Engineering Fracture Mechanics
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Module No. # 07
Lecture No. # 35
Crack Closure

We have been discussing concepts related to crack initiation and life estimation in this chapter. And we had looked at, in the last class, for the growth of a fatigue crack, Paris came out with a very elegant empirical relation, relating d a by d N and delta k. We call that as the Paris law. And I said, it is a very important contribution to fracture mechanics. In fact, the designers looked at utilizing concepts of LEFM, only when they came across the contribution by Paris. Because, that provided a mechanism for them to implement in practice and find out, whether the structure would be safe or not, by knowing how the crack propagates, due to fatigue. And I also mentioned, it is a very complex phenomenon.

And environment has a very important role to play in crack growth. The ingenuity of Paris law was he got the kernel of how the crack grows. If he had looked at all issues and attempted to arrive at an empirical relation, he may not have arrived at an empirical relation at all. And now, we will see in the discussion today, how modifications of Paris law are utilized to explain various phenomena observed in actual experimentation. And, in fact, before this chapter, we had also discussed modeling of plastic zone at the crack-tip. That knowledge would be useful, to understand the phenomena observed in the experiments.

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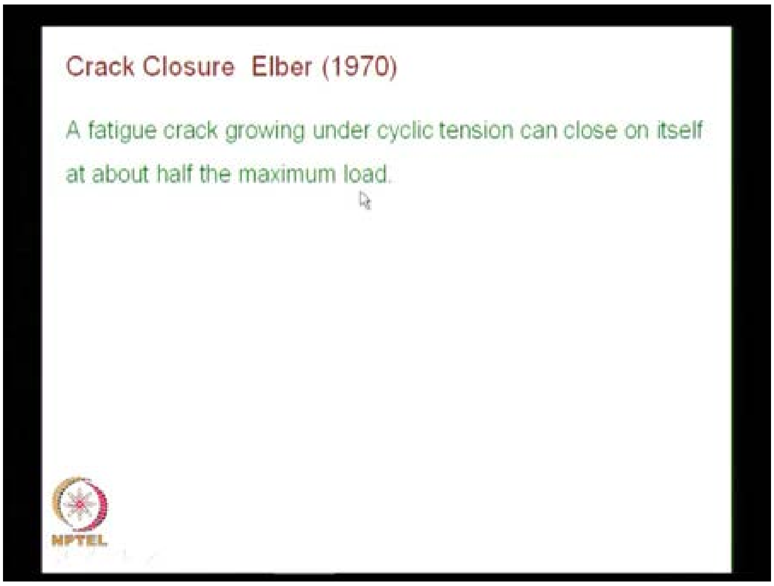

So, that is the reason, why that chapter was covered earlier. And now, we will take up a very important contribution by Elber. This was done in 1970. He brought out a concept called crack closure. And what he observed was, a fatigue crack growing under cyclic tension can close on itself at about half the maximum load. And this was purely an experimental observation; based on the experimental observation, he arrived at this kind of a very interesting result. This effect becomes significant, for crack growth calculations under random loading.

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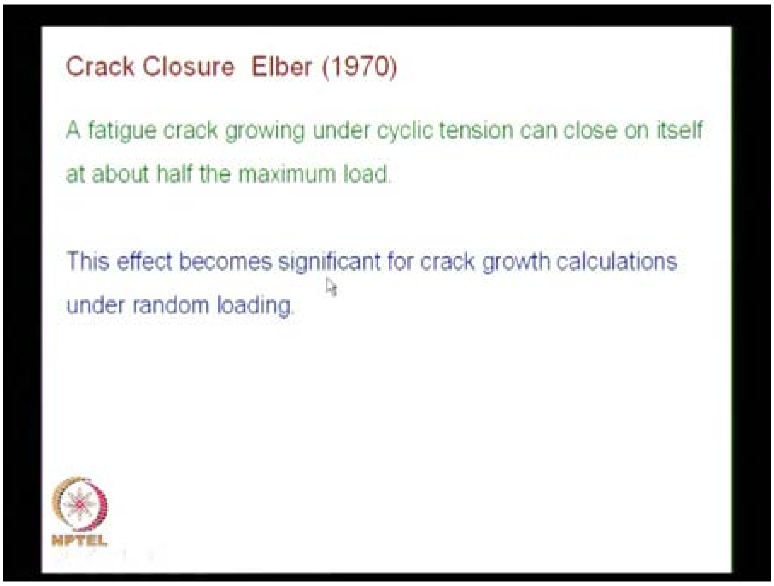

See, you will have to keep in mind, fatigue is one of the most difficult type of loading that acts on the structure. You will invariably have, variable amplitude loading. And one of the difficulties is, finding out the actual loads spectrum; it is not simple. There are methodologies available and you have to study the literature and then only calculate the actual loads spectrum. So, invariably, you will have variable amplitude loading. From modeling point of view, we may study, under constant amplitude loading, how the crack grows. But if you want to take the knowledge to the field, you will have to account for random loading. And, if you go for random loading, you will have to bring in the observations of crack closure into your life estimation calculation.

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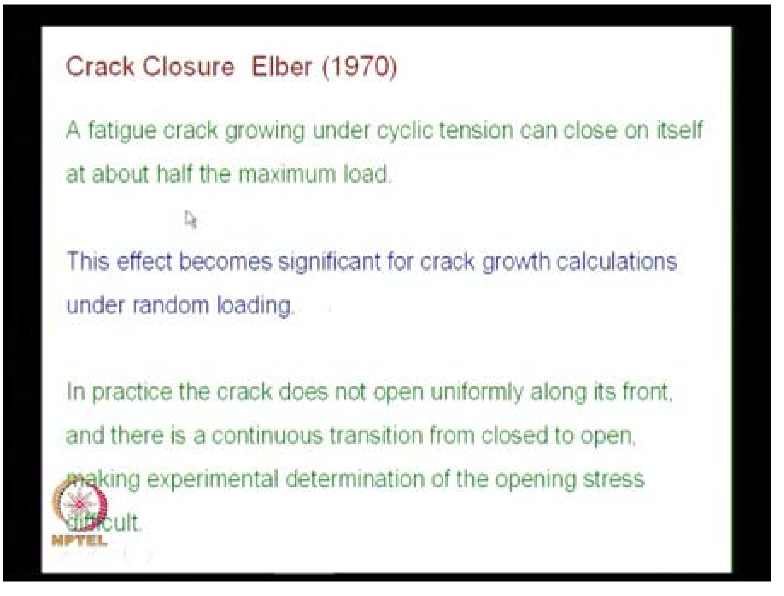

And what we find is, the crack closes about half the maximum load. And, how does the crack open? What is the difficulty in the experimentation? In practice, the crack does not open uniformly along its front. And there is a continuous transition from closed to open, making experimental determination of the opening stress difficult. See, unless you make certain observations by experiment and then identify, these aspects have to be modeled by a numerical model; you cannot develop a numerical model. So, initially you need to have experimental observation for such complex phenomena.

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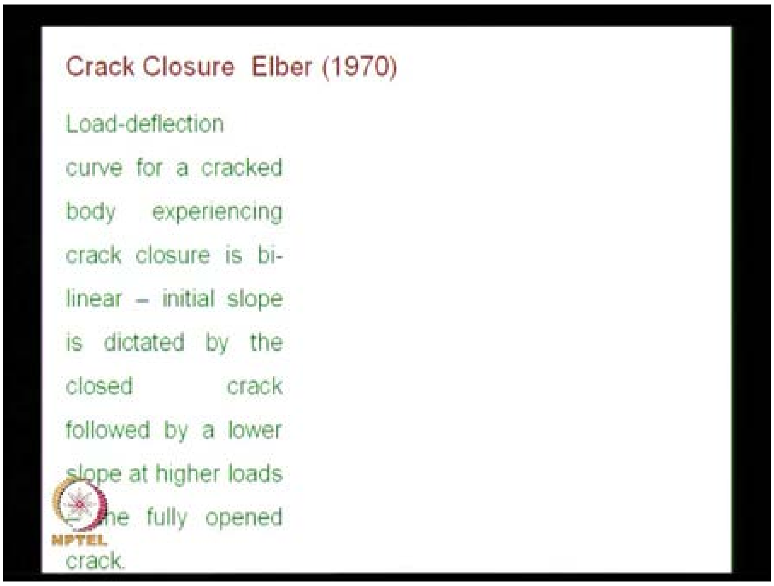

So, one of the difficulties is, how to see whether the crack is fully opened or not. And, how do you judge, whether a specimen displays crack closure phenomena. People also have methodologies to look at that. And one of the simplest way is, to get the load deflection curve. So, if I get the load deflection curve, for a cracked body experiencing crack closure, is bilinear. This is the observation. And you have the displacement versus applied stress. And if you look at, the stress displacement relationship is bilinear.

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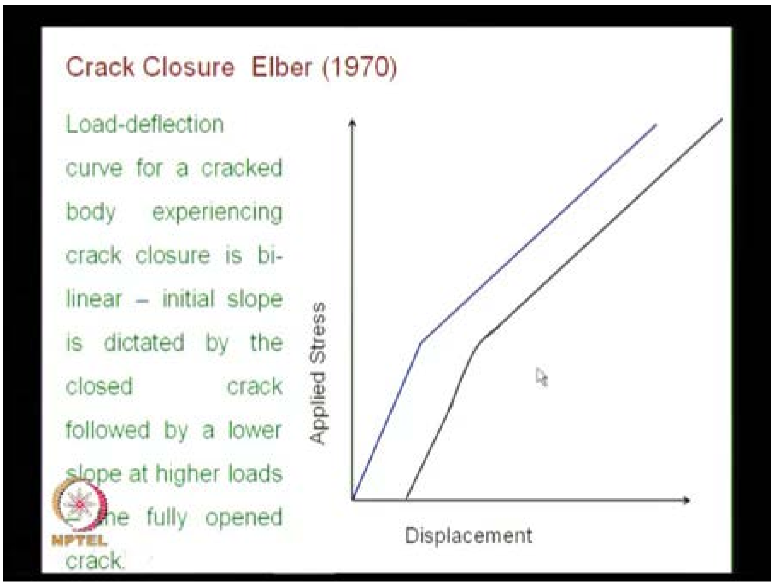

So, you have slope changes, from the initial to later part. Initially, it is more stiff; initial slope is dictated by the closed crack. So, that means, crack behaves as if it is smaller in length. Then, you have a slope which is different. At higher loads, because the crack is fully opened, the slope is also different. So, if a fatigue cracked specimen experiences a stress displacement behavior as bilinear, which is the model again. In practice, you will have a smooth translation from this to this; and that is also depicted here.

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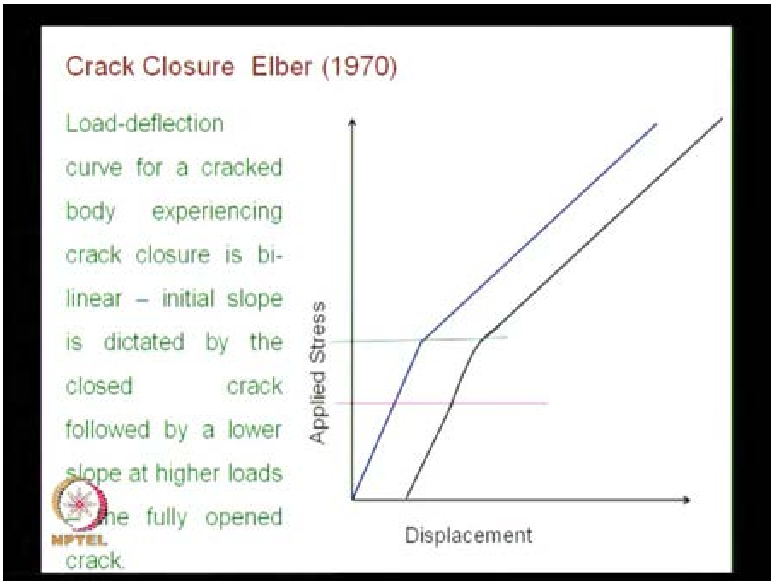

And the question here is, how will you identify when does the crack close and when does the crack opens. This is a very difficult to measure experimentally. And people have found various ways to do that. One way of describing this is, you say at this stress, when there is a translation of the slope, the crack is fully opened and when you have a curved portion becomes straight here, this you say, that crack is closed; part of the crack is closed. So, this is one way of estimation. You know, at this stage, it is also wise to look at, what was the history, in the case of crack closure. Initially, people did not believe that crack closure could happen. This was done by Elber. There was an article written by him in Current Contents in 1983, and in that, he says, the original discovery of the crack closure phenomenon was made, when I was a graduate student at the University of New South Wales in Sydney, Australia. Rather than being the result of years of directed effort, it was a split second observation made, while cutting up a fatigue cracked plate. See, this is very important.

You know, sometimes you do a very systematic study, yet you do not get a result. Here, he admits it was not a systematic study; he chanced upon an interesting observation. He was a good experimentalist. He did not miss out the details. See, this is very important. Experimentalists have to be alert. Look at anything deviating from your basic understanding, record it; and scrupulously honest. Because, if you report what you observe, in all its totality, new theories can develop. That is what happened in crack closure concept. In fact, when he reported, it was not accepted by the community; it was rejected also. I will read his quote for that. He says, because of the geographic location, I was not aware, during the ensuing months, during which I wrote up the discovery into my thesis, that I was going to contradict many of the existing teachings in fracture mechanics. Or how much that phenomenon contributed to the resolution of the complexity of crack growth, under variable amplitude loading. And what is interesting is, the next few sentences, it says like this. After having my theses initially rejected by the first American reviewer, I went to the Federal Republic of Germany, to continue my research' and the story goes like this. You know, poor fellow, he found a very nice observation and when he reported, because the conventional thinking was not able to accept that crack can close, so, they rejected the thesis. Then, he carried on; he joined NASA Langley research center. Now, it is an accepted practice. In fact, what I am reading is from Current Contents. This says, this weeks citation classic, it was published in December 12, 1983, where his paper on the significance of fatigue crack closure - damage tolerance in aircraft structures, published as A S T M- S T P- 486, had a very high citation of 240, I think. That is what I remember. 240 citations and it is discussed as a citation classic, which was originally rejected, it became a citation classic. And we will further see, what the crack closure concept is all about.

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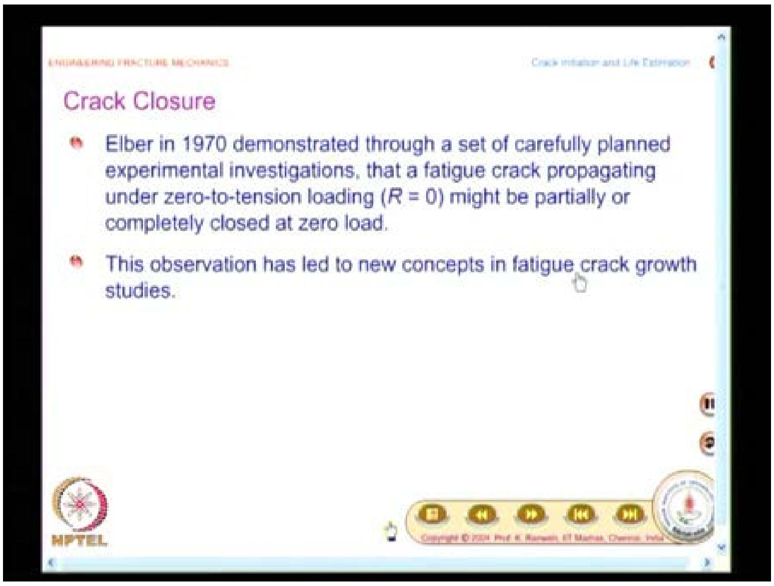

So, this is what I had mentioned, which is summarized again. Elber in 1970 demonstrated through a set of carefully planned experimental investigations, that a fatigue crack propagating under zero to tension loading, that is R to R equal to 0, might be partially or completely closed at zero load. And, this observation has led to new concepts in fatigue crack growth studies. Obviously, after initial hesitation; people were not accepting is theory, initially. Once it was convinced, people said how it could be modeled. The ultimate usage of this observation is, how do you modify your crack growth calculation. There is a designers' approach. We will see that designers' approach first.

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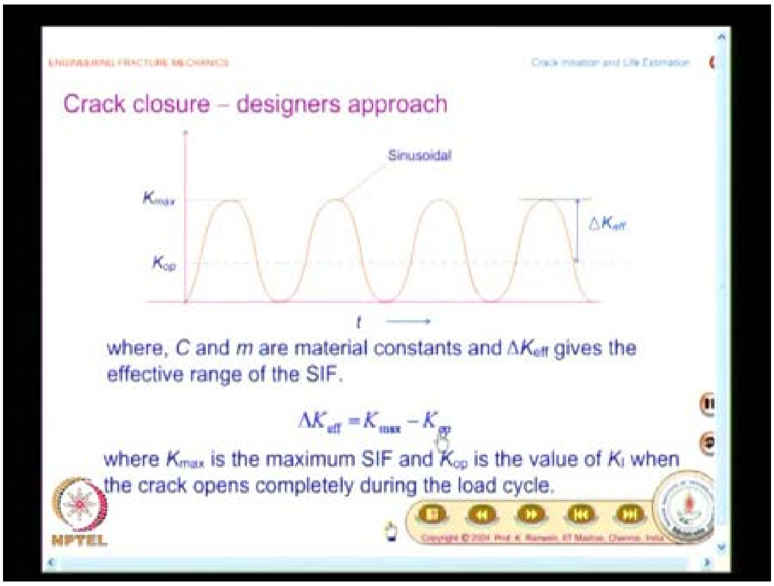

So, what they have done is, to account for crack closure, designers employ a modified Paris law which is as follows. Normal Paris law will have d a by d N equal to C delta K power m. Here, the delta K term is put as delta K effective. And how do you define, what is delta K effective. Suppose, I have a sinusoidal variation; and I have K max and K minimum is 0. But you have, at a particular stress only, the crack starts opening. Afterwards, you know, only the crack becomes effective. The crack closes by this value of K op. So, the effective delta k is between K max and K op. So, people could embed the phenomena of crack closure, by taking delta K effective. Now, the question is, how to find delta K effective. That is where the challenge lies. It is not a simple task. Conceptually, the crack is effective, only part of the cycle.

So, you look at, what is the effective delta K, rather than the actual delta K. So, you have to find out, how to get delta K effective. Several models have been proposed. People have done experimentation and calculated it. And you will also have to bring in, the effect of stress ratio R. As a function of R, how does delta K effective changes. So, this is how research proceeded. And we will see, how they developed delta K effective. So, delta K effective is K max minus K opening, where K op is the value of K 1, when the crack opens completely during the cycle.

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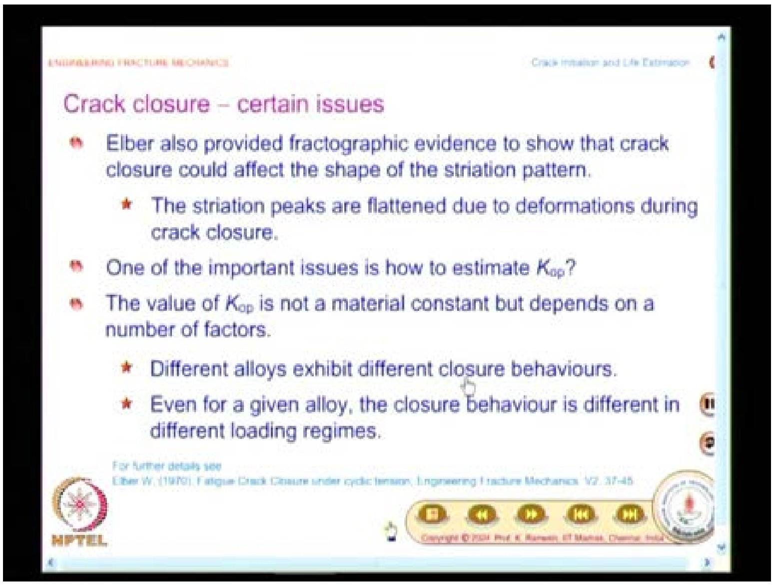

And this is difficult to evaluate. But we have to do that, if I have to embed crack closure, in our crack growth calculation. There are empirical relations reported. And, in order to ensure that crack closure indeed happens, Elber also provided fractographic evidence to show, that crack closure could affect the shape of the striation pattern. See, if you look at the history also, whoever reports the discovery, has to justify, 'yes, his discovery is correct'; if he does not do that homework, that concept does not get recognized. We had seen in the case of Dugdale model; he had performed the experiment, but he did not report the result. So, there was skepticism among the practitioners, not to accept Dugdale model, in the initial stages. Once Hahn and Rosenfield provided the experimental evidence, people started using Dugdale mode. So, in the case of Elber, when he reported the crack closure, later he also provided fracto graphic evidence to show, such thing happens. And the effect is, the striation peaks are flattened due to deformations during crack closure. And as I mentioned, one of the important issues is, how to estimate the stress intensity factor corresponding to fully opened crack.

And, you have to note, unlike the fracture toughness, K opening is not a material constant; but depends on a number of factors. So, that makes the problem difficult. Different alloys exhibit different closure behaviors. So, this is one observation. So, you have to do a lot of tests, to find out. Even for a given alloy, the closure behavior is different in different loading regimes. So, that makes the problem much more complex. And I think, you have the paper by Elber. This is the paper he published in 1970, ‘Fatigue crack growth under cyclic tension’. This was published in Engineering Fracture Mechanics. This is volume number 2, between 37 to 45. You can go and have a look at it, how he has presented his work.

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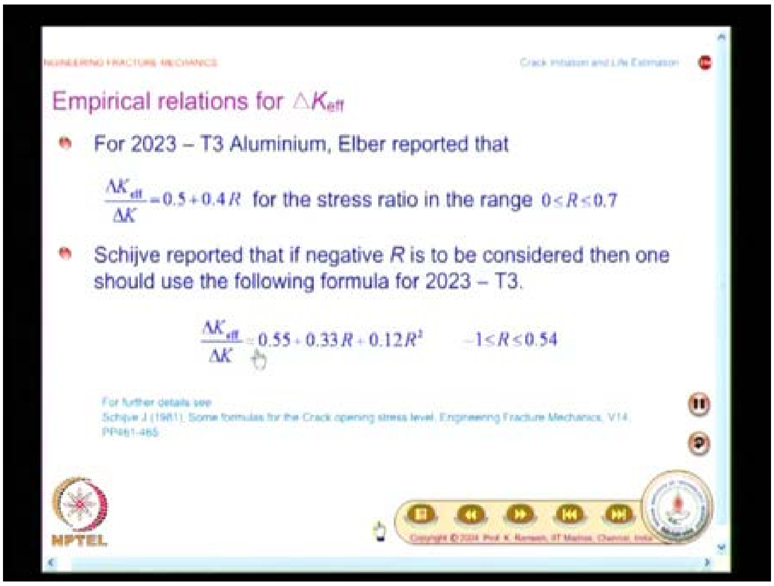

So, you have empirical relations to find out what is delta K effective. And you know, for all aerospace structures, aluminum is very important. So, you have a 2023 Titanium, that is T 3 aluminum. Elber reported that, delta K effective over delta K, can be written as 0.5 plus 0.4 R, where R is the stress ratio.

And he had obtained the relation for the range 0, R, R is between 0 and 0.7. And there were also other researchers, who have extended this range of R. But the point to note here is, you need to have some way of finding out, what is the value of K effective. If it is necessary, I can also enlarge it. So, this is the equation that you have. And another researcher reported that, if negative R is to be considered, then one should use the following formula, for the same alloy. This is a little more elaborate. We will also have a look at it. See here, whichever way it goes out, but you can write the expression, delta K effective divided by delta K equal to 0.55 plus 0.33R plus 0.12R squared. This is from R equal to, between minus 1 to 0.54. R lies between minus 1 and 0.54. You have another expression. See, the point here to note is, you have empirical relations available.

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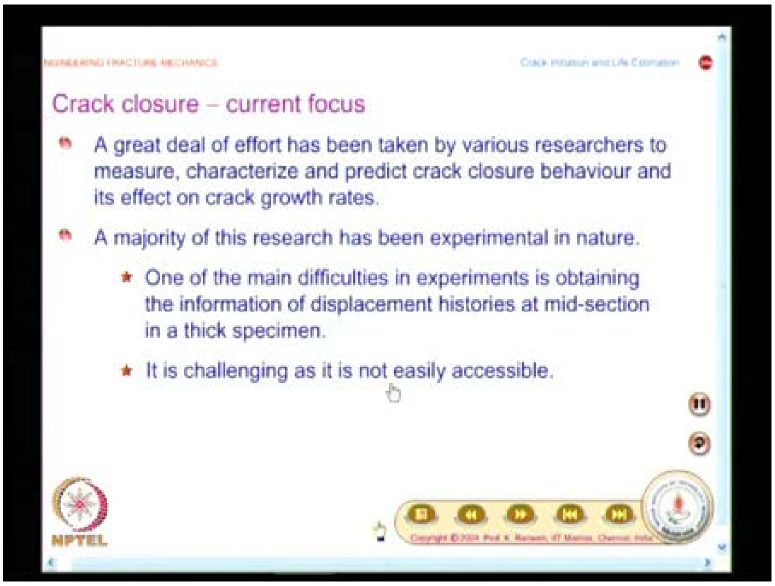

So, look at the literature, find out for your given alloy, what is the empirical relation to use. Then, you will be able to use concept of crack closure. And what is the current focus of crack closure. Because, initially, people were not ready to accept crack closure; once they have started accepting, people also have done additional work. A great deal of effort has been taken by various researchers to measure, characterize and predict crack closure behavior, and its effect on crack growth rates. A majority of this research has been experimental in nature; that is what I said, the phenomena is complex. So, initially, you have to understand what is happening through experiments; with such inputs it is possible for you to develop numerical models.

And, what is the main difficulty in the experiment? One of the main difficulties in the experiments, is obtaining the information of displacement histories at mid-section, in a thick specimen. See, I have already mentioned, the crack opening is not uniform along the front. So, you have to ensure that, if I have a thick specimen, the in between portion whether it has opened or not, I have to have experimental measurement. Only then, it will be more realistic. That poses a challenge. It is challenging, as it is not easily accessible. I suppose, you have been able to write these points; time is sufficient for you to write.

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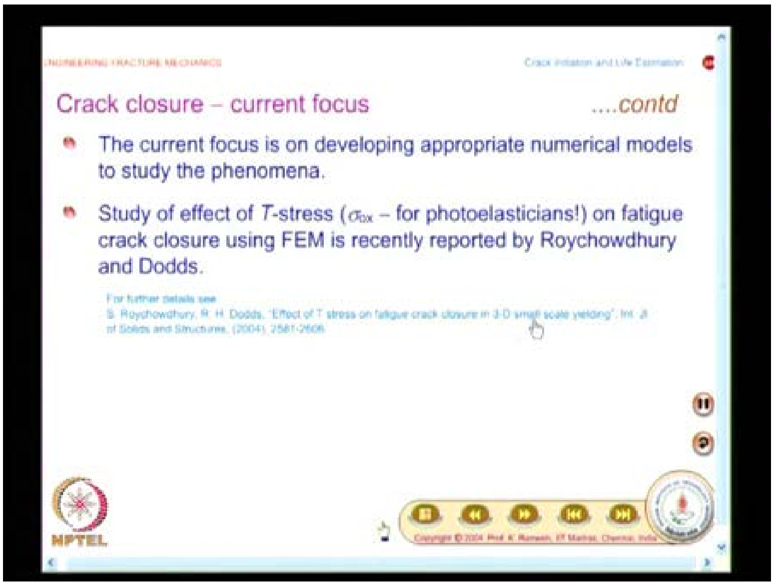

Having seen that the experiment is difficult, the current focus is on developing appropriate numerical models to study the phenomena. Not only this, they also want to find out the effect of T stress, that is sigma naught x for photoelasticians. We have spent ample time in understanding this, when we discuss crack-tip stress and displacement fields. So, they do not want to restrict only to the stress intensity factor K; they also want to go to the second term. And such an attempt is made using finite element methods, which is recently reported by Roychowdhury and Dodds. It is a paper in 2004. I think, I would read this for you. This is by Roychowdhury and R. H. Dodds – 'Effect of T stress on fatigue crack closure in 3 D small scale yielding'. This is on International Journal of solids and structures, published in 2004; you would be able to see the details of the paper.

And before we move further, and I would also like to share the another note, which came in the citation classic, which Elber wrote. He mentions like this, the time from the initial discovery of any phenomenon to a profitable application is often long; in this case, which means crack closure, the inclusion of crack closure in numerical crack growth calculations add significant cost and complexity for gains in accuracy, only required in the space and aeronautic industries; many industries will continue to use simpler and less accurate theories, leaving the continued work in crack closure as an area of basic research.

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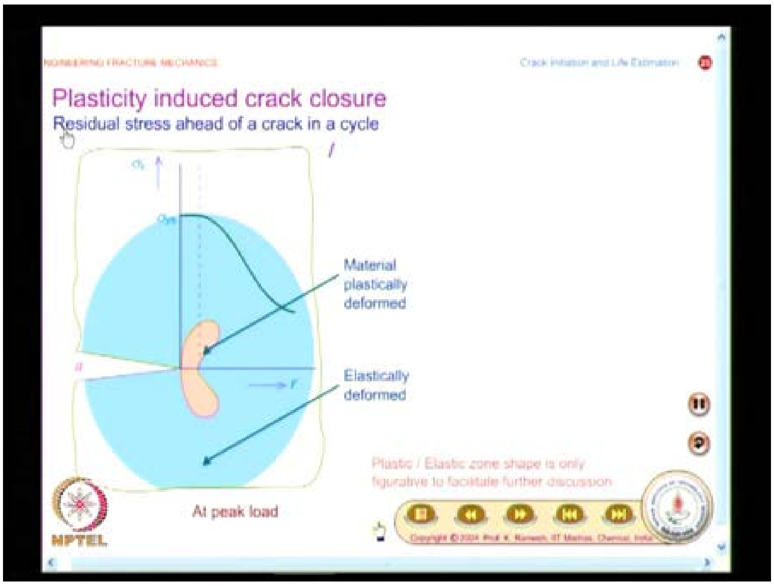

So, you will have to keep in mind, you know, if you want to employ a new phenomena, it does not come cheap; it comes to a added cost. Now, what we will do is, we will try to go and understand, what is crack closure. There are various categories of crack closure. And, we will first see, plasticity induced crack closure. And, first concept that you have to learn is, what is the residual stress ahead of a crack, in a cycle? And what is shown here, in a tiny fashion, is actually, when you have a sinusoidal load, you are actually looking at one half of the curve there, where you are going from 0 to maximum. So, when I go to 0 to maximum, we would see what happens at the crack-tip.

So, you go from 0 to maximum, and then come down. And make neat sketches of this, as much as possible. So, I have this as the crack. I have the axis as r; this is a distance measured and in this axis you have, stress is marked. So, you have tresca yield criteria. I have marked this as sigma x. So, this will be horizontal and there will be some variation of this, along the crack axis. And we have already seen, the plastic zone, in the case of mode one, will be something like this. And what is shown in blue, is the elastically deformed neighborhood. The slight light brown color, shows a material plastically deformed.

So, what you have to recognize is, even though your external loading is much smaller, because of the presence of a crack, when the load is increased gradually from to 0 to peak, invariably, you will have plastic deformation near the crack-tip. So, this happens during loading. And what is our focus is, we want to see, what is a residual stress ahead of a crack, in a cycle. This also you have to note; we are just looking at a virgin material; we are loaded it for the first time and you have a plastic zone, surrounded by an elastic region. And when I reduce the load, what would happen? Unloading takes place. Whatever that has happened to the elastic region, it will spring back to its original position. If there is no plastic deformation at the crack-tip, the spring back would be uniform all throughout. But because you have plastically deformed zone near the crack-tip, the elastic recovery will go and compress this zone.

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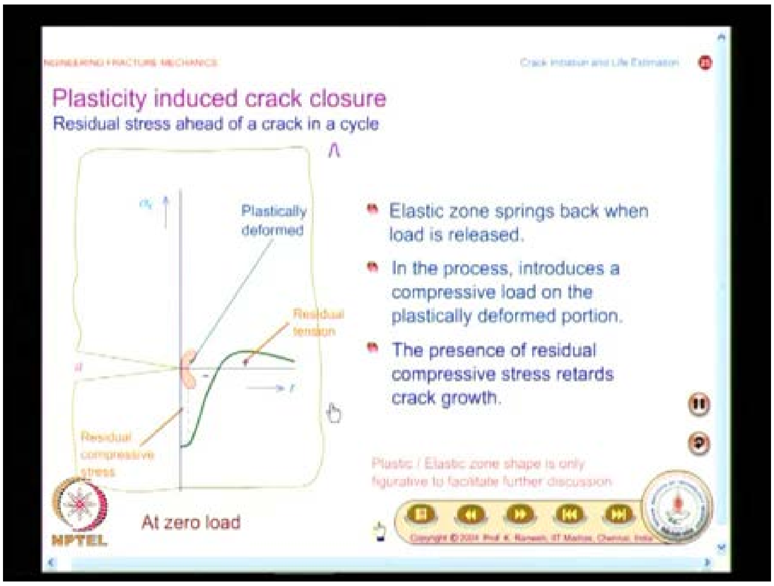

Let us see, what happens pictorially. So, you will have a zone, where this is still under plastic zone condition; much smaller in size, because of the elastic recovery of the neighborhood. And in a sense, you will have a residual stress form at the end of one cycle. You know, this concept, you will have to appreciate. At the end of one cycle, invariably, you will have a residual stress pattern, which is compressive, near the crack-tip and tension ahead. And let us see, what are the steps. Elastic zone springs back, when load is released; in the process, introduces a compressive load, on the plastically deformed portion.

So, you will have a residual compressive stress and a residual tension. You know, if you understand this, many other discussions, what happens when there is a overload, what happens if there is a crack closure, for all of this, you have to recognize, because the load is increased and decreased, you have a residual stress form, ahead of the crack-tip. It is a first term all of you should get. So, what I will do is, I will replay this animation again. You have carefully drawn this sketch as much as possible. And what we will do now is, we will replay the animation, where I am showing, what happens when load is increased from 0 to the peak value. So, you will have a plastic zone; and this is the elastic region; and this is the approximate shape of the plastic zone, at the crack-tip, surrounded by elastic region. And this is schematic; do not think, only a circle around the crack is elastic; no, it is not so.

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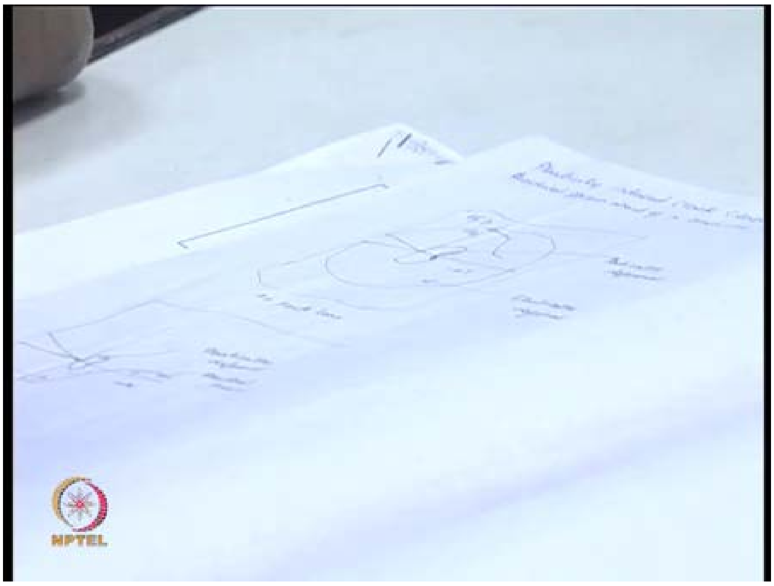

Just for visualization purpose; because we have to feel, something goes and compresses it. You know, that visualization is very good, when I show something in blue color and then it works. Now, what I am going to do is, I am going to reduce the load. This is the story when load is increased from 0 to peak load; and this plastic zone is subjected to tension. It is a value of sigma y s. When there is elastic spring back, and the load is reduced, you have a plastic zone, where the stress is minus sigma y s. And, that is what is shown here.

And you have a compression, followed by residual tension. If you understand this, all our future discussions, it becomes easier to discuss. Now, you have to see, if I have multiple cycles, the crack will go through a plastic wake; we will see that also. I have very nice set of animations and we will see through that. In a growing fatigue crack, behind the crack-tip, a plastic wake is developed. You know, I have initial crack length; and what is shown here is, this is indicated that you are having a cyclical loading. You are seeing, at the end of one cycle what happens; after several cycles, what happens. You know, the crack is growing in length.

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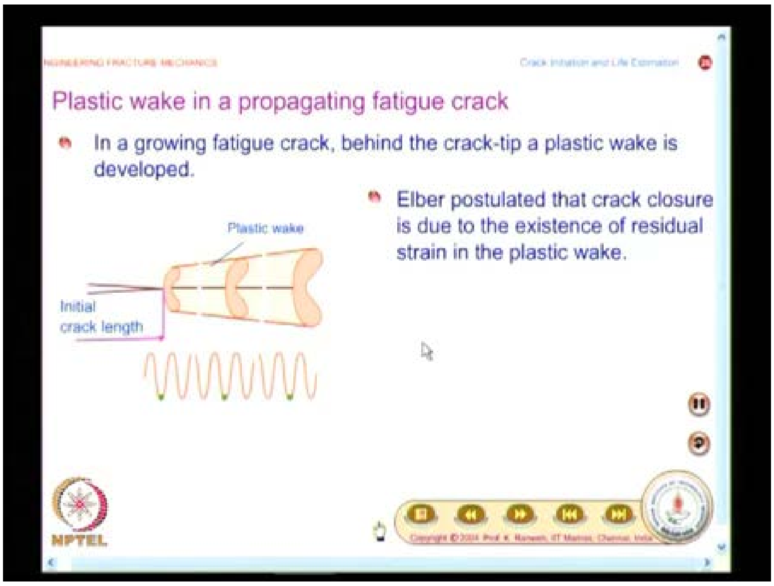

So, when the crack grows in length, the plastic zone also will grow, in shape. It is not going to remain in same size; it is going to increase. That is what is depicted in the animation. And you observe it. It is done for a sufficiently long distance, and the crack goes through plastically deformed portion, which is called as a plastic wake. And what Elber postulated was, that crack closure is due to the existence of residual strain in the plastic wake.

See, if there is no fatigue loading, you will not have a residual stress formation. Because the load is increased and decreased, there is residual stress formation. Because of the residual stress, the crack has to wade through that; because of the existence of residual strain, in the plastic wake, crack also closes. We will see how. I have an animation which explains that. But you have to recognize, as the crack grows, the plastic zone sizes keeps increasing, and also formation of plastic wake. And this goes with your cyclical loading, that is what is depicted below.

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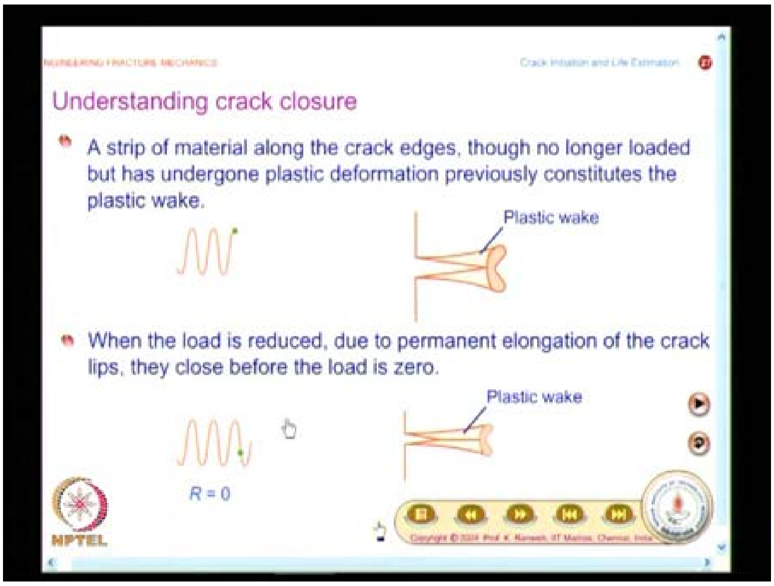

You know, you would have noticed that, there was an oscillation. So, I think, what I can do is, I can quickly zoom it. So, you will see, this only shows, that I am looking at a crack specimen, after this kind of loading. So, what you find here is, a strip of material along the crack edges, though no longer loaded, but has undergone plastic deformation previously, constitutes the plastic wake. We had seen it in the previous slide.

And what is depicted here is, I have a cyclical loading; and I am discussing, what happens, at this point, that is, after few cycles, at the peak load, the story of the crack would be like this. I would have a crack fully opened, but it would have a plastic wake. A strip of material along the crack edges, though no longer loaded, but has undergone plastic deformation previously, constitutes the plastic wake. So, you have the plastic wake shown. Suppose, I reduce the load, what happens? When the load is reduced, due to permanent elongation of the crack lips, they close before the load is 0. So, that is what is shown here. Ideally, only when the load point comes to this, the value of stress is 0.

But even at this stage, you find the crack is closed. So, the effective SIF has to be calculated from this point to this; it cannot go from 0 to K max. So, that is the issue, that is being discussed. So, what you are really looking at is, because of cyclical loading, if the load is increased and decreased, there is residual stress formation. If I have multiple cycles like this, the crack has to go though the plastic wake, and the plastic wake makes the crack faces to close, before the load reaches 0. So, that means, only part of the loading is really felt by the crack; that is why we have K effective. And this was the model proposed by Elber and later similar models have been proposed for various situations.

So, what you have here is, the crack is fully open, when the load is at its peak, but the crack is closed, even before the load has reached 0. So, that is the issue that, you have to keep in mind. And this explains, how to understand the phenomena of crack closure. And you can see the animation again. For this case, you have the plastic wake. And this explains, what is the plastic wake; and this shows when the load is reduced. You find when the load is still having some value, the crack is closed. And, you also have, the typical residual stress pattern at zero load. And I think, I can magnify this for you.

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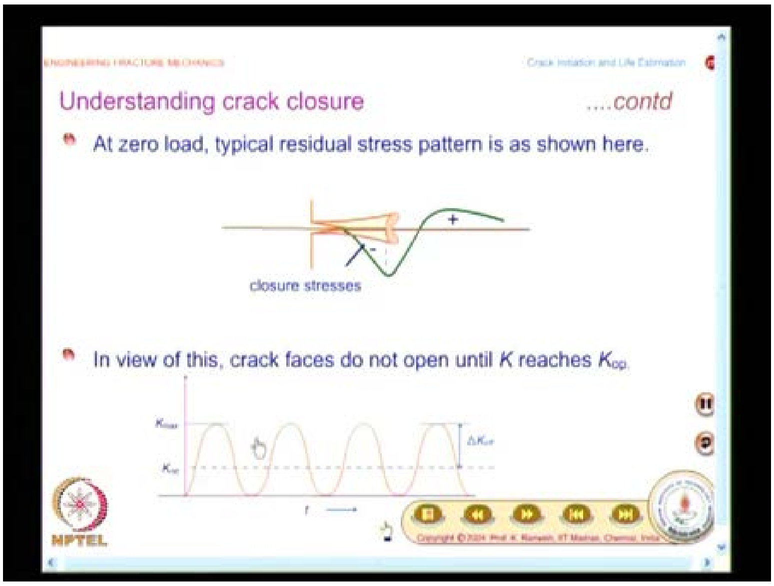

So, I have this as closure stresses; and you have a residual tension here; and because of this, the crack surfaces close; and this needs to be appropriately modeled in your d a by d N calculation. And that is what is given in the next slide. In view of this, the crack faces do not open, until K reaches K opening. You have already drawn this graph; this is just to emphasize, that you have to use a delta K effective. And we have also seen, this becomes significant, when you have random loading. For constant amplitude loading, probably you may not want to use it, but for space structures, you will have to definitely do; this is the recommendation by Elber. And it comes with a cost; it does not come cheap.

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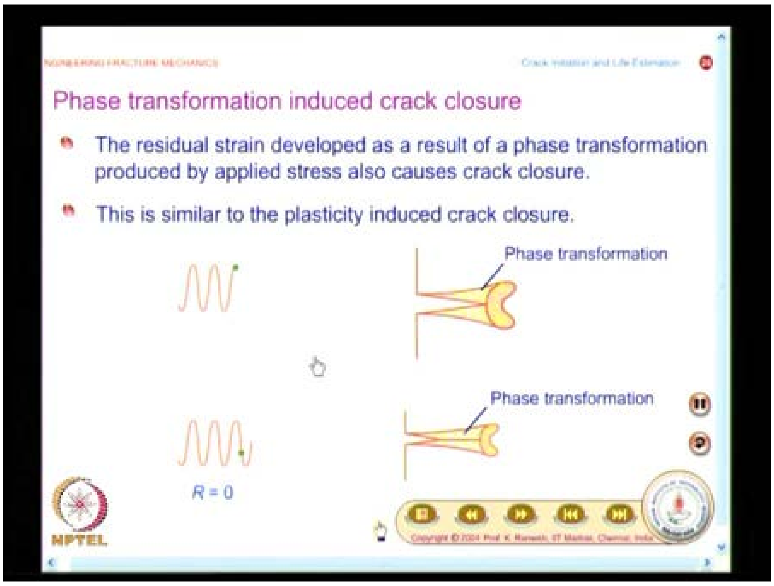

Once you have appreciated this, there are also other models, that are given, which are easy to understand, on a similar logical frame work. Crack closure can also happen because of phase transformation. The residual strain, developed as a result of a phase transformation produced by applied stress, also causes crack closure. The physical appreciation phenomena is very similar to plasticity induced case. Instead of the plastic wake, here it would all be dictated by the phase transformation. Otherwise, there is comparison between the two. And that is what is mentioned here. This is similar to the plasticity induced crack closure.

Here again, you look at, after few cycles, at the peak load. At the peak load, the crack is open. Just to distinguish it from plastic wake, here the shading is different, but the pictures are essentially same; and this is because of phase transformation. And now you see, when the load is reduced, before it reaches 0, the crack is closed; it is because of phase transformation. So, that is what is indicated here.

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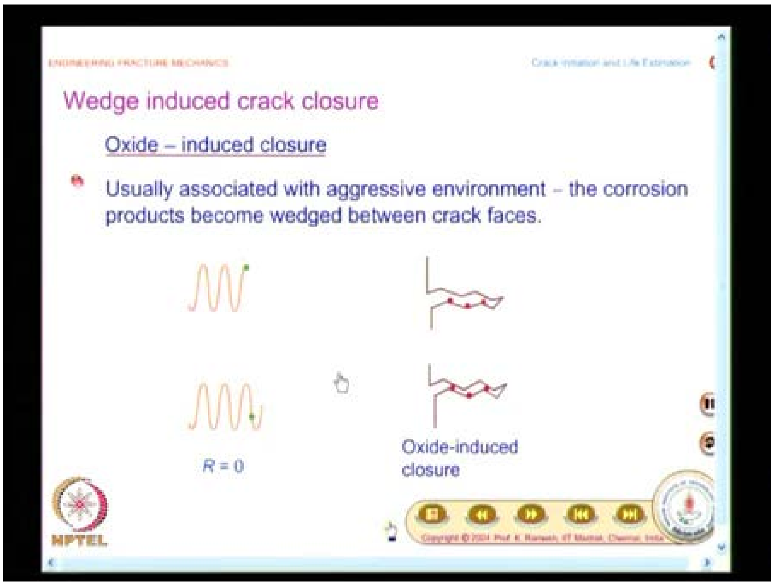

So, once people understood the concept of crack closure postulated by Elber, several other mechanisms have been identified. One is because of plasticity induced; here it is because of phase transformation, and the third case is, wedge induced crack closure. It could be due to oxide, usually associated with aggressive environment; the corrosion products become wedged between crack faces. That is what is shown here. I can show it little bit.

Because the corrosive action, you have corrosion product shown. And when the load is reduced, you know, because of corrosion products, it appears as if the crack is closed. It has a similar effect of crack closure. And this is called, oxide-induced closure. So, this is observed in aggressive environment. In every case, discussing on crack closure, we show the cyclical loading. You see what happens at the peak load and what happens when the load is reduced; and when the crack faces are closed. So, that is the way you have to interpret, what is given on the left and what is given on the right; left shows the loading and the green point shows, in the loading history, at which point, we are really looking at the crack faces. This is at the peak load, this is at a reduce load.

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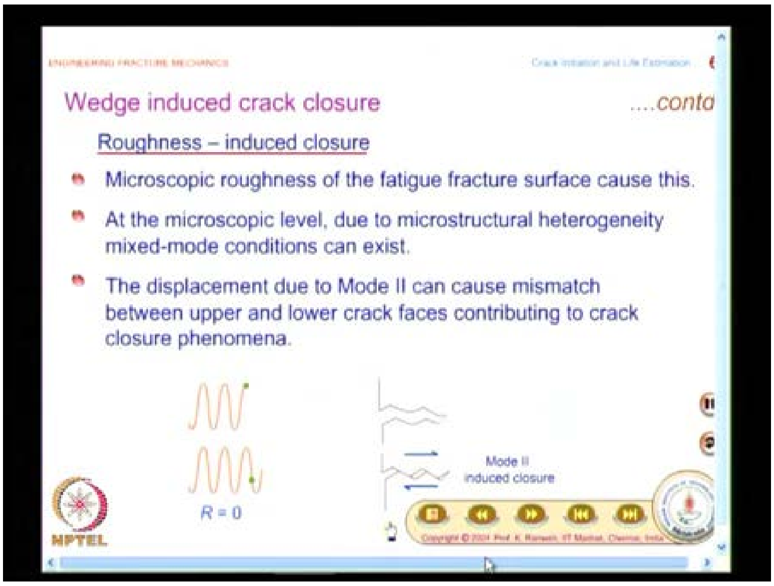

Then, you also have roughness induced closure. This is another phenomena. And what is reported is, microscopic roughness of the fatigue fracture surface causes this. At the microscopic level, due to micro structural heterogeneity, mixed-mode conditions can exist. You know, once you come to here, too much of material science knowledge you need to have. It is all highly exaggerated pictures, I am going to show. So, do not come and say, why you put the crack faces are jagged. At such high magnification, even a straight edge would appear jagged only. So, you have to look at, these are not macroscopic pictures; highly magnified microscopic pictures. That is the way you have to look at it. And what is mentioned is, the displacement due to mode 2, can cause mismatch between upper and lower crack faces, contributing to crack closure phenomena.

And we will also see, that has a picture. Here again, you see the load history. At the peak load, you are looking at it. And at the reduced load, you have a picture like this. I think, I could magnify it for you. And, this is called a mode 2 induced closure. And, this mode 2 loading comes at the microscopic level. So, what you will have to keep in mind is, crack closure is an accepted phenomena. Though it was not initially accepted, later people have recognized its role, and many other mechanisms also, people have observed and categorized.

So, in all those cases, you should have a methodology, how to find out delta K effective. So, you have to perform more tests and look at the literature and find out, how to get the delta K effective. Only then, you will be able to use the crack closure model, find, for finding out the d a by d N. See, there is also another important aspect that you have to keep in mind. Suppose, I have an overload, is it beneficial from fracture mechanics from point of view or not. See, if you recall, in my early discussion, I have said, plasticity is a friend of fracture mechanics.

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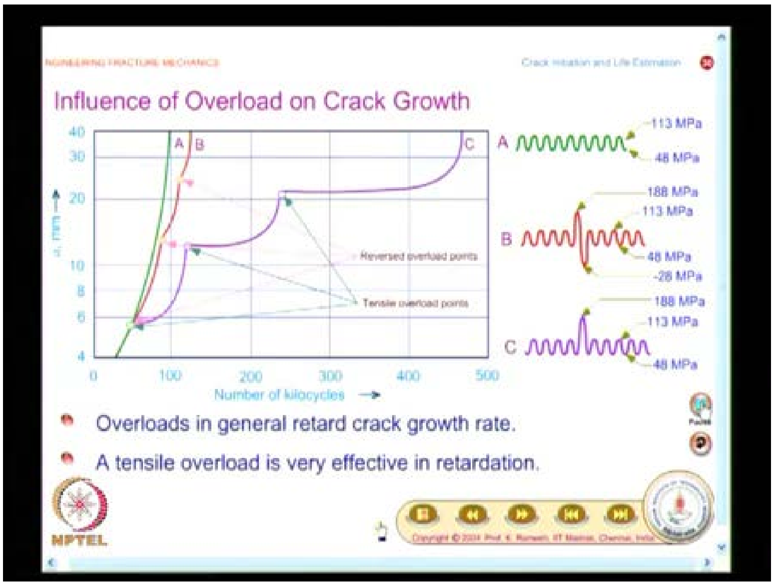

In fact, you would argue based on that only. And people have noticed, if you do proof testing occasionally, the life of the structure is improved. In proof testing, what do they do? They load the structure up to 125 percent of the load, for which it is designed. So, you are really putting an overload, at periodic intervals. People have observed from actual practice, that it has helped, the life of the structure. And, you will have a proof based on plastic deformation, that overload really helps and overload of what kind, that is another question. So, we will look at a graph like this; on the x axis, you have a number of kilocycles; on the y axis, the length of the crack is put. It is a log graph on the y axis and initially, you take a constant amplitude.

And let us see, how the crack growth for this case is. You also have some numbers given to this. It is varying from 48 MPa to 113 MPa. And you find, the crack growth is something like this. Suppose, after sometime, I have an overload, which goes from positive to negative; it goes to 188 MPa to minus 28 MPa, for the purpose of this graph. And, how does the crack growth looks like. So, definitely the rate at which crack grows after an overload, going from positive to negative, is definitely smaller than what you see, when you have constant amplitude. But it does not really benefited the structure. Then, we take another case, where I have an overload only in the positive direction.

How does the crack growth rate changes? And, what you find here is, when there is overload only in the positive direction, there is definitely retardation in the crack growth. And I have another overload here; another overload here; you have to know why is this so. Nevertheless, it is beneficial. So, the wisdom, people gained by sheer experience of periodic proof testing of the structure, has helped its life, you have an answer from fracture mechanics point of view. What way we are going to look at is, the plastic zone would be much larger, when I have a overload, the plastic zone would be much larger; that has helped this retardation.

So, your modeling of plastic zone is very important, and recognizing, when you have a cyclical load, you are going to have a residual stress ahead of the crack-tip. And, when I have an occasional overload, I will have a very large plastic zone and this crack has to wade through that plastic zone. So, that, in a sense, retards the crack growth. And you have a model for that. And this model is proposed by wheeler; the details of it, we would see in the next class. Again, he will go back to the basic kernel of how Paris reported d a by d N; there would be modification to that, which will accommodate the effect of overload. And this is very important, in the fracture literature - Influence of overload in crack growth.

So, in one graph, you see for constant amplitude, how is the crack growth; for a reversed overload, how does it retardation take place; for overload in one direction, the retardation is quite significant. So, in this class, we have essentially looked at concepts of crack closure proposed by Elber. We also saw, how he arrived at crack closure; how the result was initially accepted by the community; after a sustained effort, he has been, able to convenience a scientific community to accept this phenomena. Now, it has become a routine, in advanced life estimation calculations. People have embedded that, and the softwares like Nasgro and such softwares, use this as part of their life estimation calculation. Thank you.