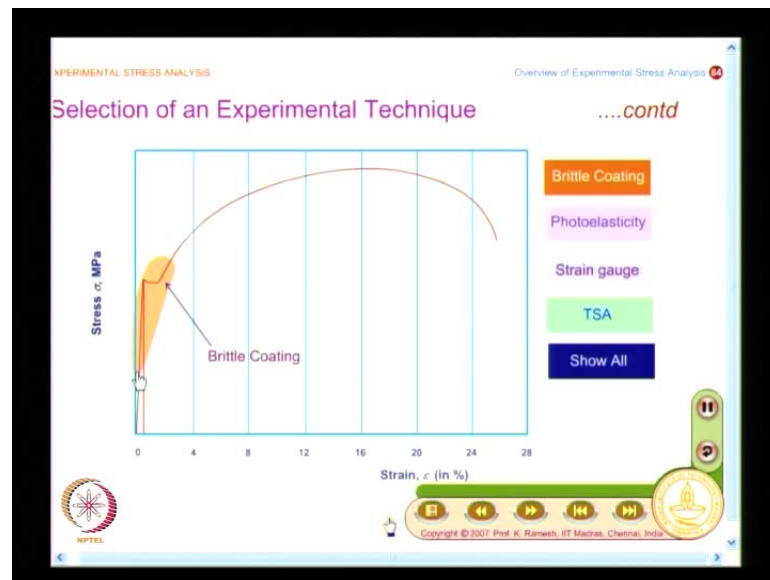
So as one improves the experimental set up, the accuracy can be improved and there is no upper limit for accuracy. The deciding factors to improve the accuracy are the cost and the time available for analysis. So if some designer wants a quick answer you could give a number with certain level of accuracy and can always use a factor of safety to carry on with the design and have a very precise analysis. See normally when people want to make a helicopter to fly they will make it fly first, optimization will come later

where you need refined analysis. When you are talking about accuracy I would like to caution that when you compare your experimental results with analytical solution, for example, if I want to find out the free end displacement of a cantilever so you have $\frac{pl^3}{3EI}$. What you will do is you will find out what is the displacement from your experiment. For the moment you consider that there is no shear deformation, the depth is very small but even then when I make my analytical calculations I need to feed in the value of moment of inertia and moment of inertia is $\frac{bd^3}{12}$. If you do not measure the depth of the beam correctly, then what will happen? You will make an error on analytical calculation of the displacement because I have $\frac{bd^3}{12}$ it will have a very high error and $\frac{pl^3}{3EI}$ will not match with experiment. So you should not always blame the experimental technique when analytical and experimental techniques do not match.

In your analytical calculation have you provided correct numbers for calculation is also very important. See if I give a ten specimen to a student he uses a scale and measure the thickness. You have to use a vernier and measure the thickness. When thickness and small distance is to be measured, you should use the precision and this many people ignore when they compare analytical method results to experiment. Even for analytical calculation you need to feed in some measurement by making actual measurements on the component of object. So these measurements need to be precise. You should use methods of principles of statistics, take few measurements, and take an average of all that you should employ. People never use statistics they go and measure once and then say that this is the value. You will also always have an averaging procedure which is also equally important.

So when we say experiments there is no upper limit. For accuracy, you should also cautious when you are doing an analytical calculation; those calculations have to be accurate enough. What we will look at now is based on the stress strain history. We will look at a broad classification of suitability of different techniques for different ranges of strain in the next slide.

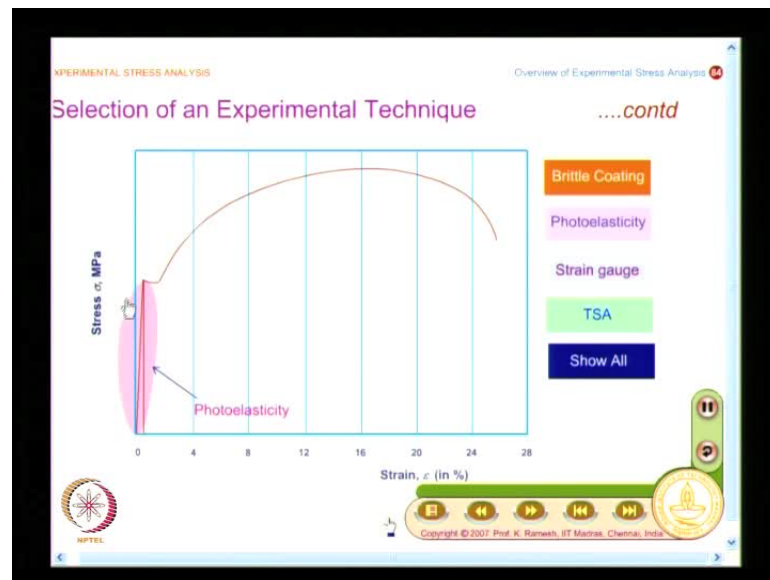
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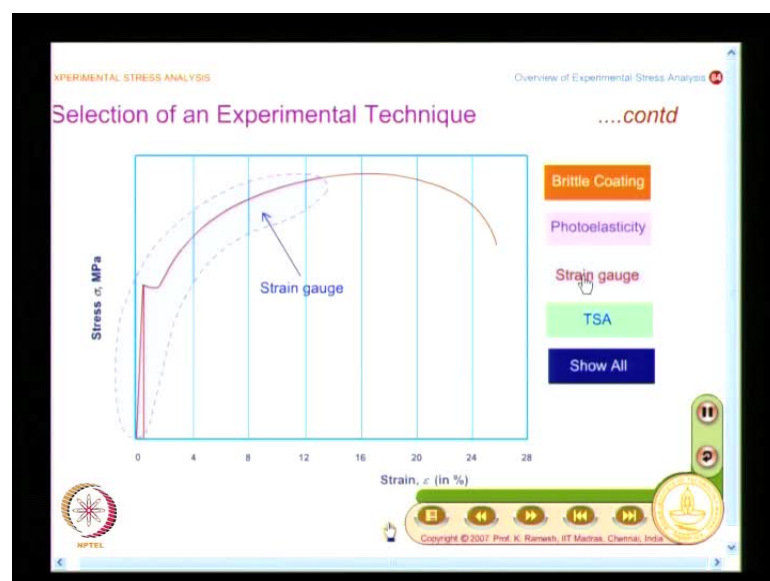
I have taken a typical stress strain graph to show that I have a very small elastic region and a very large plastic region. What we will look at is the range we can use for the curve in brittle coating, photo elasticity, strain gauge and thermo elastic stress analysis as visual representation always helps in identifying in what region I am working elastic or plastic. In brittle coating we all see that the failure strain of the coating decides the lower limit and we saw that it is about 300 to 500 micro strain. As newer materials are developing, you may be able to bring it down but the threshold limit is about 300 to 500, so you will have brittle coating in this zone.

So when you are operating in this colored zone as shown in the Refer Slide Time: 22:31, the stress strain is about 300 micro strain and when it has the elastic point, it is about 2000 micro strain. So in brittle coating which is used to identify plastic deformation, we cannot go to lower strain levels. This is the typical range where brittle coating can be employed.

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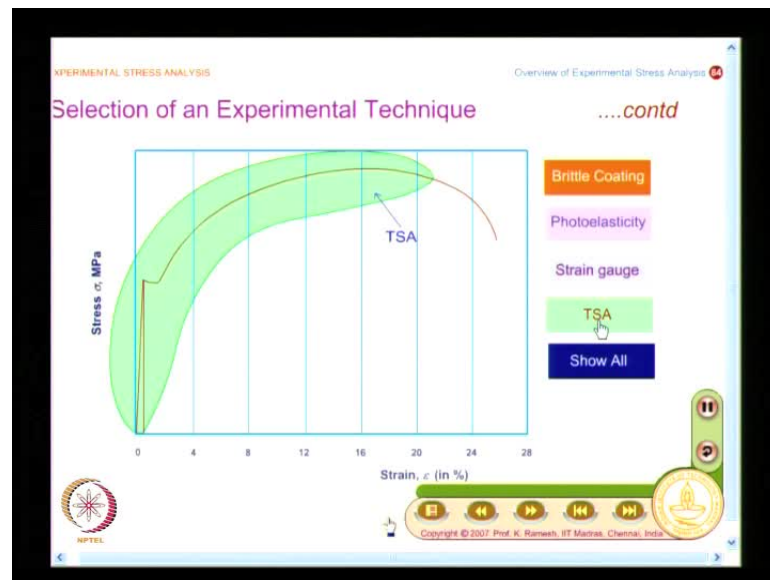


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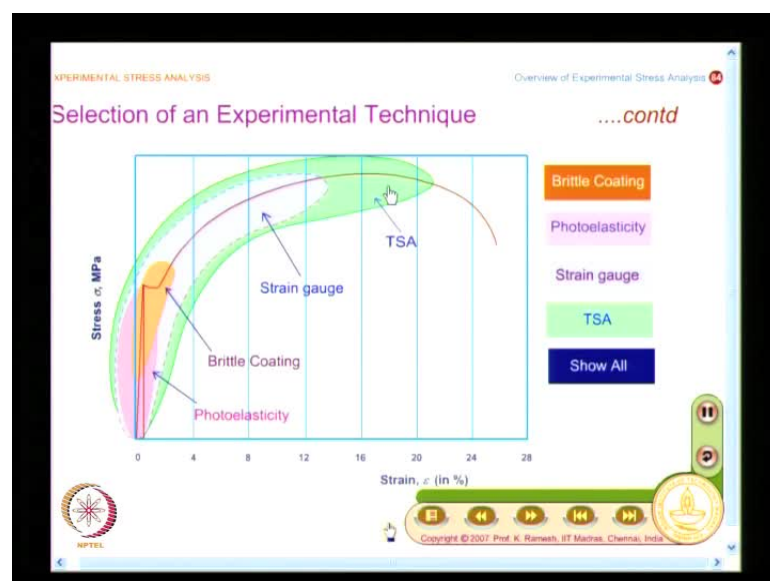


What you need to worry about is the end point and the start point. So in photo elastic analysis will confine only in the elastic region. The moment you come to strain gauge we have seen currently you have techniques which can measure even 0.5 micro strain which is almost close to zero. You can also go to the plastic region about 10 to 12 percent of strain which can be easily measure with strain gauges. That is the reason we say that strain gauge is a versatile technique.

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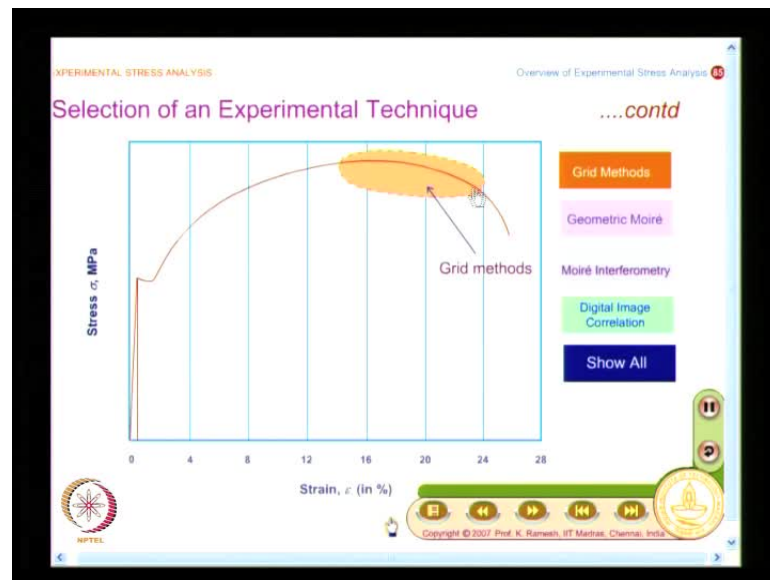


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So it has a longer range where I can go from elastic to plastic comfortably and thermo elastic stress analysis is proposed as a whole field strain gauge technique. The strain gauge instrumentation is simpler when compared to TSA, but both the instrumentations are expensive. Now you have relative zones for different techniques as shown in Refer Slide Time: 26:29 which will help to find out which way you will select the existing technique.

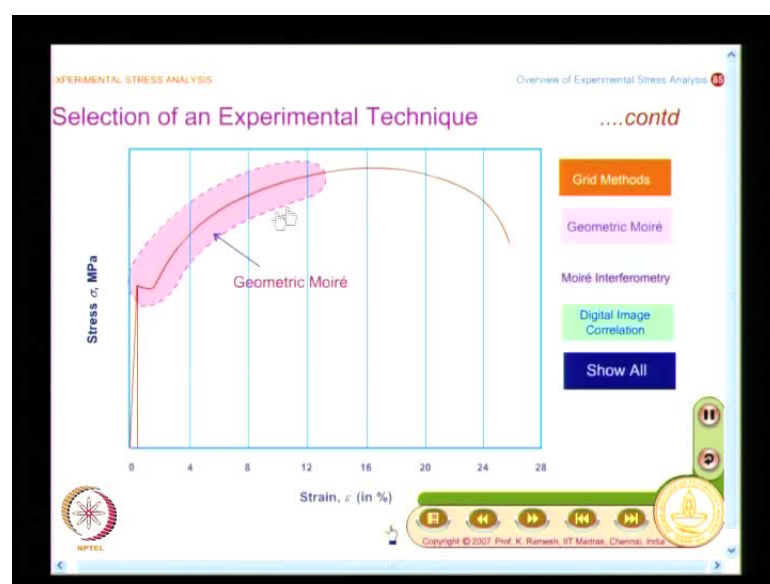
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Here we will have similar graph for other four techniques which are grid methods used for very high plastic region, geometric moire, moire interferometry which is the refinement of geometric moire and digital image correlation which has large range but does not have accuracy at lower strain levels.

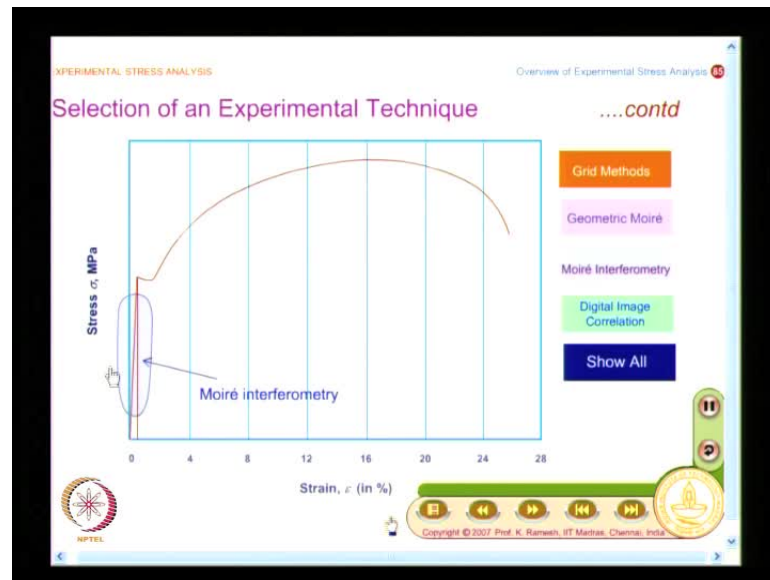
So when you look at grid methods it is applied only in the highly plastic region. The range is shown in Refer Slide Time: 26:51.

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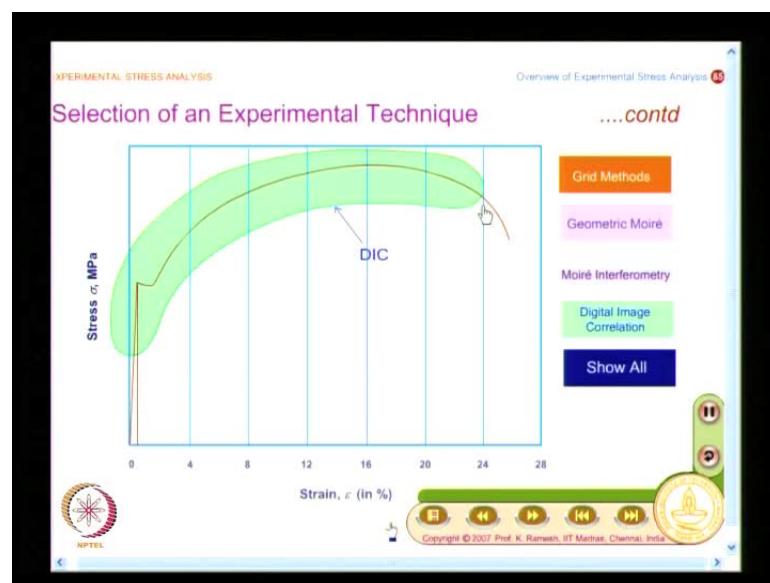
When you come to geometric moire, it would be here as shown in Refer Slide Time: 27:47 and if you want to lower it then you can go for moire interferometry. What you need to worry about is the end point and start point and the issue here is the range of strain that you are really looking at.

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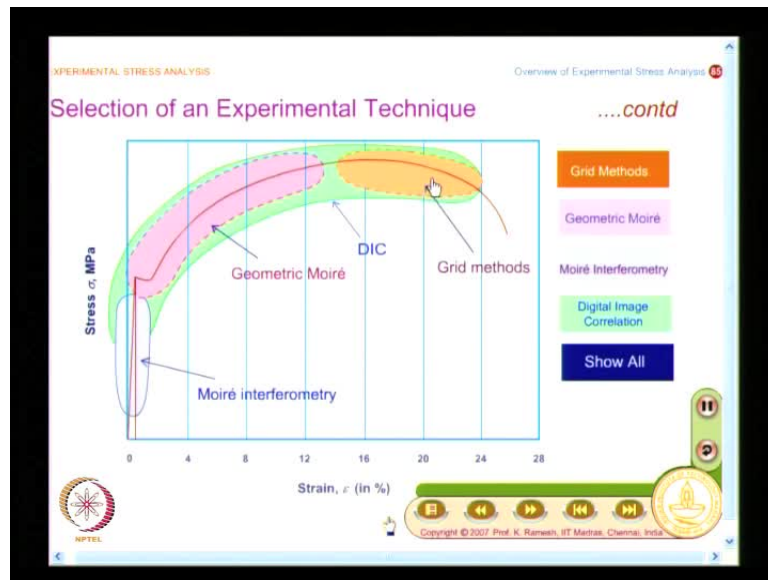
Moire interferometry cannot measure very small strains it can go down but it cannot go close to zero as shown in Refer Slide Time: 28:24.

(Refer Slide Time: 28:48).



And when I come to digital image correlation I can go from one end to another as shown in Refer Slide Time: 28:48. It can be brought down to as close as zero by improving the technology on which the researchers are working. Their claim is now about 50 micro strain and it is only a question of time that they improve the technique further.

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Now you have relative appreciation of various techniques as shown in Refer Slide Time: 29:23 to decide the techniques based on the strain range.

(Refer Slide Time: 29:54)

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And now we come to the important step of looking at what are all the references that you need to have a look at. You have a recently published third edition of Springer handbook of experimental solid mechanics which has a large number of techniques compiled in one volume and it is edited by W N Sharpe. In this handbook, we have a chapter on photo elasticity which covers the modern development of photo elasticity and it also give a nutshell variant of photo elasticity.

Then you have a very famous book optical methods of engineering analysis by Gary Cloud published in 1998 who has given a series of articles in experimental techniques. He is writing articles on various optical methods for displacement stress and strain measurement which are very interesting.

The next you have is a book on digital photo elasticity written by me. This completely covers the development of photo elasticity from conventional techniques to digital photo elasticity and exhaustively gives you various methodologies including phase shifting techniques, Fourier transform techniques, color image processing methods and also application to fracture mechanics and stress separation.

Then you have handbook of moire fringe technique published in 1993 and high sensitivity moire published in 1994. Here the focus is more on applying this moire technique for thermal stress analysis problem. There is also a separate chapter that was written in the book on experimental analysis for mechanics and materials.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

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This is the last slide for this chapter. To go to next/other chapters navigate through the main menu button.

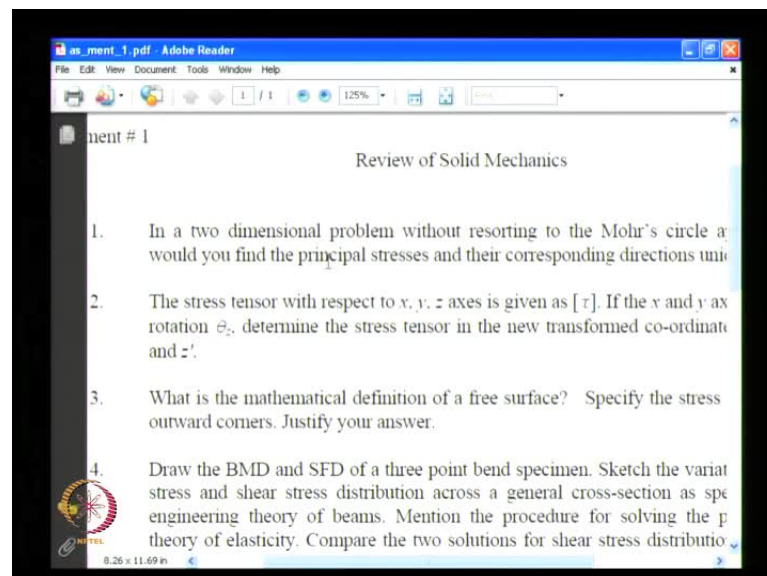
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Next you have two references on holography. I have a book on holographic interferometry by Rastogi published in 1994 and the engineering applications of lasers and holography by Kock published in 1979. For each of the techniques and sub techniques you have separate books available. We have another book on holographic interferometry written by C M Vest.

Then you have a book on speckle methodology a group of interferometric techniques by R S Sirohi. He had a very good lab at IIT Madras and several of his students have made significant contributions in the area of speckle methodology.

And finally you have a book by Asundi who has done more computer processing had come out with how MATLAB can be used for photo mechanics. We have already seen when we looked at trends in experimental mechanics that the current trend is on finding out phase shifting techniques which are used in several interferometric methods. All these interferometric methods use CCT camera as an electronic eye and the image processing is done. For many of the image processing application MATLAB could be advantageously used. That is why Asundi has come up with the book on MATLAB for photo mechanics.

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Then what we will look at is review of solid mechanics. The important aspect here is, in a 2 dimensional problem without resorting to the Mohr circle approach how would you find the principle stress and their corresponding directions uniquely. If you look at, in

your learning your focus was mainly on failure analysis and finding out truss kyle yield criteria or one mises yield criteria where you need only magnitudes of principle stresses. As we are not bothered about the direction theta we will not look at what are the (σ) in finding out the direction. In fact if somebody says if you have an expression for theta whether it represents maximum principle stress direction or minimum principle stress direction you cannot say with the simple calculation of trigonometric equation.

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Isoclinic evaluation

Conventional Method

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

Eigen value – eigen vector approach

$$(\sigma_x - \sigma_1)n_x + \tau_{xy}n_y = 0$$

$$\tau_{xy}n_x + (\sigma_y - \sigma_1)n_y = 0$$

$$n_x^2 + n_y^2 = 1$$

σ_i $\begin{Bmatrix} n_x \\ n_y \end{Bmatrix}_i$

Let us look at that equation which is given as theta equal to tan inverse 2 tau xy divided by sigma x minus sigma y multiply by half. As this expression is a trigonometric expression it will give you multiple values of theta. How will you associate it is in Mohr circle if you know how Mohr circle is drawn and interpreted then you will be able to fix up the theta which corresponds to sigma 1 direction. This theta what I have calculated plus 90 degrees will give me sigma 2 direction. When I use this standard analytical expression this expression cannot give it.

So you need to do something extra. When I do an experimental technique to find out the sigma 1 direction and sigma 2 direction, I use auxiliary methods to convincingly find out whether it is direction for sigma 1 or sigma 2. This was a problem in digital photo elasticity. So here you need to pose the problem differently and do it as an Eigen value Eigen vector approach. Here I get principle stresses as Eigen values and the direction

cosine n_x and n_y as the corresponding direction. So I have the Eigen value and Eigen vector.

(Refer Slide Time: 41:09)

Isoclinic evaluation

Conventional Method

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

Eigen value – eigen vector approach

$$(\sigma_x - \sigma_1)n_x + \tau_{xy}n_y = 0$$


$$\tau_{xy}n_x + (\sigma_y - \sigma_1)n_y = 0$$

$$n_x^2 + n_y^2 = 1$$

$$\theta = \tan^{-1} \left(\frac{n_y}{n_x} \right)$$

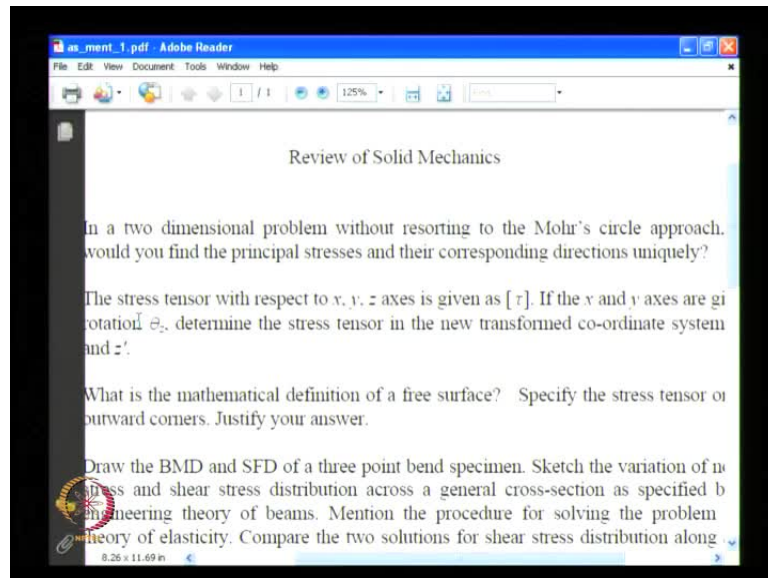
$$n_x = \frac{-(\sigma_y - \sigma_1)n_y}{\tau_{xy}}$$

$$\left[\frac{\sigma_y - \sigma_1}{\tau_{xy}} \right]^2 n_y^2 + n_y^2 = 1$$

$$n_y = \sqrt{\frac{1}{1 + \left(\frac{\sigma_y - \sigma_1}{\tau_{xy}} \right)^2}}$$


When I repose the problem as Eigen value and Eigen vector it is possible for me to find out value of sigma 1 and corresponding theta. The greatest one is sigma 1 the smallest one is sigma 2 and sigma 3. Now the question is that how you will associate theta to sigma 1 direction and sigma 2 direction as you get 2 values of theta. You can do if you reformulate the problem as an Eigen value and Eigen vector approach. So when you do that it is possible for you to find out n_x and n_y from simple arithmetic.

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So analytically also we need to do special efforts to find out this. When I do by experiment particular by photo elasticity, you will see that I have to use some kind of a calibration to associate the direction obtained to sigma 1 or sigma 2 and the emphasis here is more on finding out the principles of direction.

Next we have is finding out the stress tensor in a transformed coordinate systems. In fact in the whole of strain gauge analysis if you know the tensor transformation law you can comfortably understand that stress and strain are a tensor of rank 2. So once you know the transformation law whole of strain gauges can be handled.

Then we have what is the mathematical definition of a free surface when the stress tensor on free outward corners is specified. Now let us get in to the details of this.

(Refer Slide Time: 43:16)

Mathematical Definition of a Free Surface

If n is the outward normal of a surface then the stress vector on that plane is obtained as


$$\{T\}^n = [\tau] \{n\}$$

On a free surface, stress tensor need not be zero but stress vector is necessarily zero.

A corollary of this is that the stress vector direction on a free surface can at best be tangential to the surface.

$$\{T\}^n = \{0\}$$

Note that stress vector cannot cross the free boundary.



So what I have here is a mathematical definition of a free surface and we have to find out the stress vector. In solid mechanics we have learnt state of stress at a point gives you totality of all the stress vectors on all the possible infinite planes passing through the point of interest. A surface is defined by an outward normal and if n is the outward normal of a surface then the stress vector on that plane is obtained by multiplying stress tensor with the direction cosines.

So what you have here is $T n$ equal to τ into n where n is the direction cosine vector, τ is the stress tensor at the point of interest and $T n$ is the stress vector acting on plane which is defined by outward normal. On a free surface, stress tensor need not be zero but stress vector is necessarily zero. So the mathematical definition of free surface is $T n$ equal to zero.

We will see this by an example. So if I have to find out the stress vector on any specified I can go to Koshish formula and make a statement that on a free surface, stress vector is necessarily zero. The corollary of this is that at the stress vector direction on a free surface can at best be tangential to the surface. In other words stress vector cannot cross the free boundary which is what I get from looking at Koshish formula and the mathematical expression. Finally we have an axiom that stress vector cannot cross the free boundary.

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For simplicity, consider point B.
The stress tensor at that point is as follows

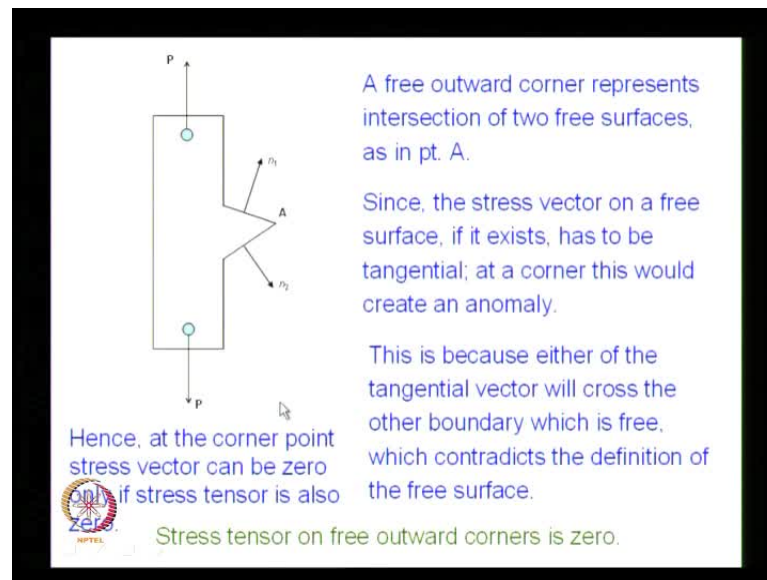
$$[\tau] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It is obvious that the surface $\{n_i\}$ is a free surface where $\{n_i\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$

$$\{T\} = [\tau] \{n_i\} \quad \{T\} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Let us take a simple example problem where I have a simple tension specimen and here let us consider a point B and find out what is the stress vector on this surface. If it turns out to be zero then from the definition of free surface, the stress vector on that surface is zero will be indirectly proved. Then we will take the stress tensor at that point which is given as shown in Refer Slide Time: 46:51. So as I have the y direction on this and if I know what is the cross section I can find out the stresses which is some magnitude a. So as I know the stress tensor I can define the outward normal n_1 equal to 1 0 0 as shown in Refer Slide Time: 46:51. Now you have a Koshish formula, a stress tensor which is not zero and which have an element. So when I find out tau into n I get a null vector which invariably satisfies the equation $T n$ equal to zero on the free surface. As I said earlier we have a corollary which says that on a free surface stress vector can only be tangential. Suppose I take a beam which has a free surface then $T n$ will be zero and if I cut it perpendicular I will have stresses perpendicular to that which will be tangential to the boundary. If I take a cantilever beam and apply an end load then it becomes the cantilever and you will have a shear which is a parabolic variation. In parabolic variation you have top and bottom fibers equal to zero. Both the top and the bottom fibers are zero as shear exists in pairs and it cannot cross a free boundary which is an unloaded boundary in free surface. A generalization of that is $T n$ equal to 0 and this is a very useful concept.

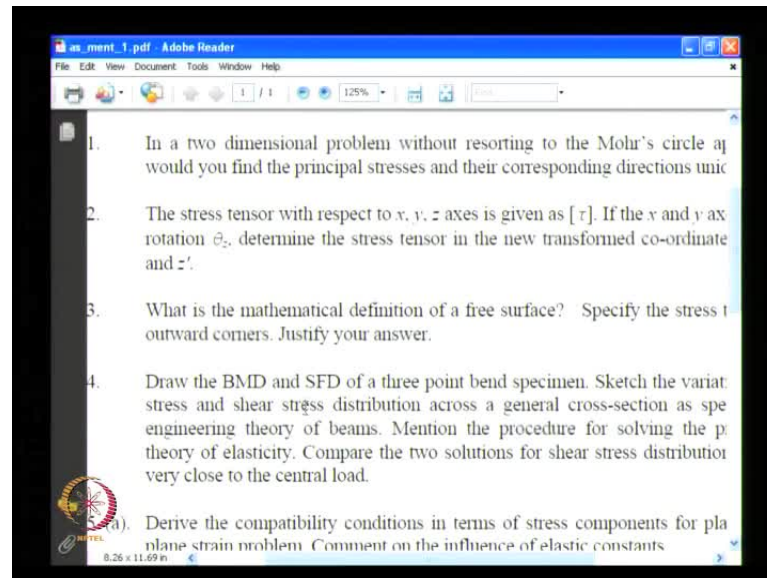
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Now suppose I take another problem where I have a projection and I would like to know what is the stress magnitudes at the sharp corner A. Suppose I have an outward corner and here I can go from my knowledge of stress vectors and show that the tip stress tensor has to be zero. A free outward corner represents intersection of two free surfaces as in point A. I have one free surface given by outward normal n_1 and the other free surface given by outward normal n_2 . Since we have that the stress vector on a free surface can only be tangential, this would create an anomaly at a corner. This is because as either of the tangential vector will cross the other boundary which is free which mean that the stress tensor needs to be zero. So this contradicts the definition of the free surface and finally we have is that stress vector can be zero only if stress tensor is also zero. In fact in photo elasticity when you want to find out the fringe orders this will be used as one of the methodologies to find out the zeroth fringe order. You can also verify when you have a finite element solution and plot it.

In this case there is also another point P where I am applying a load with a pin as shown in Refer Slide Time: 51:25. So what happens in this particular loading arrangement is that the entire surface has zero stress. But in general for any free outward corner you will have stress magnitude and both the stress vector and stress tensor will be zero.

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So the next problem is you have to find out the shear distribution on a three point bend specimen. Suppose I have a beam and I put a sharp load on it. So what I need to do is to find out the shear distribution over the depth. For shear the variation is a parabola but when I come closer to the loading point the variation will no longer be a parabola. I can solve shear distribution by theory of elasticity for this problem but that would be very complicated. Here the shear is still zero at the top fiber because as we have seen shear cannot cross a free boundary. You will have a maximum value near the surface and it reduces to zero. So if you do not put reinforcement your design will fail.

So that is the pulse of the third question and this is interrelated to the earlier question. You will find out in photo elasticity that shear variation can be easily identified without much effort by a simple photo elastic analysis.

So this brings to the conclusion of overview of experimental stress analysis. In fact I followed a different approach wherein we had looked at what are the advantages of analytical, numerical and experimental methods, what are the differences and specific features of each of these and then we moved on to the results given by an experimental technique. Later on we saw how to name an experimental technique, what are the trends in experimental mechanics and we also had a reasonable discussion on selection of an appropriate experimental technique and we also looked at it as a function of range of strain. With whatever we have discussed if you are interested you can always go to any

one of the references, read and try to find out how those techniques could be employed

Thank You.