Design and Optimization of Energy Systems

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Lecture No. # 08
Successive Substitution Method and Newton-Raphson Method

Good morning. We will continue with our study of successive substitution. Today, we will see one last example. This concerns the movement of a truck uphill. There are two characteristics involved: you have load characteristics, you also have engine characteristics. These two you have to match; you will get the operating point. So, the problem we are going to consider is for a particular transmission setting. For different transmission settings, we will have different curves and we will have different operating point. That is why, we say that, when you are climbing uphill; when you are in fourth gear; suppose when you are in fourth gear or sometimes even third gear; we will get some odd sound; that means the engine is telling you that, you have to downshift. Then, you reduce to the appropriate gear and then when these two match… So, you stay very close to the operating point. So, this example will highlight how we do the successive substitution for a two component system.

As is our practice, I will first dictate the problem, and you will start working. Meantime I will take attendance. Then, the same problem has been written up on PPT; I will show it. And then, we will work it out on the board. Then finally, I will show it on Microsoft Excel. Problem number 7; is that right?

Student: 8

8. Problem number 8. A truck is climbing up a hill. Problem number 8. A truck is climbing up a hill road slowly. A truck is climbing up a hill road slowly. Given below are… A truck is climbing up a hill road slowly, given below are: 1 – the torque-speed characteristics of the engine. A truck is climbing up a hill road, a hill road slowly. Given below are: 1 – the torque-speed characteristics of the engine; and 2 – the torque-speed characteristics of the load. Given below are: 1 – the torque speed characteristics of the engine; and, 2 – the torque-speed characteristics of the load. The truck continues to drive

uphill… The truck continues to drive uphill… The truck continues to drive uphill at the same transmission setting. The truck continues to drive uphill at the same transmission setting. a – using successive substitution… using successive substitution, determine the operating condition of the truck… Using successive substitution, determine the operating condition of the truck for two possible information flow diagrams. Using successive substitution, determine the operating condition of the truck for two possible information flow diagrams. b – examine the stability… b – examine the stability of both the operating points; examine the stability of both the operating points.

For the benefit of the people who came late, problem number 7 – it is on successive substitution. You can write down in telegraphic language. A truck is climbing up a hill road. Torque-speed characteristics of engine and the load are given. a – determine the operating point using successive substitution. b – examine the stability. So, you have to do it for two information flow diagrams. I will give you the equations.

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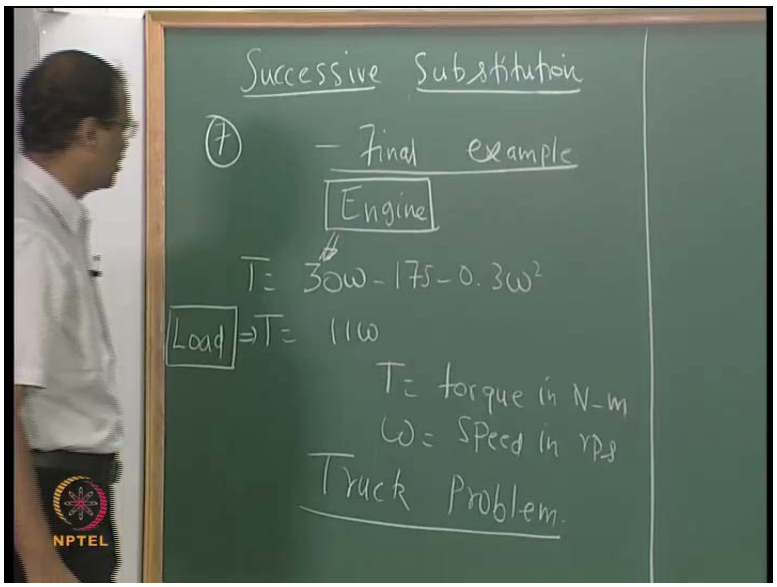

This is the problem number 7. We call it as the truck problem. Omega is the speed in rps and T is the torque in newton-meter. Torque T is given by 30 omega minus 175 minus 0.3 omega square and T equal to 11 omega. So, this is the engine characteristic and this is basically the load.

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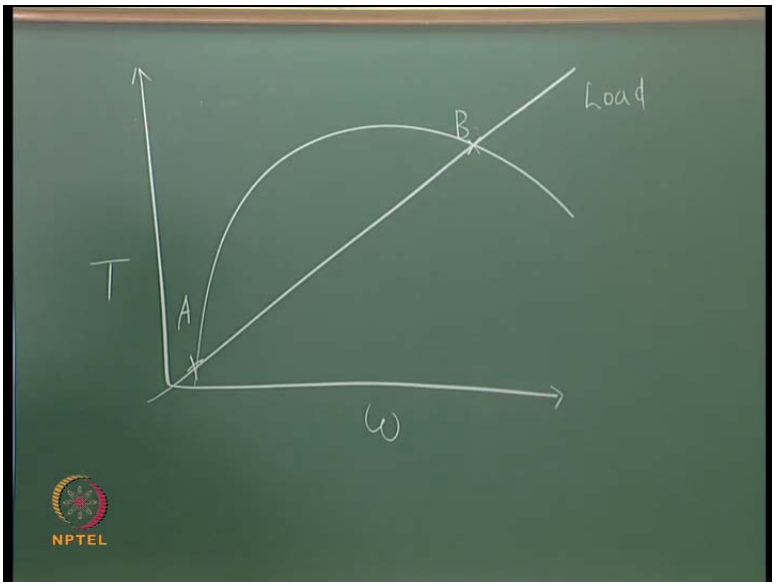

So, you can expect that; one is going like this. What is this? If you ask me sir, how do you know it is going like this; it is a parabola; it is quadratic. It is quadratic in omega. Therefore, it should have two operating points. So, we can use two information flow diagrams; that is, this information flow diagram can be used to determine T from omega or you can write omega is equal to T by 11 and use it to determine omega from T. If you use equations as such, it gives you one information flow diagram; I mean one of the two equations you have to change or you can invert and then have the other information flow diagram. So, the two information flow diagrams should lead to different solutions, but both are possible solutions. But, later on, we will discuss whether you want the truck to operate at point A or point B. So, basically, the torque-speed characteristics will be something like this. It is better to have torque-speed characteristics such that the torque is basically more or less uniform with respect to the speed, so that you get a high torque for most of the speed.

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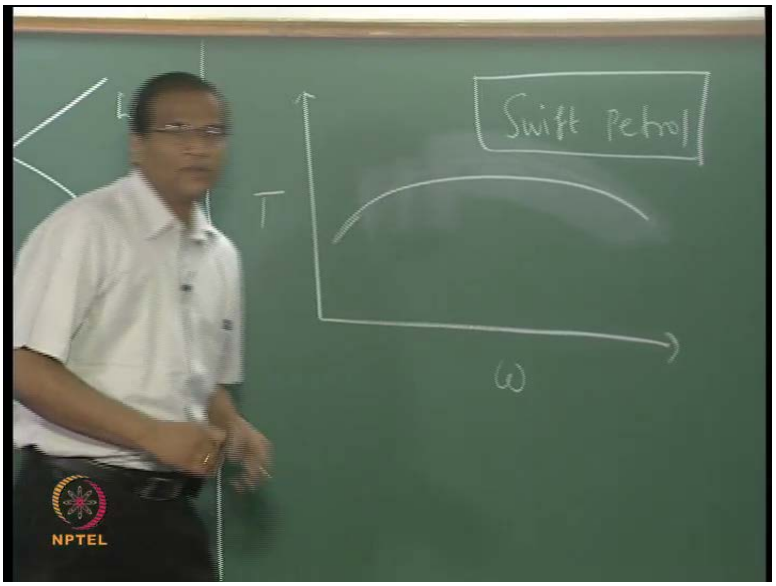

For example, if you look at… For example, the swift petrol has a torque-speed characteristic like this. The swift has a... See this is an excellent torque-speed characteristic, because at high speed also, it gives good torque. Torque gives you the ability to accelerate. So, it would not be sluggish. So, it will have the zip throughout. Once torque-speed characteristic shows a trend like this; once you cross this; then, there is a tendency to have less torque. But, the goal of any automobile designer is to have as flat a torque-speed characteristic as possible, but subject to other conditions and so on.

You can just go to Maruti and then go to torque-speed engine specifications; and, you can get the torque-speed characteristics of many of the automobiles. You can critically compare all these. Of course, for the layman, this is all too much. They would not even know the difference between torque and power. What is a point in having high power when you are starting at the signal? What do you want when you start at the signal? You want high torque; you want to accelerate; you want to get pass the others. So, the torque is very important in those situations. So, you please start solving. For the benefit of the people who came late, I will now show this (Refer Slide Time: 09:10) on the screen. Start solving. In the meantime, I will take attendance. Initial guess value – 40 rps; is it all right? What is it in rpm? 40 rps is how much?

Student: 2400 rpm

2400 rpm is a good speed for the engine? It is all right. It is not a bad guess. You should not start with 2,40,000 rpm for example. You should not start with 24 rpm. So, what is idling speed of your Hero Honda, for example? Hero Honda will also bring some 50,000. Cars – when it is idle, it will be about 1000. Now, they also have a tachometer. All the modern cars also have the tachometer. So, they say that, whenever… Even if you go to 100 kmph or 120 kmph, they are always asking you to look at the speed. You should not try to go at 4000 rpm, because the strain on the engine goes up, if you try to increase the rpm. There is no… You can go fast, that is not a problem. But, when the speed also goes up; I mean the rpm also goes up; that means you are not at the appropriate gear. For example, if you go at 70 kilometers per hour in third gear; then, it will show a wild rpm; that means it says change to fourth and fifth. So, it is for the benefit of the user.

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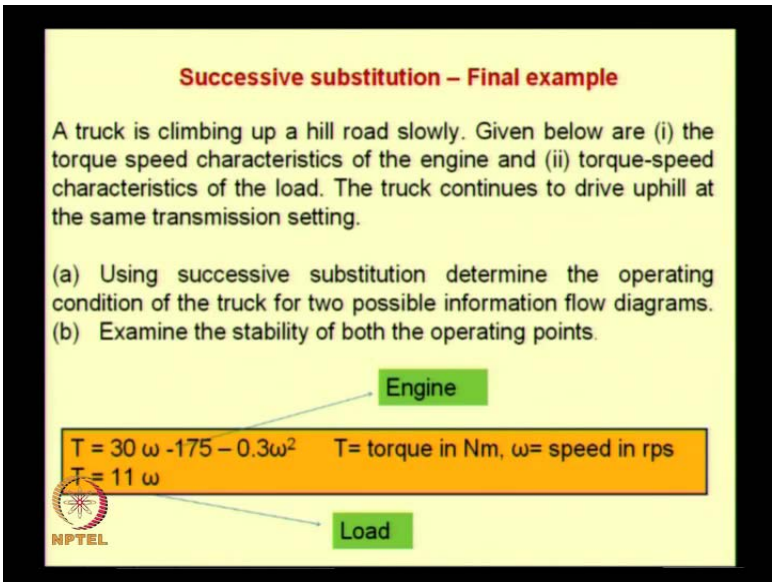

For the people who came late, this is the problem. I have given both the equations. One is a quadratic; one is a… I do not want you to make the whole problem silly like substituting for T here and solving it as a quadratic. That can be used for checking. Basically, I told you, this is successive substitution. From one equation, input omega; get T. From T, you again get omega till it converges, because this is a methodology I am teaching you; which will help you to handle multi-component systems all right. So, I think the basic idea of asking you to solve the problem with two information flow diagrams is to ensure that you get both the operating points. So, we will have to… We will first draw the information flow diagrams. See this is for a particular gear setting. For

different gear settings, some characteristics will change. That is why I told you that, when it is climbing uphill, if we change the gear; then, we are in trouble. Maybe it cannot be solved in a classroom. Not that it cannot be solved; maybe in a classroom environment, it may be difficult.

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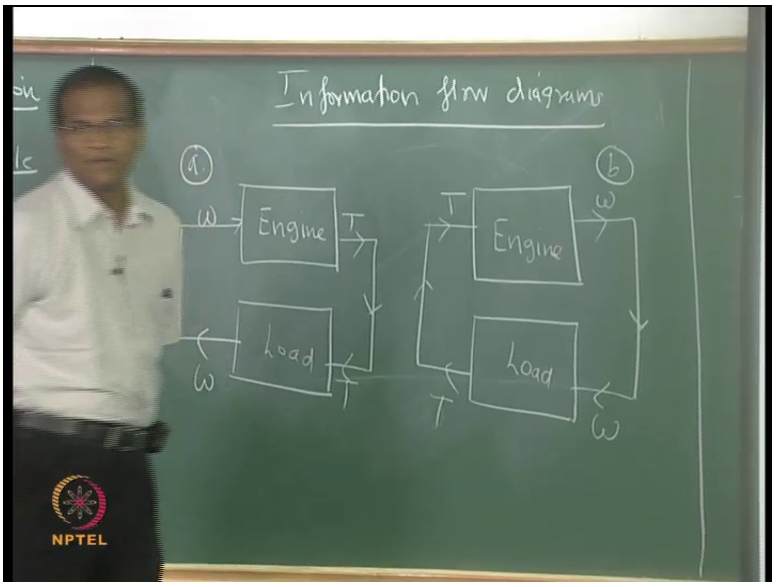

Information flow diagrams – What am I doing? Engine is one information flow diagram. The other information flow diagram is... So, the first step is to draw the information flow diagrams. And then, appropriate to the information flow diagram, you start, you draw out the tabular column – serial number or iteration number, omega, T; then, decide on an appropriate stopping criterion and proceed with the iteration.

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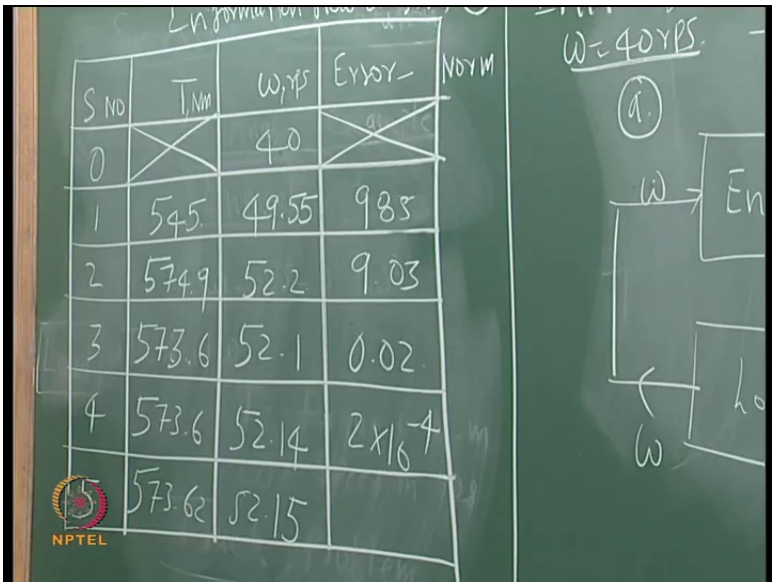

Let us start with… Which do you want me to start off with? Start off with first one? Serial number… Initial guess… What is an error criterion we want to use? Omega i plus 1 minus omega i whole square plus T i plus 1 minus T i whole square. So, I will say now, iteration 0. This is the way to start off with. Iteration 0; I do not calculate anything; I just start with initial guess. So, I do not fill up two columns: column pertaining to T and the error. Now let us go to iteration 1. So, what is a… From omega, I have to calculate (Refer Slide Time: 16:38) T, is it? So, what is the T you are getting?

Student: 545

545; that is good. T is Newton meter. 545 divided by 11= 49.55.

Now, is it possible to calculate error now? You have to wait for some time. The error on this is not known; 49.55. You again substitute here; 49.55 into 30 (Refer Slide Time: 17:21) minus 175; you can set your calculator, so that you can do it fast. 574. What is this?

Student: 52.5; the equation will be…

No; now, tell me this. With these two; this minus this, this minus this; how much?

Student: 1000

49.55 minus 40 whole square plus 574.9 minus 545 whole square. How much is it?

Student: Around 1000

800, 900; how much is it?

Student: 1000.

1000? Exactly 1000?

985.2. Do not be lazy Deepak. So, 574.9. Now, you do this – divided by – it goes to 52 or what is it? 52.2. Now, you have to again do this. 578.6. Now, 9 plus 1 – 10; how much is it? Varun?

Student: (( ))

Deepak, tell me.

Student: Sir, we need to calculate error only when they are closed together.

No, you do not try to minimize your work.

Student: 9.03

Then, you will do the same thing in the exam and then you will argue with me. 573.6 and this is? 52.1. Now, 573.6. Now, you please calculate this. 0.01 is it, already? (( ))

Student: 52:14

52.14, because you are cutting down to two decimals. You are getting the same thing? 573.6; 52.1? 52.15. So, please report the error here. So, we got the… 2 into 10 to the power minus 4.

Fine. So, we use one of the two information flow diagrams namely, information flow diagram a; it was pretty straightforward. You did not have to solve the quadratic equation. It is pretty straightforward. Successive substitution – got it to the answer of 52.15 multiplied by 60; it is about 3100 rpm. But, nobody will go up the truck with 3000 rpm on a hill. This is something which we have to avoid. We will go in a lower gear. Generally, you do not exceed 2000, 2500 rpm. Therefore, it is imperative. No, I am not saying that, these are wrong solutions, correct solutions, or whatever. This is one of the

two solutions. Anyway it is a quadratic; a quadratic cuts a straight line. So, there should be two points. So, we should turn around and find out the other operating point.

Yesterday, Vinay asked me a question – when I tried to plot all these on excel, I think some people misunderstood that as me, you employing the successive substitution method on excel. There is no way of, I do not know of any way of employing the successive substitution on excel. What I did yesterday was just plotting of the two curves and intersection. I graphically depicted how the solution looks like, so that your understanding of the situation is complete. I did not put the algorithm in Microsoft excel and solve. I just took some arbitrary values of the parameters and found out the fan. I use the fan curve and I use the duct and then we solve when these two were intersecting. But, Abhishek told me there is a way of doing it and we can actually put into recursive relation and actually show it on Microsoft excel.

Now, shall we go to the other information flow diagram? So, this is the correct answer. Omega is 52.15 revolution per second – rps and torque is 573.62? 573.62 newton-meter. Now, we will go to the other information flow diagram. In real life or in the exam, if you get the other information flow diagram… Anyway I have asked you to obtain both the operating points. It is incumbent upon you to draw the other information flow diagram and extract the other point also. And in some cases, the other information flow diagram may completely diverse. In that case… In such a case, you have to draw the information flow diagram, which you originally thought will not work; and then, rework the problem and finish off. Generally, I will check. Before giving you a question, I will check that the problem has a real solution. So, one of the two information flow diagrams will usually work. Why am I saying one of the two information flow diagrams? Usually, we are looking at two component systems. And generally, for these things, two types of information flow diagram alone are possible.

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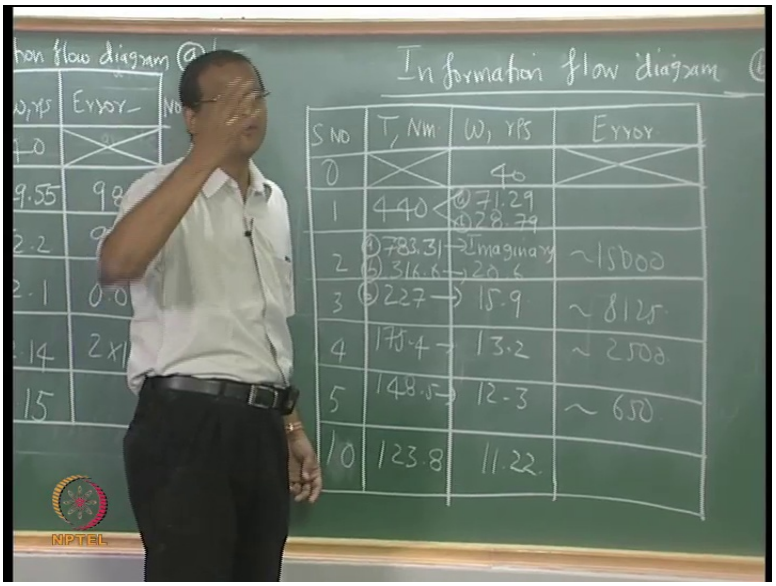

Information flow diagram b – first step is the same. Now, we proceed to… Now, what do you do? You substitute 40 here; and then, T is (Refer Slide Time: 25:20) how much did you get?

Student: 440

4?

Student: 40

Good. So, T is 440? No. What did you do in the first one? You did that. Now, you have to go to this. Now, you will get 440. Good so, 440. Unfortunately, with this 440, you will get two values of omega, because it is a quadratic. One of the two will come; we will see. Hang on, hang on, hang on; shall solve this; solve this.

Student: I have taken some other value.

No, we cannot do that. First, we will allow things to fail.

71.?

Student: 21

Student: 28.79

I still cannot calculate this, because I have not completed it. So, I work with 71.29; corresponding to 71.29, what is the value of omega I am getting?

Student: (( ))

So, I will say a and b. a – 783.31; b – 360.?

Rajesh , tell me? Venu?

Student: I (( )) starting guess; starting guess is 5

Why?

Student: (( )) smaller

That is cheating. You must have got it in 3 or 4 iterates. No, because I do that plot. No, that is ok. But, generally, since I want the class to… I want all of you to go through the steps; you better stick to 40 rps. Sowmya, what is the matter? Things are not working out? Koustub, tell me? So, 783… What is the other one you are getting?

This one? This leads to what? 75? Imaginary. Please look at me now. Please stop working. So, you are getting minus b root of minus b square minus 4ac by 2a. So, the moral of the story here is when you minus b plus or minus root of b square minus 4ac; if you consider that b square minus 4ac by 2a, you will get two roots. One of the two roots will automatically diverge. Therefore, subsequently, you take a chance that, you look at which of the two roots diverges; and then, that root I will not consider in the future calculation. So, anything which goes beyond 50, 60 or 70; you just ignore the other root. So, look at the other root, which is proceeding towards the lower value of omega. That will be the true solution, because you already know by plotting. But, that is again cheating. So, even if we… Or, you can use the plain vanilla approach; you do not worry about anything; you just keep on working out the two solutions and discard the solution, which leads to imaginary roots. Ultimately, you will land up with the operating point a.

316 will lead to?

Student: 20.6

Now, you can work out the error. 440; I will do it approximately. So, it is this about 120. 120 whole square is how much? 14400 plus 400; around 15000. So, 15000. Now, using this 20.6, use the b. Now, you will use 20.6 into 11; 223 is it? 2?

Student: 227

227, good. So, from 227?

Student: 15.9

Good; 15.9. Now, you can find out. 316 minus 227; this is about 90. 90 whole square is how much? 8100. 8100 plus 25; about 8000. But, still it is large. Using this 15.9, you can again work back. How much are you getting?

Student: 175…

175?

Student: .4

175.4. With this… So, this will be about 50, 2500. So, about 2500. You can see it is rapidly decreasing. 13.2 will give you about 150?

Student: 148.5

11?

Student: 12.

12.?

Student: 3

12.3. So, how much is this? 140, 150, 25, 625; 650. It is still way half; but, it is converging much more slowly compared to the other one. So, I have worked it out. By the tenth iteration or so… Tenth iteration – I am getting 123.8; 11.22. So, this is more like it. 11.22 into 40, how much is it? Around 500; 11.22 rps. That is about 500 rpm. So, generally, all…

Have you seen trucks climbing? All the other vehicles will easily overtake them. They absolutely cannot climb. So, even the small vehicles can overtake them. Maruti 800 can overtake a very big truck. But, as soon as he comes down; and then, on a level road, since its power is more, after a couple of miles, he can overtake. But, if you see actually in those days, when many bikes were there; the Kinetic Hondas were the first to come out of the signal, because of the excellent torque. You can accelerate; even now, the Honda Activa and… The Hero Honda is four stroke; Hero Honda will struggle; Splendor, this thing; whatever. I am not talking about the big ones, which have got 150 cc and all those Zwamy, Swankey – new bikes. The old classic CD 100 – it has got only about 7 horsepower, 6.5 horse power originally. It will struggle. But then, after it stabilizes, when you come to fourth gear; then, you can overtake. Initially it will not; it is only the Honda Activa, Honda Dio and all those guys, who will really zip past all the others in the signal. So, this is the story as far as this problem is concerned. I will now, work it out on Microsoft excel; or, shall me skip it? Who wants me to do it? Even if there is one person, I will have to do it. Fine; that is good.

Now, have are to examine the stability of the operating point. Now, let us… For the sake of… In the interest of time, let us do the stability analysis only around point A. All of you please examine the stability around point A. How did you approach point A? You approached from 40. Venugopal, already approached it from 5 and got the answer; but, we will check it now. So, please approach it from 0.9 or you start with 9 rps or 10 rps and do two quick iterations using information flow diagram b and get yourself convinced that, problem – that it will hit the same solution of 11.2.

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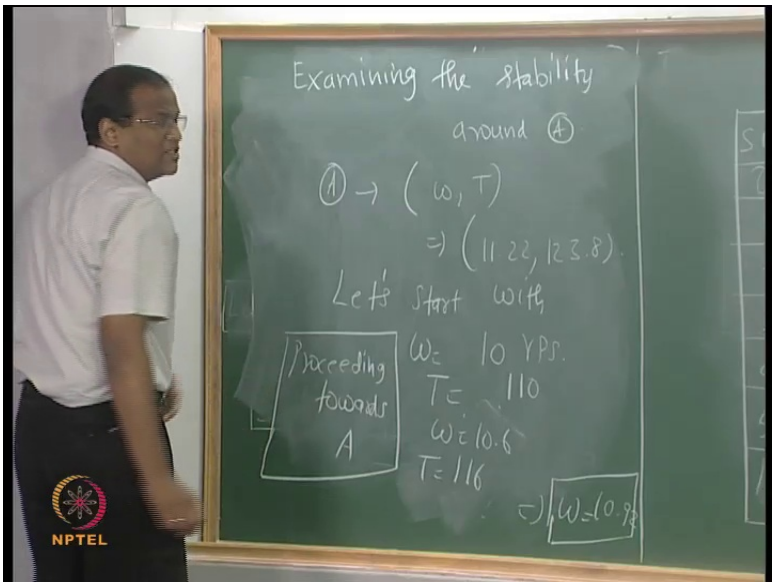

Examining the stability… Do you have the next class? Please do this. We start with 10, what do you get, Torque? What is this T? 110. Omega?

Student: 10.6

10.6. Then. T?

Student: 116

116; omega? So, we will just suffice it to say that, proceeding towards A. Since we got the solution starting from 40 and you get the right solution – 11.22, when you start from 10 also, it is proceeding towards A. Therefore, it is a stable operating point. So, you can go back to your hostel rooms and also investigate the stability at operating point B. How did you approach B? We started from the left side of B. Now, you start from the right side of B and let me know in the next class, whether it is coming back to the point B. Any doubts? Yes, Venugopal has it.

Student: (( )) omega is equal to (( )) We get an answer…

No; but, I told you the other derivative will be very small. So, even if one of the two… Yesterday also, there was lot of confusion. All this will be resolved once we look at the Newton-Raphson method, where we use the information on the first derivative to approach the problem. Anyway, I will just take a minute and graphically tell you what

the Newton-Raphson method is; and then, we will… I will just take two minutes; and then, in the next class, we will work out the algorithm.

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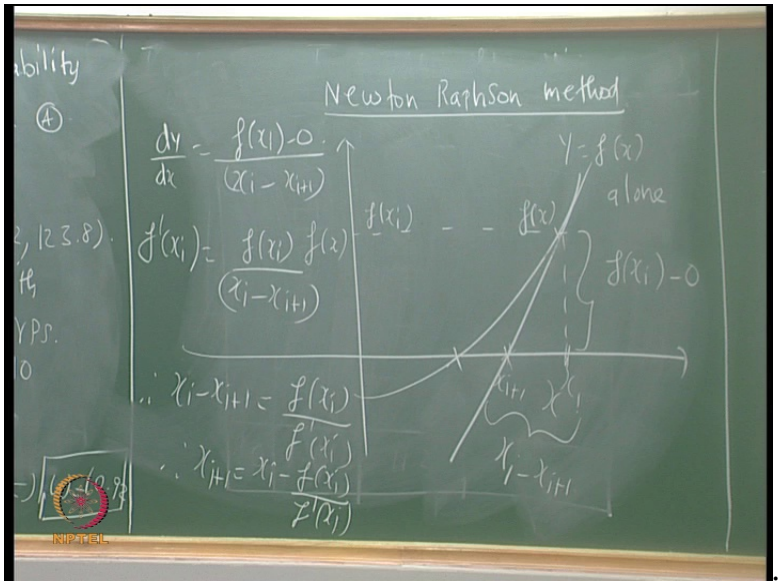

Now, let us consider y is a function of x? It is a one variable problem. So, this is f of x. I want the roots of f of x; that is, I want to find out where f of x becomes 0. That is what we are trying to do. Using successive substitution also, making this f of x is equal to 0. And then, that is how we get the operating point. Now, this is the root, because the equations are non-linear or transcendental; there is no direct way of finding the roots. Therefore, what we do is we start with an initial guess x i; and then, we draw a tangent at this point. And, wherever it cuts the x-axis, we call that as x i plus 1. So, this will be x i. So, this will be f of x i. So, what is this? f of x i minus 0, correct? What is this? x of i minus x of i plus 1, correct? Therefore, dy by dx at x i is equal to f of x i minus 0 divided by x i minus x i plus 1. If you do this… This is the Newton-Raphson method. So, I will derive the same equation using Taylor series expansion. So, we will do the derivation; I will repeat the derivation – graphical method. We will also do it using Taylor series. We will solve a couple of problem, some simple transcendental equations involving trigonometric function – problem number 1. Then, problem number 2 – we will do the electrically heated wire and then we will revisit either the… The choice is yours. We will either visit the fan and duct problem or the truck problem; one of the two problems. We will solve as a Newton-Raphson method in multiple unknowns.