Engineering Fracture Mechanics

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Module No. # 04

Lecture No. # 22

Multi-parameter Stress Field Equations

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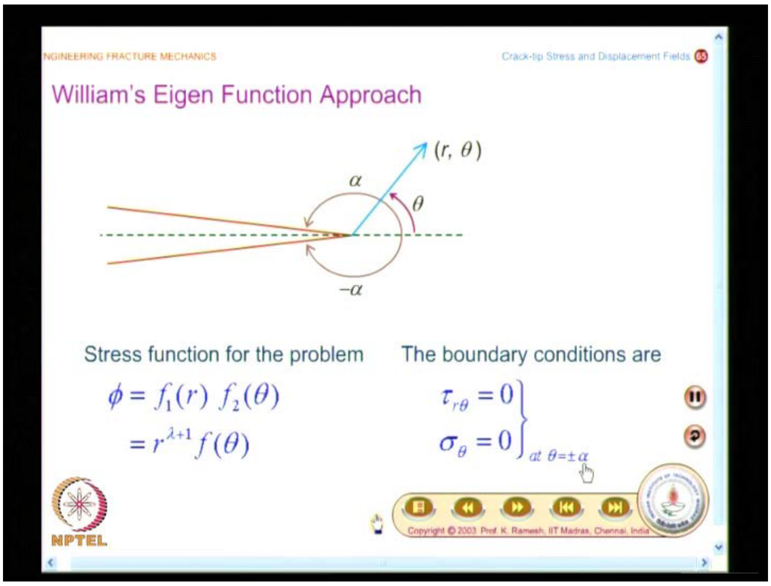

In this class, we will look at multi-parameter stress and displacement field equations. In fact, two classes earlier, we had discussed generalized Westergaard equations for mode 1 situation, and the last class, we had taken up Williams Eigen function approach; in the case of Westergaard's approach, it used complex variables while Eigen function approach by Williams used real functions. And what Williams considered? He considered a wedge, and he mentioned that the stress function phi could be taken as r power lambda plus 1 multiplied by f of theta, and what we did, we applied the boundary condition on the crack phases; the crack phases are free, this is depicted by the boundary conditions tau r theta is equal to 0 and sigma theta equal to 0 at theta equal to plus or minus alpha.

The problem becomes a crack problem when alpha becomes pi; and you should also note one more aspect- see if you look at Ingli's solution, what he did, he solved the problem of an elliptical hole in a tension strip. His focus was only on the elliptical hole; in the limiting case he was getting how a crack will be more dangerous from this analysis.

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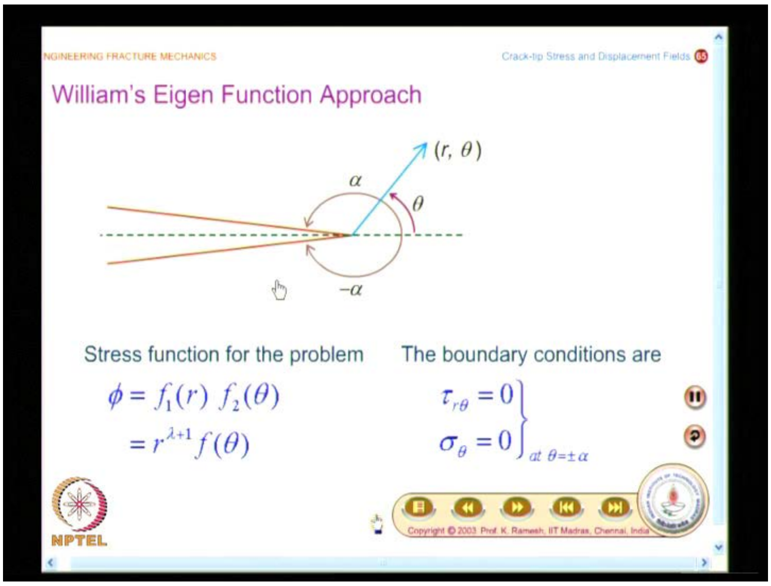

In a similar way, what Williams did, he actually took a wedge problem; by changing the value of alpha you could get solution for variety of wedge angles, and when you make alpha equal to pi, it becomes a crack. So, both had some similarity, in the limiting case they considered the problem of a crack; and how the crack phases? The crack phases are free. This is a very important observation when you look at the Williams Eigen function approach.

We would see in the next chapter at Westergaard's generalized stress function approach can accommodate loaded crack phases; they are also very important, from an engineering analysis you may need to find out what happens in the case of a riveted hole? So, the rivet sitting there will apply a force on the crack phases.

So, one of the advantages of the Westergaard's stress function approach is, one can also steady loaded crack phases.

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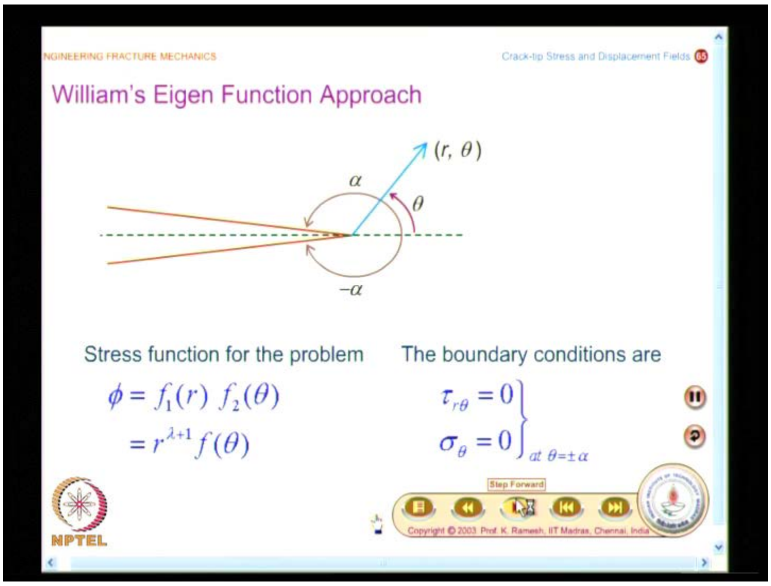

Now, we will continue with William's Eigen function approach. We had looked at what is the stress function form, and we have looked at what are the boundary conditions; we are not talking anything about what happens at infinity, we are only saying the crack phases are free.

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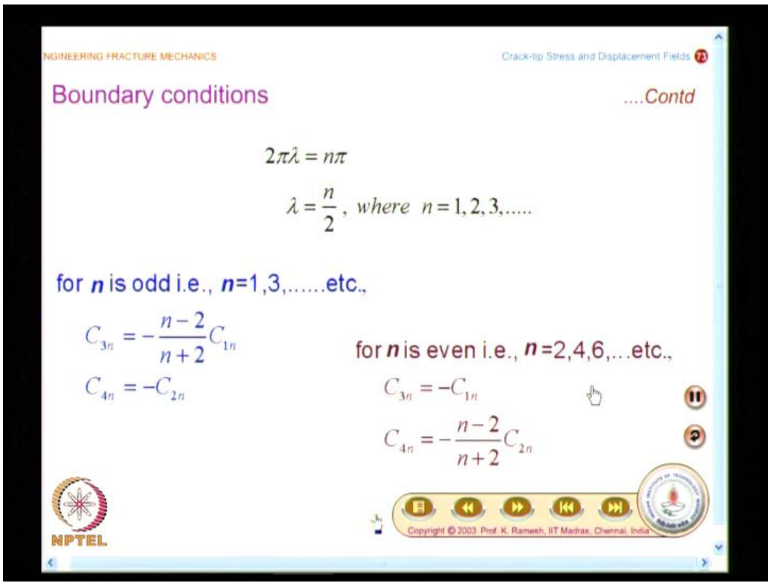

At infinity, you only say, you have a smoothly varying loading situation. And you know we had a long discussion, and then finally obtained what are the Eigen values- The Eigen values have to be positive integers; we have also looked at the reasoning for it. Then we also commented, among the four coefficients interrelationships exist, there is interrelationship between C 3 and C 1, and C 4 and C 2.

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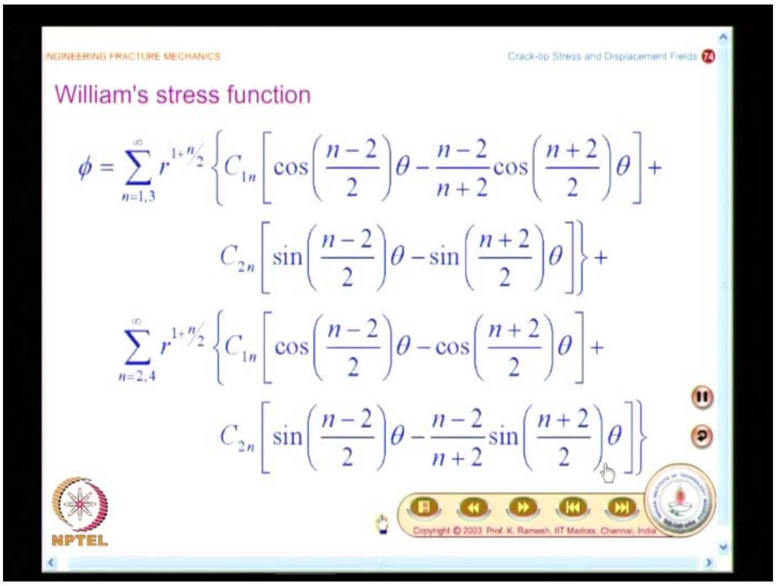

We have looked at this for n equal to 1, 3, 5 and so on we have also looked at it for n equal to 2 4 6. So, now, we are ready to write the stress function completely. Whatever the exercise that we did in the last class has helped us to write what is the stress function for this class of problem, and the stress function turns out to be like this... Mind you in this class you have to write long expressions, there is no other go.

So, what I would do is, I would also read it for you. The stress function phi has 2 summation series; one series when n equal to 1, 3, 5, odd numbers; another series when n are even numbers, 2, 4, 6 etcetera. And the first series is like this, r power 1 plus n by 2 multiplied by c 1 n cos n minus 2 divided by 2 theta minus n minus 2 by n plus 2 cos n plus 2 by 2 theta plus C 2 n multiplied by sin n minus 2 by 2 theta minus sin n plus 2 by 2 theta.

And the second series for n equal to 2, 4, 6 etcetera., you have r power 1 plus n by 2 multiplied by C 1 n cos n minus 2 divided by 2 theta minus cos n plus 2 by 2 theta- in fact, while reading, I am not reading brackets, by looking at the figure on the slide, it is possible for you to put the brackets appropriately- plus C 2 n sin n minus 2 by 2 theta minus n minus 2 divided by n plus 2 sin n plus 2 by 2 theta.

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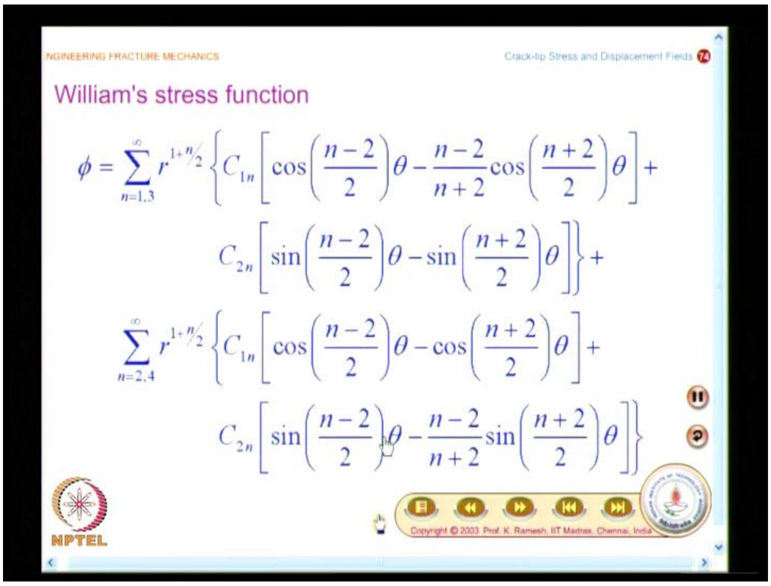

And what you have when you have a stress function in this form? You have a symmetric as well as an anti-symmetric part- this we had not seen in the Westergaard stress function approach; we solved mode 1 problem separately, we solved mode 2 problem separately. And what you find here, you get the stress function in the form of a series that is the observation number one. Observation number two is, you have cosine terms as well as sin terms. I could group all the cosine terms involving C 1n, this is symmetric, and this is the stress function for the mode 1 loading; and you have a anti-symmetric part, you have a sin function, whatever the terms associated with C 2 n will address crack problems involving the mode 2. So, in one go you get the solution for mode 1 as well as mode 2 in the case of William’s Eigen function approach, this is observation number two.

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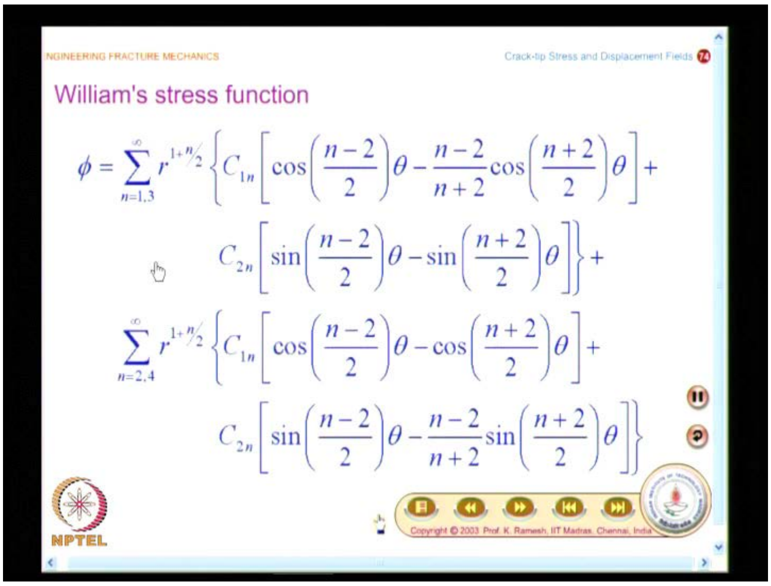

And we will also have one more observation. See, if you look at the second term in the mode 1 loading, that is nothing but n equal to 2, if you write this expression, I will have this as r squared C 1 to cos 0 so that it will become 1 minus cos 2 theta. So, that would be the second term of the stress function. Mind you, you are looking at the stress function, And we have also had some discussion what are the forms of stress function.

So, if you look at the form of this stress function, r squared multiplied 1 minus cos 2 theta, it actually represents a uni-axial stress field in the x direction- this was also observed by Williams when he reported the solution in 1957. You know, if he had said, when you are analysing crack problems, take as many terms in the series as possible for modelling, people would have looked at his solution differently and also used it differently.

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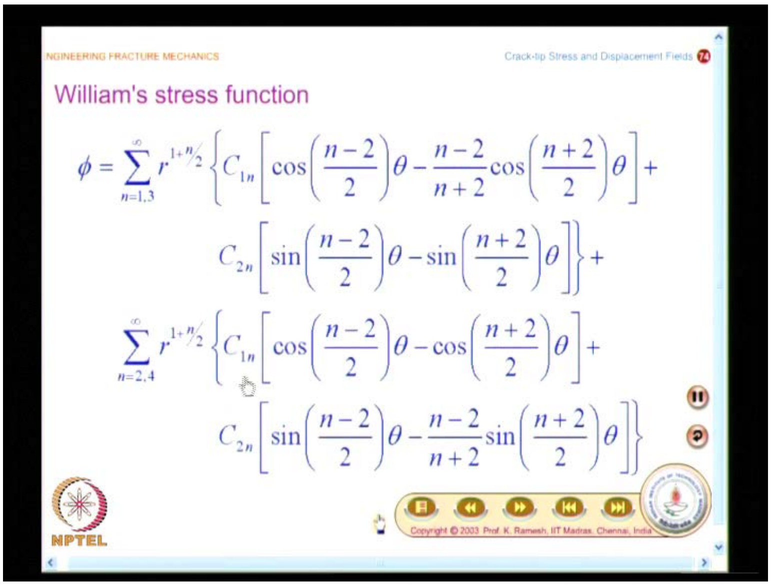

But what he said was, suppose you have a uniaxial loading situation when the vertical edge is free, he made a comment that the constant C 1 2 should become 0 to satisfy the boundary condition, this is the very unfortunate statement. If he had not made that statement, people would have realized the role of second term in the series right then, just because he had made that statement, and people are focusing more on what happens at the crack-tip, they were concentrating only on the singular field.

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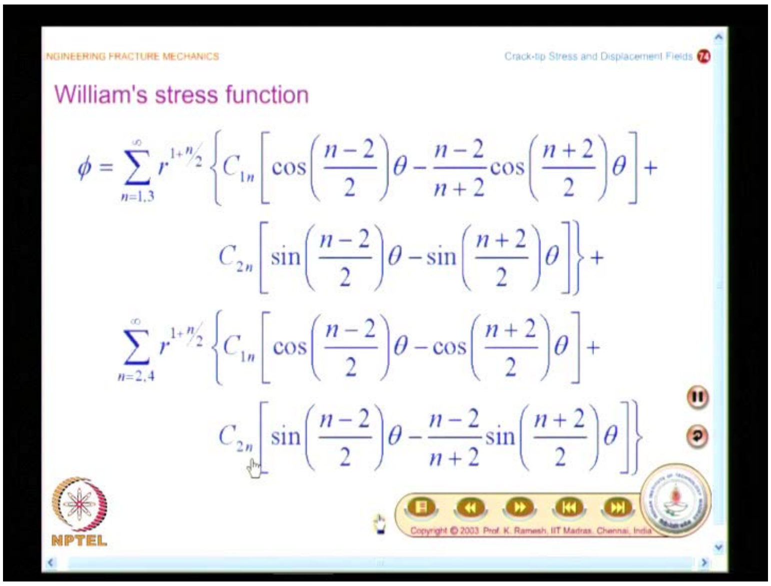

What you, now you'll have to look at is, when I have a stress function in series form for any given problem, depending on the kind of situation, you may have to evaluate the coefficients; as you take as many coefficients as possible that would satisfy the far field boundary condition.

When you have a free edge, it is not only C 1 2 should go to 0, combination of several coefficients can satisfy that boundary condition; that is how any general solution has to be looked at, just because people looked at circular hole in a tension strip, elliptical hole in a tension strip, for crack problems also they were focusing on uniaxial loading. Whereas, in the Westergaard solution, the basic approach was on biaxial stress field, it is not for uniaxial stress field. But after looking all that, we have on our own understanding that, when u say z naught is very close to 0, very small compared to the crack length, you are essentially focusing on a z1 very close to the crack-tip.

So, what really happens is, it is only the crack phase displacements that is important in the case of the fracture mechanic; whether it is the opening mode, in plane shear mode or out of shear plane mode, you have to look at only on the crack phase displacements, the far field loading is secondary.

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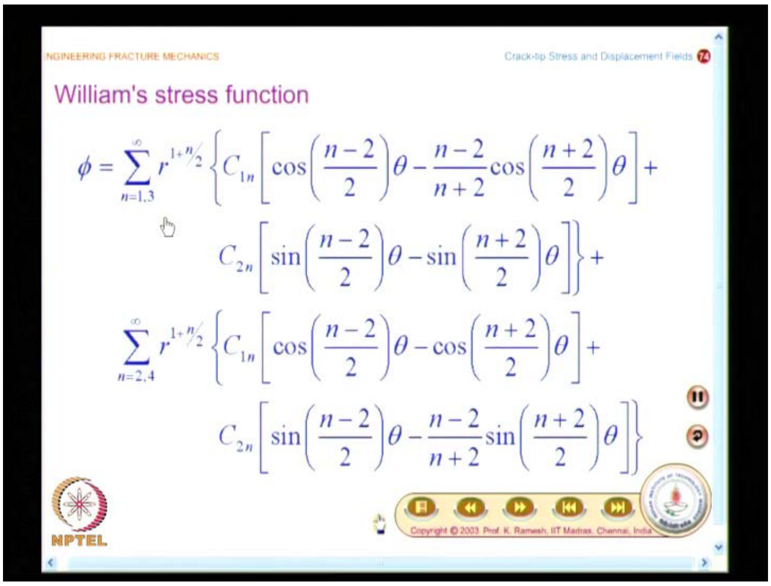

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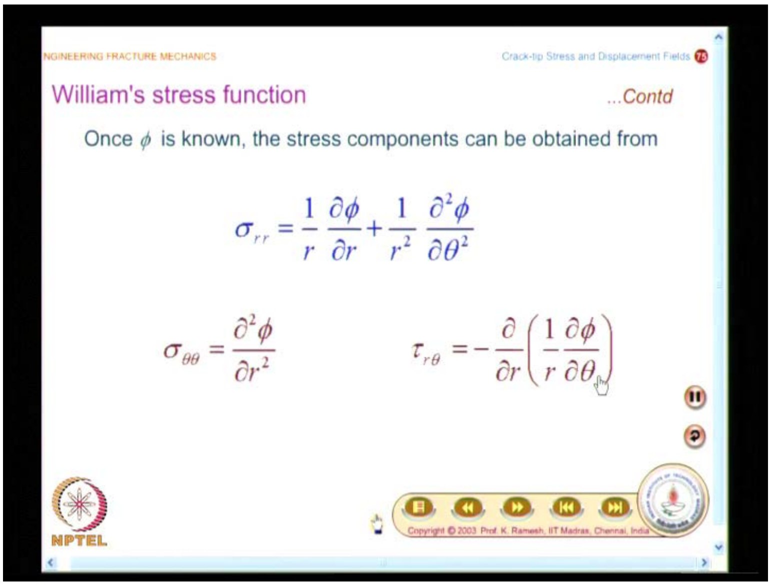

So, once you take up a multi-parameter solution, from now on we will have to say, take as many terms that would satisfy your problem under consideration. Once you know, see, what is the next step, you can easily find out the stress field. And in the case of polar coordinates, once you have sigma phi is known, sigma r r is defined as 1 by r tau phi by tau r plus 1 by r squared tau squared phi by tau theta squared, and sigma theta theta equal to tau squared phi tau r squared. And tau r theta equal to minus tau by tau r of 1 by r tau phi by tau theta.

In fact, I would like you to try out at least sigma theta theta... you have the function phi, you do not take many terms just take the first term, and you will only worry about C 1 1. You have the Airy's stress function given, for C 1 1 you find out sigma theta; that is the simplest one to find out. And in this case what we you are going to get? You are going to get polar components. You are actually looking at sigma r sigma theta tau r theta, and they would also be expressed in terms of r n theta coordinates. So, the expression should be different than what you had seen in Westergaard solution. So, do not come to a wrong conclusion, I have expressions for the stress field, they are different from the Westergaard, they have to be different, because here you are looking at sigma r sigma theta and tau r theta, when you transform this to Cartesian coordinates the solution will be identical.

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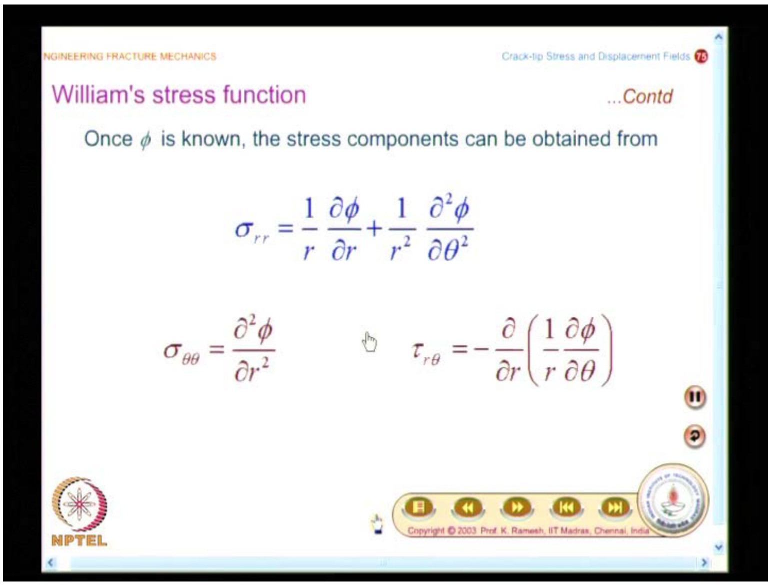

Because this is again, I have been emphasizing it, in the case of Westergaard solution when I have Cartesian stress component, express in terms of r n theta. So, please work it out. So, that way this will help you to follow the class. And what we will do is we will see the singular solution as well as multi-parameter solution.

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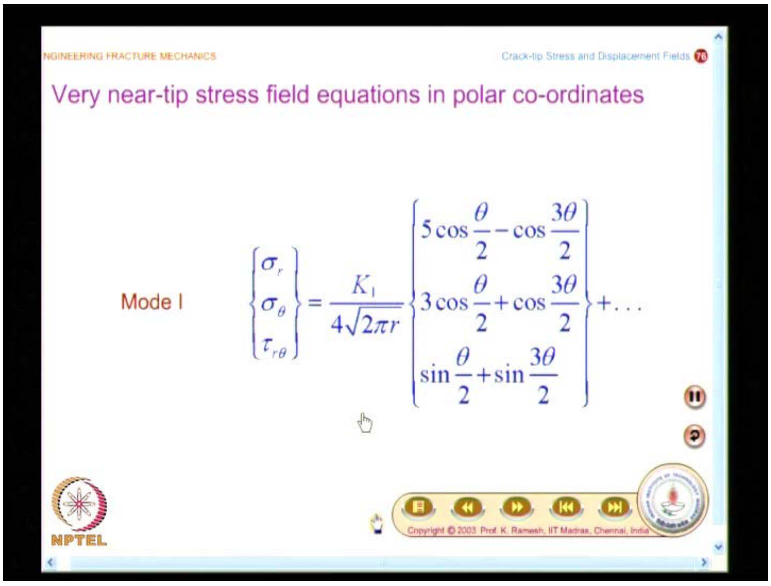

You have to work it out. Anyway, for the benefit of those who have already did this, we get the very near-tip stress field equations in polar co-ordinates. And I have also written the expression in a fashion that I have a term one, I have put plus, some dots I have put, this indicates that you have higher order terms.

And you have sigma r sigma theta tau r theta, the common that is taken out is K 1 divided by 4 root of 2 pi r, and you have a function of theta for sigma r, the function is 5 cos theta by 2 minus cos 3 theta by 2, and for sigma theta it is 3 cos theta by 2 plus cos 3 theta by 2.And you have sin theta by 2 plus sin 3 theta by 2 for tau r theta. See, we had made some observations after we looked at Westergaard's singular solution- we had looked at the stress field, we have also looked at how sigma x and sigma y varies.

And what is that we noted at the crack-tip? We noted at the crack-tip, sigma x is equal to sigma y, and you do not have a shear stress at the crack-tip 0. So, sigma x and sigma y are principle stresses. So, when two quantities are equal, every direction is a principle stress direction. So, what does it imply? What way you should have sigma r and sigma theta at the crack-tip?

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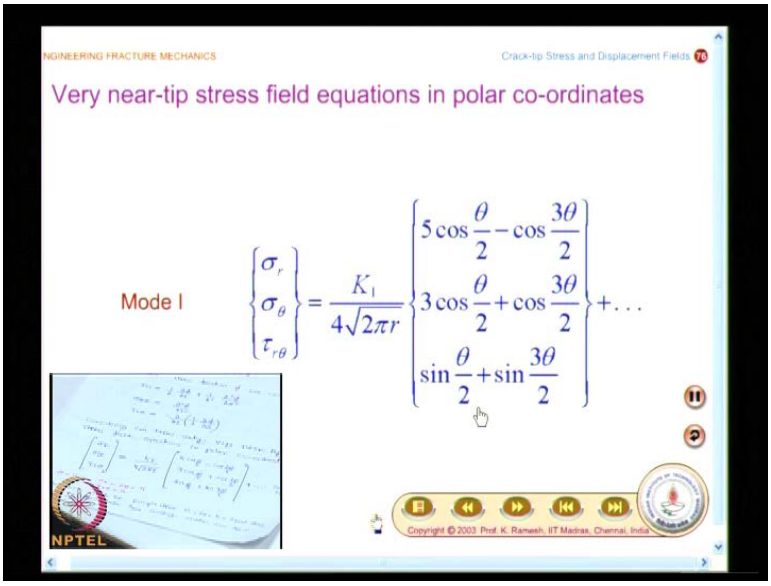

They will also have to be equal. Are we having it as equal? We will have a look at the solution. So, at the crack-tip you will say theta equal to 0, so when I say theta equal to 0, I have this as 5 minus 1, so this is 4, and this will be 3 plus 1, so this is also 4, and your tau r theta is going to be 0. So, now, you find that principle stresses are equal.

And what is the kind of stress field is this? Particularly when you are looking at a plane strain situation, that is interior to the object, you will also have a sigma g(z) component, and what people have noted is- you have a tri-axial loading situation, and you have hydrostatic type of loading. From your understanding of solid mechanics, what you find when you have hydrostatic type of loading, you cannot have plastic flow; so, the material has to follow some other kind of failure mechanism. So, this is the reason Williams also attributed while discussing his results. Because you have hydrostatic type of loading near the crack-tip, in view of it, there may not be possibility of plastic flow; the material fails by cleavage fracture.

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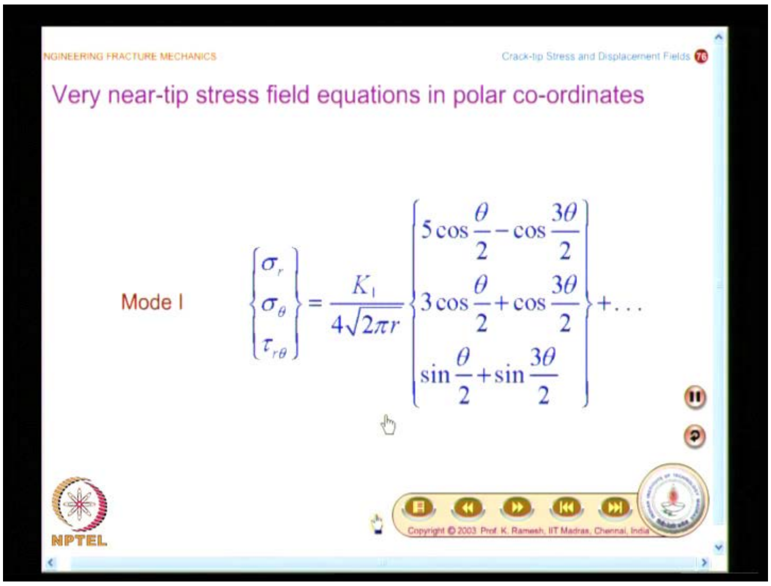

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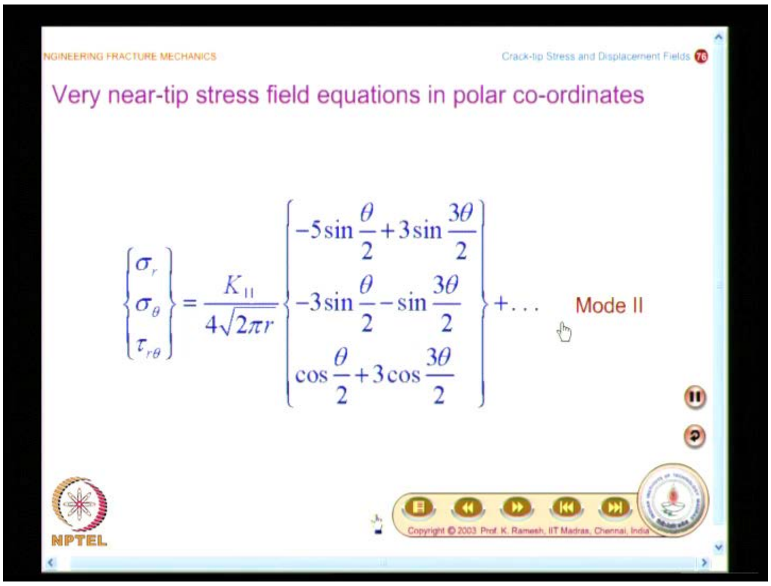

It is a very important observation, that you have hydrostatic type of loading, the stresses are equal, this is a very important observation. And later on we will also look at what is known as triaxiality constraint. So, all that we will look at. So, this stress field is not innocent, so, you find when r goes to 0, stresses become singular, and this is what we have obtained for mode 1 situation. On similar lines, you could also obtain this stress field for mode 2. And mode 2, it is like this: So, I have sigma r sigma theta and tau r theta, it is given as K 2 divided by 4 root of 2 pi r multiplied by minus 5 sin theta by 2 plus 3 sin theta by 2, this is for sigma r and for sigma theta, the function is minus 3 sin theta by 2 minus sin 3 theta by 2, and you have for tau r theta cos theta by 2 plus 3 cos 3 theta by 2.

Even while writing it, I write it in such a fashion that you have higher order terms. See, the higher order terms are explicit when you follow the Williams function approach.

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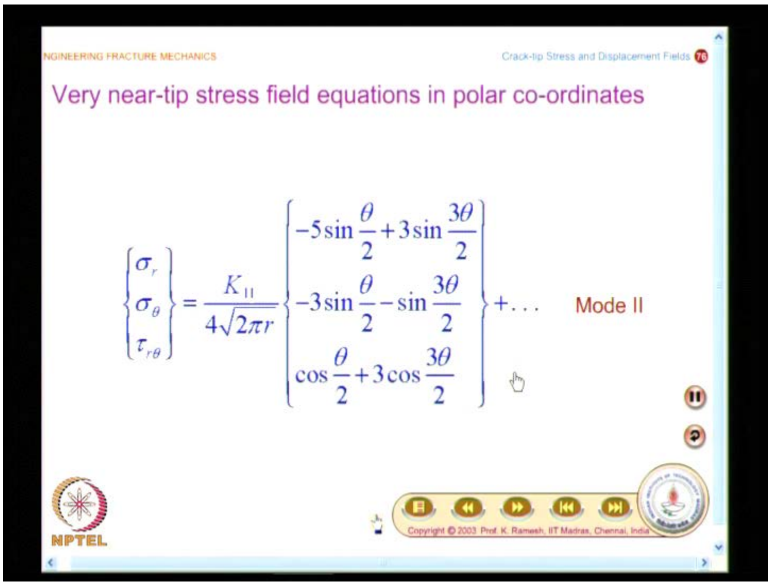

When you do the William's Eigen function approach, the stress function is in a series form, and you get several terms in the series, you are looking only at the first term which happens to be the singular term, and if you really look at experimental literature, when people have used method of caustics, they have directly taken up the stress field in polar co-ordinates. When you go and look at any paper on caustics these equations were found to be convenient to handle.

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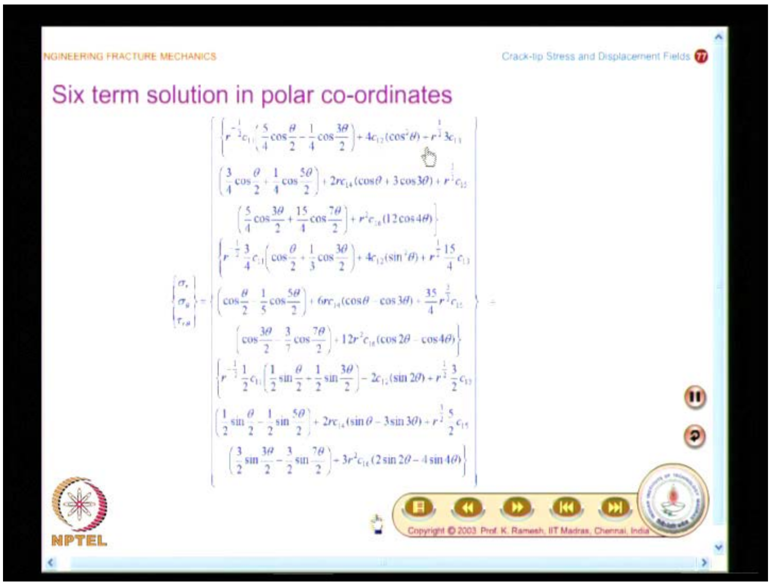

Now, what we will do is, we will not stop at the first term, we would see a multi-parameter solution, we will not see several terms, we will see a 6 term solution. You know the letters are very small, what I would is even if I magnify, you may not be able to see the complete set of stress field. So, I would read them, and my reading speed and your writing speed should match, if there are any corrections please advise me to follow it slow. And you know these equations are very important, the reason why I am saying this is you will not find them in books. So, at the end of the course you should have rich collection of important results. So, you start writing with me.

For sigma r I have six terms, and it starts... at the first term is, r power minus 1 by half multiplied by c 11 and you have a set of terms, 5 by 4 cos theta by 2 minus 1 by 4 cos 3 theta by 2 plus 4 C 12 cos squared theta plus r power half 3 C 13 multiplied by 3 by 4 cos theta by 2 plus 1 by 4 cos 5 theta by 2 plus 2 r C 14 multiplied by cos theta plus 3 cos 3 theta plus r power 3 by 2 C 15 multiplied by 5 by 4 cos 3 theta by 2 plus fifteen by 4 cos 7 theta by 2 plus r squared C 16 multiplied by 12 cos 4 theta.

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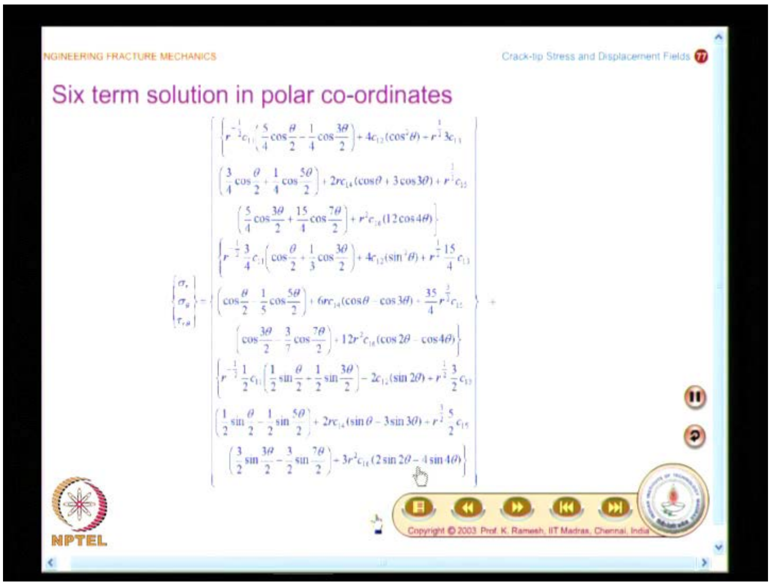

So, what we have here is, look at the second term- the second term is a constant term in the sigma r also. So, you have this additional term which was misunderstood earlier, and you have to take this into account, and you have the expression for sigma theta that turns out to be r power minus half 3 by 4 C 11 cos theta by 2 plus 1 by 3 cos 3 theta by 2. The second term is 4 C 12 sin squared theta plus r power half fifteen by 4 C 13 multiplied by cos theta by 2 minus 1 by 5 cos 5 theta by 2; the next term is 6 r C 14 cos theta minus cos 3 theta plus thirty 5 by 4 r power 3 by 2 C 15 multiplied by cos 3 theta by 2 minus 3 by 7 cos 7 theta by 2 plus 12 r squared C 16 cos 2 theta minus cos 4 theta. So, this is for sigma theta.

Then you go for tau r theta: tau r theta is r power minus half multiplied by 1 by 2 C 11 multiplied by 1 by 2 sin theta by 2 plus 1 by 2 sin 3 theta by 2 minus 2 C 12 sin 2 theta plus r power half 3 by 2 C 13 multiplied by 1 by 2 sin theta by 2 minus half sin phi theta by 2 plus 2 r C 14 sin theta minus 3 sin 3 theta plus r power 3 by 2 5 by 2 multiplied by C 15 which is multiplied by 3 by 2 sin 3 theta by 2 minus 3 by 2 sin 7 theta by 2, and the last term is 3 r squared C 16 2 sin 2 theta minus 4 sin 4 theta.

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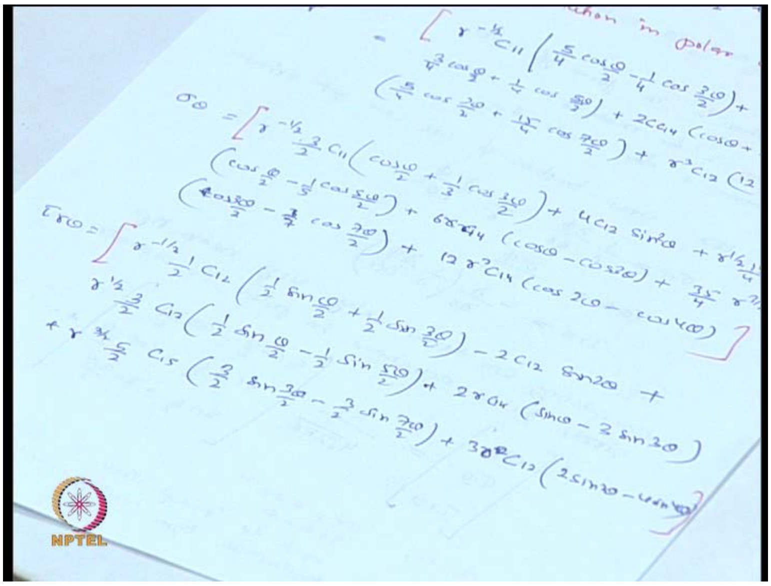

You know, the expressions are very, very long. This is a 6 term solution, for what? This is for mode 1 loading, because Williams gives you mode 1 as well as mode 2. So, I am going to have another set of expressions for mode 2. Why I want to you to write is, we would see another elegant representation. You would appreciate that representation is elegant only when we write down this complex expressions. So that exercise also will help you to do that.

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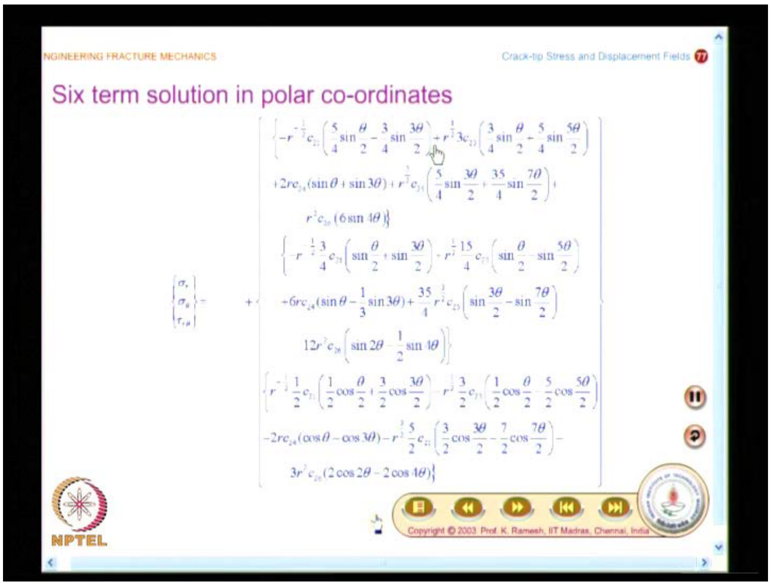

So, now I have for mode 2, for all the mode 2 terms you have the coefficient starting as c 2. So, for sigma r it is minus r power minus 1 by 2 multiplied by C 215 by 4 sin theta by 2 minus 3 by 4 sin 3 theta by 2 plus r power half 3 C 23 multiplied by 3 by 4 sin theta by 2 plus 5 by 4 sin 5 theta by 2.

Mind you the second term is 0 here. I had already mentioned, even in the case of generalized Westergaard equations, the singular solution for mode 2 reasonably matches with experimentally observed fringes. The role of second term was not influenced, and I mentioned that second term is 0; even when you considered a series solution for mode 2 the sigma x stress component, the second term is 0.

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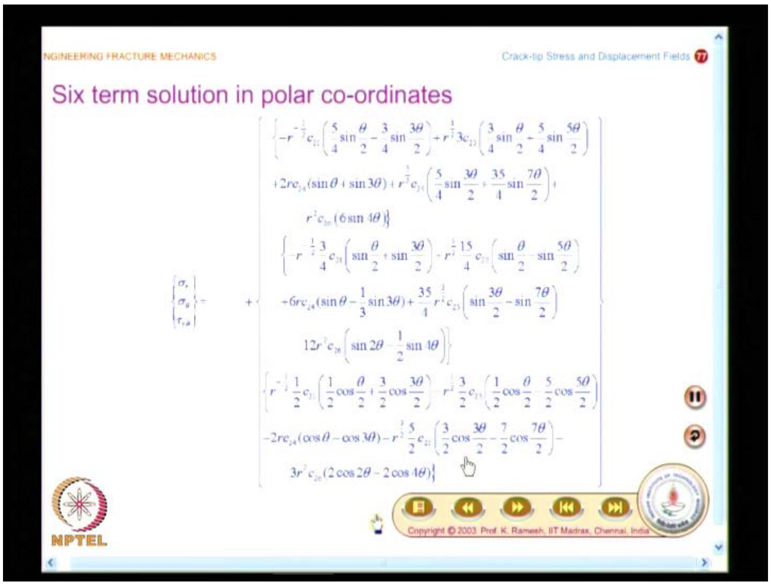

A similar thing is also observed in the polar component; here you have to be very careful, you have to transform the polar, whatever the polar component to Cartesian component. And you have the... third term is, 2 r C 3424 sin theta plus sin 3 theta plus r power 3 by 2 C 255 by 4 sin 3 theta by 2 plus 35 thirty five by 4 sin 7 theta by 2 plus r squared C 266 sin 4 theta.

And for sigma theta, you have this as minus r power minus half multiplied by 3 by 4 C 21 sin theta by 2 plus sin 3 theta by 2 plus r power half 15 fifteen by 4 C 23 sin theta by 2 minus sin 5 theta by 2 plus 6 r C 24 sin theta minus 1 by 3 sin 3 theta plus 35 by 4 r power 3 by 2 C 25 sin 3 theta by 2 minus sin 7 theta by 2.

And the last term is plus 12 r square C 26 sin 2 theta minus half sin 4 theta. And you have the shear stress term, r power minus half 1 by 2 C 21 multiplied by 1 by 2 cos theta by 2 plus 3 by 2 cos 3 theta by 2 minus r power half 3 by 2 C 23 multiplied by 1 by 2 cos theta by 2 minus 5 by 2 cos 5 theta by 2.

And I have this next term as minus 2 r C 24 cos theta minus cos 3 theta minus r power 3 by 25 by 2 C 23 multiplied by 3 by 2 cos 3 theta by 2 minus 7 by 2 cos 7 theta by 2. And the last term is minus 3 r squared C 262 cos 2 theta minus 2 cos 42. So, now, you have a six term solution in polar co-ordinates given by Williams Eigen function approach. You know, this is very clumsy, you know, if you have this kind of a form the solution would not have become popular.

You know, there was a very nice work done by Atluri and Kobayashi, they simplified and represented all the multi-parameter stress field equations in a very elegant fashion. We will look at that solution shortly. Before we go into that, let us also look at what is the kind of stress function for the generalized mode 2 Westergaard stress function that we have look at.

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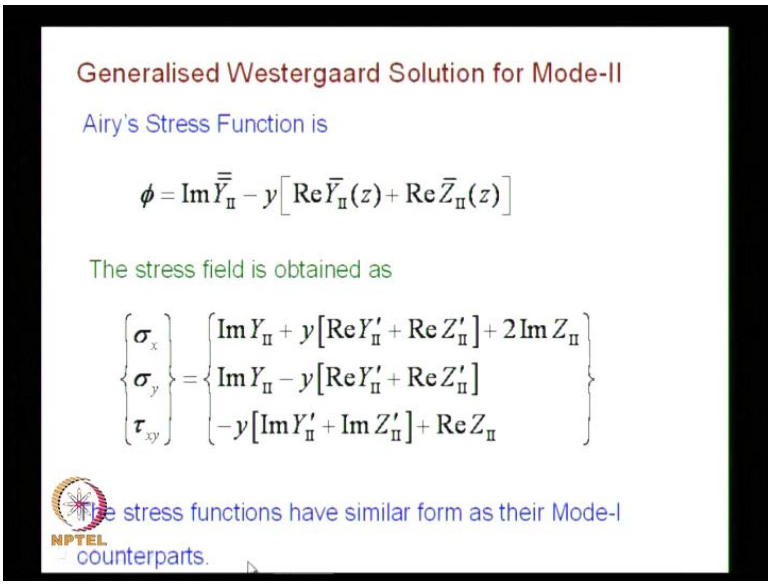

We have only looked at at length what do for mode 1 situation, we have not looked at how the Airy's stress functions changes in the case of mode 2. We will have look at that. I am just going to give you the stress function and the stress field; I will not get into the mathematical details. And thus, Airy's stress function takes this form... imaginary part of y 2 double bar.

Since we are discussing mode 2, I have put a subscript 2. When we discussed mode 1 situation, we simply gave capital g(z) and capital y, we did not give the subscript 1 there, since we are discussing specifically the mode 2 situation we are putting the subscript 2 here.

So, I have stress function phi as imaginary part of y 2 double bar minus y real part of y 2 bar plus real part of z 2 bar. If you recall, the singular solution which we obtained had only this minus y real part of z 2 bar, these two terms are addition.

Once Airy's function is defined, it is possible for you to get the stress field. And I would write the stress field in the form of stress functions only. I have sigma x sigma y tau x y, and sigma x is given as,, imaginary part of y 2 plus y multiplied by real part of y 2 prime plus real part of z 2 prime plus 2 imaginary part of z 2. And you have sigma y is given as, imaginary part of y 2 minus y multiplied by real part of y 2 prime plus real part of z 2 prime. And the shear stress tau x y is given as, minus y times imaginary part o f y 2 prime plus imaginary part of z 2 prime plus real part of z 2.

So, now, you have the generalized mode 2 Westergaard solution. So, if you look at the form of the stress function z 2 and y 2, you can construct a series solution; and what is mentioned is the stress functions y 2 and z 2 has similar form as their mode 1 counterparts. And what are the mode 1 counterparts?

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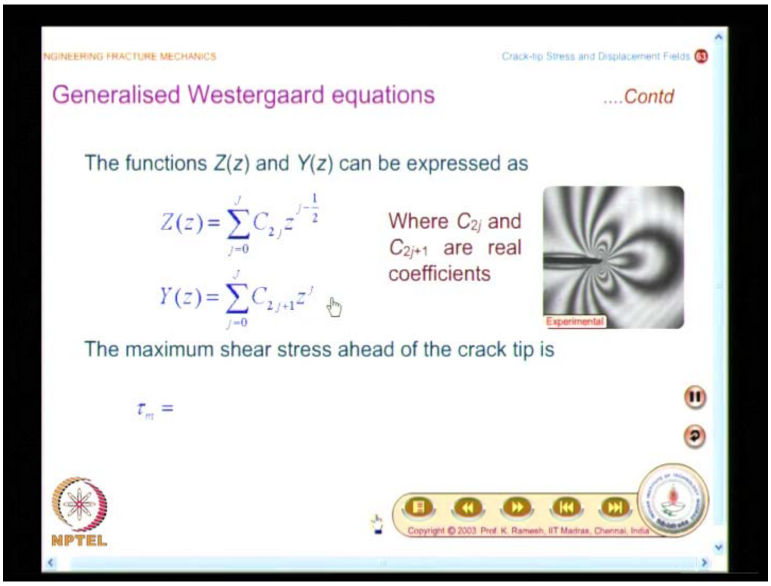

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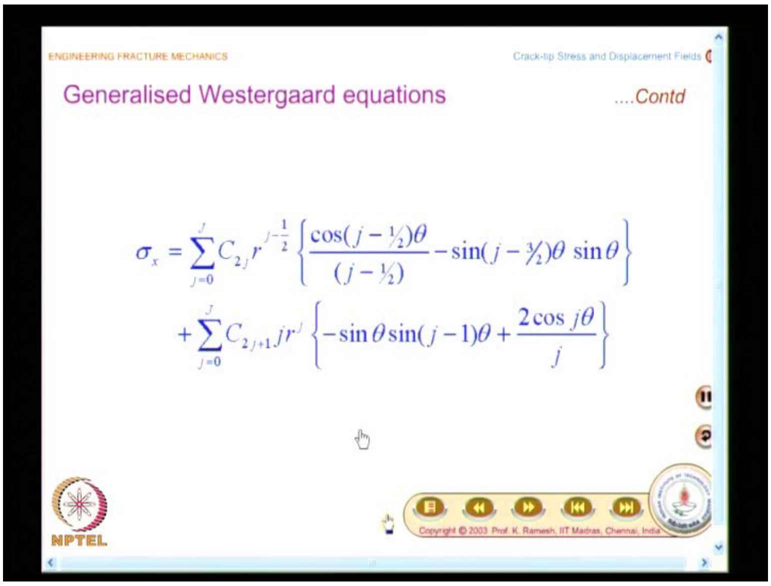

In mode 1 counterparts, you had this function z and y given as a series solution. So, you need to go for a series solution like this. And you have to ensure that z 2, real part of z 2 is 0 in the crack phases to satisfy the boundary condition there, similar to mode 1 situation, here also imaginary part of y 2 should be 0 on y equal to 0. And we also look at the kind of stress fields that we saw for the mode 1.

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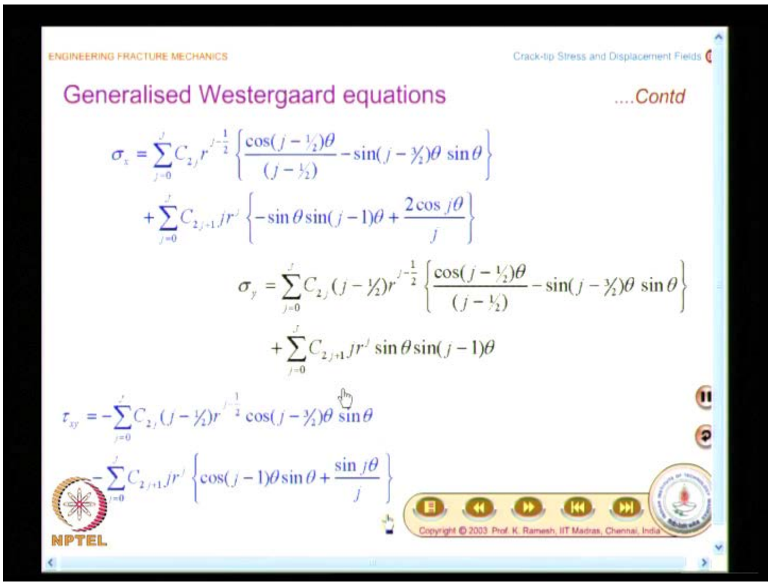

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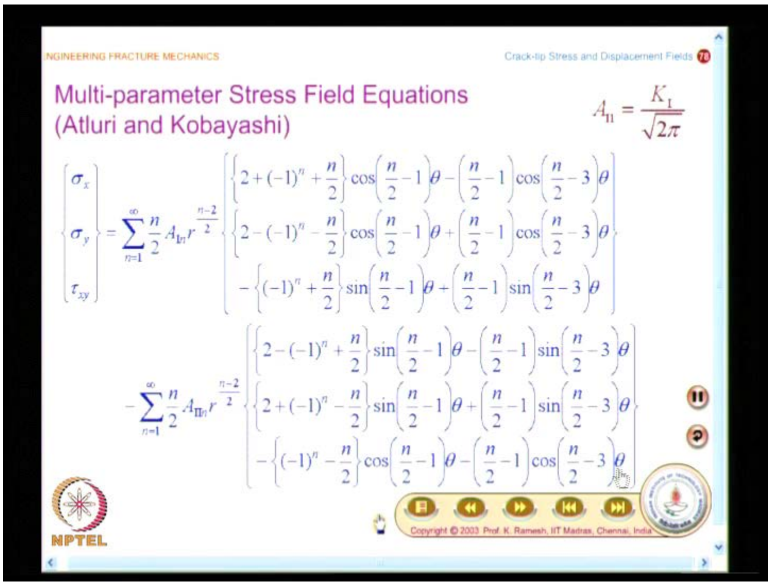

Something similar to this I would like you to work it out for mode 2. And the expressions are like this. You also wrote down these expressions, and I said they are clumsy, we have also written down Williams Eigen solution results for six terms, and they were also very difficult to write. Now, we will go and see the solution given by Atluri and Kobayashi, and they are the most elegant form of stress field expression, and you have to write this down very carefully.

I have sigma x sigma y tau x y. So, these expressions give only Cartesian stress component, and this is given as summation of two series- the first series involving the coefficient capital a 1n the second series involves capital a 2 n.

So, the first series is meant for mode 1 loading where as the second term in the series is meant for mode 2 loading, and n varies from 1 to infinity. I have n by 2 a 1 n r power n minus 2 divided by 2, and for sigma, for x I have this as 2 plus minus 1 power n plus n by 2 cos n by 2 minus 1 theta minus n by 2 minus 1 multiplied by cos n 2 minus 3 theta.

And for the sigma y stress term is, 2 minus minus1 whole power n minus n by 2 multiplied by cos n by 2 minus 1 theta plus n by 2 minus 1 multiplied by cos n by 2 minus 3 theta. And for shear stress, it is minus of minus 1 power n plus n by 2 multiplied by sin n by 2 minus 1 theta plus n by 2 minus 1 sin n by 2 minus 3 theta. See, you'll have to notice one thing, I have the mode 1 stress field as well as mode 2 stress field in terms of generic expression; when I change n, I would get the second term, third term, fourth term and so on. I can easily computerize the set of equations, that is advantage number one and another aspect is, it is easy for computerization.

And what you'll have to look at is, this solution was reported in handbook of experiment mechanics first, and it had one or two small typographical errors, even as change in minus or plus sign or a sin to cos can cause havoc. So, these expressions are edited, they are as accurate as possible, and these are used to reconstruct as fringe pattern for complex problem situation which we would also see in this class. So, take some time in writing these expressions as accurately as possible

Because these equations are verified free of any typographical errors. For mode 2 this is minus of summation n equal to 1 to 1 to infinity n by 2 a 2 n r power n minus 2 divided by 2 and for sigma x it is 2 minus minus1 power n plus n by 2 multiplied by sin n by 2 minus 1 theta minus n by 2 minus 1 sin n by 2 minus 3 theta. And for sigma y it is 2 plus minus 1 power n minus 1 by 2 n by 2 multiplied by sin n by 2 minus 1 theta plus n by 2 minus 1 sin n by 2 minus 3 theta minus of minus 1 power n minus n by 2 cos n by 2 minus 1 theta minus n by 2 minus 1 multiplied by cos n by 2 minus 3 multiplied by theta.

And you can again look at the expression; ensure that no sign change or sin or cos change is done in your expression. And how the coefficients are defined? See, we are interested in one or two coefficients, because you know we want to find out what is k 1 and k 2, and the second term in the sigma x stress field for mode 1 situation, we label it as sigma naught x.

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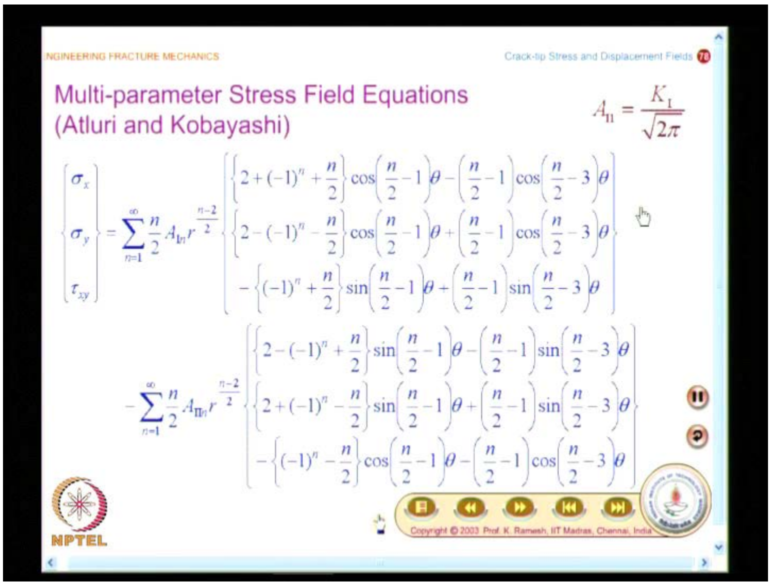

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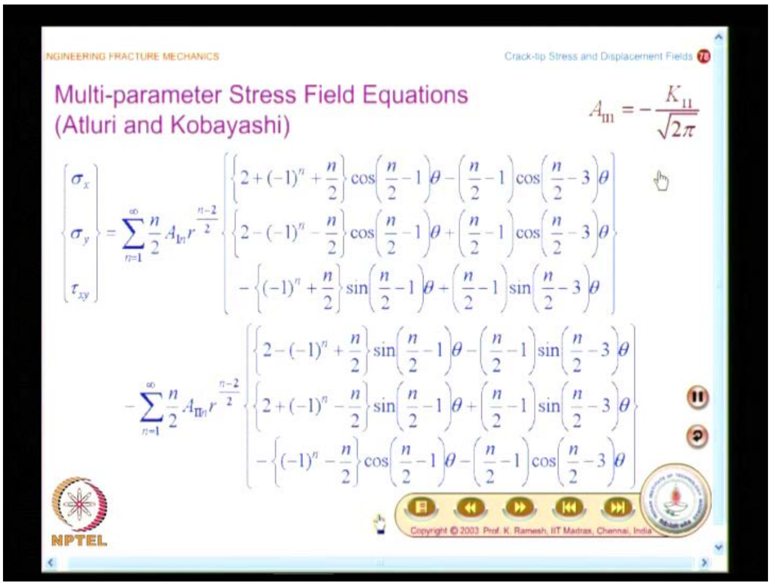

How the coefficients are related to the quantities that we already know? So, that is given here, I have a 11 is given as K 1 by root of 2 pi. Please, write this down. And as I mentioned earlier, the terms involving a 1 corresponds to mode 1 loading, the terms involving a 2 corresponds to mode 2 loading. And a 21 is related to minus k 2 divided by root of 2 pi, this minus and minus becomes plus, that is how it is written. So, I can find out if I know the coefficients, as part of the solution I can find out what is k 2 and k 1.

And the second term in the sigma x series is related to sigma naught x in this fashion, 4 a 1 to equal to minus sigma naught x.

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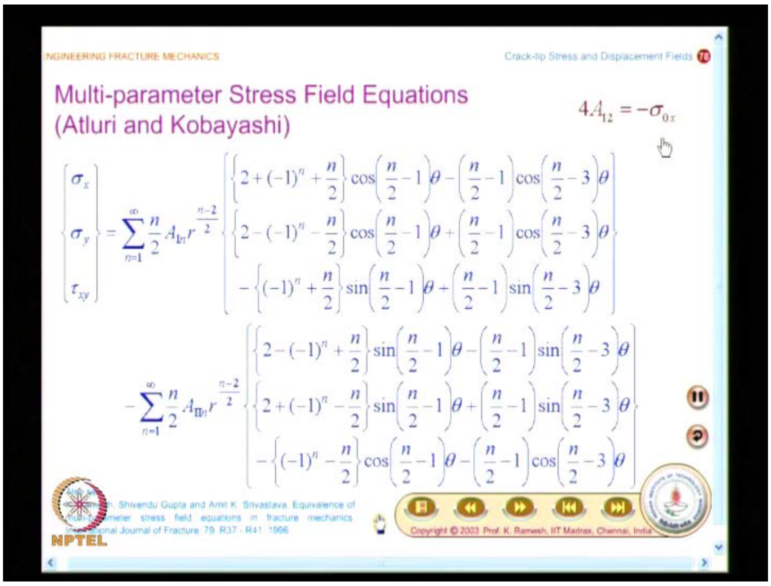

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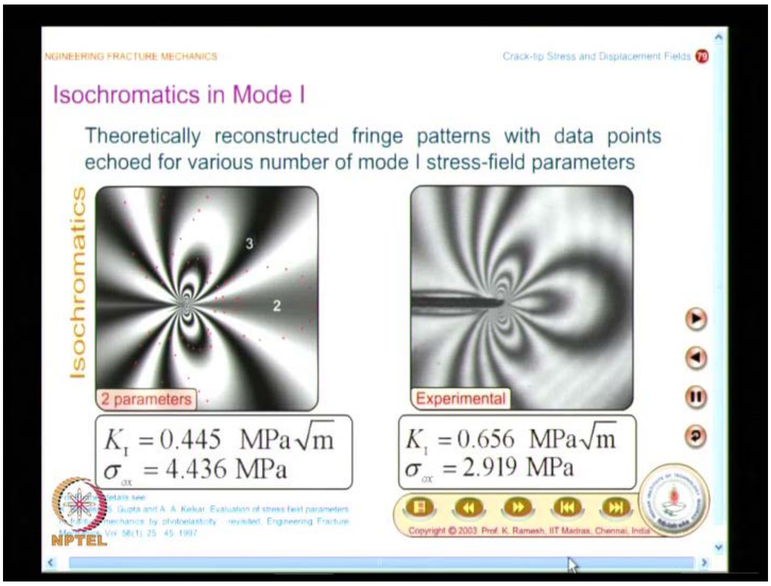

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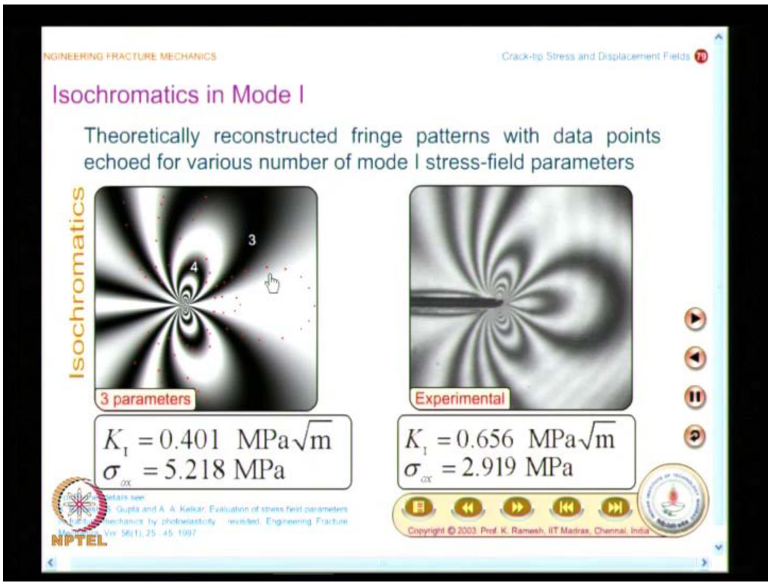

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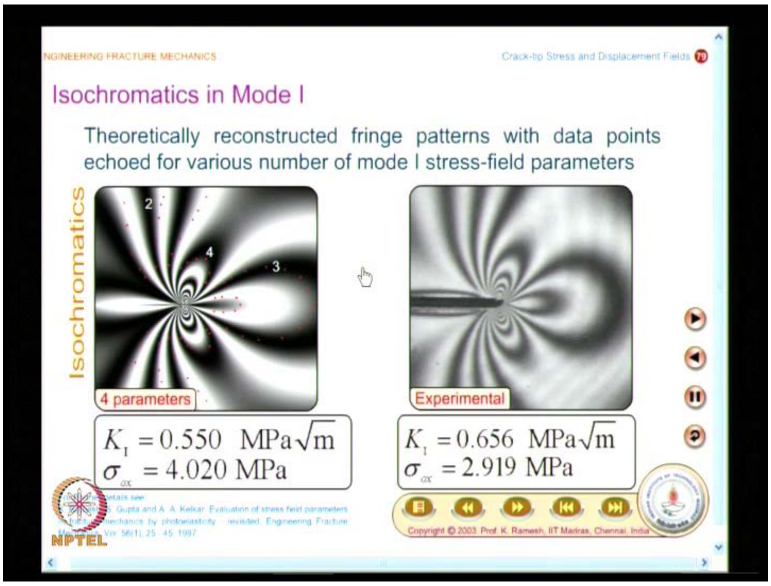

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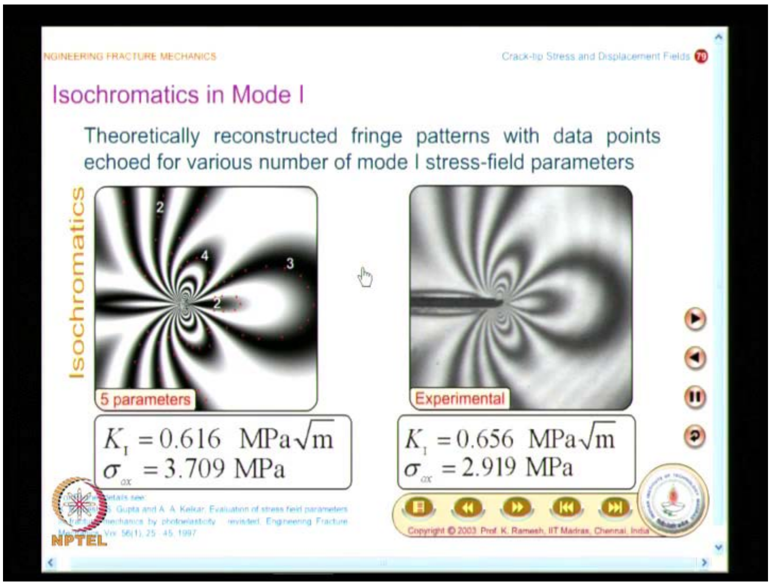

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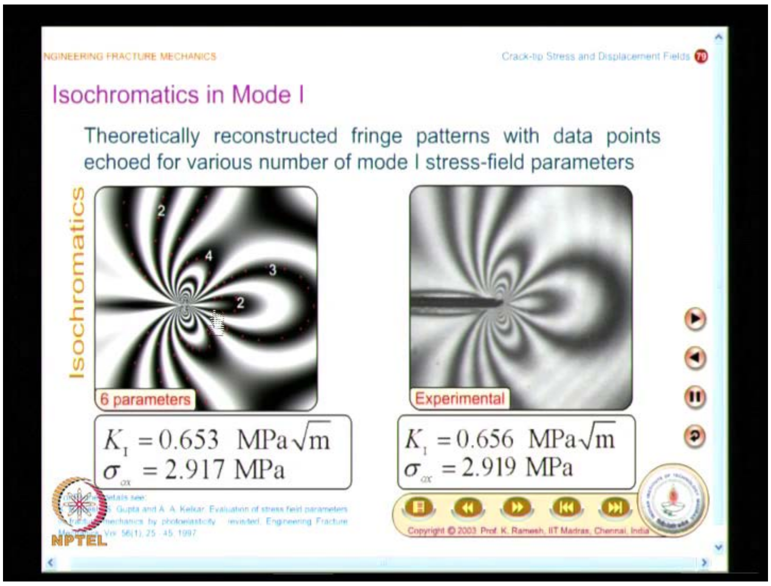

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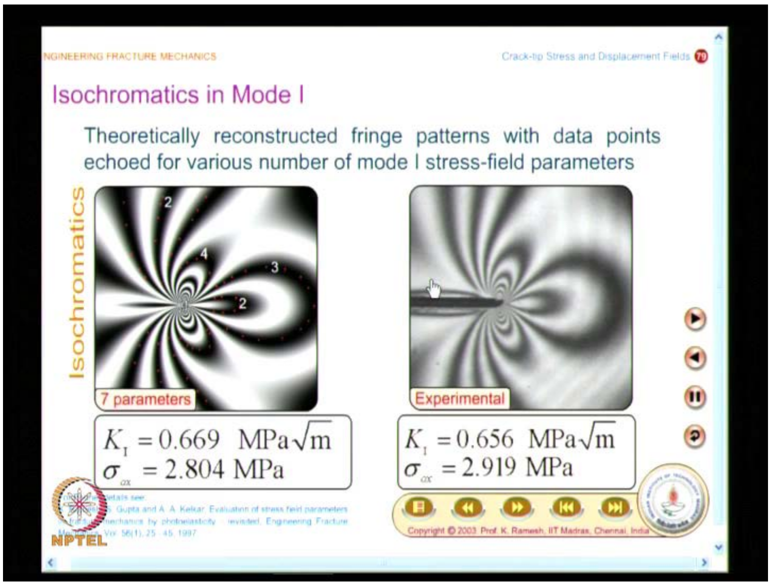

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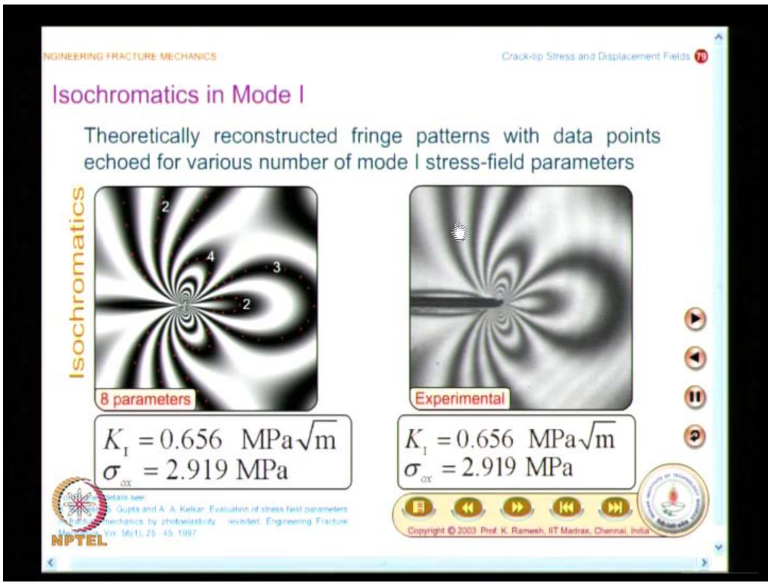

And we would see the utility of these expressions. Now showing such a long expression is not going to interest you. See, you just make an observation these animations, and what you see on the right side is the experimentally obtained fringe pattern, I have a crack, I have forward tilted loops as well as fontal loops, and on the left side you have the solution obtained from experiments which are done by a over deterministic least square methodology.

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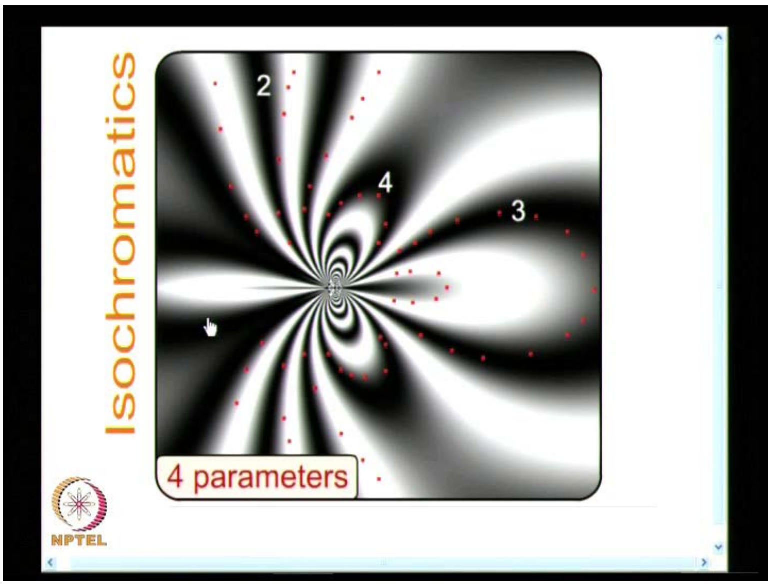

And the number of terms in the series solution is incrementally increased. You would notice, it is the four parameters, five parameters, six parameters, seven parameters, and eight parameters. And you can also notice that the values of k 1 and sigma naught x change as a function of number of parameters. Now what I want you to look at is, I will magnify this picture- you know you see red dots, these red dots are the points selected from the experiment. So, that is the input to your algorithm, if time permits may be towards the end of the course we will look at the algorithm otherwise you can look at the reference and know what the algorithm is. So, the idea is these are all the experimentally obtained points, and from these data you evaluate the series coefficients.

Using the series coefficients re-plot the fringe pattern, because you know how to calculate a maximum shear stress, and you also know what is the material stress fringe for value for the given problem, using that it is possible to reconstruct the fringe pattern.

The idea is, the reconstructed fringe pattern should closely match the experimentally obtained data. So, what I want you to make an observation is, as the number of terms changes how the geometric features of the fringe change, and as the number of terms are increased you will find the geometric fringe pattern match with the experimental data.

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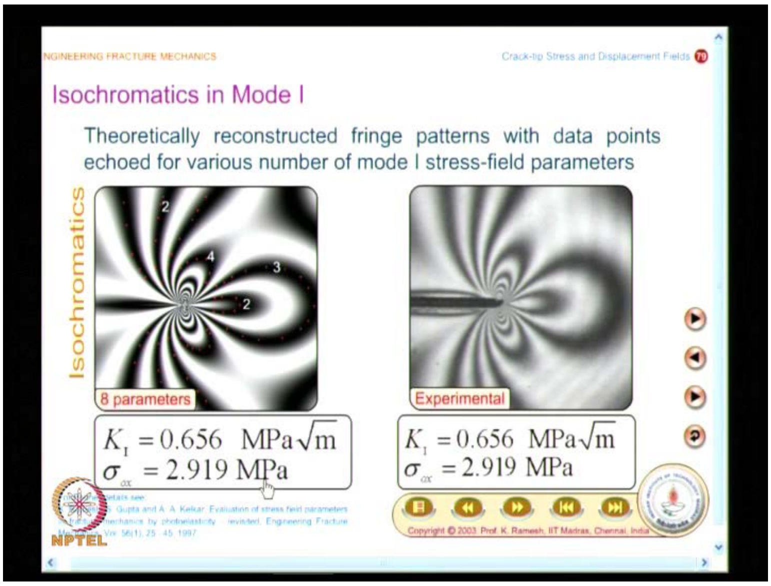

Simultaneously, also notice the variation, and the values of k 1 and sigma naught x; these are 2 pertinent parameters in the case of a mode 1 situation. So, I will repeat the animation. So, it is going for five parameters, at six parameters you know the contours match, you find sever and eight, there is hardly any change.

So, beyond six parameters whatever the complicated fringe contours which you observe in an experiment is very nicely captured from your post process results. And the final value of k 1 and sigma naught x are given here, k 1 is 0.656 Megapascal root meter and sigma naught x is 2.919 Megapascal. And we look at what happens in the case of 2 parameters solution.

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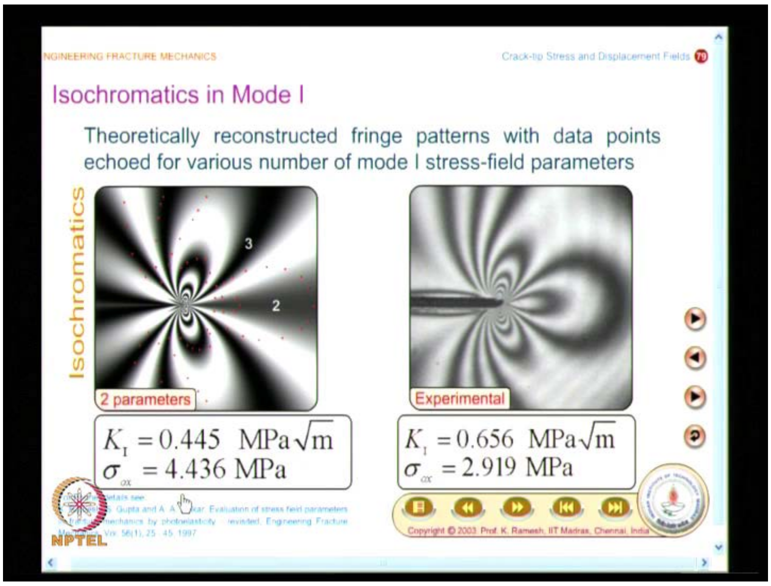

We start from a two parameter solution; we do not start from Westergaard singular solution. We said even for short cracks I cannot rely on Westergaard singular solution, I have to have minimum of 2 parameters. So, when I have minimum of 2 parameters, what do I observe? I only get forward tilted loops, and the fringes are not matching with the experimental data.

You see red dot several places, and what is the value of k? K is only 0.445 where as sigma naught x is quite high. See, if I take a two term solution, and if my k is same as what I am going to get with the six term solution, I do not have to worry, but what you find here is, a six term solution gives k as 0.656.

Here, it is for eight term solution, which is higher than the k for 2 term solution. So which definitely says that you need to go for high order terms, otherwise k 1 evaluation is erroneous; not only this, we have seen in sufficient discussion that people have looked at the second term in various ways, and we have been saying, you should get the second term as part of your solution, whatever the solution gives, you may have to take it. If I take two term solution, I get sigma naught x is 4.436, but it does not remain constant, two term solution is not sufficient to model the situation. So, interpreting sigma naught x equivalent to the far field stress field, that kind of discussions are not completely correct.

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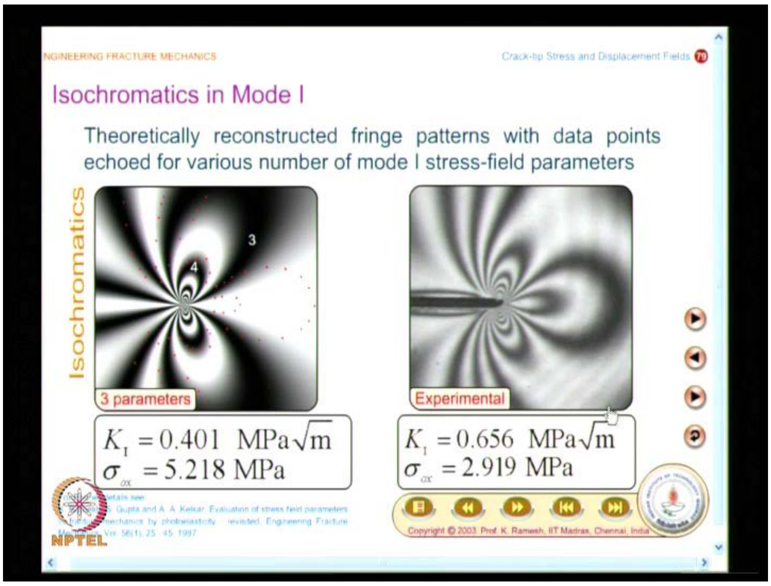

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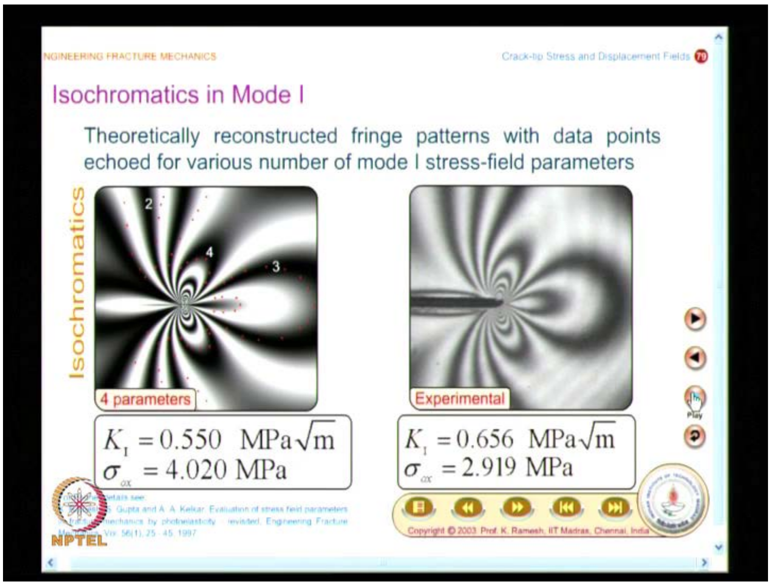

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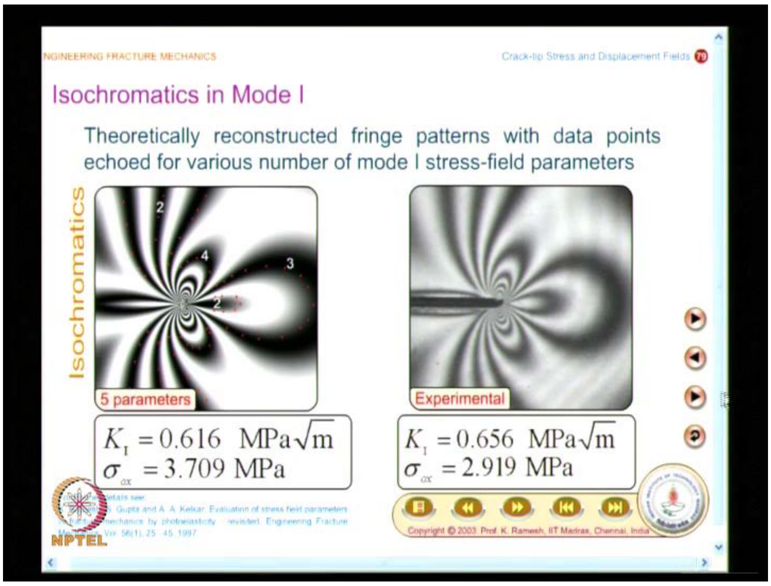

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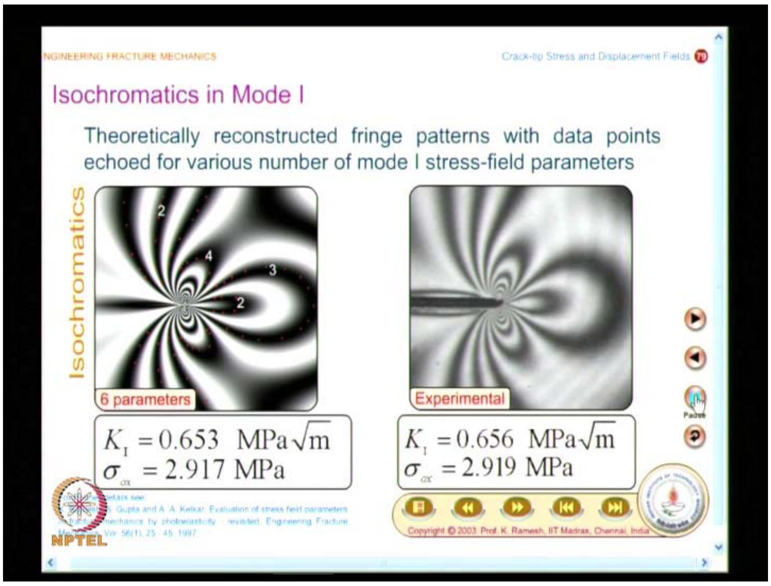

Let us look at what happens to three term solution. So, for three parameter k changes, sigma naught x changes and you find the frontal loops are not at all matched. You go to four terms, there is semblance of formation of frontal loops, and you find k is steadily increasing and sigma naught x is diminishing. Five terms it is slightly better, and if I have six terms all the fringe features is captured.

And beyond 6 terms what do you find? The solution stabilizes. And you know this is done by a non-linear least squares analysis. The coefficients a 11, a 12, a 13, a 14, a 15, a

16, they were calculated based on the experimental data. It is a very sophisticated algorithm, based on that you re-plot, and you find for this problem at least 6 parameters are required to model. And when you are having a least squares approach there is no guarantee that your final solution is correct.

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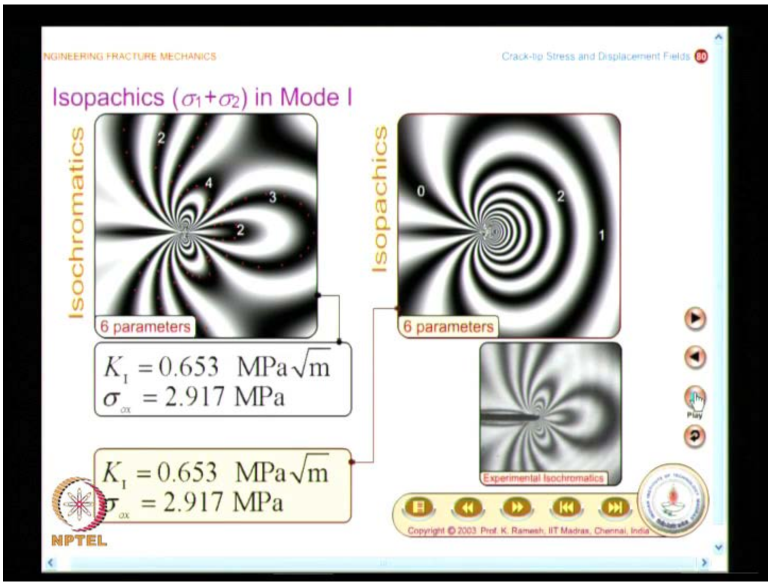

Here, in order to ensure that my final solution is matching with the experiment, the fringe patterns are reconstructed, and you ensure that we have converged to a proper minimum, so that the solution is indeed representative of the stress field. Now, you find there is quite a variation in the geometric features. We will also have a look at what happens in the case of holography; you look at what way the fringes change for two parameters three parameters, four parameters, five parameters and six parameters; and what do you find?

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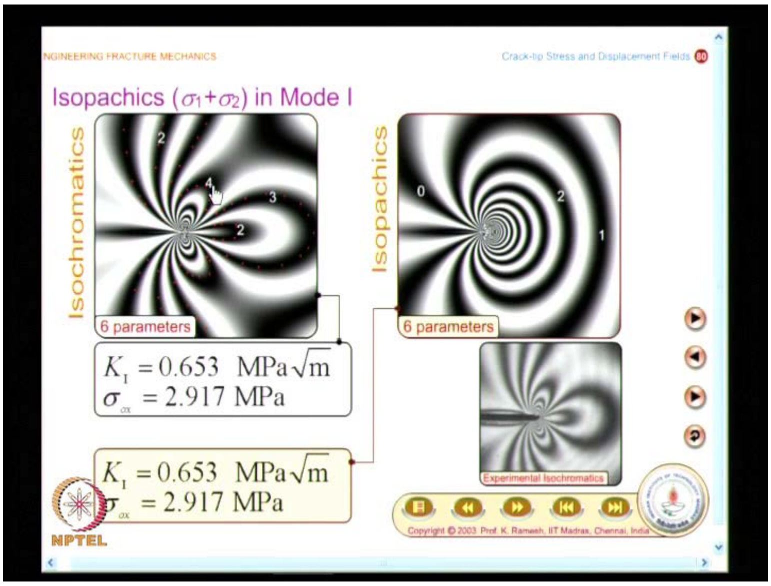

You do not find a great change in the geometry. Say, if, we have been discussing for the crack problem, we wanted tau x y to be 0 on the crack axis, Westergaard solution turned out a surprise, that tau maximum have also 0 which is incidental, we did not want it. Whereas, the experiment shows there is a variation of maximum shear stress along the crack axis which is beautifully captured when you go for a six term solution.

So, the final understanding is, it is desirable that you go for multi-parameter stress field solutions, and evaluate the coefficients based on the stress field. And that is how we will have look at the equations in fracture mechanics. And we will also see more than one example in the next class. We would also see the multi-parameter displacement field equation. And you have seen Westergaard guard, generalized Westergaard, then William's Eigen function, then Atluri-Kobayashi, they cannot be different uniqueness, theorem says you have to one solution for one problem.

So, we would also look at the identity between the coefficients. In fact, in the next class I would like you to sketch some of this fringe patterns. You also need to have these fringe patterns as part of your notes. So, in this class what we had looked at was, we had looked at multi-parameter stress field equations although we wanted to see displacement field, we will postpone it to the next class. Lot of writing you had to do, and these equations are very important, and you may not find them in books. So, these equations will come in handy when you need them, thank you.