Design and Optimization of Energy Systems

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Lecture No. #07

Successive Substitution Method Contd…

So, good morning, so we will, we will continue our discussion on successive substitution. As you may recall, we are trying to now work on successive substitution for two component system. This, this specific system we have in mind is the fan-duct system.

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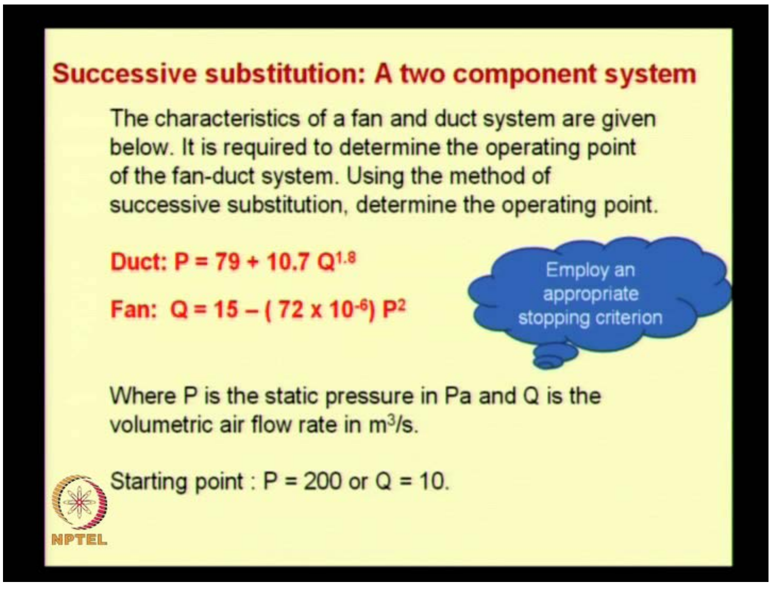

So, the equations for the duct and fan are given. So, I have already dictated this problem to you. So, there are two variables in this problem, one is P, which is the head, which is in for the pressure, which is in Pascal. So, the other one is the volumetric air flow rate, which is in meter cube per second. So, these two are (( )) by, these two by, the two respecting equations. Now, you want to solve and get the operating point.

I do not want you to, to, from your previous knowledge you can try to substitute for Q and solve Q as one variable problem and make it very, make the whole thing very trivial.

That is possible, it is possible for you to do this because it is just a two equation problem. In principle it is possible to keep on substituting back into the respective equations such that you finally end up with one equation, but that, that is not the intent. I mean, it is not, that somehow we solve the problem; that is not the goal. We want to see how optimally we solve the problem given a multi-component system. That is why, it is important for you to recognize, that you have to essentially solve this as a two variable problem, right. So, qualitatively how the characteristics will look like, are shown on the blackboard.

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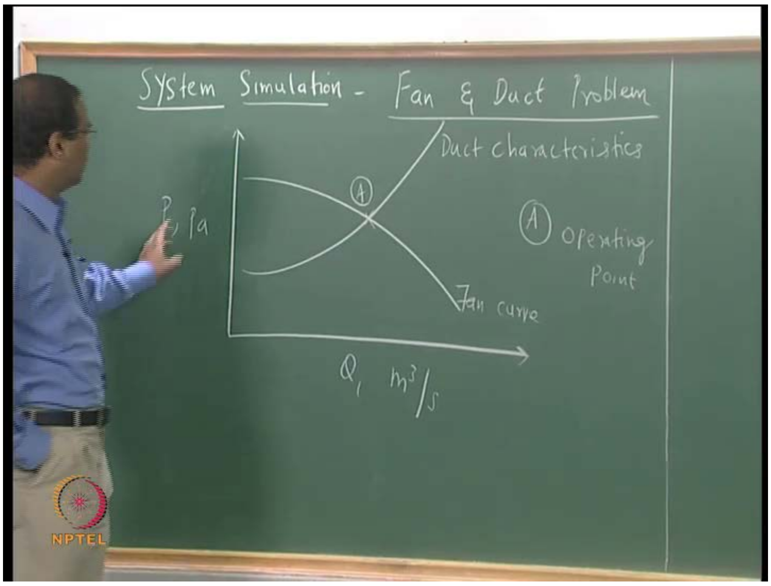

So, one is the duct characteristics where basically, the pressure rise, which is Q into 1 to the power 1 point, Q to the power of 1.8. This 1.8 is a consequence of the (( )) equation, which gives you the head-loss flv square by 2gd. f itself is 0.182 Reynolds to the power minus 0.2. The 2 and minus 0.2, they combined to give 1.8. I have already discussed this in the last class.

The fan curve will essentially, the fan curve will essentially go like this, right. The, the pressure will, the pressure and the discharge will be related by this, right. So, if you want, if you want lot of discharge, then you cannot have high pressure rise, which is accomplished, but if you want high pressure rise, then you have to compromise for a lower discharge because there, there is a finiteness to the capacity of the pump.

Now, we want to, we want to find out where these two curves intersect. In fact, they intersect at the point A. what are the coordinates at the point A?

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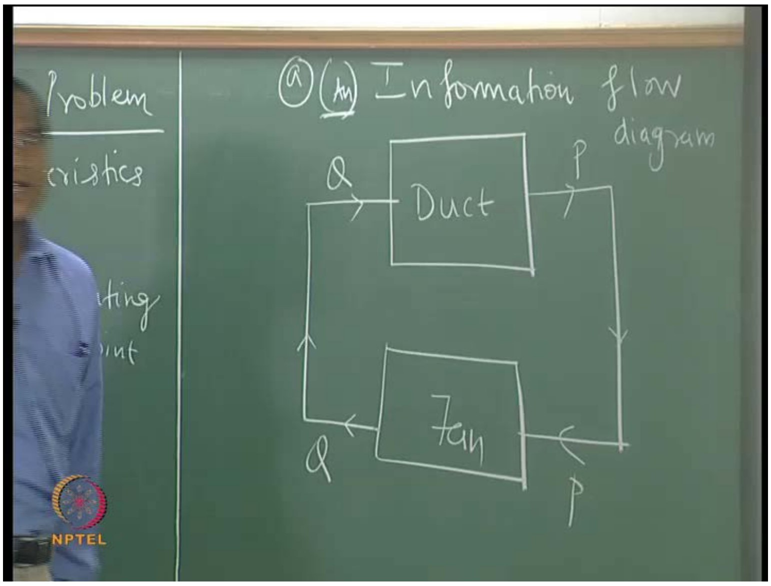

So, we can use information flow diagram. We use first, we first draw the information flow diagram. So, so this is the duct. It is possible for you we use Q as input and get P as the output. For the fan give the P as input and get Q as output. This is A. So, you can qualify, it is not the information flow diagram, it is an information flow diagram because there is no law, which forbids you from rewriting this equation as Q equal to P minus 79 divided by10 to the power of 7 whole to the power of 0.55. This is an information flow diagram.

Now, try, let us try to solve this problem with this information flow diagram, in order that all of us proceed along similar lines. I have also given you the starting point. You can either start P equal to 200, correct. Q is equal to 10 meter cube per second or P equal to 200 Pascal, right. How many of you have already finished? Is there anybody who has already finished? Why do not you start making the table, right.

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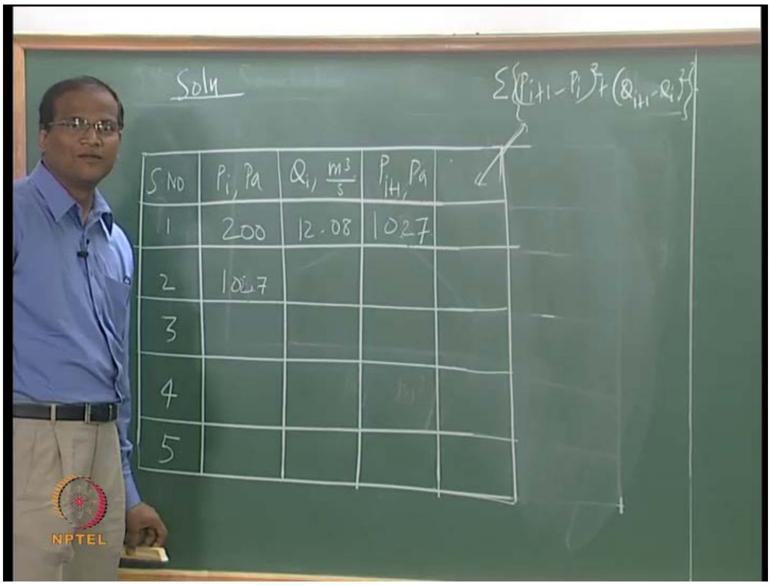

So, I told you, please employ an appropriate stopping criterion. Let this be there. The serial number, P of i, Pascal, sorry, what will be the stopping criteria?

Student: (( ))

But the units are different, right. Is it ok? Ok, we will start with this. Let us say, if it is like…

Student: (( ))

Normally, (( )), then do what. Let us stop, let us look or you normalize and have a separate criterion each of which has to be less than 0.1 percent or something. So, nobody will tell you what the appropriate stopping criterion is. It is for you to, it is for you to figure out. So, what, whatever is appropriate, whatever you deem fit you use, right, but it be reasonably, should be able to convince somebody, that this is adequate. There is no need to proceed with more number of iterations, right.

So, let us start with this information flow diagram. So, should we start with 200? So, we are starting. So, we are starting from the fan equation, right. We are starting here, so this is 200. We can substitute here, we can get the value of Q. What are we getting? 12.08, that is ok. So, with Q i you are getting P i plus 1.

Student: (( ))

(( )) Is it diverging?

Student: Yes sir.

Ok, 1000, 1037. Now, there is no need to put this additional column, right. Is it required? Anyway, you are going to use this again, right. So, or you want this? Now, I can start with 1037 here, right, 1027, ok. So, we will cancel this, not required. Yeah, I did you some, now from 1027 if you want to go to Q i, minus, it is gone. So, does not work. So, we are first to use the information flow diagram, right, and then we will see whether it works. If it works, then we will turn around and find out why this information flow diagram did not work. Of course, you can realize, that there is a Q to the power 1.8, there is P to the power (( )) soon. So, it is not linear, it is non-linear and apart from this power you also have minus sign and all that. Then, something is 15 minus something whole square. It is very dangerous, right.

There is every possibility that it can turn negative, but you know, that both P and Q ought to be non-negative variables, right. You cannot have a negative flow rate in this case and all that, right. So, therefore, it is meaningless to negative numbers, negative values, so the solution is diverge.

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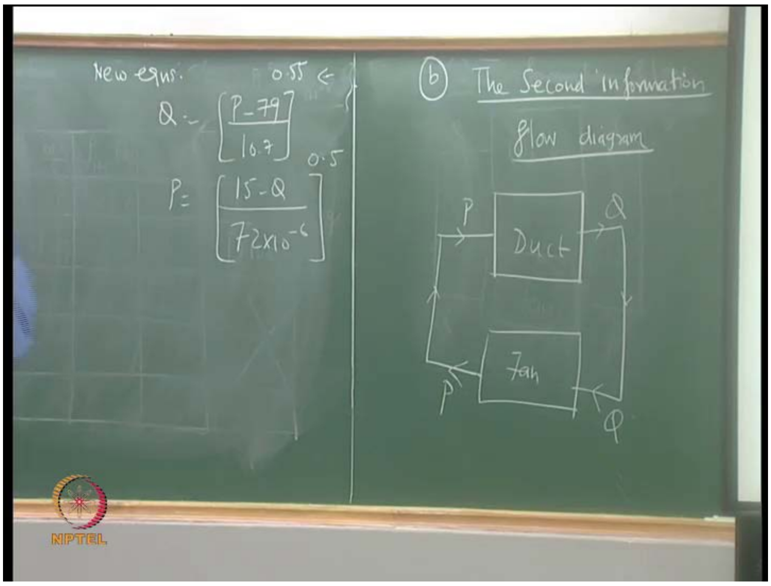

So, let us go to the B. Yeah, now tell me, I will use for the duct. P is the input and for the fan use Q as input. Is it ok? Please correct me if I made some algebraical errors. Is it ok?

Now, try, I will give you 10 minutes, please try, it will work. Now, you have to draw the tabular column again.

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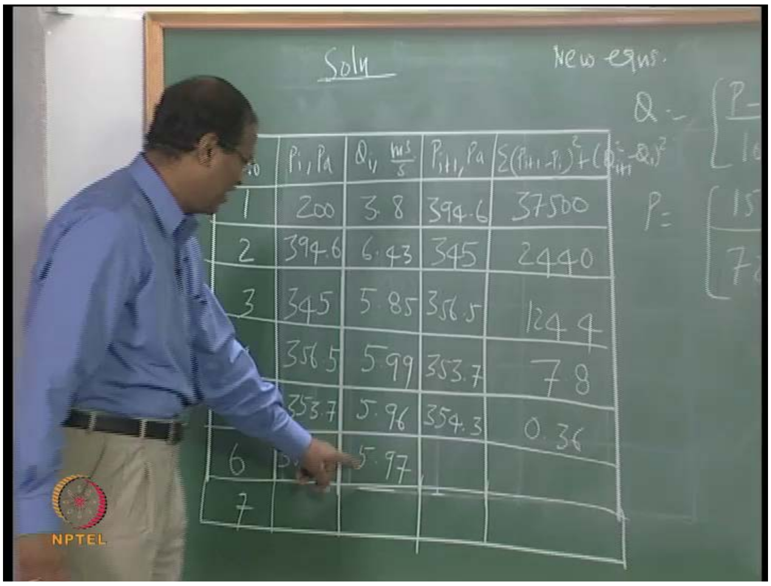

Yeah, we start with 200 again. So, what is Q? 3.8; P i?

Student: 394.6

So, we did not calculate Q i plus 1. So, we will wait, right. Or you can (( )), we cannot calculate this now, right. You have not seen change in both the variables, let us hang on. So, now, let us put 394.6, ok. (( )) now, 394.6 you can, 6.43 calculated. Now what is this, please calculate that, yeah do not be lazy, (( )), one second, the (( )) is 344.

Student: (( )) 37500

Oh, too much man. Yeah, go ahead with your doubt.

Student: (( ))

Yeah, yeah, does not matter. No, no, why do you always, suppose somebody says P is equal to, that is what I am saying, P is equal to 79 plus 10.7, why do we always think this equation should be used to find out P given Q. Who told you that? I never told you, that it is not like a teacher saying, that y is equal to mx plus c, where c is the independent variable, y is the dependent variable, both are two, both P and Q are variables in the problem, there is no teacher in like… So, there are two variables, two equations are there, somewhere you have to start.

So, we started (( )) last time. You go ahead, there is a problem with one of the two equations, that is because of derivatives are growing rapidly, that we will see. Whenever you try to solve for g of x equal to 0 and x is equal to alpha, g dash of alpha must be less than 1. That is a condition, I am going to state that. So, if the derivatives are, the derivatives lead to very high values, then this scheme will become unstable. So, now 6.43.

Vikram are you working on this or (( ))

Student: (( )) 345

We are, we are almost (( )) here, we have to wait. Now, now you have to tell me, now you have all the values, please tell me this. I want all of you to get this right. Quizzes will have problems of this level, right, or more, not lower than this. I will not give you single component system. Two-component system you have to do, simultaneous, I may ask you to do either this or the Newton-Raphson method, which I will teach you tomorrow. So, we will start with Newton-Raphson method for a single variable in tomorrow's class and you remember, that electrically heated filament of electrically heated wire, that problem we will solve by Newton-Raphson method and we will solve this same problem by Newton-Raphson method.

Student: (( )) 2440.

It is not a bad rate, but you are still way, you are still way off from our target. You are almost, you are almost there. Now, you tell me. Abhishek tell me.

Student: (( ))

You can take a root of the whole thing and say it is norm. It is called l l 2 norm, it is, always it is square. Square root is called RMS, it is called a norm. Norm is always ever square. You can go for the 4th power also, I have no problem, but 4th power means, it will be, you can take an appropriate stopping criterion. Yeah…

Student: (( ))

Error is coming down rapidly, 124.

Student: (( ))

Yeah, go ahead, point…

Student: 0.7 and 5.96

See the way it is rapidly converging. If it converges, the successive substitution will rapidly converge. If it diverges, it will rapidly diverge, success or failure, failure is guaranteed immediate. It does not agonize, it does not agonize you. Yeah, what is your value? Now, I think you have got some, you have got some decent value, 7.8. Now proceed, I think I will put one more, 354.32 and put 354.3 here?

Student: 5.96666667.

No, I will put 5.97. We are there almost. What is this k (( )) 10 to the minus…

Student: 0.36

Still 0.36? Yeah, one more there with 5.97.

Student: 354.2

Point 2, time to stop point. So, we should have some realistic expectation. If you put10 to the power of minus 8, then you should not commit suicide. I mean, you put 10 to the power of minus 8, it would not, otherwise you should program it, you should not round off to second decimal and have 10 to the power of minus 6, right. You start writing up to the fourth decimal and then you have 10 to the power of minus 4 or minus 5. When you are rounding of the second decimal, 5.9697, is always have 0.01 error in it. Also, you should understand the importance of, I mean, number of digits, significant numbers and significant digits and all that, right.

When you say the error associated, when you say the temperature is 36.54, when you say the temperature is 36.5 degree c or the temperature is 36.50 degree C, there is a difference. When we say the temperature is 36.50 degree C, you are, you are declaring, that you know for sure the second decimal place is also 0. That means, you have a, you have a measurement, which can resolve that. Therefore, when you are having an instrument, which can resolve only 0.1 degree centigrade, you are taking the temperature measurements repeatedly, once you get 36.5, once you get 36.4, once you get 36.2 and all that, you take 10 or 15 readings and average. You should not write the averages 36.546. The instrument cannot measure that. The average may be a number, which has got three decimal places, but common sense tells you, that you, to round it off to the value, which is normally shown by the thermometer. This is engineering, right. There is no, just because your calculator gives four numbers, you should not put four number, 36.5454. If you want to design a thermometer, which will, which will be so exact, you will get PhD. So, that is the, you cannot design alright, that is the importance of… Now, this information flow diagram works. So, we are happy, we can go home.

Now there are 27 more minutes so, but, now we will have to figure out why one information flow diagram works, and why one information flow diagram does not work. Apart from silly things, like whether I take the wrong information flow diagram (( )) exam or let us leave all that. In this case, even if you go to the incorrect information flow diagram in the sense, in the, in the sense, that you go to the information flow diagram, which does not converge, you know that immediately within two iterations. It does not converge; you go back, so the wastage is only 5 minutes.

Now, let us examine from numerical point of view, why, that the other information flow diagram did not work. So, everybody is through with this?

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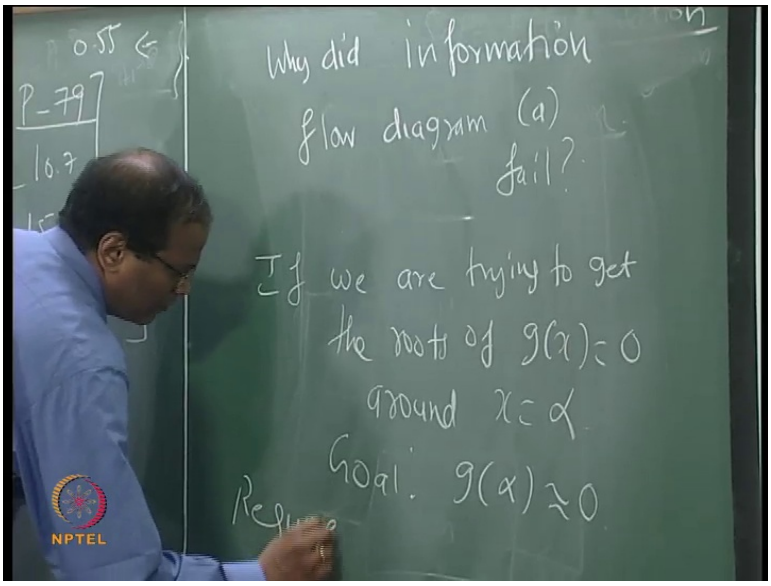

So, what did you call it, information flow diagram A or 1?

Students: A

So, if you are trying to get the roots of the equation, x is equal to 0, around x is equal to alpha, the goal is basically, g of alpha must go to 0. You want it to go to 0. Then, alpha is a root, I say it is approximately 0, I mean, it need not be 0.000, you know, that within limits it is alright. So, the, the requirement now is, I can do a Taylor series expansion and then take a new, take an example and show you why g dash of alpha is greater than 0. The whole thing will diverge, but this is not a course in numerical methods, Even though for the sake of completeness I, I am just trying to give you an idea of, some people were perplexed and some people were bamboozled why one information flow diagram is working, the other is not, basically because the culprit is, because of non-linear power.

Now, you go back and see, for, for the information flow diagram A, for the chosen initial condition whether derivatives is indeed greater than 1. So, we do that, let us do a quick exercise for 5 minutes and in the mean time, I will put this numbers on Microsoft Excel and I will show you the operating point on Excel. While I work with this, you guys will work with that.

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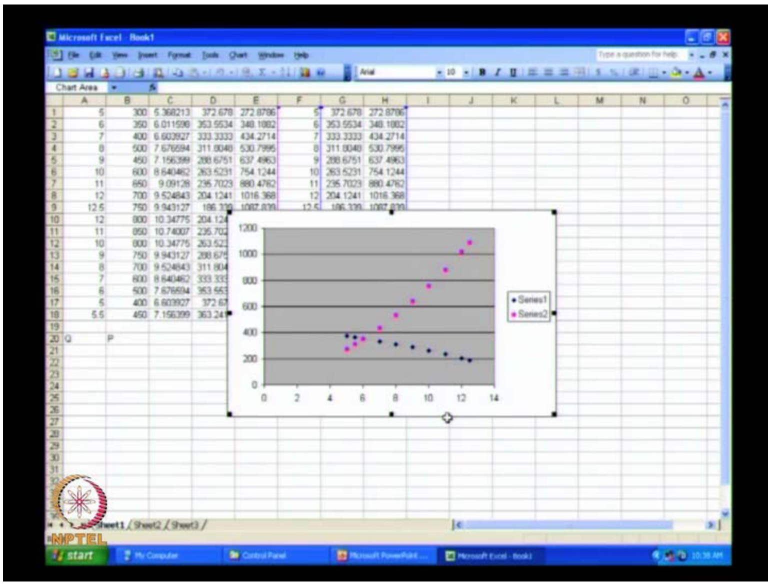

Take the information flow diagram A and get dou r by, not duo p, by duo q and duo q by duo p are the initial case, whether it is 200 or 10 meter cube per second and see, whether you are able to get it. Let it be that. It will go up to 1000. That is very dangerous, know. Oh, I put some numbers, which are reasonable. These are equations, we use second k. So, this is; what is this? Now, Q, this Q, correct, this P, correct. Now, I will, so this is Q, this is P, so is it 0.5555, too many 5s. So, the numbers are decent, formula is correct, is it correct? 15 minus A1, is it correct?

Now, what is the big deal? You plot to the two equations, how do we solve, how do we show it now?

Student: (( ))

I should use again this equation from A1, I should be able to get the P right from A1. I should able to get a Q, what is, A1 is P.

Student: Q

Tell me what is A1 is?

Student: Q

A 1 is Q, ok and B1 is P. Now, I should be able to get P from A1, that is, starting from, starting from Q. So, using the, which equation it be used here? From, from the other equation I have to rewrite. What are the other equations? So, P equal to 10.7, alright. What happened? Paste special? Tell me Abhishek, paste special, now what is this? This space is alright, that is (( )), where we go, should we make it bigger? (( )) is dicey. So, where are the insert shapes and all that you know, not picture, where are the arrows?

Student: (( ))

Text box?

Student: (( ))

So, this is the, this is basically operating point. So, this is basically P, correct. X-axis, P or this Q in meter cube per second, this is your P. I want to call somebody who is good in Microsoft Excel. I want somebody to do an exercise live now, that is, the duct characteristics are the same, but suppose, the fan characteristics are displaced 10 percent above and 10 percent below, your operating point will be the shifted. So, can we, can we take this equation, the equation given in blue. Suppose you want to multiply by factor of 0.9 or factor of 1.1, then graphically it will depict how the operating point changes. Who is going to volunteer? Can we draw two more curves? Can we draw two more curves on this?

Student: Yes.

With the same value of, with the same x, X-axis values, I want to, I want to generate two more fan curves for the same duct curve and related to the original fan curve. Are you getting the point? This curve, if it gets shifted here, what are the operating point? You know, that goes up just graphically seeing that. Can somebody do that or I will just give up and go to the next problem?

Student: Give up

Give up, that is good, that is the spirit. So, this is how you did on this, how you do it on Microsoft Excel. There is no need to save the work; we can always generate the work. More than two variables it is difficult to show. You are supposed to answer one question. What happens to d dash of alpha? So, this is the solution to the problem.

So, people who have completed, completed this, let us do the last part of the problem. Examine the stability; examine the stability of the system at the operating point. This is problem number 6, right; problem number 6. Whatever you have done so far take it as part a, part b, examine the stability of the systematic operating point; examine the stability of the system at its operating point. What do you mean by that? So, if I start with an initial guess, which is close to the correct answer, but which can be left of the correct answer or right of the correct answer, whether it is proceeding to the same operating point, that you will have to numerically test. That means, you have to go back to the same equations and see whether we over shoot. Are you getting the point?

If you over shoot, suppose you start with, you go to 360. When you go to 360, after 1 or 2 iterations, does it come to 354 and 5.97? That is one way of checking it. The other is, instead of you approach from 5.99, 5.96, so you approach from 5.8 or 5.6, whether it goes. Just complete this, do this exercise, so that this solution is complete. How to determine the operating point of a system using the method of successive substitution? How do we show this graphically? Why one operation, one, why one information flow diagram is not working, because of the first derivative, and then, how to examine this stability of the system and its operating point?

Yeah, but now for the sake of completeness you finish it. I know, that we have started from 3.8 and, but I want you to do specifically, examine very close to that. Instead of 5.97 you can start with 5.85 or something and go to 6.15. We start with 5.85 and 6.15 or you quickly coming back to 5.97 or you start from 340, you also start from 360, whether you are quickly coming back to 354.2.

So, so it just takes 5 minutes for the notes is also complete, so that your understanding is complete and then, we will go to one more problem, that will be the last problem on successive substitution and tomorrow we will complete that and start the Newton-Raphson method. Please complete this. So, I will write the question on the board.

So, that we still get the same answer. Done?

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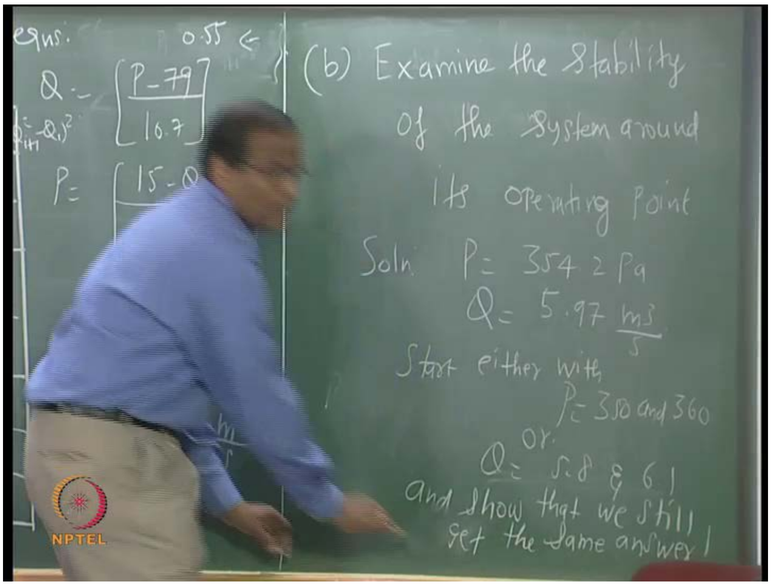

Yes Deepak, P of x equal to..

Student: (( ))

No, no, you will, what are the other information flow diagrams? Let us finish this, this is also being recorded, so…

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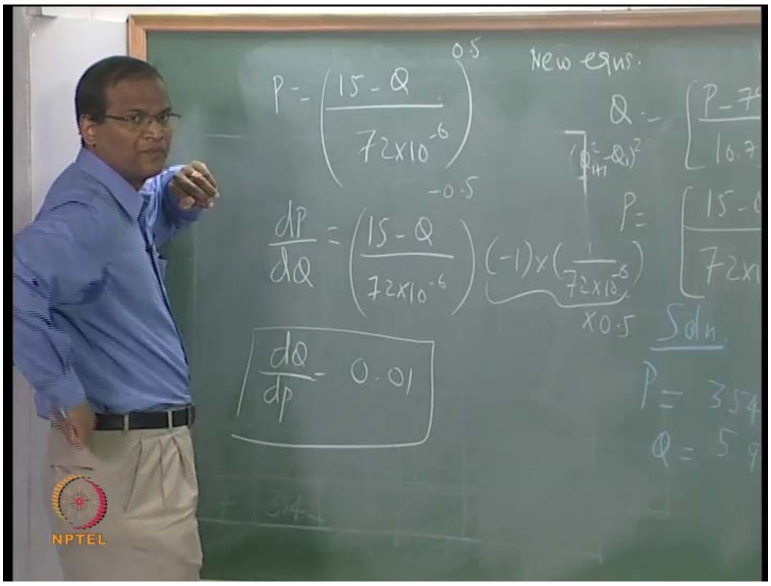

So, you are ok with this space, I am, I feel that we are going little slow, but it is ok? As people are examining we will also try to answer the other question, why the other information, why this information flow diagram for example, work? What is your dou P by dou Q or dP by dQ, right? So, you will just say 15 into minus 1, right, that is, it into that, that is there into which is multiplied, yeah is this coming? Please evaluate this, then…

Student: (( ))

Yeah, you have to take Q at the initial starting one, 26, let us see, then dQ by dP, there should be only one value. For this second one is…

Student: (( ))

So, initial condition is, whatever, what did you assume starting point? If you use P is equal to 300, then this Q must be equal to that, whatever you get from that. Yeah, one of the two initial conditions you have to use. Now, if you do this what are you getting? The other one is 0.01. So, what are the derivatives is very small compared to 1, it will not grow. The other one is to find to create some mischief, but eventually, after few iteration, they will reach some balance. But, if you look at the other information flow diagram, I think you get very, very large value for both the derivatives. I got some 119 or something; it is very, very high values.

So, that is, this is not an exact mathematical justification of why it is not working. This is, but this gives you an understanding, this gives approximate understanding or why certain premature flow diagram will not work. As you know, because P and Q are continuously changing to every iteration, these things will fluctuate. It is highly non-linear and so on, but one of the derivatives you are showing promise. But, if you go to values, which are closer, but we are actually reaching the correct answer of, if you up, if you work this, work this derivative out for the 3rd or 4th iterate, both will be less than 1 and that is why you are eventually getting converted.

Student: (( ))

No, one is 119.8 or something.

Yeah, but, what I am telling you, you are to do this for each of the iteration and see how it progressing, right. So, the previous information flow diagram, when it is diverging, if you look at the derivative, it will be enormous, that is why, it is diverging. But, here those first one or the two derivatives are, was more than 1, subsequent derivatives, it will die down, it will settle down.

Student: (( ))

Yeah, actually, already there is, already we screwed system by not allowing the error to propagate because you are raising something to the power of 0.5. When you raise something to the power of 1.8 or 2, I mean, mischief is guaranteed unless you are so smart, that you start at 354, you start at 354 Pascal and 5.97 meter cube per second. Then, you can use, I do not know whether information flow diagram works when you start very close to the initial, when you start very close to true value, right. So, so that is why, so this is the good exercise in the sense, that it tells you how to rewrite the governing equations in such a way, that even when you way off from the final answer, because you are raising something to the power of 0.55, so it constraints or it bounds the errors. Are you getting the point?

So, you can work out, you can Q i and Q i plus 1 and delta, delta Q 1, the error, how it propagates, you can write it mathematically and this is why this information flow diagram works, whereas the other information flow diagram does not.

Already 10:57 we will close, tomorrow we will solve another interesting problem. Your truck is going up the hill. So, again there are two, two characteristics. There is a load characteristic, there is a load characteristic, the other one is the engine characteristic. So, these two have to meet and you are to get the operating point of the truck. So, we will solve this using successive substitution and proceed to Newton-Raphson method.