Engineering Fracture Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module No. # 08
Lecture No. # 37
J-Integral

You know, in this class, we will discuss concepts related to J-Integral. J-Integral finds application in linear elastic fracture mechanics. It is a useful parameter in non-linear elastic fracture mechanics. And, these concepts are extended for elasto-plastic fracture mechanics. So, we will see all these aspects, as part of this chapter. And, what you will have to really look at is, limited plasticity at the crack-tip was the focus in LEFM. And with that, you could solve a variety of problems.

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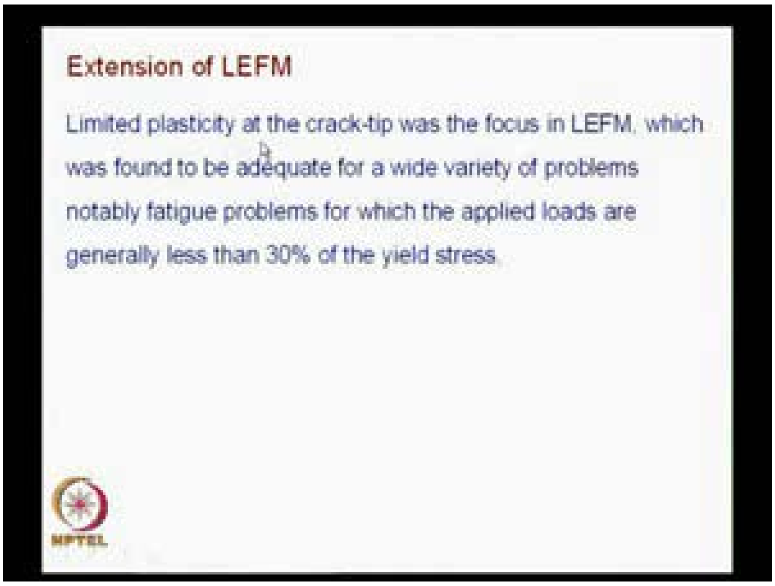

So, there is an advantage. You had certain pressing need to understand, how crack-tip structure would behave. You could immediately apply to a variety of problems. And another point that you will have to note is, notably, for fatigue problems, the applied loads are generally less than 30 percent of the yield stress. So, you are talking of structures, which are loaded far below the yield stress; and you are confining your attention to high strength alloys, and LEFM was sufficient. And, this was possible because, you had only limited plasticity at the crack-tip. You know, as design requirements expanded, this kind of analysis was not found to be sufficient. So, when you have a different combination of geometry and loading, it could lead to expansion of the plastic zone; so, the plastic zone is not confined.

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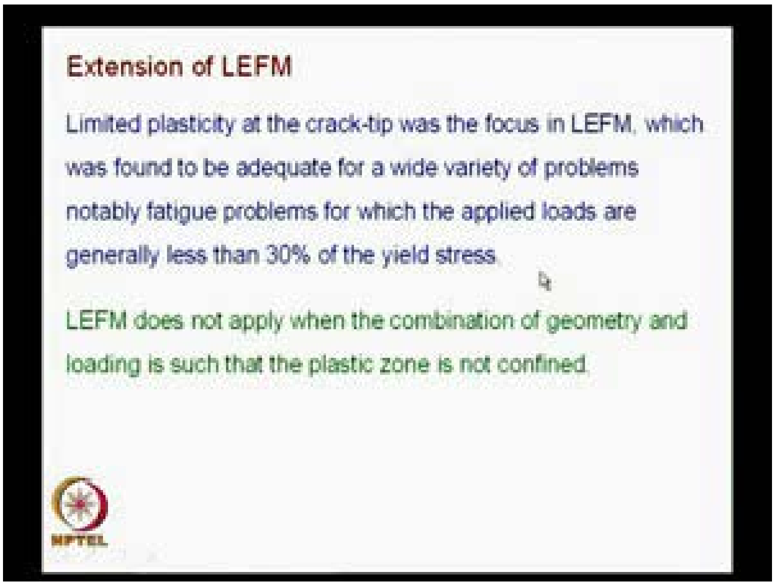

So, in such applications, linear elastic fracture mechanics would not be sufficient.

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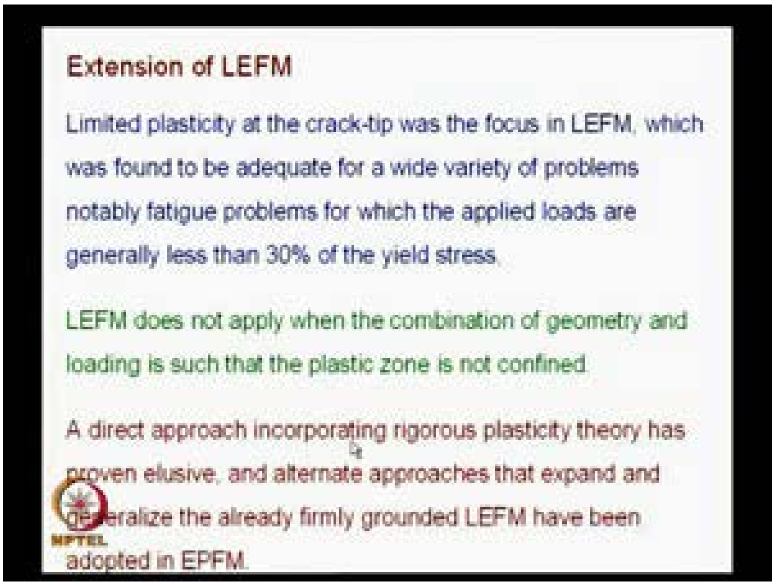

So, you have to look for newer methodologies. And the desire was, to account for elasto-plastic fracture mechanics. And, what was the difficulty? A direct approach, incorporating rigorous plasticity theory has proven elusive. See, if you look at plasticity literature, it is very mathematically oriented and even in conventional problems, people use some sort of a design approach. Because, you have to keep track of the loading history, in any plasticity problem. So, if you want to have a rigorous plasticity theory, which was not quite convenient, and people were looking at, what are the alternate approaches. And in those approaches, what you really want to do is, you have already learned concepts related to linear elastic fracture mechanics; how these concepts could be modified and adapted for elasto-plastic fracture mechanics.

So, what you will find is, like we have looked at energy release rate, we will also look at energy release rate. And, we have looked at r curve, we will also look at J curve, here. So, there is similarity between what you have done in LEFM. There you had, fracture toughness, K 1 C has to be determined; similarly, here, you will find out, J 1 C has to be determined. So, there is parallelism in this, but you need to go through certain approximations. So, before we get into elasto-plastic fracture mechanics, we will try to understand what is J-Integral. That is the way, that we are going to proceed.

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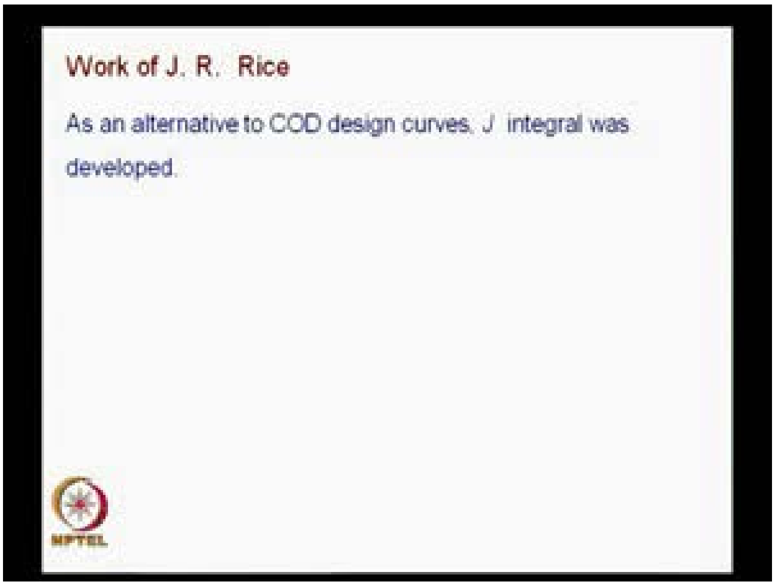

And if you look at the literature, the J-Integral is credited to J. R. Rice and historically, if you look at, the concept of COD or CTOD has been developed earlier than J; around 1961, Wells and Cottrell have reported, how to account for problems involving larger zones of plasticity. Because, LEFM was confined to very small plastic zone. So, people felt the need for developing fracture mechanics concepts to materials, which would have a higher level of plastic zone.

So, one of the earliest approaches is that for CTOD and I said, crack-tip opening displacement; you have to understand the terminology. We would postpone it for the time being, but we would definitely see, as part of this chapter. And while discussing J Integral, you cannot avoid CTOD. So, you take that as crack-tip opening displacement, for the time being. And we will first develop J-Integral, then, get back to CTOD. Historically, if we look at, COD was developed earlier and J Integral got developed later.

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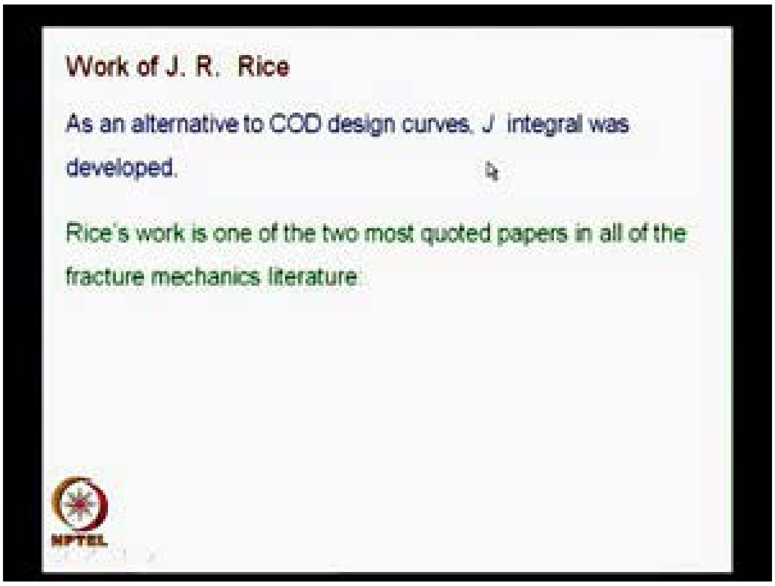

And, you should also know, Rice's work is one of the two most quoted papers, in all of the fracture mechanics literature. This is because, the contribution has been very very fundamental.

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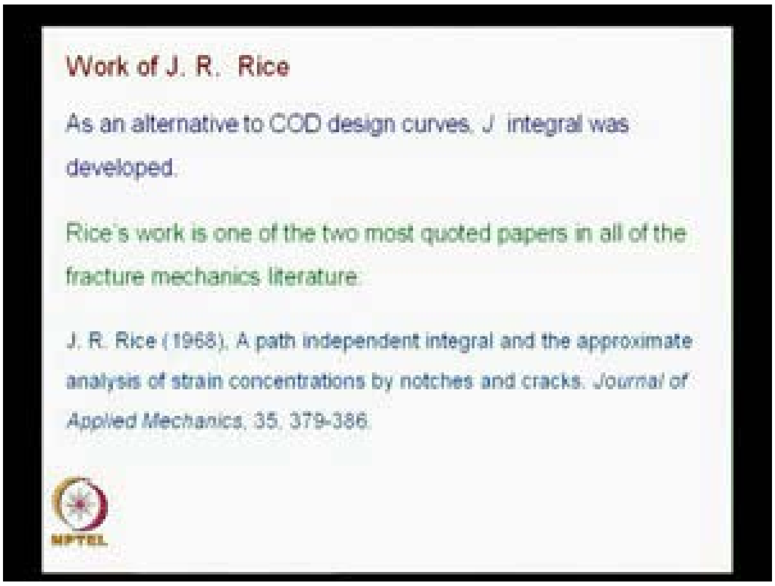

And, what was that paper? This was the paper published in 1968, 'A path independent integral and the approximate analysis of strain concentrations by notches and cracks'. This appeared in Journal of Applied Mechanics in volume 35. This paved the way for elasto-plastic fracture mechanics, at least approximately. As engineers, we will have to accept that. We will attempt for an exact solution; if you are unable to get an exact solution, at least solve the problem approximately. So, I mentioned that, there are 2 most quoted papers and what was the other paper? This paved the way for elasto-plastic fracture mechanics; the other paper was, obviously, one by Griffith, which paved the way for linear elastic fracture mechanics. He published in 1921, in the Philosophical Transactions of the Royal Society of London.

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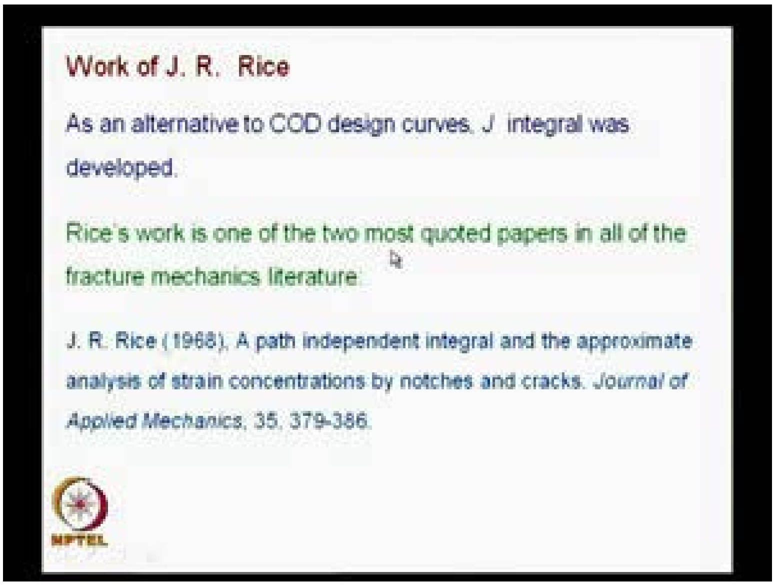

There was also another paper published in a conference in 1924. These two papers were considered as fundamental ones for linear elastic fracture mechanics. You know, even if you write the name of the author and the journal and the year, you should be in a position to search it.

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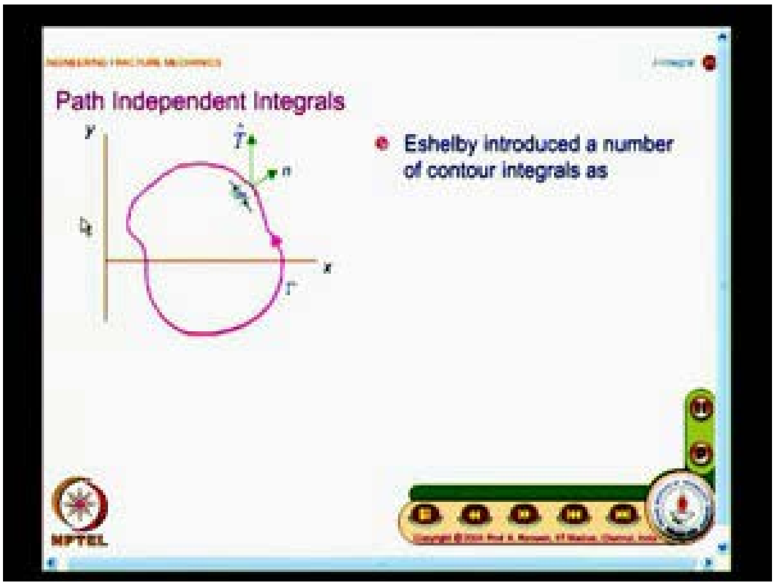

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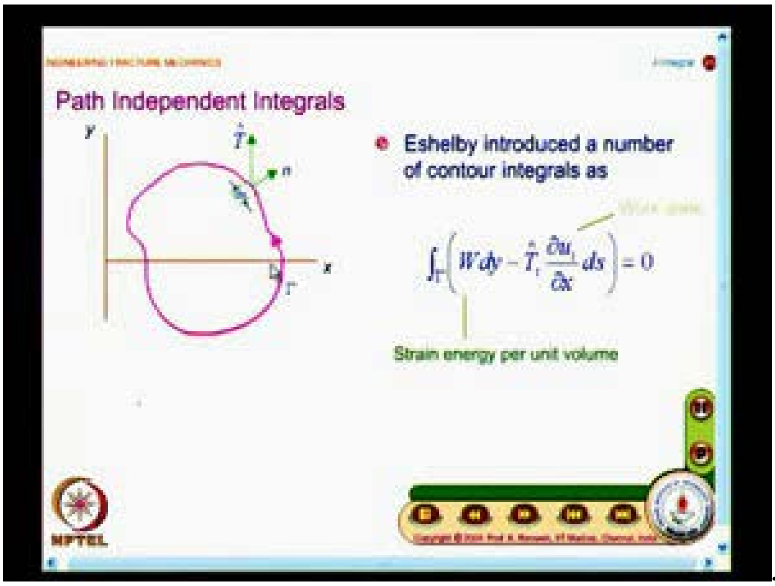

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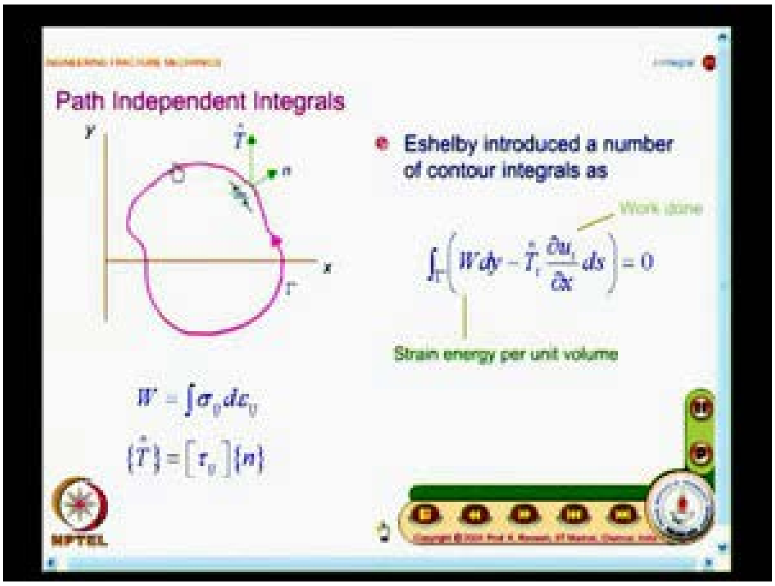

So, now, we will look at, what is a J-Integral? And, before we get into J-Integral, we have to look at, in elasticity, people have developed path independent integrals. And if you take a two dimensional problem, you consider a planar situation. I have a plane X Y and you take a closed contour and the contour shape can be arbitrary. On the closed contour, at a particular point, we have depicted the outward normal and also the stress vector acting on that plane, which is given as T n.

So, the boundary could be specified; whether we have traction on the boundary; there could traction on some portions of the boundary. And, Eshelby introduced a number of contour integrals. And one of them is picked up here. And this takes the form, like this. It is essentially a line integral, taken over the contour, as shown here. And it has two main terms. You know, in the literature, they put it as W into d y.

See, normally, we associate W to work done and it is better to change this symbol; I would also show that. The first term, essentially gives the strain energy per unit volume and the second term, you have the stress vector and this is the work done. And if you look at elasticity, you will have strain energy minus work done would be related to a potential energy. And, what you are actually seeing here is, this potential energy is differentiated with respect to the crack-length.

So, what we had seen as energy release rate, what is the energy available for an incremental crack growth, is written in a different form, which appears as a line integral, after several simplifications. That is the way you have to look at it. And if you look at the second term, this is written in indicial notation. So, I have T n i multiplied by dow u i divided by dow x. So, this will repeat. If you look at the Einstein summation convention, when I have i i, this indicates, that this would repeat. Since we are considering a planar situation, you will have the first index 1 and second index as 2.

So, both, you will have essentially 2 terms for this. And if you go and really find out what is this T n, you get that from the Cauchy's formula. We have T n equal to tau i j multiplied by the direction cosine vector n. So, if you really look at this, for a two dimensional situation, you will have 2 terms for each one of T 1 n and T 2 n; either you could put it as T 1 n or T X n and T Y n. And the first term is nothing but integral sigma i j delta epsilon i j. This we had already seen earlier, we will again have a look at it. This is the final form of the integral and if you take a closed contour, the integral goes to 0. How this is applied to fracture problems? This is what we have to look at; and, that was done by Rice.

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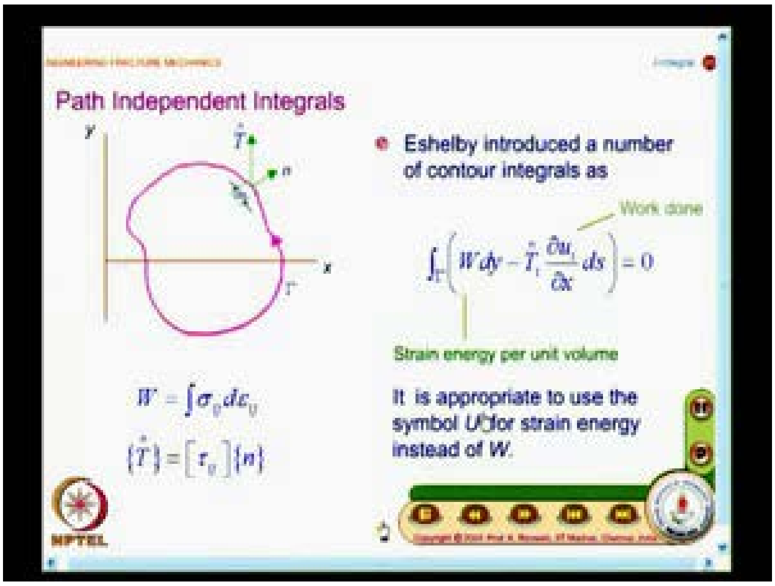

As I mentioned, since we are accustomed to looking at strain energy with a symbol capital U, it is mentioned here. As books show it like this, I thought I would show both these type of expressions. And before we proceed further, we just recapitulate, for linear elastic solids, the strain energy per unit volume is expressed as follows. You are essentially getting it for a volume. This is one half of integral over the volume, sigma x epsilon x plus sigma y epsilon y plus sigma z epsilon z plus tau x y gamma x y plus tau y z gamma y z plus tau x z gamma x z multiplied by d V. This is expressed in terms of stress and strain. If you employ the stress strain relations, you could also express the strain energy in terms of only the stress components.

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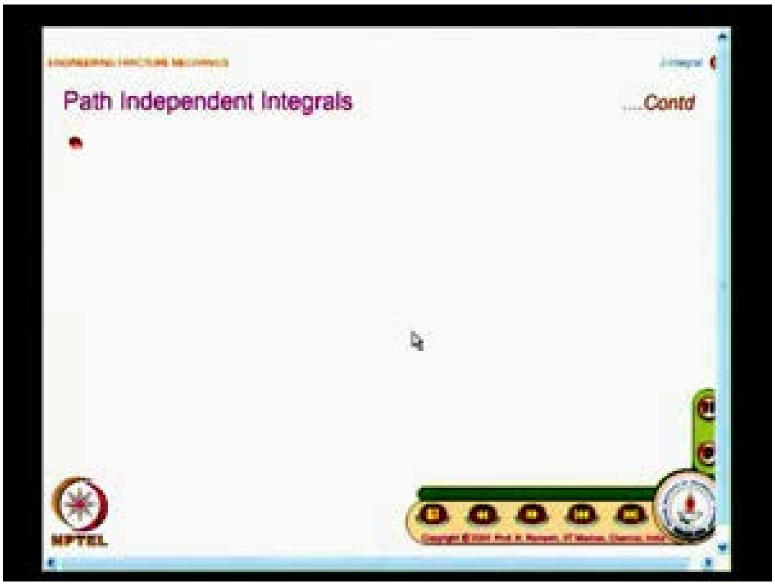

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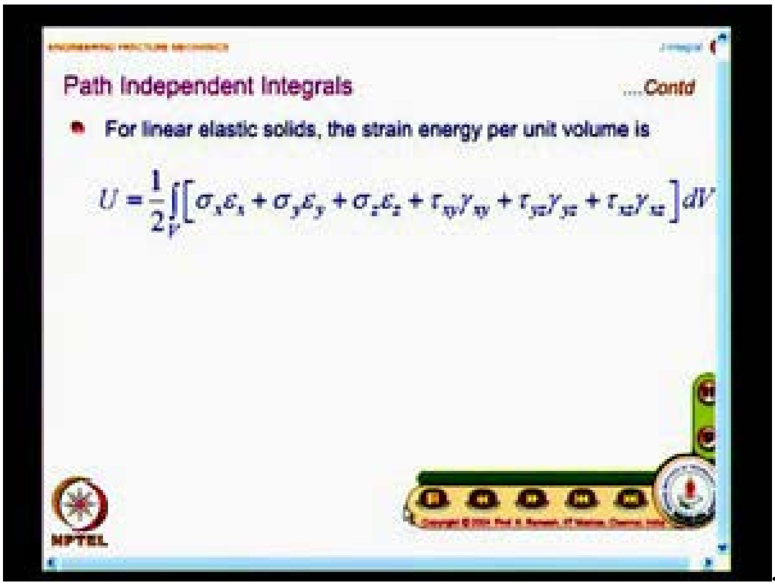

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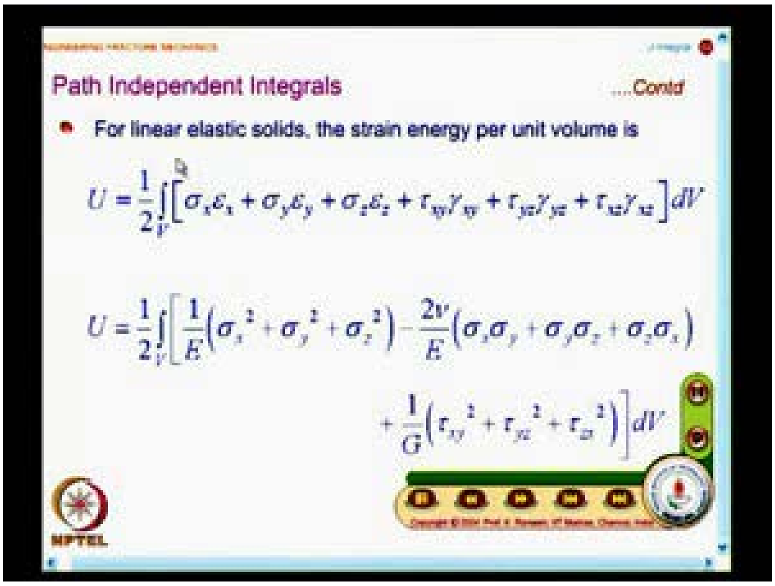

Strain energy in terms of stress components is as follows. U equal to 1 by 2 integral over the volume, 1 by E into sigma x squared plus sigma y squared plus sigma z squared minus two nu divided by Young's modulus multiplied by sigma x sigma y plus sigma y sigma z plus sigma z sigma x plus 1 by G multiplied by tau x y squared plus tau y z whole squared plus tau z x whole squared, whole multiplied by the d V. These expressions you already know. And this is just for continuity in discussion, this is shown. And what is the J-Integral? A J-Integral starts from one of the crack faces and goes in a counter clockwise direction and stops at the other crack face.

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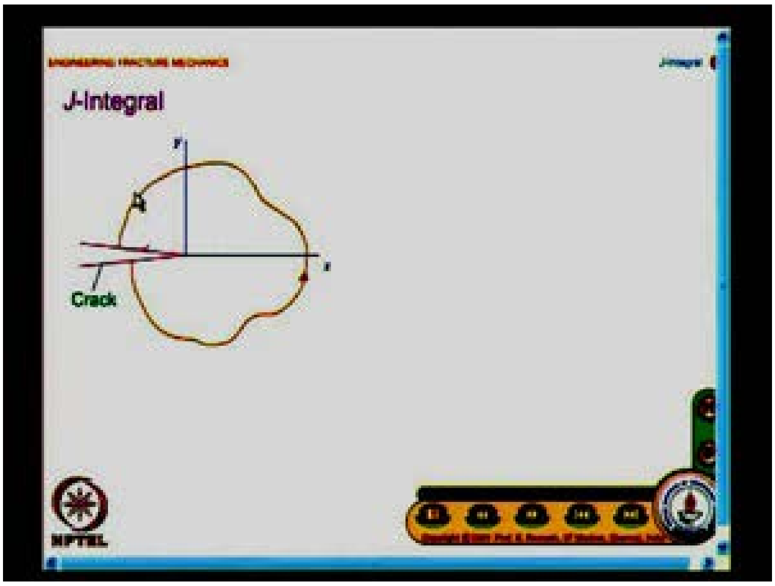

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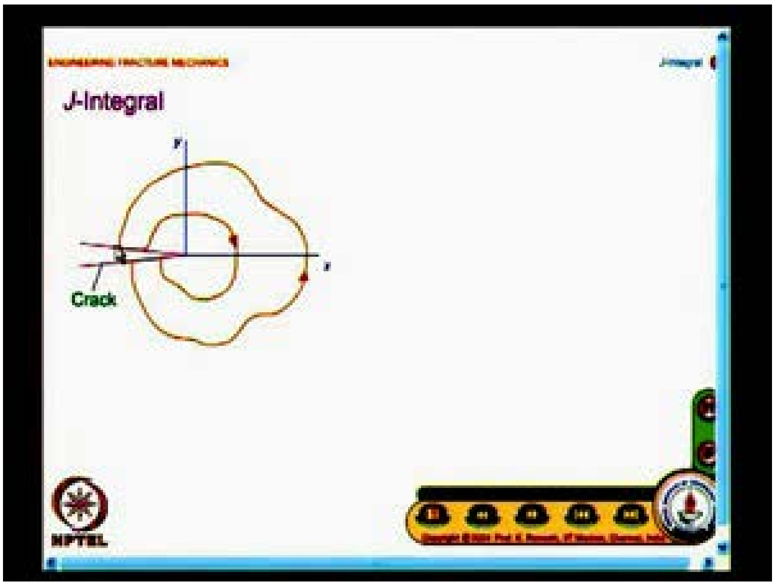

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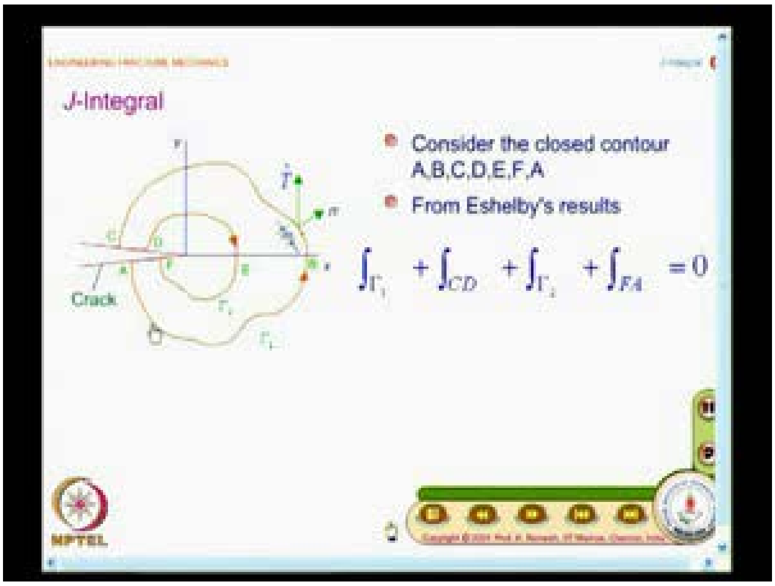

And you should also note that, this crack face is not loaded. That is a very important observation. J-Integral is applied to problems, where crack face is not loaded. And what is attempted to be shown in this is, the integral is also path independent. So, I have taken a path like this, A, B, C and to D and then, come to E, F and then close it at A. And when you do like this, this is your Eshelby's integral and what does the Eshelby's result says. If you take a closed contour, the integral whatever we have looked at, should sum to 0; and I write this as different contours; in the contour 1, you have one integral, plus on C D plus A F, plus the contour 2.

We can definitely comment about, what happens for the integral on the contour C D and integral on the contour F A, to start with. For clarity, the crack is shown as a V type of shape. In reality, it is almost like a line. So, that means, the d y term will be 0. And another aspect is, this is a free surface. On a free surface, traction is 0.

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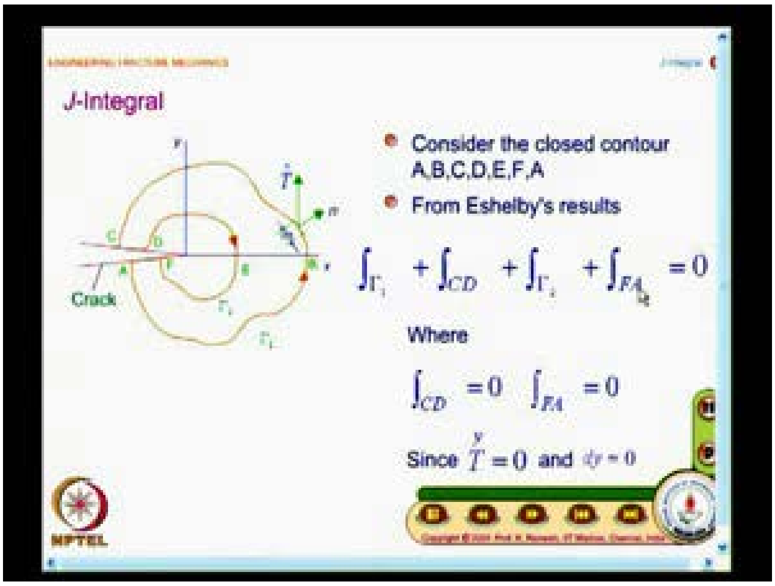

So, in essence, the integral C D and integral F A would go to 0; the reasons are stated here. Because it is a free surface, the traction is 0; and it is a very fine crack, so, dy equal to 0. Do not get misled by this V shape. This is, for illustration, it is put like this. So, when these two integral goes to 0, we get a very interesting result, integral over the contour 1 plus integral over the contour 2, equal to 0.

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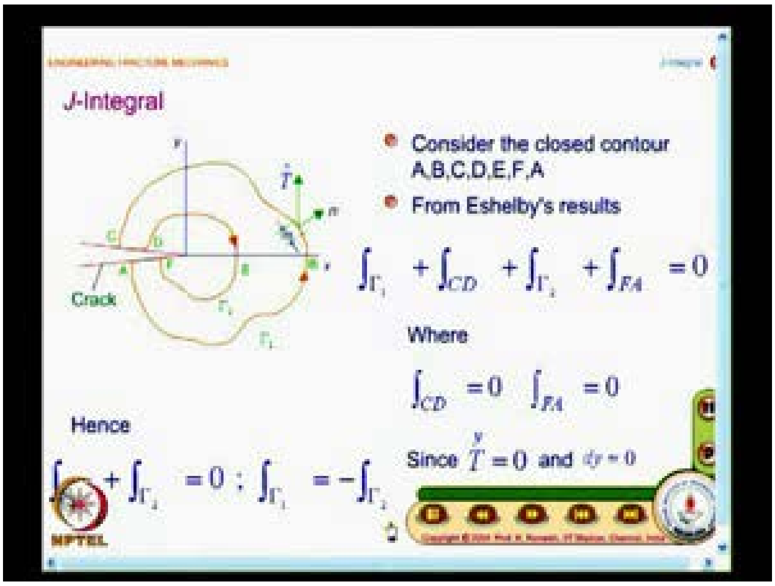

So, I get, these are equal and opposite; in one case, you are going in anticlockwise direction; another case you are coming in a clockwise direction. That is why the sign change is there. And, what do you see? We took a arbitrary contour. We have not put any restriction, which path the contour should take. So, this gives a proof, that if you take a integral from the crack surface and ends with the other crack surface, the value of the integral is independent of the path that you take. So, J-Integral is path independent. It is a very very important result. That is what is summarized here. The negative sign indicates that, the direction of line contour is different. This result implies that, in crack problems, if a contour starts from one crack face and ends in another crack face, the magnitude of the line integral does not change.

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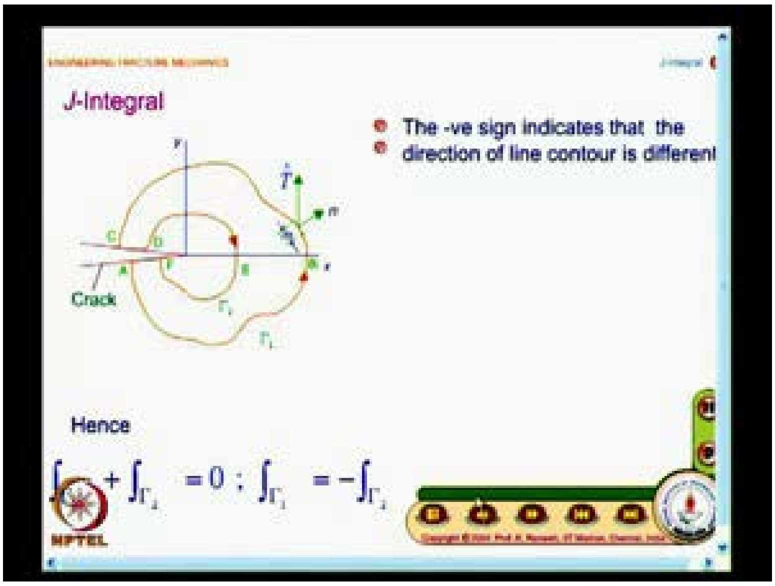

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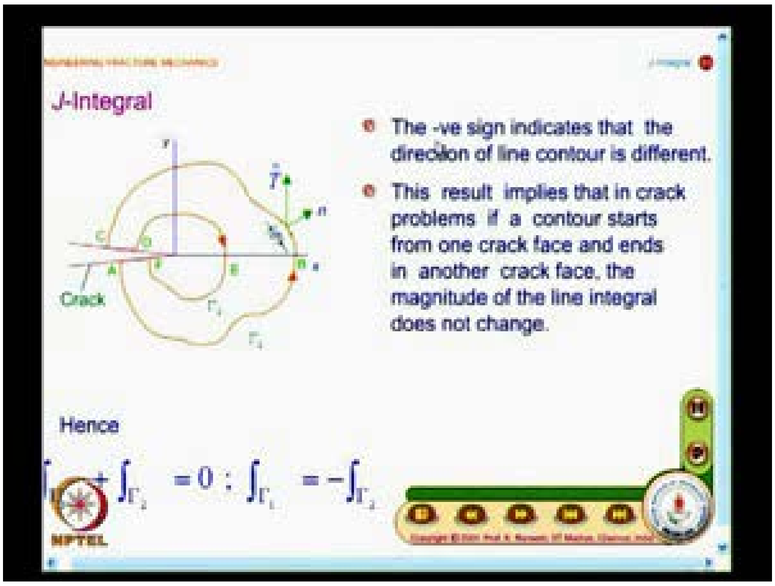

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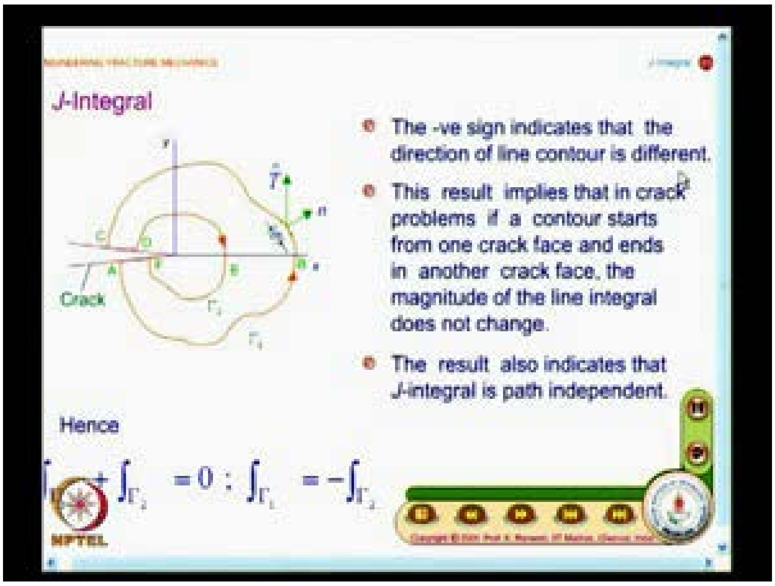

We have already seen, what is the expression that you would write for this integration. That I have, that is not written here. You have two terms; one involving strain energy; another involving traction. We will see that later.

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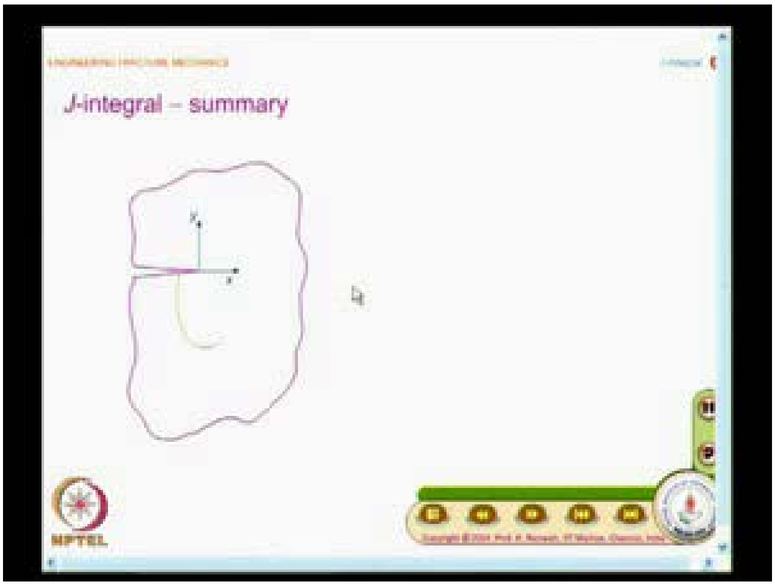

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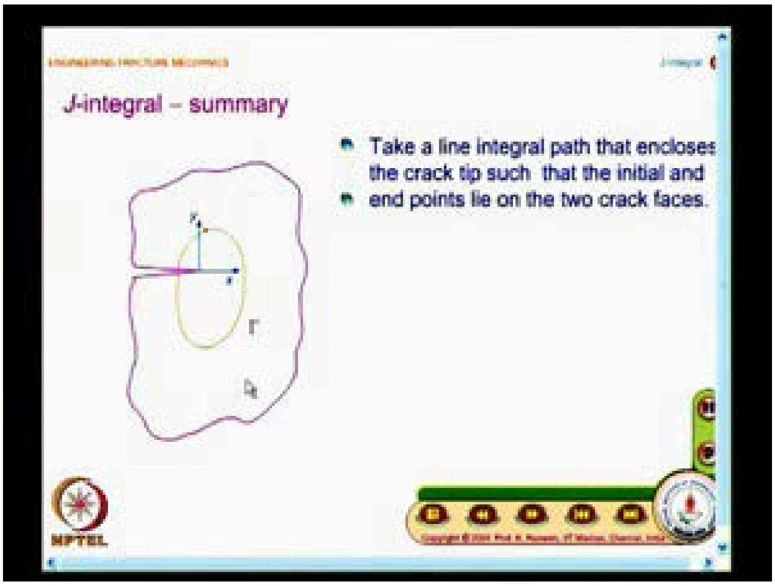

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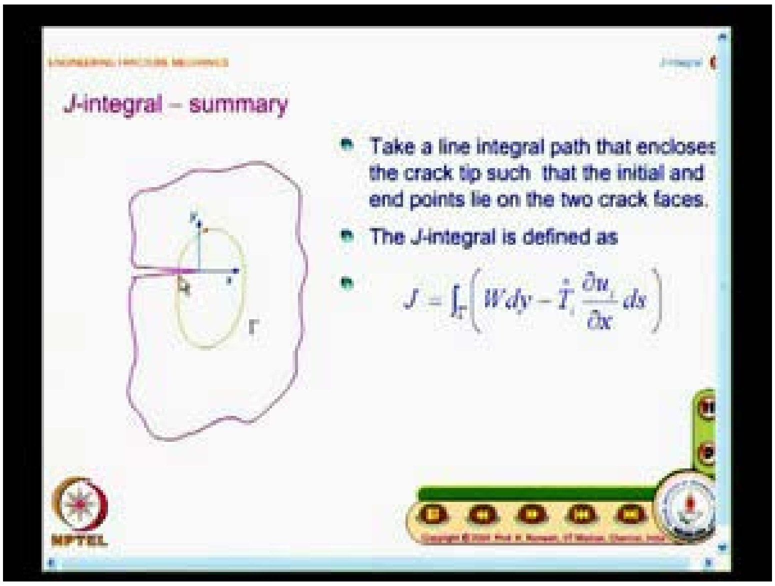

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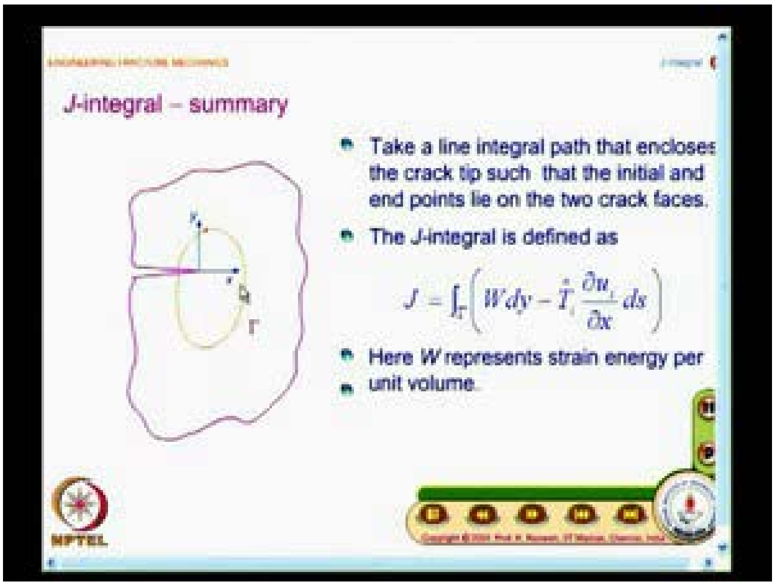

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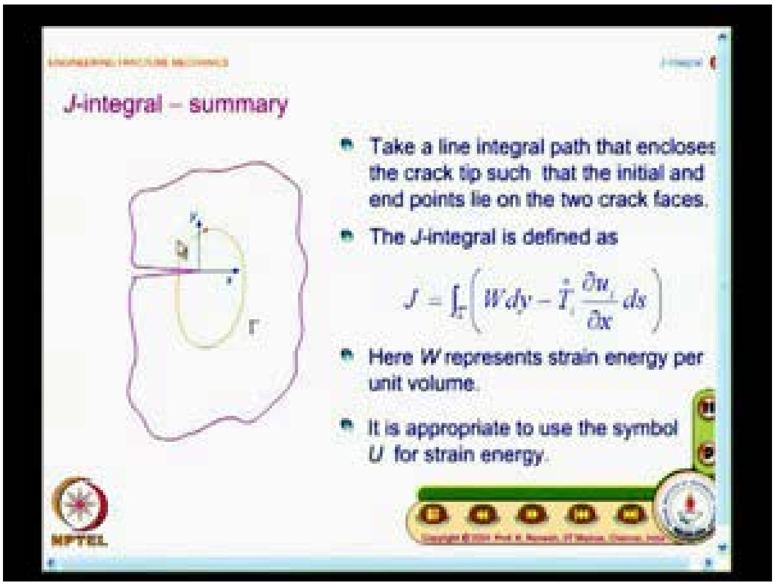

The result also indicates that J-Integral is path independent. That is well utilized in linear elasticity. If you really go to finite element computation, they use this path independence, find out J-Integral, from that, determined the value of the stress intensity factor. That is one of the uses of J-Integral. So, we will see, in a summary, what is J-Integral. I have a two dimensional crack body; I start the contour from one of the crack faces, and it ends in the other crack face and the crack faces are not loaded; and the integral of the type J equal to W into d y minus T n i dow u i divided by dow x into d s; this is how you see in books; I would prefer to draw your attention, that it could be written better this way. Instead of W, put it as U d y minus T n i multiplied by dow u i by dow x into d s.

The key point to note is, take a line integral path that encloses the crack-tip, such that, the initial and end points lie on the two crack faces. See, if it is a closed contour, then, the integral value will go to 0. Here, it starts from one crack face, goes in an anticlockwise direction, encloses the crack-tip and ends in the other crack face. Then, it has some finite value. And this finite value is the value of J. And you know, for you to understand this expression better, we need to take up a known problem and evaluate the energy. Then, you will know, how to identify the interplay of these terms. And the contour can be anything; it can follow the boundary of the object also. Any contour, that encloses the crack-tip, starts from one crack face, ends in the other crack, crack face; that is the only requirement.

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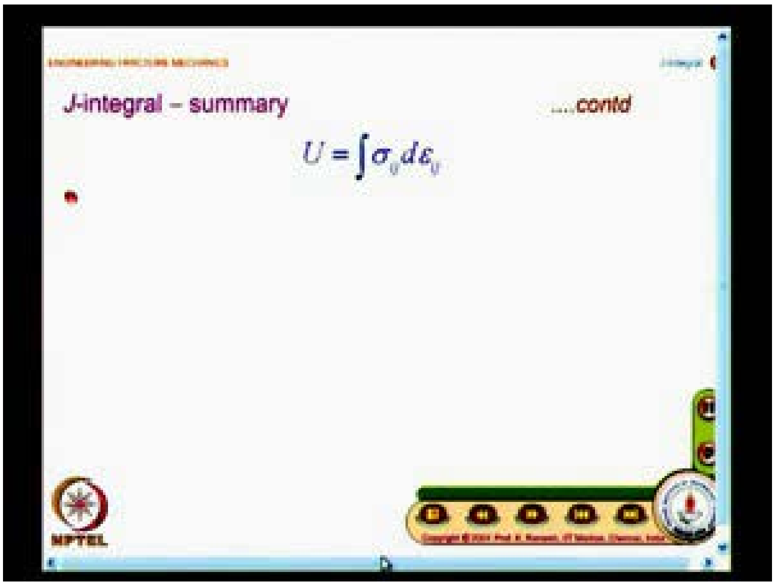

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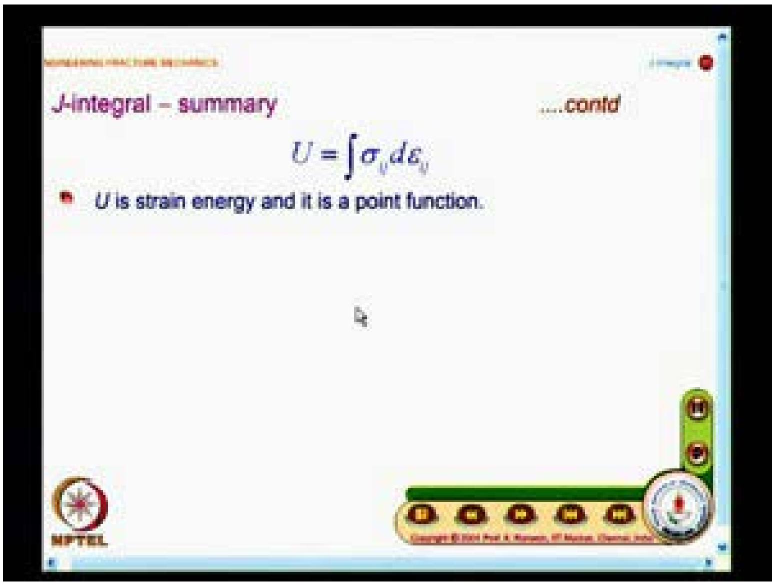

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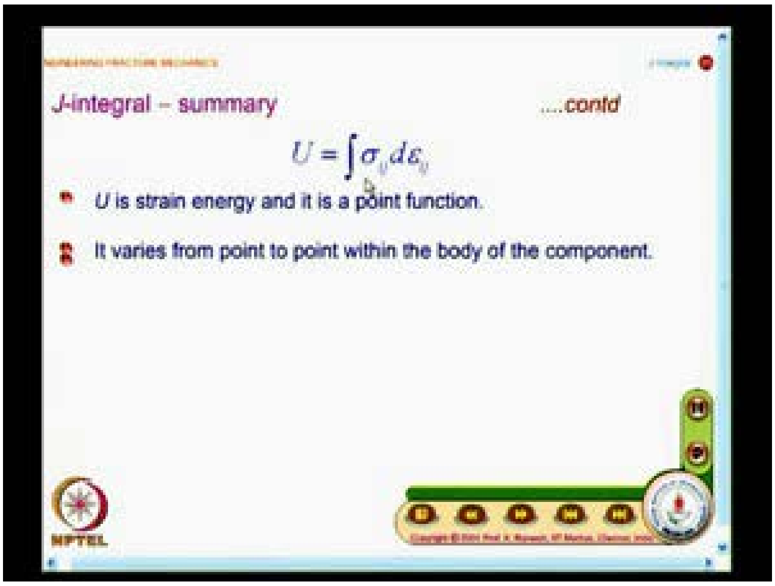

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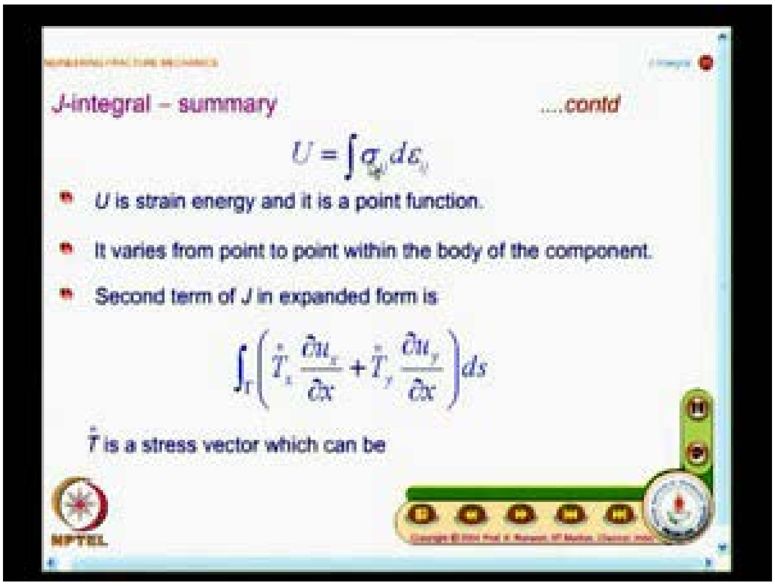

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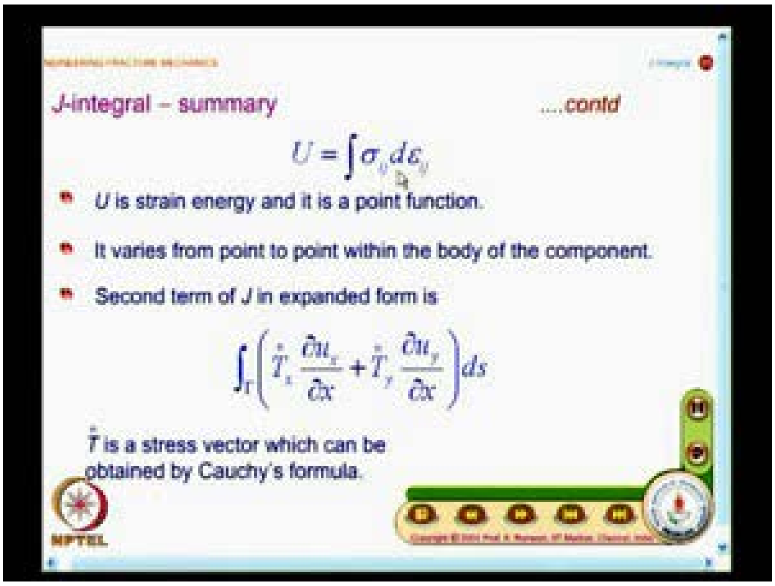

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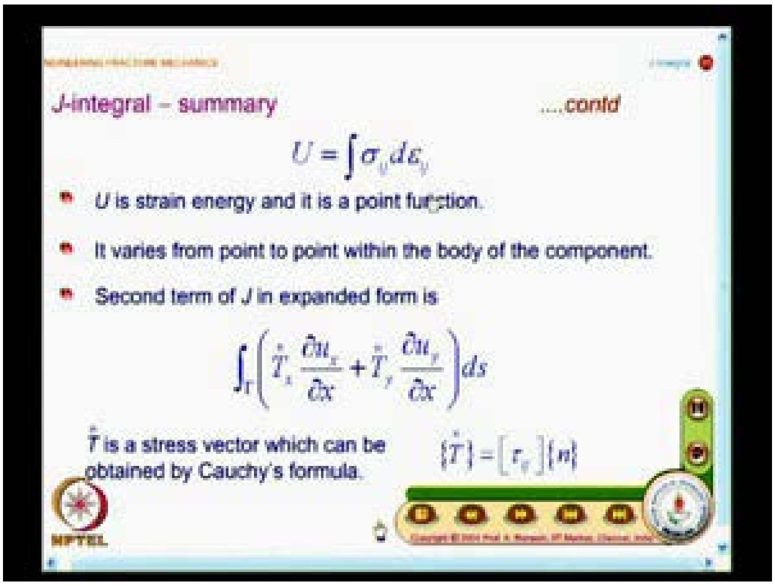

So, before we do the calculation further, let us look at the terms a little more closely. The strain energy term is given as integral sigma i j d epsilon i j and the traction term is expanded here. It is, it was written as T i dow u i dow x. Now, when you expand it, I have two terms; T n x dow u x divided by dow x plus T n y dow u y divided by dow x. And each one of these terms, would be obtained from your Cauchy's formula.

So, when you solve a problem, you would know how to fill in the quantities. You know, the line integral calculation, you have to do it carefully. What has happened is, people are using finite element calculation. There the software is so well developed, you identify a path and the software calculates the value of J, and churns out the number. So, many times, students do not even understand, what are all the terms in a J-Integral; it is taken care of by the software. It is a very bad scenario. You will have to understand, how the software calculates, what is the kind of expression that is used; it is very very important. So, we will take up a problem, which you have already solved as part of the linear elasticity. And what you have here, is a double cantilever beam specimen. You have a crack and the cantilever beam specimen is opened by the load P as shown. See, the moment you come to elasticity, you show that as a distributed loading. You are actually applying a shear, and when you want to analytically analyze, we have already seen, in theory of elasticity, when you had that uniformly distributed load of a beam, supported using simply supported supports, we had replaced the reactions as a shear; a same analogy is done here.

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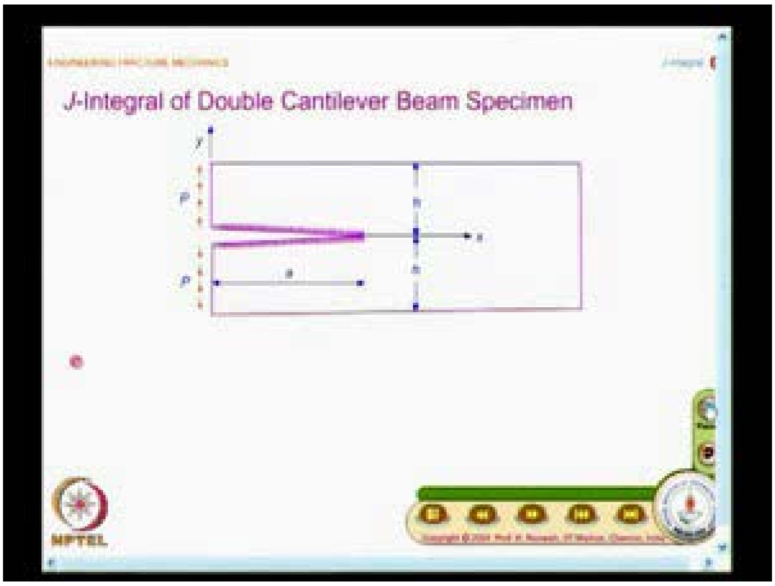

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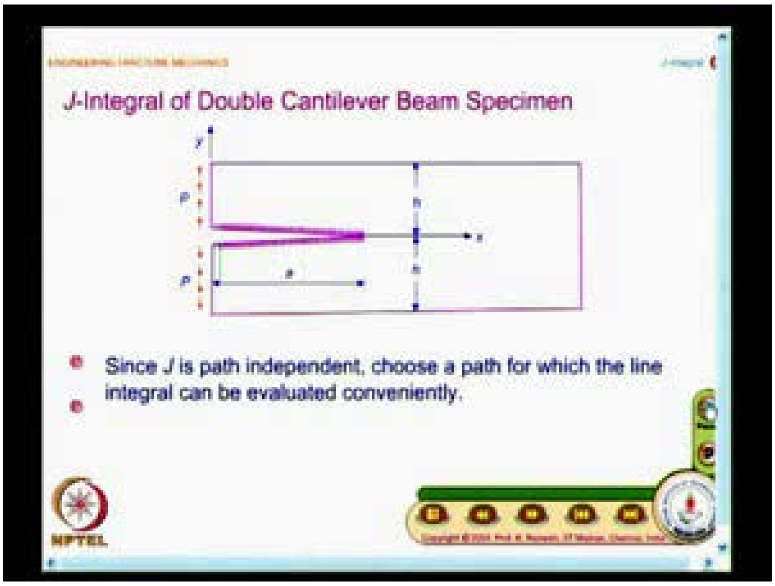

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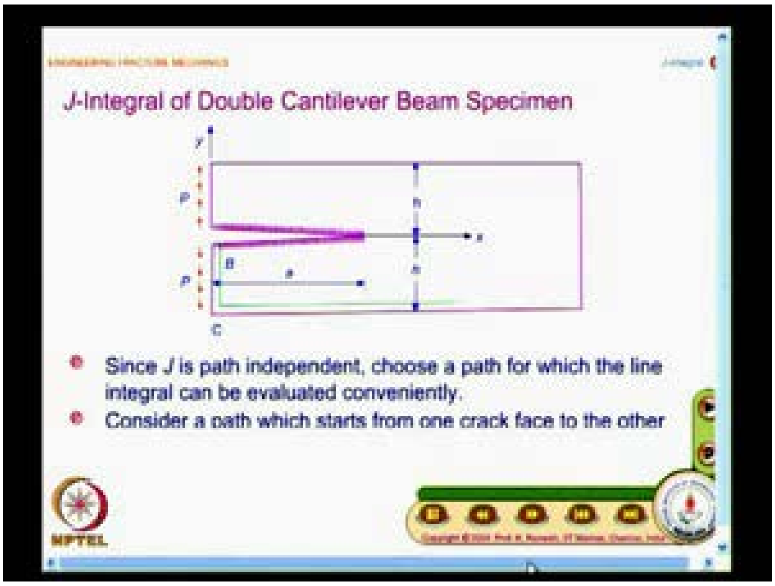

So, you replace this load by a shear. So, the integral of the traction on this, would be equal to the shear load. That is what we are going to use. So, keeping that in mind, it is shown as a shear. And you have a particular height h, and for our analysis, we are going to consider the length of the beam as a, which is the crack-length, and do the computation. And we have already noted that, J is path independent. So, choose a path, for which the line integral can be evaluated conveniently. So, we will choose a path, where the contour goes to 0 at many places. So, we have to do only computation, only at a place where contour is non-zero; the contour integral is non-zero.

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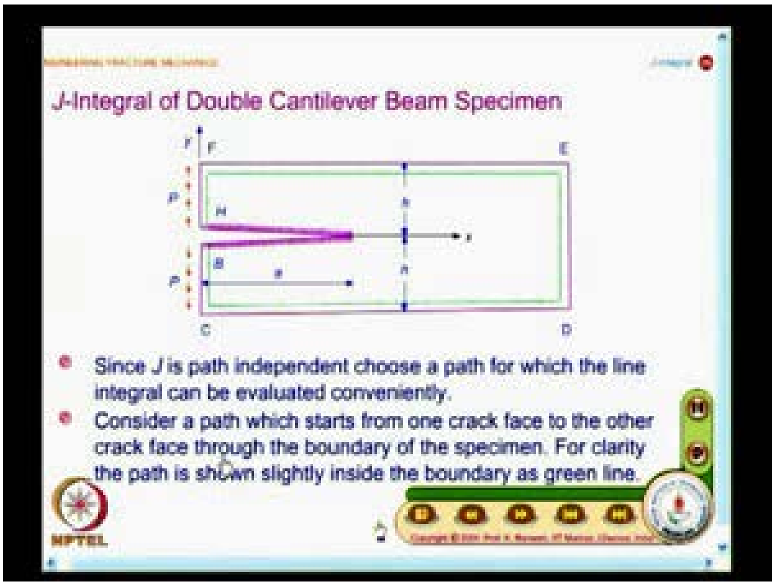

So, what we are going to take is, consider a path which starts from one crack face to the other crack face, through the boundary of the specimen. Because, if you look at the boundary, it is a free surface. So, when into the free surface, you know traction is 0. So, that way, your computation of the line integral would be a lot more simpler. For clarity, the path is shown slightly inside the boundary as a green line. So, make a neat sketch of it. So, we are taking a path which has straight edges, which has corners, all that is permissible. The only requirement is the path should be continuous. I can choose any convenient path; it should enclose the crack-tip, start at one crack face and end at another crack face.

See, even before we look at the discussion, you can easily say that, whatever happens on the face C D, D E and E F the line integral go to 0. We have to calculate the line integral only for the segments F H and B C. I have already given you the reason, that you are looking at a free surface; we would formalize it. Surfaces C D, D E and E F are free surfaces; hence traction is 0; thus the second term is 0. And second term, we have already seen in an expanded form. Because traction is zero, the whole term goes to 0.

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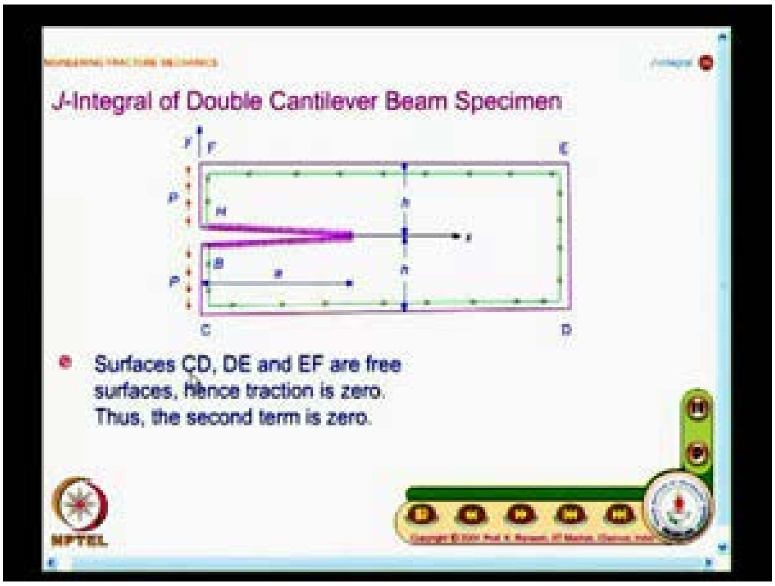

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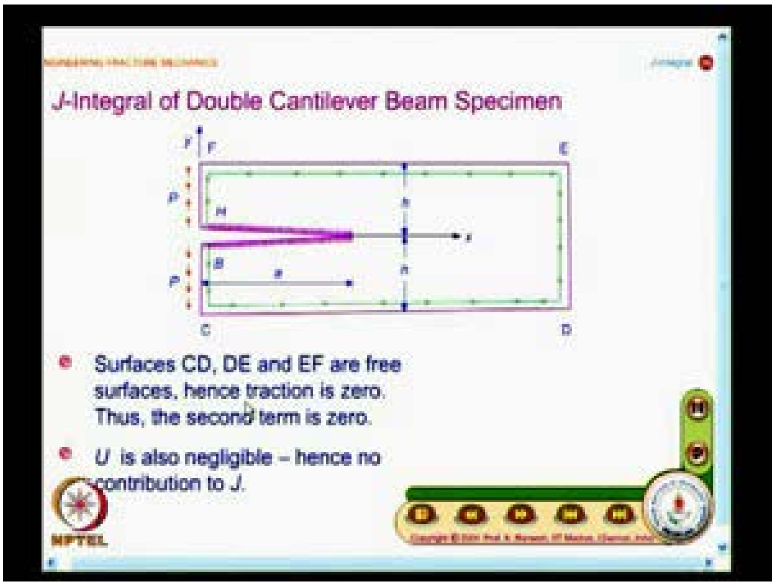

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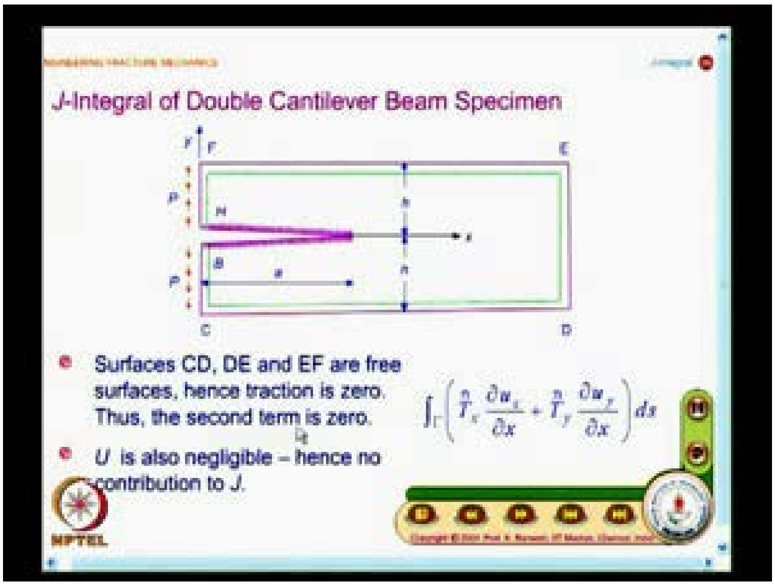

In addition, you should also recognize that, U is also negligible; hence no contribution to J. You will have d y equal to 0 in this. You will have very small value of stresses here. So, U is also negligible. So, we have to focus our attention only on the segment F H and B C. And you know, you have learned in solid mechanics, many things. Whenever we have a solid mechanics problem, if there is a bending as well as shear, we would recognize the energy due to bending better than the energy due to shear. We will always neglect any shear effects in slender beams. We will also do the same approximation here. When we are going to look at the surface F H and B C, you are having only shear.

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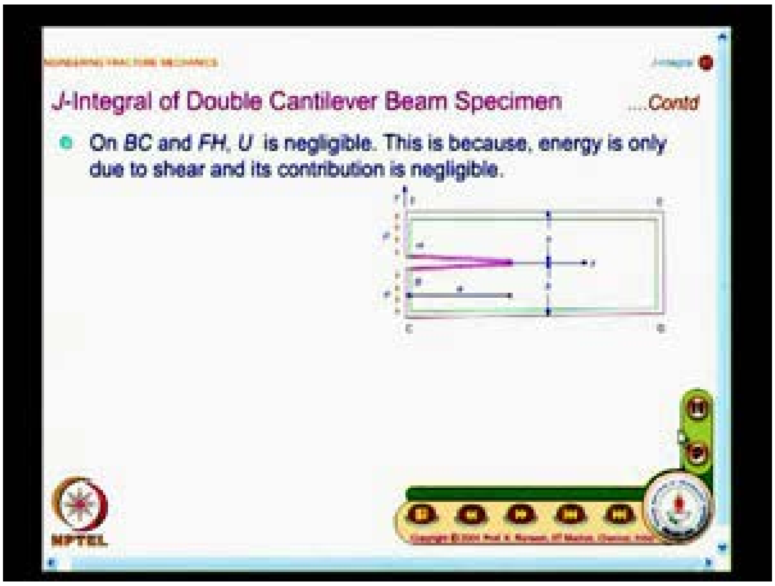

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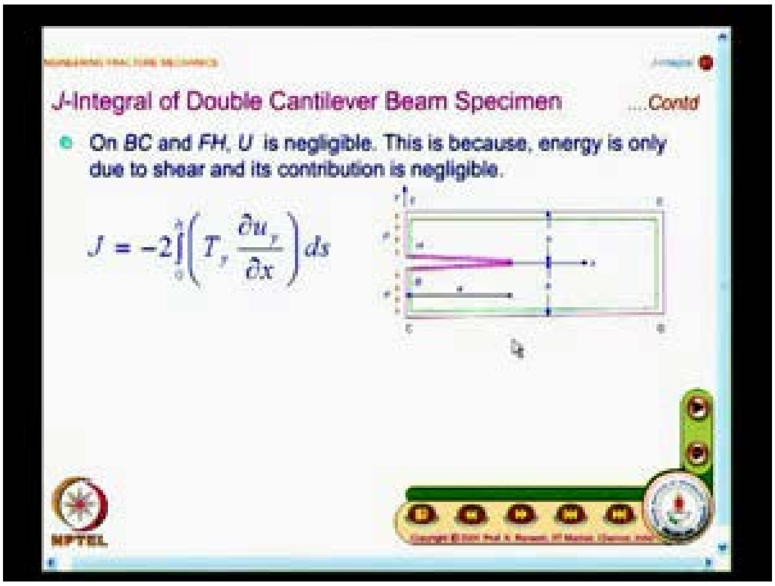

So, whatever the strain energy introduced because of shear, we would neglect it. So, essentially, we will have to find out, what is the contribution for the J-Integral through the traction term. And if you looked at, on this surface, I do not have T n x; there is no component in the x direction; there is only component in the y direction.

So, even this traction term reduces to only T y dow u y divided by dow x. And, you do this for two parts of the cantilever. So, you have this as two times that. So, you have this as minus two times integral 0 to h, T y dow u y divided by dow x into d s. Can you just recall, what is this dow u y by dow x? At the free end of the cantilever. You are actually looking this as a double cantilever specimen. You recall your knowledge of your deflection of beams; when you look at the deflection of beams, it is nothing but the slope of the beam at the free end. And, can you write the slope of the beam at the free end? If you want to find out the tip deflection, for a cantilever with the end load, a very famous formula is P L Q by 3 E a, where L is the length of the beam. Here we are taking the length of the beam as a. And if you look at the slope, it is P L squared by 2 Ea; when you adapt it for this problem, it will be P a squared by 2 E a.

And that 2 and this 2 will cancel and you have to write this moment of inertia. You have the thickness as B, and you will have this as B h q by 12 as the inertia term. So, when you substitute all this, this simplifies to this form.

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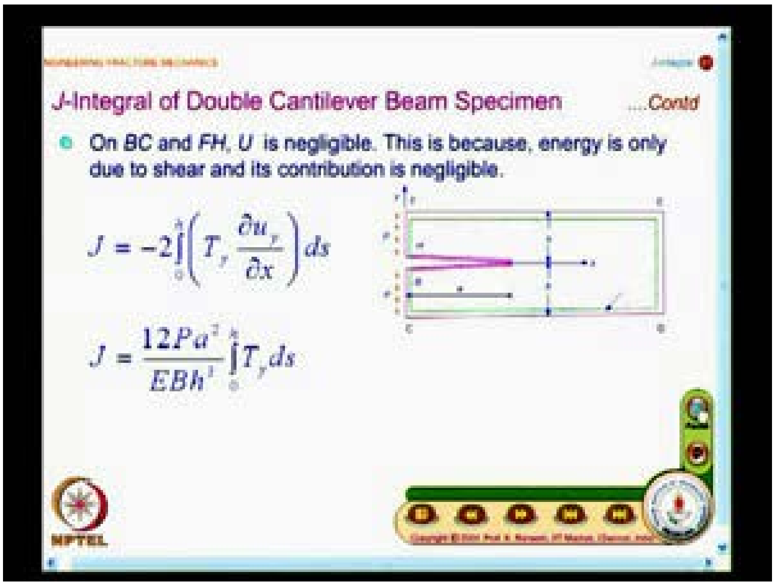

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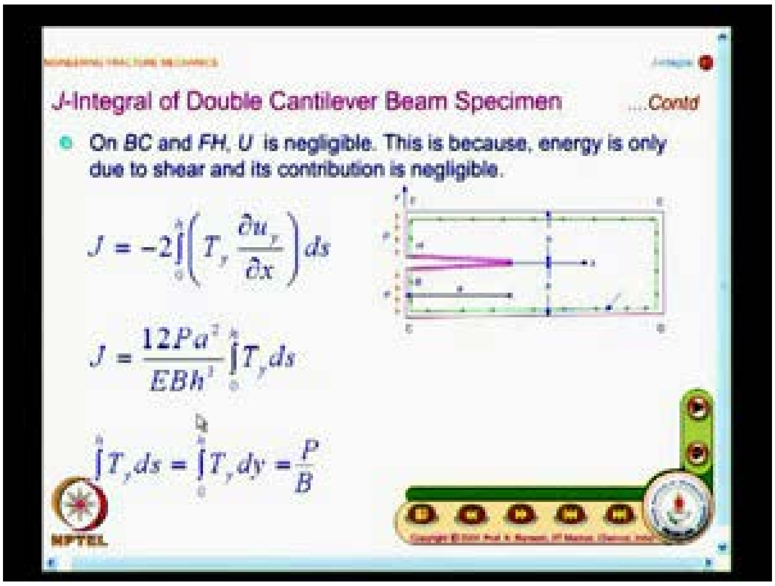

I have skipped those steps and I have written the final expression. You have this as 12 P a squared divided by E B h cubed and you have the integral 0 to h, T y d s. And I have already said, even while writing the P as a shear loading, I said the integral of the variation should go to the value P. So, you have this as integral 0 to h, T y d s, that is equal to integral 0 to h T y d y; you get this as P by B. And when you substitute this, you get the final expression, J 1 equal to G 1 equal to 12 by E a squared by B squared multiplied by P squared by h cubed.

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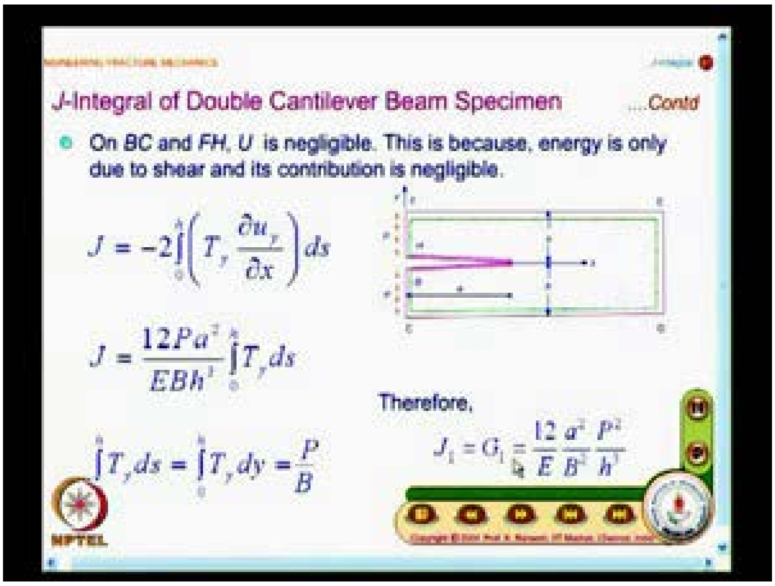

So, with this example, what you find is, if you are looking at linear elasticity, the energy release rate which is known as G there, is obtained by a line integral. And you got that as J. So, J and G are identical in the case of linear elasticity. And we also have an inter-relationship between G and K. So, J and K can be related. That is how, in finite element computation, what people do is, they evaluate the line integral and from that, find out the stress intensity factor. That is one of the uses of J-Integral, but that is not the only use.

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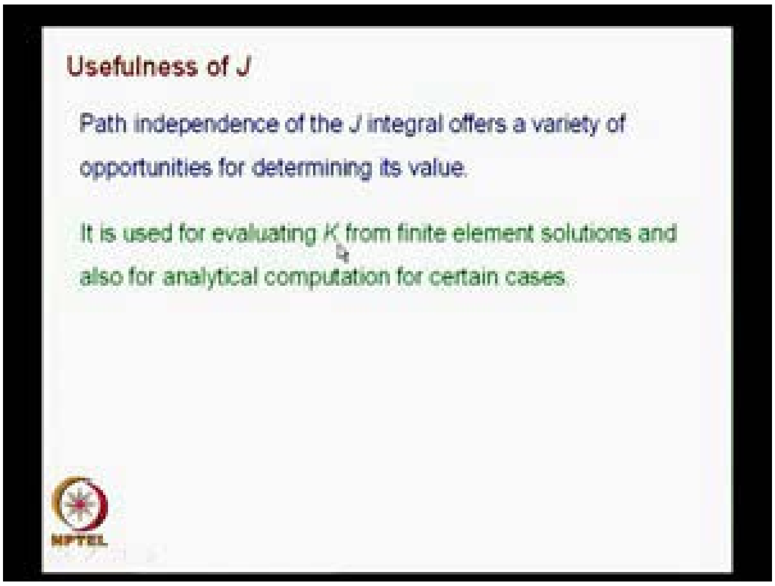

What are the usefulness of J? Path independence of the J Integral offers a variety of opportunities for determining its value. Because, I can choose the path, which is comfortable for me to evaluate. It is used for evaluating stress intensity factor K, from finite element solutions, and also for analytical computation for certain cases. And, what we have done? We have first evaluated J for a known problem, which you had solved in the linear elasticity. And, we find, whatever the expression you finally get for J, is same as G, which is nothing, but the energy release rate. And, you would see further, because I have advantage of path independence, what is the use of it, in finite element calculations.

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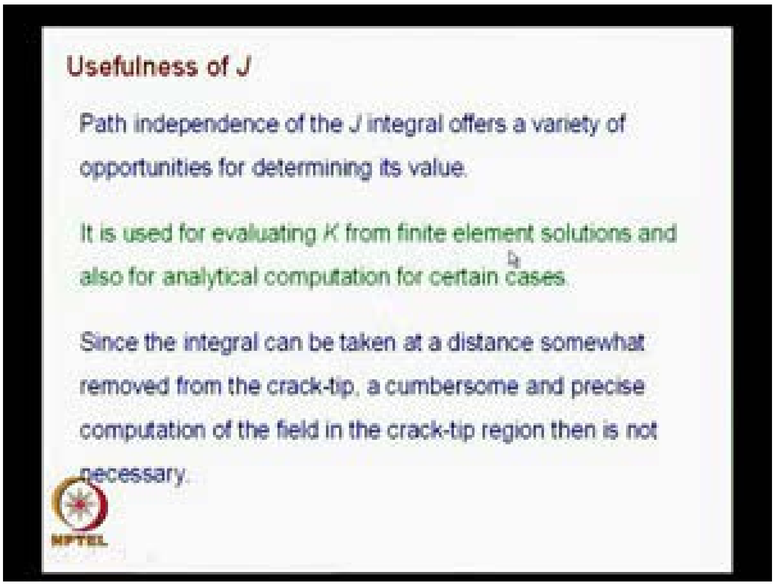

Since, the integral can be taken at a distance, somewhat removed from the crack-tip, a cumbersome and precise computation of the field in the crack-tip region, then, is not necessary. See, if you take a bench mark problem and apply J-Integral, you are not going to have any advantage. Because, for bench mark problems, you can really fill it, with very refined mesh. Suppose, I have a complex structure, even modeling of the structure is difficult, and you need several elements for variation in geometry, if you save precise mesh closer to the crack-tip, you have a enormous amount of saving; you have to view it from that perspective. Practical problems. If you can do away with very precise analysis near the crack-tip, and yet to find out the crack tip parameters, J has been useful. And, that is what is summarized here. The need for a highly refined mesh may not be required near the crack-tip.

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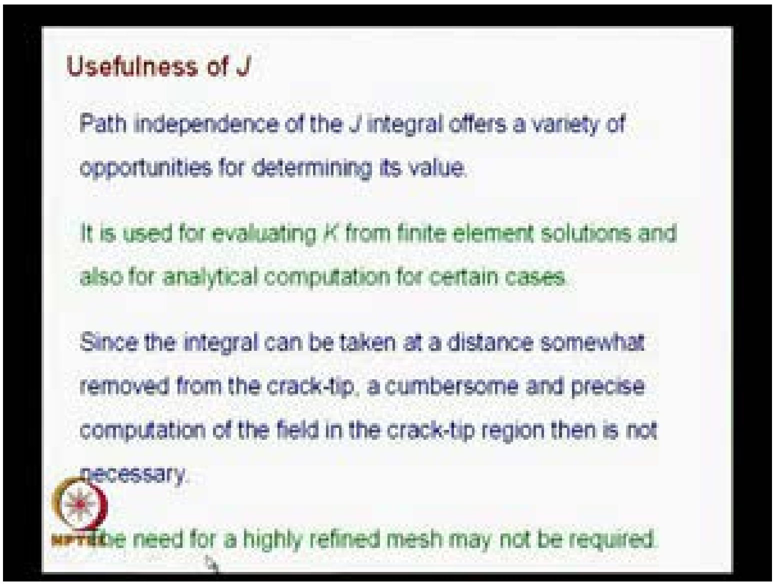

So, you can attempt to solve practical problems, when you have the advantage of J, for you to calculate the stress intensity factor K. You know, we say that it is path independent. Is it always so?

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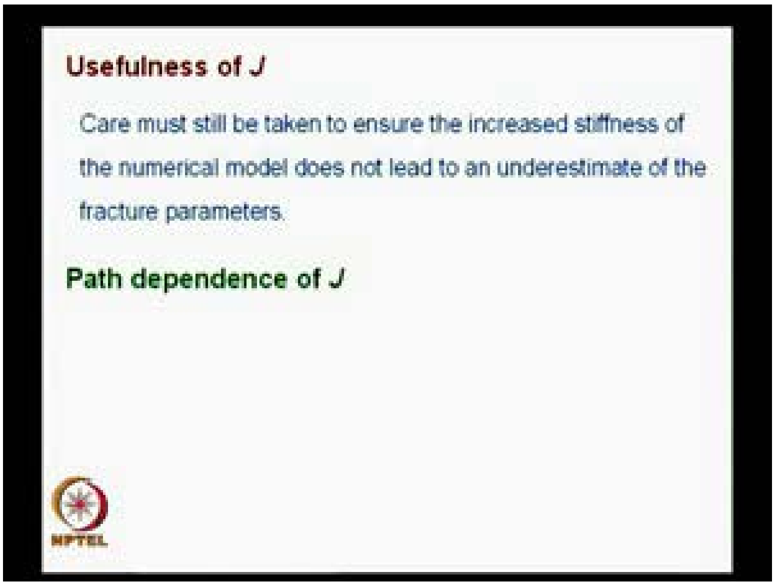

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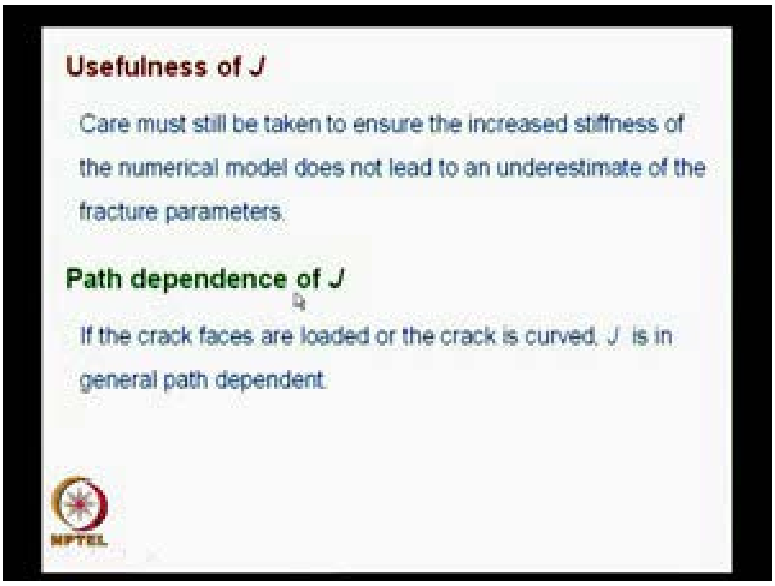

You have path dependence of J; it happens if the crack faces are loaded or the crack is curved, J is, in general, path dependent. So, excluding these exceptions, you can find J is useful. And, there is also a caution given. See, in the case of finite element computation, you say, you take a convenient path and then evaluate the value of J, you will have to keep in mind, care must still be taken, to ensure, the increased stiffness of the numerical model does not lead to an underestimate of the fracture parameters.

So, people will do reduced integration technique, a combination of that, they will do. They will go through the Gauss points, away from the crack-tip. So, there are some recommendations, that you have to follow from finite element solution developer. And based on that, you will have to evaluate J. It definitely provides you, because of path independence, a freedom to select your chosen path, but also look at the integration scheme, that you employ, so that, the approximations of finite element does not affect your fracture calculations.

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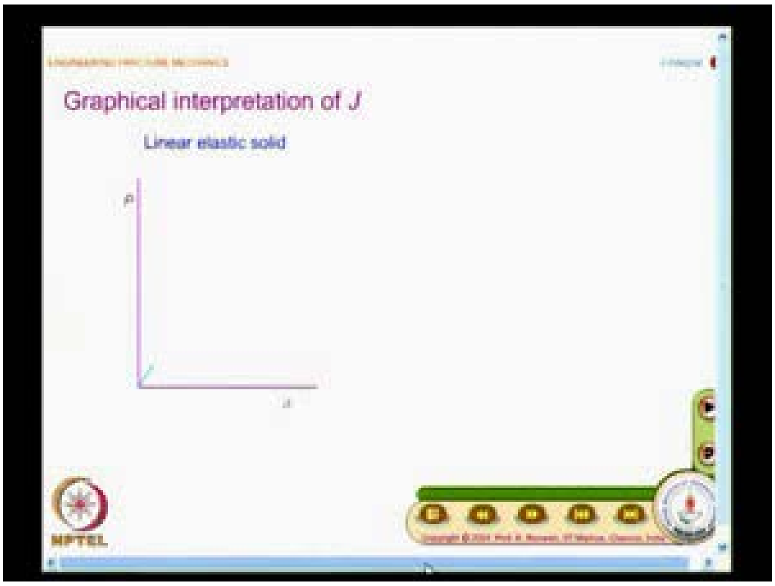

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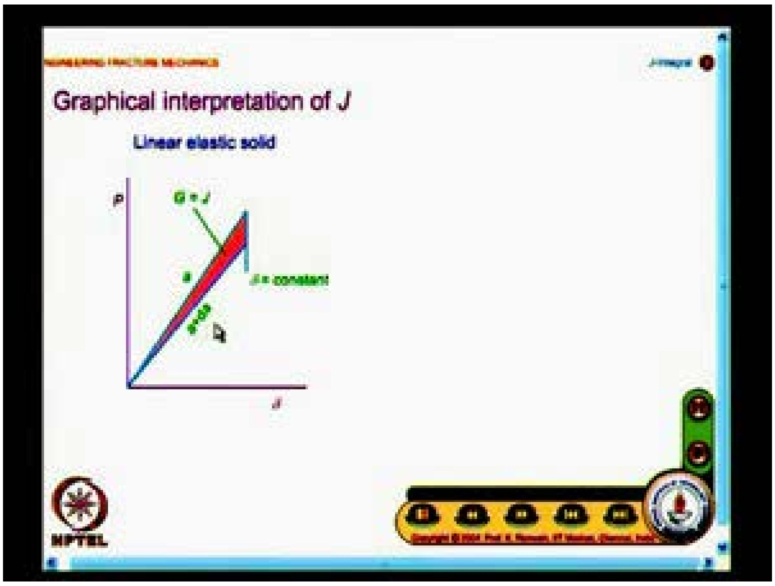

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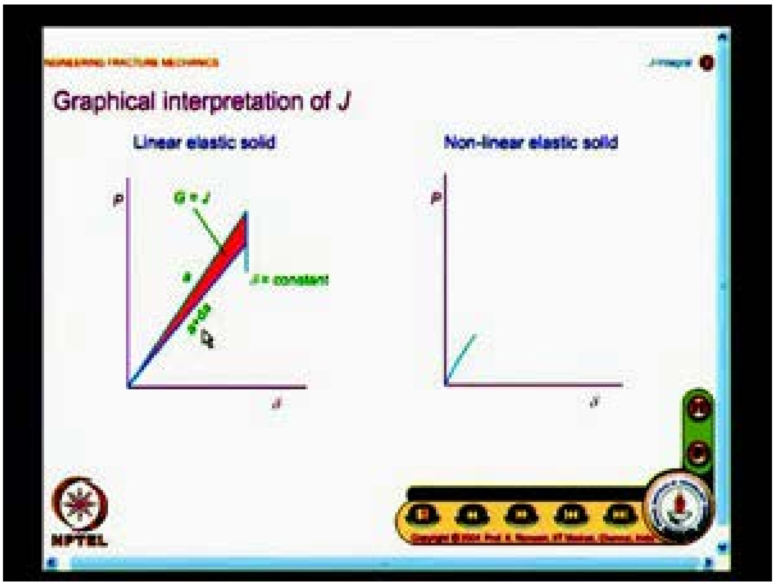

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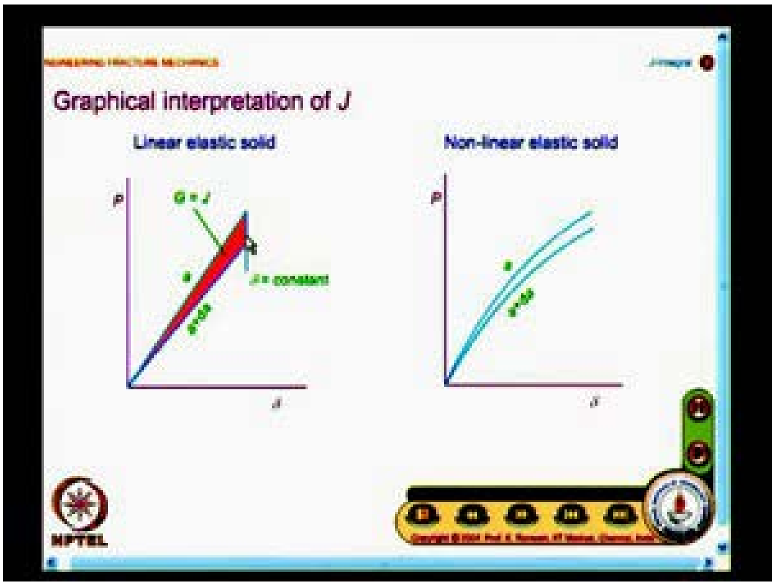

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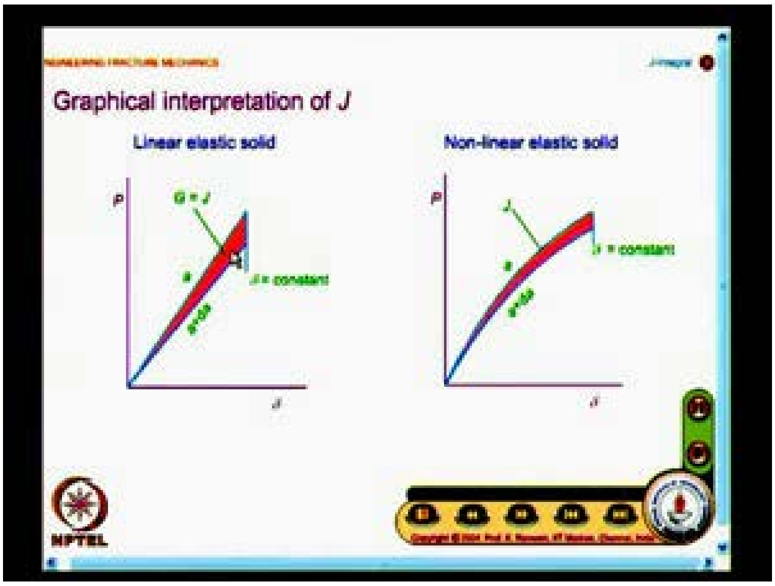

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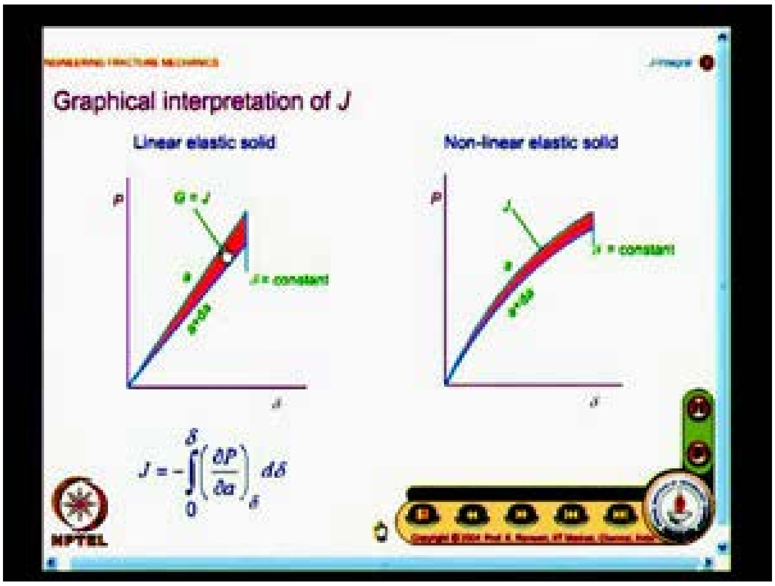

And now, you know, we have looked at, in the linear elasticity, J equal to G. And we will also have a look at, graphical interpretation of J. In linear elasticity, we know what is G. I have a specimen with crack of length a, another specimen with crack of length a plus delta a; whatever is the shaded portion, is the energy availability for crack to grow. And this, you call it as G, which is also equal to J. And we have looked at, a constant displacement and constant load; and we had also argued, whether it is constant displacement or constant load, the energy availability is same in the limit, when the incremental value of crack growth is as small as possible. Here it is shown for constant displacement.

So, this is the energy interpretation of G, which is same as J for a linear elastic solid. You could extend this for non-linear elastic solid. Here, the force displacement relationship is not linear; it is non-linear, in this fashion. And the crack extends by a small amount delta a and this shaded area can be interpreted as J.

So, the advantage of J is, it is applicable for non-linear elastic solids too. I can still evaluate this line integral and then say that, this is nothing, but energy availability for the formation of 2 new surfaces. You have got G equal to differential of potential energy divided by A, B A, in the case of linear elasticity; a similar interpretation of J as differential of potential energy with respect to the growth of the crack is possible, in non-linear elasticity. And you have this, J as, given as minus integral 0 to delta dow P by dow a into d delta.

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See, now the question is, I had raised, from linear elasticity, we can go to non-linear elasticity and J is useful. But J is also used for elasto-plastic analysis, at least approximately. So, we will have to understand, what is the difference between non-linear elasticity and elasto-plastic behavior. You just watch this animation carefully. I have this stress strain curve and this is shown for a non-linear elastic solid; and up to this, there is no problem.

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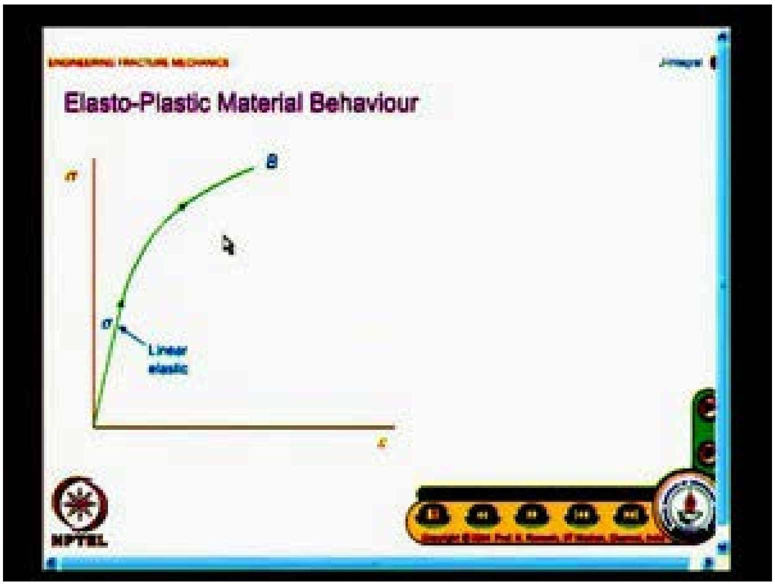

When I have a, this portion is linear elastic, and this portion is non-linear elastic. Suppose, I unload, what would happen, if it is a non-linear elastic solid. The unloading path will trace back, the same way.

In a linear elasticity, it will go up and down in the straight line; in non-linear elasticity you will follow a curve. Suppose, I go to elasto-plastic behavior; from elastic region I have gone to plastic region; when I unload, the unloading path is different; it is not same as the loading path. This is a very key, important information, that you have to keep in mind. And what is its influence? When I have a non-linear elastic curve, for every value of strain, you have only one value of stress; there is no difficulty at all.

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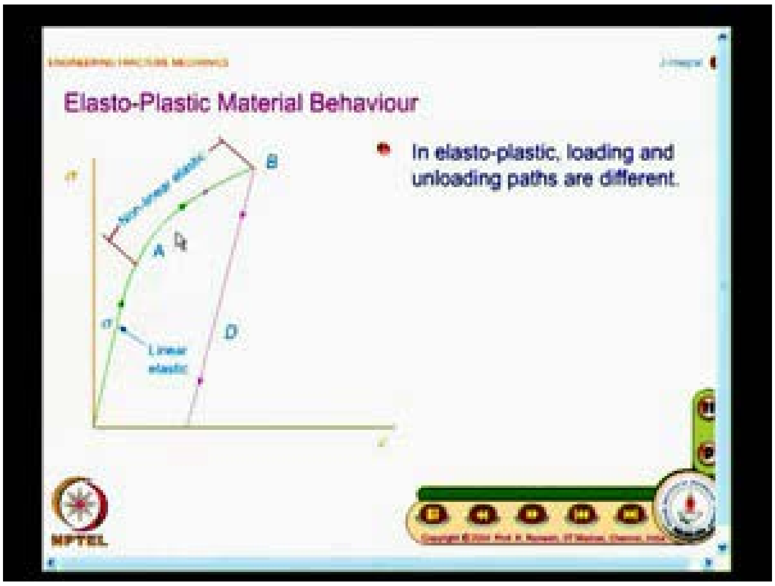

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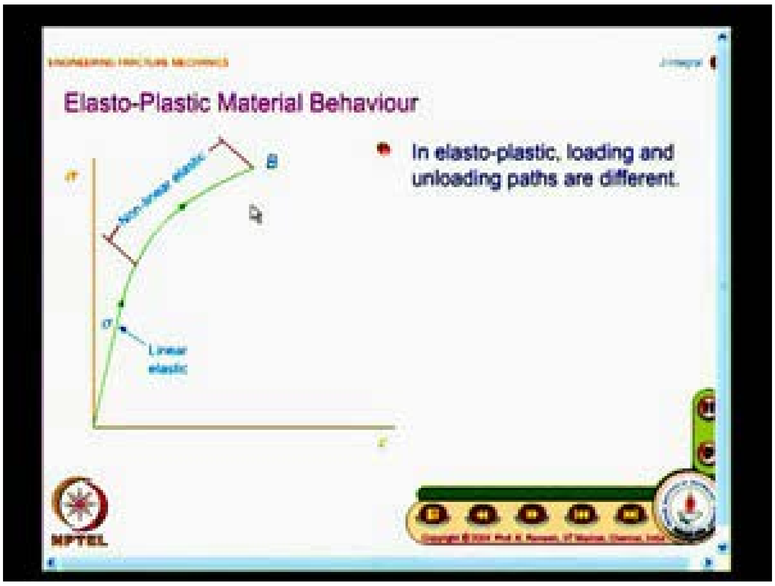

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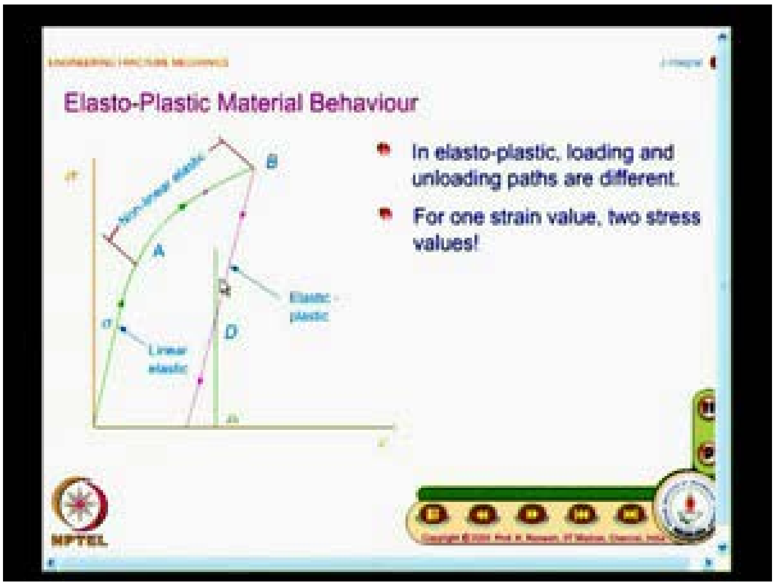

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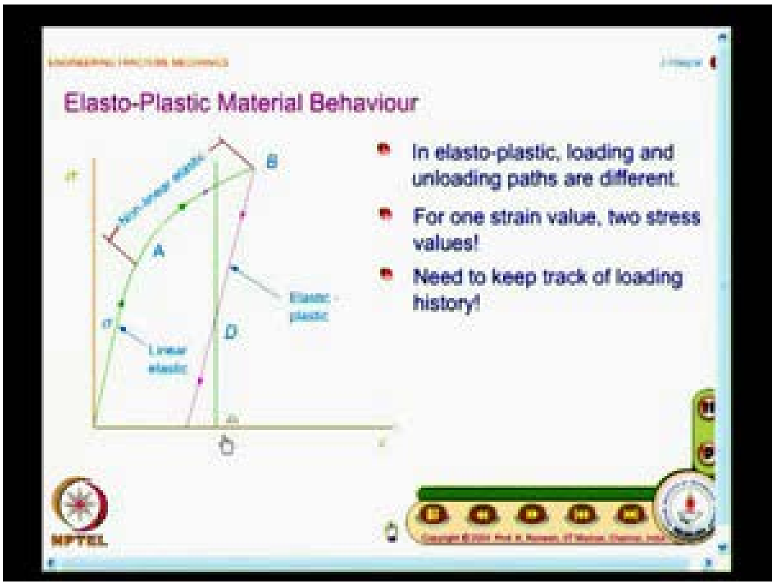

Now, I do an unload; it has reached a plastic condition and I unload; the unloading path is different. And what is its influence? Suppose, I take a strain value, two stress values are possible. You know, this is the very very key, important observation and this is where the complexity comes. When you want to go for plastic analysis, unless you keep track of the loading history, you will not be able to find out a relationship between stress and strain.

You have to keep track of the loading history. I am in the loaded path; when I am in the unloaded path, for the same strain value, I have a different stress. And that is what is summarized here. You have to keep in mind, in elasto-plastic, loading and unloading paths are different; for one strain value, two stress values exist.

So, you have to keep track of loading history. So, this is where all your approximations come, when you want to extend J-Integral for elasto-plastic fracture mechanics. For non-linear elastic fracture mechanics, J is same as g; you can also have this energy interpretation, and you, the system is conservative. And the moment you go to plasticity, system is not conservative; some energy is lost. Whatever the graphical interpretation you saw, all that energy will not be available for crack extension; some energy is lost in plastic deformation.

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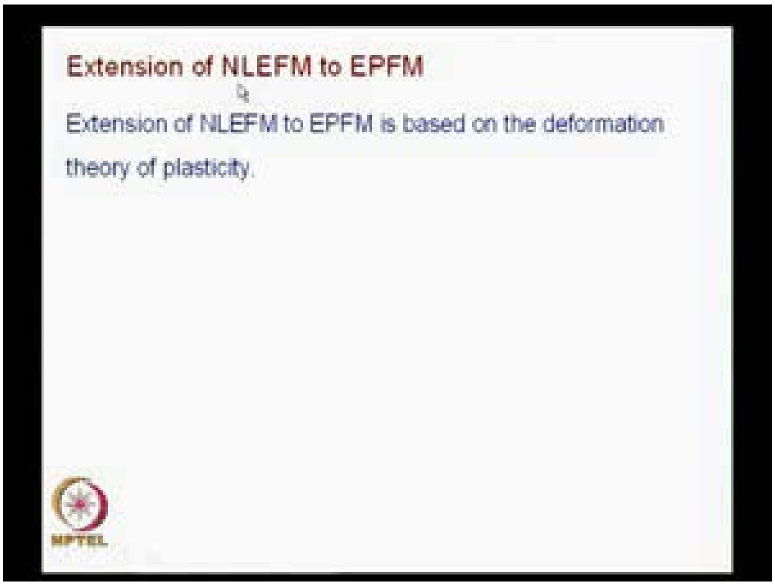

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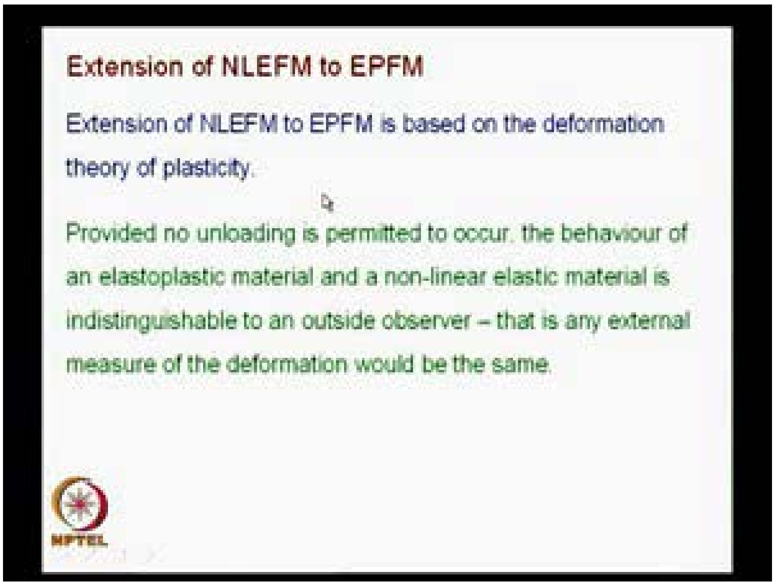

So, people have to bring in some kind of approximation. Let us see, what are the approximations people talk about. So, we want to graduate from non-linear elastic fracture mechanics to elasto-plastic fracture mechanics. And this extension, is based on the deformation theory of plasticity. And, we have seen unloading creates the problem. So, you say, provided no unloading is permitted to occur, the behavior of an elasto-plastic material and a non-linear elastic material is indistinguishable to an outside observer. That we had seen; the loading and unloading paths are same, when your non-linear elastic, but if unloading is permitted in elasto-plastic material, the unloading path is different.

So, now, you do not permit unloading. For a external observer, it will remain identical. What it means is, any external measure of deformation would be the same in non-linear elastic material as well as elasto-plastic material. But you have to keep in mind, the mechanisms going on inside the two materials are markedly different, but outwardly, there is no difference.

So, this is what you take advantage. When you have unloading, you have a problem. So, you ensure that, there is no unloading takes place, which is, we have to investigate; whether it is pragmatic approach; under what conditions unloading may not occur; so, what way you have to utilize J; these are all issues, that we have to look at. And, for unloading not to occur, or if you want to extend the concepts of NLEFM to EPFM, the stress components must remain in fixed proportion, as the deformation proceeds. This is called proportional loading. We will see that in detail.

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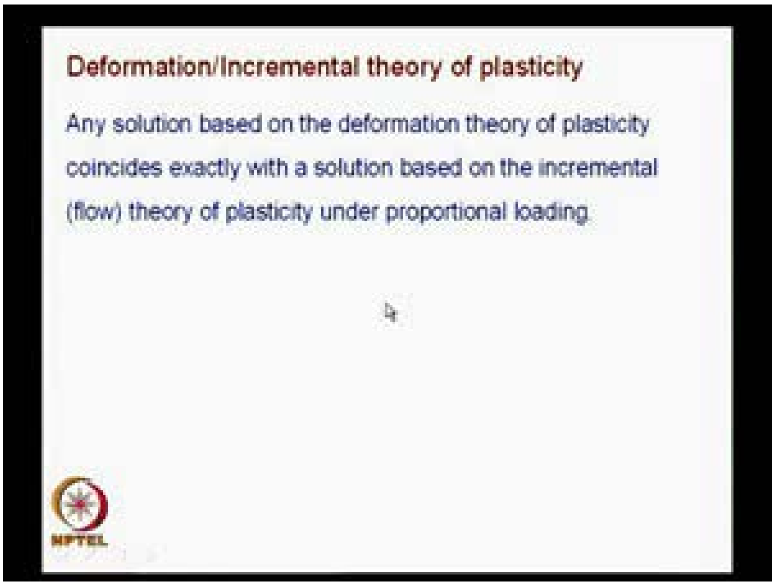

So, in all fracture mechanics literature, you will come across the terminology proportional loading. When you are discussing concepts related to EPFM. And what is proportional loading? I think, before we go to proportional loading, we have to justify, why we want to go for proportional loading. The plasticity theory, you have 2 aspects; one is the deformation theory of elasticity; and another one is the incremental theory of plasticity. The deformation theory of plasticity is easy to handle, because the mathematics is far more simpler; whereas, incremental theory of elasticity, the mathematics is very complex and highly involved.

And what you find here is, any solution based on the deformation theory of plasticity, coincides exactly with a solution based on the incremental or flow theory of plasticity, under proportional loading. So, we want to have a justification. We have already seen, utilizing plasticity theory in a rigorous manner for EPFM, is going to be very challenging.

So, we need to go in for approximate approach. Even in plasticity analysis, you can do by deformation theory of plasticity and incremental theory of plasticity. We find deformation theory of plasticity is mathematically simple and this is applicable, when you have proportional loading. And you have also indirectly seen, if you avoid unloading, you can extend the concept of linear elastic fracture mechanics, non-linear elastic fracture mechanics to elasto-plastic fracture mechanics. So, that is how the story goes. And under proportional loading, stress components change in fixed proportion to one another, for which no unloading is permitted at any point of the plastic zone. That is a key point. We have already seen, what is the kind of complexity, when you have unloading.

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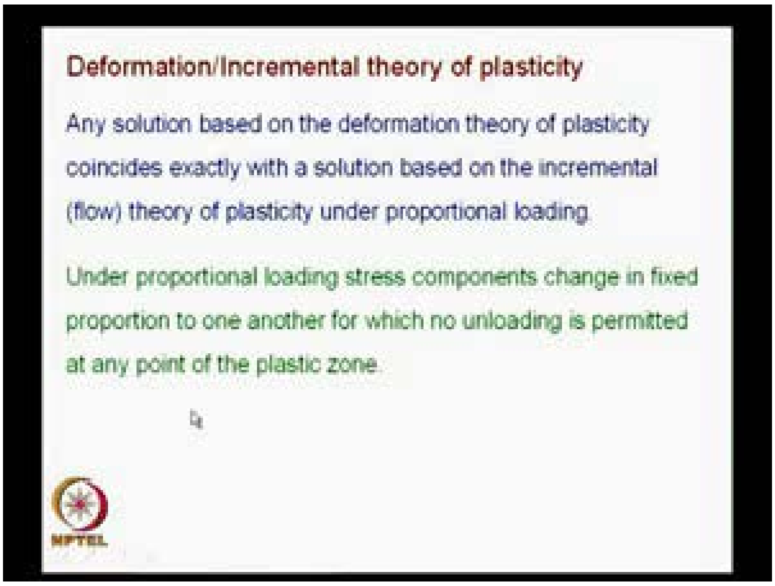

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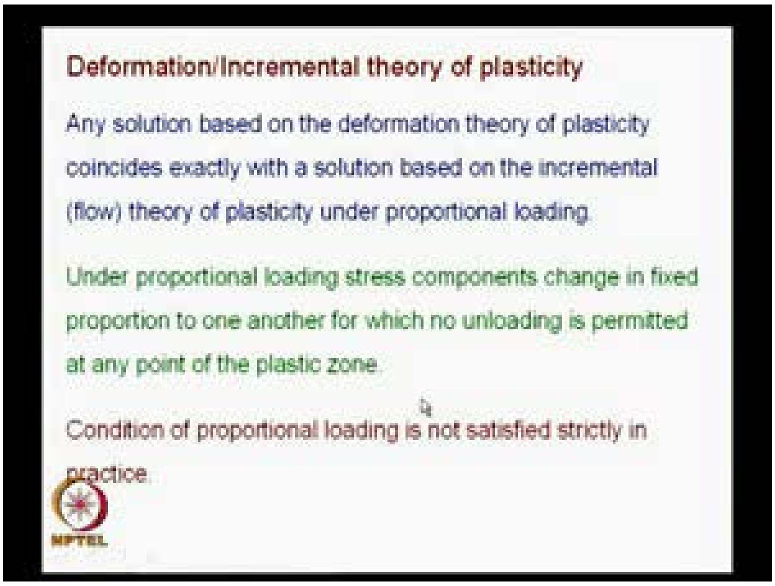

So, in proportional loading, you ensure that no unloading is permitted. And, this also you have to keep it at the back of your mind. Condition of proportional loading is not satisfied strictly in practice. As engineers, we approach the problem. We want, at least, an approximate solution. So, that is how we want to proceed and you have justification.

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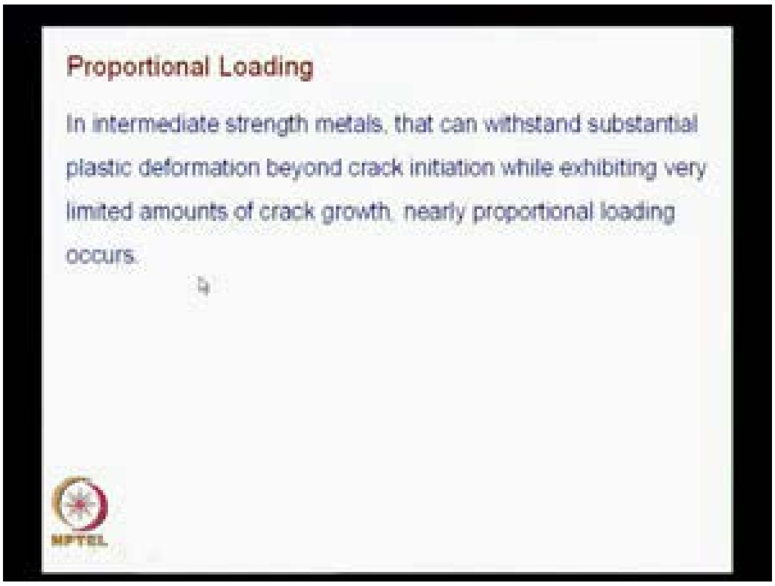

You know, in intermediate strength metals, that can withstand substantial plastic deformation beyond crack initiation while exhibiting very limited amounts of crack growth, nearly proportional loading occurs. The moment crack grows, unloading takes place. Because, you have formation of, 2 free surfaces. So, when you have 2 free surfaces, the stress has to be redistributed. And what you are trying to say here is, certain materials they can withstand substantial plastic deformation. So, essentially there is no significant crack growth. So, nearly proportional loading is possible. That is the way we have approximation.

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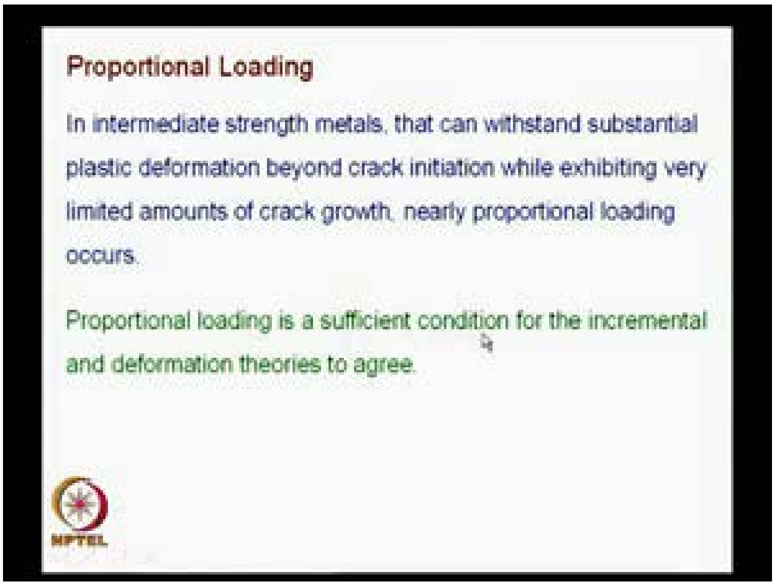

And this is again reemphasized. Proportional loading is a sufficient condition for the incremental and deformation theories of plasticity to agree.

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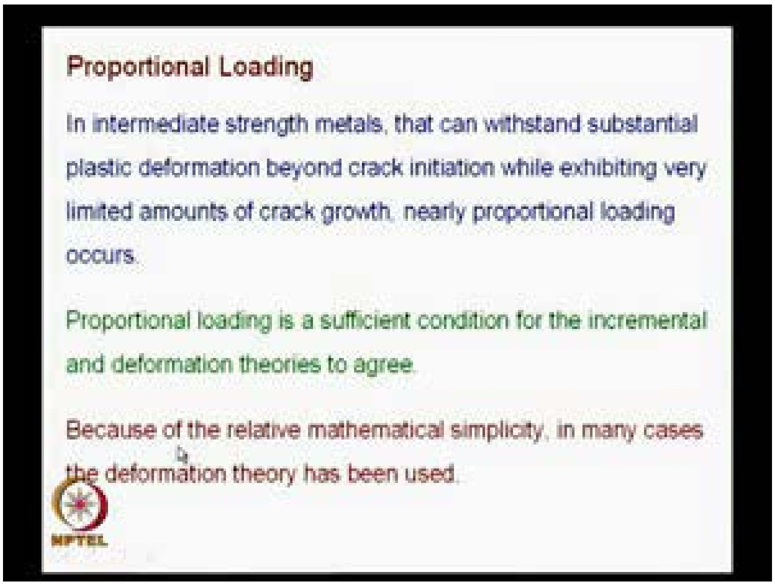

Because of the relative mathematical simplicity, in many cases, the deformation theory has been used. See, what we want to look at is, we want to have the mathematics as simple as possible; a comprehensive to capture the phenomena.

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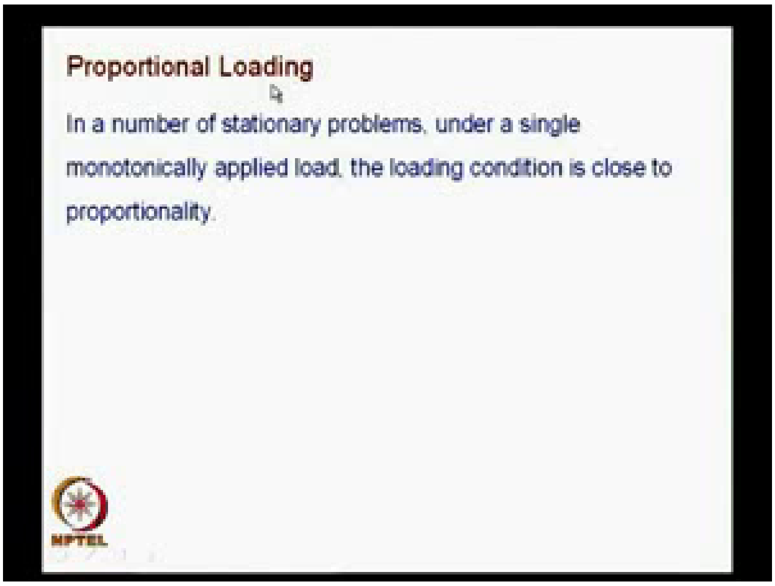

So, we have to necessarily make an approximation. In certain materials, you are able to see, that you will have more plastic deformation and very little crack growth; then, this is applicable. And here, you have a little more expansion on the terminology proportional loading. In a number of stationary problems, that means, crack is not growing, under a single monotonically applied load, the loading condition is close to proportionality.

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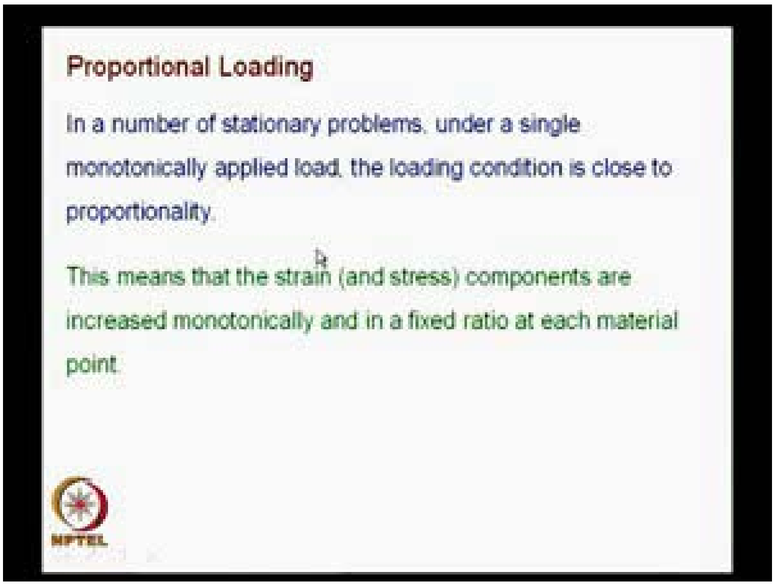

This means, that the strain and stress components are increased monotonically and in a fixed ratio at each material point. And this is how, you justify your approximation.

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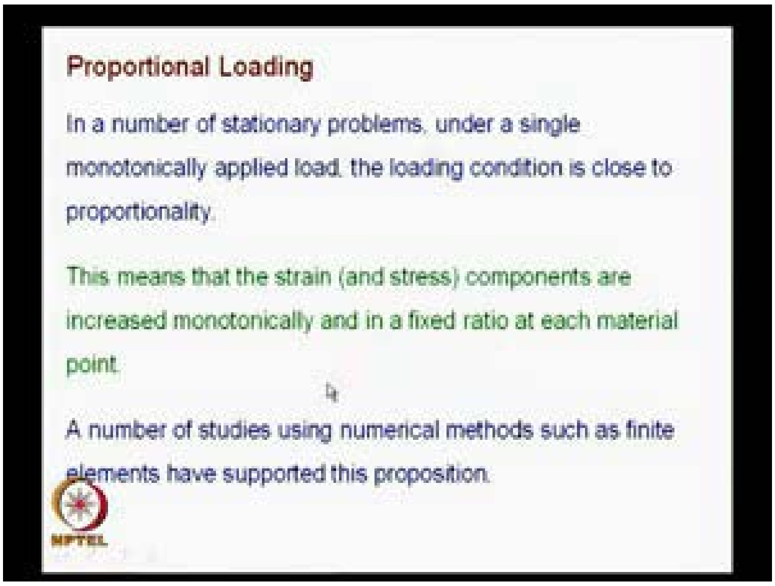

A number of studies, using numerical methods such as finite elements, have supported this proposition. See, the whole of elasto-plastic fracture mechanics, you have to look at proportional loading; that is how you justify the applicability of J, at least approximately. That you have to remember; it is not an exact representation. Approximate solution is good enough; as long as I am able to solve the engineering problems, even approximations are welcome.

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You have to take it with a pinch of salt. In elasto-plastic fracture behavior, stable crack growth is usually observed. And I have said, if there is a crack growth, unloading takes place. So, you have to find out, to what extent you can apply, the concepts of J.

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So, what we want to do is, the focus of EPFM for practical applications is limited to the ability to describe the initiation of crack growth and also handle a limited amount of actual crack growth.

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As long as you are able to predict the initiation, you are happy; you are not interested to study further. Some crack growth, you can possibly, analyze approximately. There is a crack growth, you have a problem; unloading takes place; these approximations are not going to be valid. And many concepts have been developed in EPFM. Of the many concepts developed, two have found general acceptance. They are J-Integral and CTOD or COD.

So, in this class, what we have looked at is, we have looked at J-Integral. We have seen its path independence and I said, this path independence is exploited in linear elastic fracture mechanics to calculate stress intensity factor, from finite element solution. And we also saw a graphical interpretation of J. Like in linear elastic solids, in non-linear elastic solid, the J is same as the energy availability for the formation of two new crack surfaces. Then, we looked at, what is an elasto-plastic material behavior. The moment unloading takes place, we found for a single strain value, there could be two stress values and you need to keep track of the loading history. This is a challenging aspect. But we want to extend our knowledge, from non-linear elastic fracture mechanics to elasto-plastic fracture mechanics.

So, by bringing an approximation, as long as no unloading takes place, you can still extend the concepts of J, at least approximately, to handle crack initiation elasto-plastic fracture mechanics. We have seen the energy interpretation of J in this class. We would see J as a stress parameter in the next class, and we will also see rudiments of CTOD. Thank you.