Design and Optimization of Energy Systems
Prof. C. Balaji
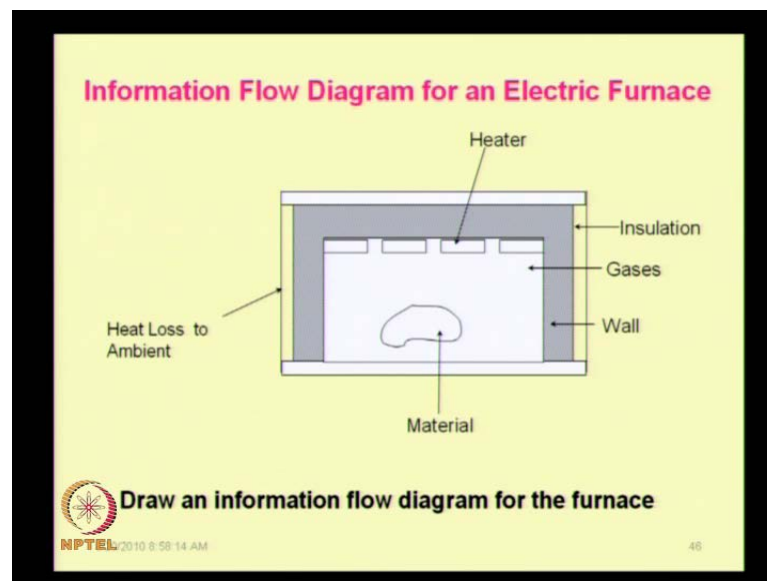**Department of Mechanical Engineering**
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**Lecture No. # 06**
Successive Substitution Method

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Good morning. So, yesterday we were talking about this information flow diagram. We considered a problem where we looked at the information flow diagram for an electric furnace.

So, I think this was the problem where you got a material, which is to be, which is to be heated or heat treated or whatever. So, you have heaters, then you have got this wall material insulation and I asked you to draw an information flow diagram for the furnace without having to go through all the equation. Just identify the various blocks, various components of the system and then using arrow marks, I asked you to identify the heat transfer processes, which are taking place.

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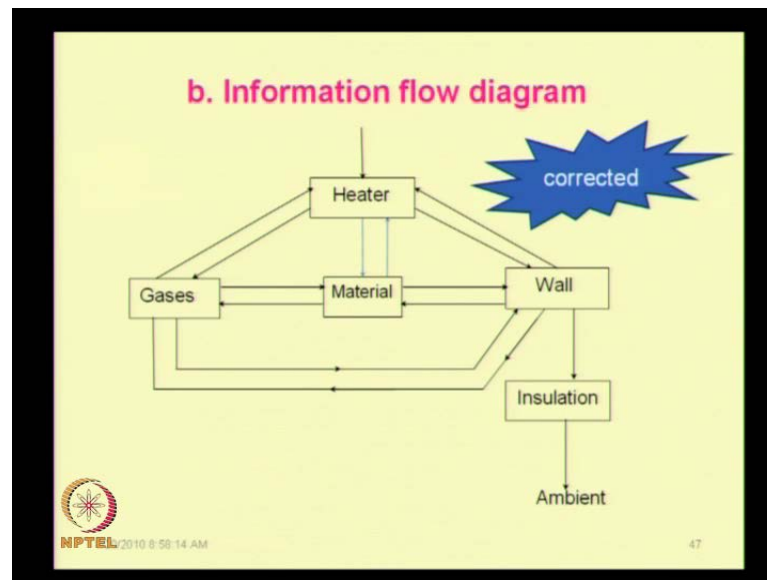
There were some issues in yesterday's class, so we will correct it. We have the correct or the corrected information flow diagram. Please, let me know if this is fine.

So, I think there were two basic problems. So, the heated to wall, the heated to the material, the heated to the material, you can have radiation, you can have radiation. And since all bodies above 0 Kelvin radiate heat, radiation should always be indicated by a two-way arrow. Radiation will see only something approaches 0 Kelvin. Unfortunately zero Kelvin has not been reached and 0 Kelvin is reached, we will have other problems, like we cannot define properties. Insulation to ambient…

Yeah, yeah, about that is ultimately going on, but you can take it back, there is no problem. But generally, the ambient is to indicate, that nothing is coming back when it is coming back. So, that means, suppose you are considering a satellite or something, then solar radiation or cosmic radiation coming, it will indicate that. I think, without any loss of generality we can say, that it is alright, wall to insulation, wall to insulation is only conduction, right. So, we indicated by one-way arrow. Then, there is a convection and radiation from the insulation to the ambient.

So, whenever you design an insulation system, there are other issues. For example, when you are carrying hot water, when you are carrying hot water or steam in a pipe, there are some regulations. When you design insulation system, the outside temperature should not exceed more than 60 degrees or 65 degrees that is an industrial standard because if

somebody goes and touches, then we will get scalded or burnt and so on. So, the, so you fix the outside temperature 65, then you turn around and calculate what will be the, what will be the insulation thickness and if you decide on a particular type of material and then there is an optimization, which is possible for various values of thickness of the material, for various types of material, you can work out the insulation. Some materials are banned, for example, asbestos is carcinogenic and so on. So, there are lots of issues, which will be outside engineering when you have finally decided on what is the appropriate system you want to use. Are you getting the point?

Now, we will proceed with our successive substitution, but I will first dictate the problem, you take it down and then, I will show the same problem on the PowerPoint. You start working and I will also work it out on the board, so that we are able to convergent on a particular solution.

Problem number 6, system simulation. Consider an electrically heated wire of emissivity 0.6, consider, consider an electrically heated wire of emissivity 0.6, full stop. Under steady state conditions, consider an electrically heated wire of emissivity 0.6, under steady state conditions the energy input to the wire, under steady state conditions the energy input to the wire is 1000 watts per meter square, under steady state conditions the energy input to the wire is 1000 watts per meter square of surface area of the wire. The energy input to the wire is 1000 watts per meter square of surface area of the wire and the heat transfer coefficient, and the heat transfer coefficient to the ambient is, the heat transfer to the ambient is 10 watts per meter square per Kelvin, 10 watts per meter square per Kelvin. The ambient temperature is 300 Kelvin; the ambient temperature is 300 Kelvin.

1, what is the energy equation governing the steady state temperature of the wire? 1, what is the energy equation governing the steady state temperature of the wire? What is the energy equation governing the steady state temperature of the wire? B, B or 2, using the method of successive substitution, using the method of successive substitution, determine, using the method of successive substitution, determine the steady state temperature of the wire, using the method of successive substitution determine the steady state temperature of the wire.

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So, people who have not been able to be follow this or people who came late, whatever I dictated now has been put up on PowerPoint. For the sake of completeness, I added one more part to decide on an appropriate stopping criterion. To the critical data, it is losing heat by both, convection and radiation. Surface emissivity is given, h is given, surface area is not required because I have given power density as watts per meter square.

So, first draw a sketch, there is no need to draw an information flow diagram. If you want you can, it is only a single component, it is not very difficult. You can go ahead and draw the information flow diagram or a sketch of the system. Indicate all the heat transfer processes, which are taking place. Write down the governing equation, and then start solving.

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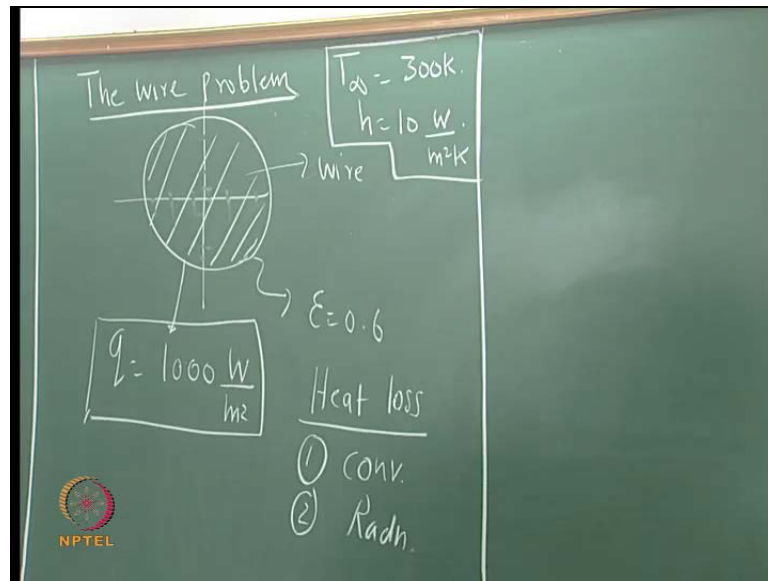
Identity the two physical processes, heat transfer by convection radiation. The data is given, the heat flux is 1000 watts per meter square, emissivity is 0.6, this is the wire, and the ambient data T infinity is 300 Kelvin, heat transfer coefficient between the wire and the ambient is 10 watts per meter square per Kelvin.

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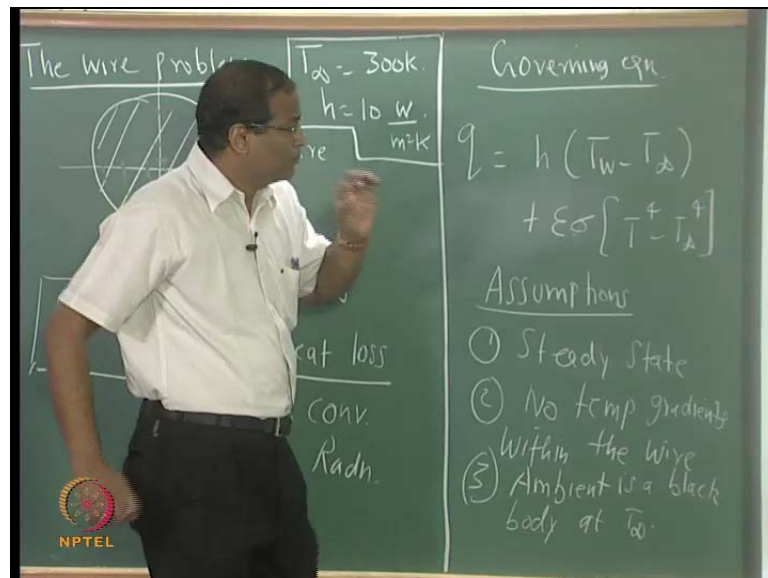
So, what is the solution of the first part? What is the governing equation? q equal to (( )), h…

When you write an equation like this, what are the inner assumptions in that? What are the inner (( )) assumptions? How could, how could you just write it like that? Steady state is alright, steady state, number 1 then. Vinay…

Student: (( ))

Wire

Student: Wire as (( ))

No, temperature gradient within the wire, which is which is not really correct because it is generating heat at the rate of so many watts per meter square. So, where will the wire be the hottest? Center, but we are not considering, all that is not central, because this is a problem on successive substitution. We are not getting into heat transfer; we are not getting into heat transfer debate. No temperature gradients within the wire. Anything else, what about the radiation?

Student: (( ))

Emissivity of the surroundings is 1; that is how you get this. The emissivity of the surroundings, that is, the surroundings the ambient is considered to be a black body at T infinity. The most important thing is, the other most important thing is, watch carefully, the ambient temperature for free convection is the same as the ambient temperature for surface radiation. Often times, without realizing we just take this for granted. It is possible, that in a room like this you may have T infinity for the convection, but because of reflectors and all these things, you may have a different surrounding temperature in, in, so far as radiation is concerned. But generally, unless otherwise stated, the T infinity for radiation can be considered to be the T infinity for convection. This is not always true.

So, already I have written this, ambient is a black body at T infinity. Here epsilon term, that is why, if you know heat transfer properly, you will know why that epsilon is coming and epsilon is only of the body. What is your doubt?

Student: (( ))

There is no epsilon, that epsilon is only for the, epsilon is only for the, for the body. People who take in conduction and radiation, those I spend an hour, half an hour as so, I, explaining how this formula, why this formula is the most abused formula in heat transfer. Simply, people say radiation epsilon sigma T to the power 4 minus, I derived this from first principle using two body enclosure and I proved in my class, in my conduction radiation class, that this, so many assumptions are there. So, I do not have the time now, we are also live on this thing, but if you want I can show it to you at end of the class. So, you have to use a two body enclosure, you can just show by simple physics, so that epsilon is equal to 1.

His doubt is, since epsilon is outside the bracket, he is, he is having a problem whether there is an epsilon of the surroundings also. There is no epsilon in so far as, surroundings is concerned. See, this problem can be solved either using the network theory or using what is a radiastic radiation method. I teach only the radiastic radiation method using the radiastic radiation method in 15 minutes. You can show this is the formula you get for, for a small, small body in a large enclosure, alright. Now, go ahead and knowing fully well or knowing fully well, what the assumptions are, go ahead and start doing the iteration.

Serial number T i instead of T w, you can just take it as T. T i, T i plus 1, T i plus 1 minus T i whole square, that is error criterion decided, decided on an appropriate stopping criterion. And I can tell you, that two types of information flow diagram are possible and in one of the information flow diagram the solution will diverge. Some of you will, will tell me, Sir, this will, it is not converging. Then, we will have to turn around and use the other information flow diagram. Please start.

So, if you recall we are still trying to study system simulation. I told you, that I will teach you system simulation by looking at two or three techniques to solve the system of non-linear algebraic equations. We started with a method of successive substitution. Yesterday's class, we started with a simple mathematical problem x equal to 2 sine x. Now, we proceeded to a problem in thermal systems or most, more specifically a problem in heat transfer, which has got one component. After you finish, as soon as you finish this we will go to a thermal system, I mean, a fluid thermal system with two components, so, so that will reinforce the, once that will reinforce your understanding of this method of successive substitution.

Now, what do you want to?
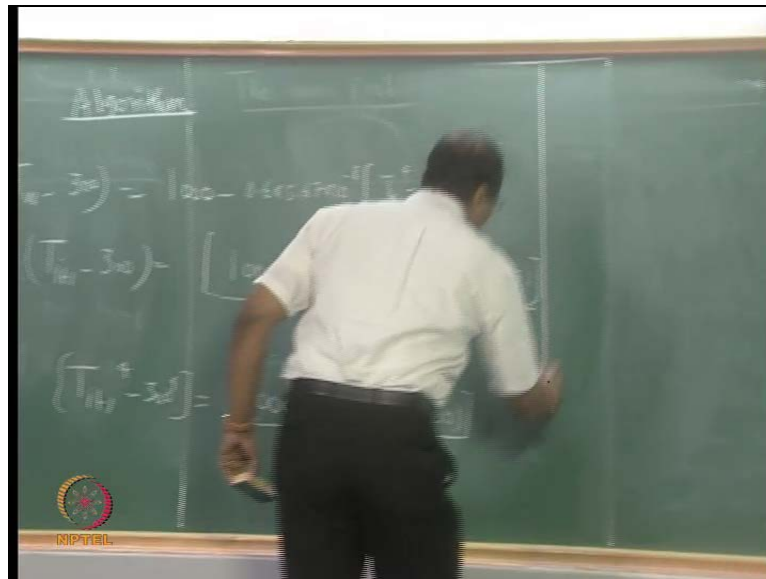
So, what you want the algorithm to be? I just…

Is it ok? Or…
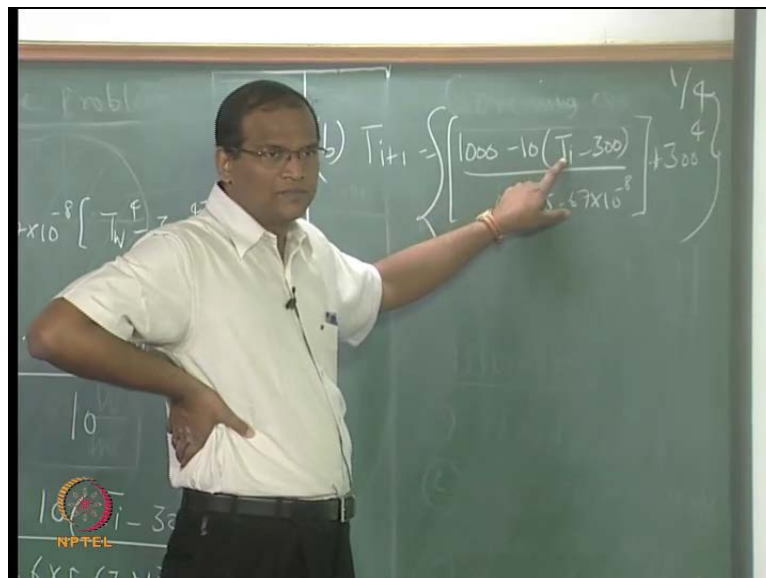
Is this correct? But let me guarantee, that one of the two algorithms will fail. Negative, yeah, Deepak is having trouble, Vinay are you having trouble? It is a highly non-linear problem, why is it not linear? Right side you got T to the power of 4, radiation is highly

non-linear. I do not think, you have solved problems like this in heat transfer course are, it is very unstable. So, one of the two algorithms will cup, we will not investigate why it is so, why one of the two algorithms is not working and so on. But suffice it would to say, that because of T to the power of 4, one of the two algorithms will not work. So, because the errors will propagate, but you know, that it is a physical problem.

A wire is cooling, it has got emissivity, there is natural convection, there convection and radiation, so it has to attain a physical temperature. If at all you get negative temperature, it is not because of Physics, 1000 watts per meter square is a reasonable flux, emissivity is reasonable, ambient temperature is reasonable, h is reasonable. Therefore, you should get the reasonable temperature as the answer. If you get an unphysical solution, that means, a problem is not with the Physics, but the problem is with your algorithm. So, we can say, that this is a, this b. Deepak which one fails?

First one fail, second one fails, second one fail. So, let me, so people who already started, actually I want it to try both, you will realize that. Now, why you are smiling? It is not a painful problem. So, you, you can very well realize, that even one component is not so simple as you thought. Suppose, I have two component system, if it is non-linear, it will give lot of trouble. Two component system if you give you in the exam quiz, whatever, it will take half an hour, 40 minutes.

Suppose, I give many terms in the equation, and it is going to take a lot more time. Of course, it is eminently, it is, it is eminently doable by using a script file Matlab or writing your own code and all that, but, but for the purpose of exam you have to use calculator and we have to use calculator and pen and proceed, alright. Yeah…

Student: (( ))

Is it?

Student: 366

366

366.4 or something. Let us solve this. Vikram got it? Which one, which algorithm you use? First one, how many of you use a second one? What it, what happen? What is the problem there?

Student: (( ))

That is the problem. So, because of this, this, if the T increases, what is a maximum possible for this? The moment it exceeds 400 it is gone. The moment it exceeds 400, then (( )) algorithm is in struggle because it is over shooting. I mean, if you, if you actually take the slope, another things, you can investigate mathematically, but that is not central to our our discussion here.

So, if you solve this problem using the method of successive substitution, if you encounter a difficulty, if you get an answer, which is negative or is diverging, you are not able to proceed further. You should not say, that this problem has no solution because from what, from the data given to you, given to you, it is so clear, that there is a solution, which is the problem. Immediately, you should think of the other information flow diagram.

What, what, what is the information flow diagram I am talking about here? Basically, whether q convection is equal to total q minus q radiation or q radiation is equal to total q minus q convection, these are the two information flow diagram. So, here I am saying, q convection is q total minus q radiation. Here, I am saying, that q radiation is q total minus q convection. It so happens, that one of the two is not working. So, when one of the information flow diagrams is not working, go to the other information flow diagram.

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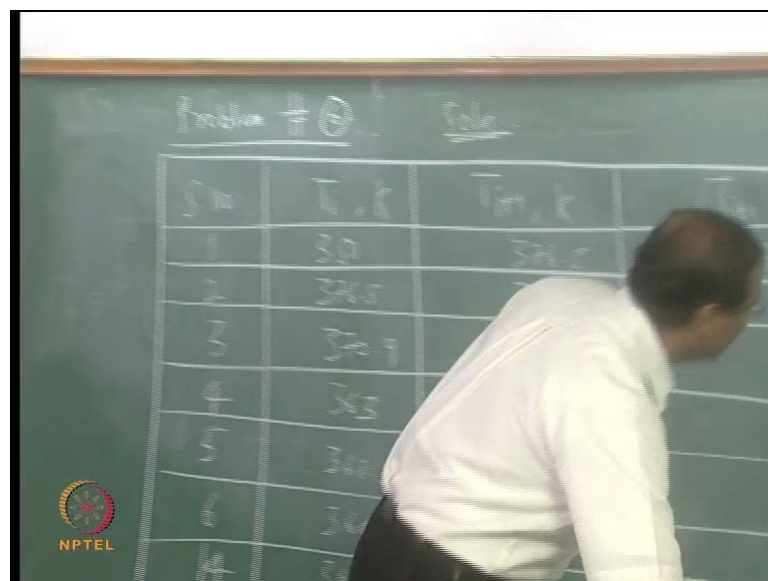
What is 376.5? How many of you started with 350 and why did you take 350? The reasonable, 350 appear to be reasonable. Why are you… It cannot be at 1000 degree, 1000 Kelvin or something. 350 appear to be reasonable. So, so we should put Kelvin. What about this unit? 350, you get 376.5. What you get? How many of you are troubled? Why are the, why is the class so silent? Venugopal, you got it?

Student: (( ))

Vaibhav got it? Vikarm has got it, you? So, 376.5, what is it now, is too much, what is this? 700. I worked it out last night, so I have the numbers. 359.1, I am allowed not to write this now because I am the teacher, but you have to. So, you just, because I know it is not converging, 359, 4. So, do not feel lazy, you have to fill up all this. Number 6 was 368.5, 364.8, 14. Did somebody proceed beyond the 14th iteration? 366.4, it is like 90, 93 degree Celsius, alright. (( )), for those people whose information flow diagram corresponding to algorithm 2 or algorithm b cup, you please go back and substitute this 366.4, and see whether now it works. Exactly at that point it will work, just check, it should work, right. Any other point?

It is very unstable algorithm that is the problem; it is very unstable. That is basically, because if you take f of x, sorry, f of t equal to q minus total q minus q convection minus q radiation, if you take dou of f of x or d of f of x by d 2, that is, if you plot, that fellow is a very strong function of temperature because it is 3 t cube and all that. So, he is not behaving properly, he is misbehaving, that is why you are not able to. So, if you want to investigate easily, you can go to Matlab and investigate why it happens.

You just take the first derivative, it is highly oscillatory. So, you have to, you have to always guess in the right region and it will not allow because of t to the power of 4. Once you start with, unless you are, unless you start a 366.4 or 366 itself, it will swing to the right side or the left side. And if you are so, if you are so sure, that you have to start with 366.4, then Swamiji (( )), this is the answer I mean. So, you do not require method of successive substitution because you know everything.

So, that is why, yeah, now 366.4, did you substitute in to the other one? Please, please check that. So, why x equal to 2 sine x? It was the very simple problem and I wrote, we wrote the algorithm yesterday and solved, there are no issues; note, there are issues of divergence already.

So, all the information flow diagrams are not the same, some are more equal than the others. It is possible for you to get a solution with one information flow diagram while it is perfectly possible for you not to get a solution, the other information flow (( )). In the exam if you give a highly non-linear problem, you start with an information flow diagram and you do not get the answer, you are unlucky. Then, you have to switch over to the Apriory, I do not know whether to, otherwise you have to take the derivative and all that. The time taken for that, you might as well start with this information flow diagram, get (( )) and proceed to the point at which the solution diverges and turn around and say, that this this information flow diagram does not work, so I will have to go back to the other information flow diagram. Is it ok?

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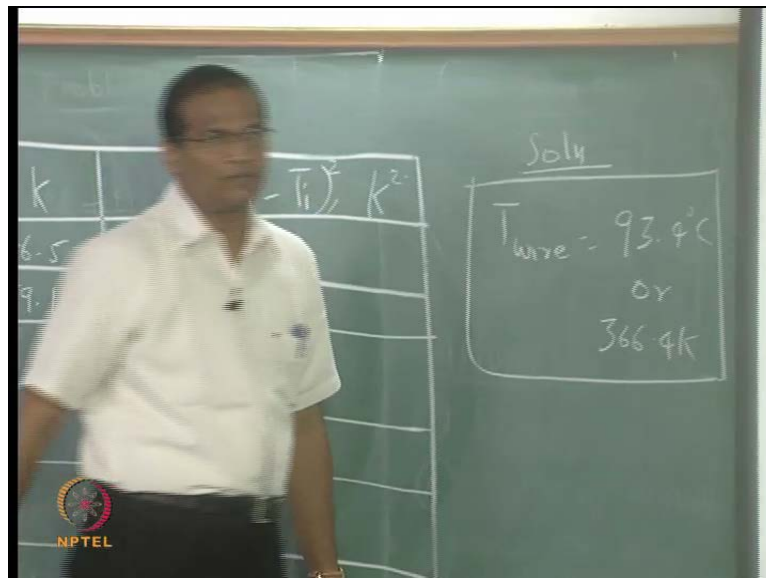
So the solution is, 90…

Does it work? Well said, so if you are started, probably 366 point, 367 or 366, may be it work, may be it would have work. If you start with 350 or if you start with 400, it may, anyway, you cannot start beyond 400 because it becomes negative. So, these are the issues associated with this, fine.

Now, we will go to a problem where we take a real fluid thermal system, namely the fan, and namely a fan and duct system. So, this is going to take about 40 minutes, you are not going to able to solve this in today's class completely, but nevertheless we will start.
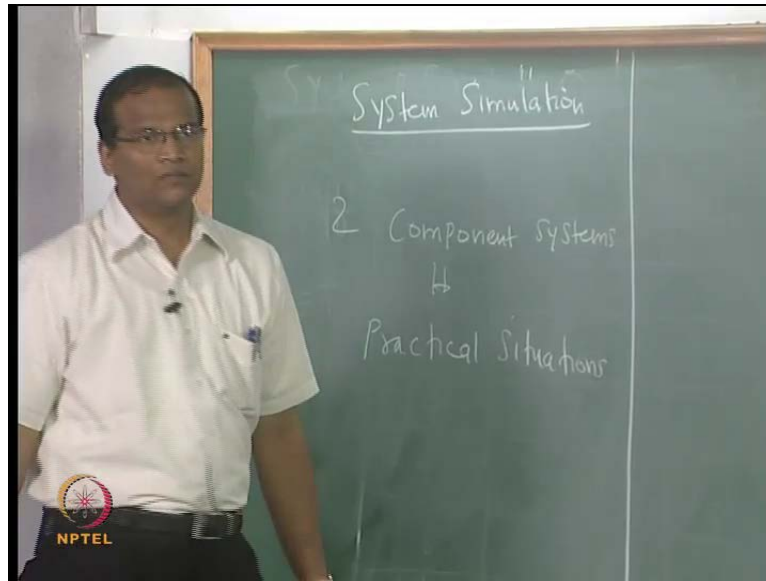
So, let us go to system simulation. Let us talk about some practical situation. You are designing a ducting system for a, you are designing a ducting system for an air conditioner on auditorium for the ICSR building. You are deciding about the fan capacity. I am not talking about the chiller capacity, and how many tons of refrigeration you want and all, you want to send chilled air. You have to decide on the ducting, fan and ducting system, correct.

Now, it is possible for you to calculate the total length, total length of the duct, how may bends are there. Then, flv square by 2gd, and then k flv square by 2gd and all, you can add all these and put everything as a functional velocity or discharge and get a formula for calculating a total pressure drop. If you have the total pressure drop multiplied by the discharge it is, and if you know the fan efficiency, if you know the approximate efficiency, you will, you will have an idea of how much of power is required. But for overcoming a particular head, for overcoming a particular head, you also, you also will know what is the discharge corresponding to that head from the manufacturers operating characteristics or these are called fan curves, you must have studied in turbo-machines.

Now, if you eventually have to decide have to design the fan and ducting system, the fan characteristics of the manufacture have to match the load characteristics of a particular design. When these two intersected at particular point, this is called the operating point that means, you have done the system simulation to determine the operating point.
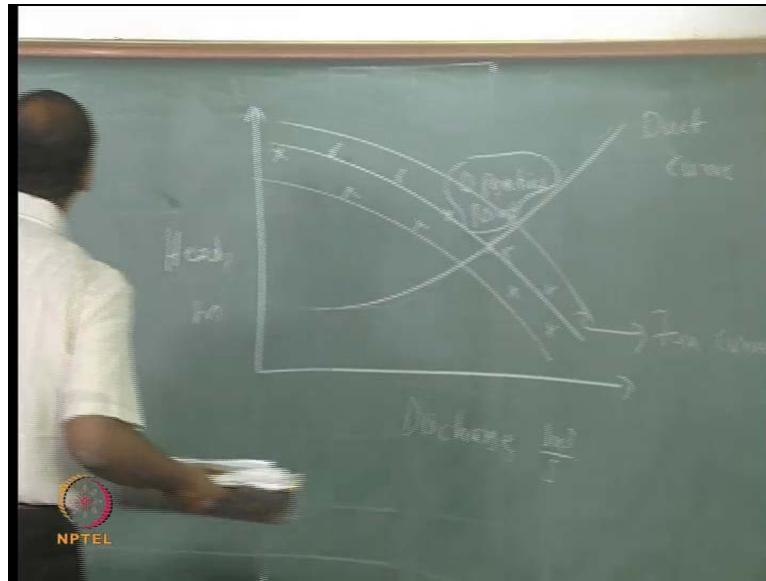
So, graphically, so if you consider, you, is it like this correct. So, this is what is, what is this? Fan curve, what about that curve? This could be meters or it could be, whatever, this will be meter cubes per second. So, this is the operating point. Using system simulation, we want to calculate this operating point. This can be easily calculated because this is based on a formula, right. If you know the friction factor, if you know the friction factor and all this, you can calculate. How do we get this?

So, this you have to get from the manufactures catalogue or whatever. How, how will he have got this? So, basically you have to do experiments. You will, you will first do experiment with no discharge, 0 discharge, then will slowly increase the discharge and you will find out the head and you will find, do an experiment connecting this. And so, therefore, you may get some points like this.

But if you want to do simulation, you just have point; it is no good for you because you want a curve, right. So, you will convert all this points into an approximate, into a, into a best fitting curve using principles of regression. That is why I said, regression is very important for system simulation, but if start teaching regression upfront before teaching system simulation, you will see, you will think what this course is all about mathematics and we thought we will learn design and optimization is only regression.

So, last year I first start regression and then went system simulation. This time I am swap. I will teach you system simulation, so that you get a feel, that regression is very,

very important to get this. You are getting the point? Once you have these two curves, then you determine the operating point, then you have to determine, you have to determine the sensitivity of your operating point. For example, it is possible, that with time or something there could be multiple fan curve can go up and down. This can also go on up and down depending upon the age and other things and friction factor may increase or the efficiency of the pump may decrease and so on. So, you will have to study the sensitivity of the performance of the equipment of the system with respect to the change the design operating conditions, alright.

Now, shall we solve problem on this. It is required, it is required, it is required to determine, it is required to determine, the operating point is required to determine the operating point of a fan duct system, it is required to determine the operating point of a fan duct system.

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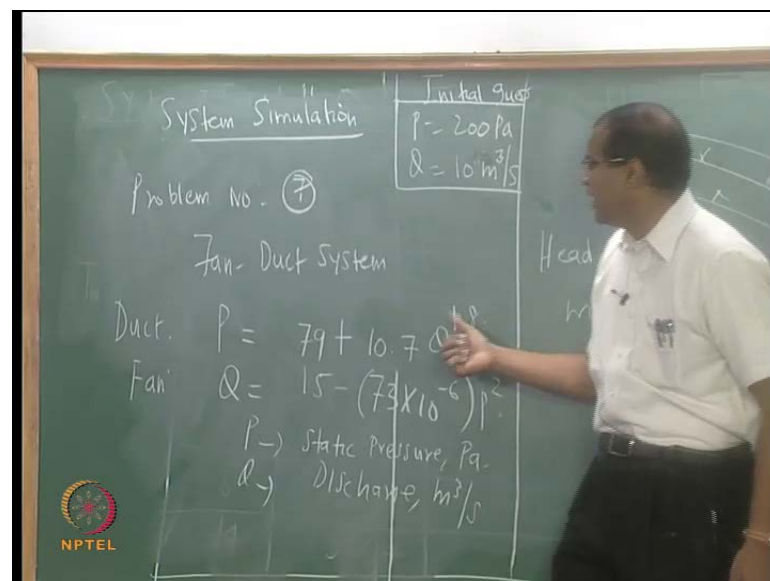
The equations to the two components are duct…

I do not know, I will just check, whether I have written 79 or 80. P is 79 plus, P is 79 plus 10.7 into Q the 1.8, Q equal to 15 minus 73, either minus 6 P square, where P is a static pressure in Pascal, Q is going to be the discharge good in units, good meter cube per second. A, draw the information flow diagram for the above system; A, draw the information flow diagram for the above system. B, using successive substitution, using successive substitution, determine the operating point.

An important thing here is the initial guess. Of course, using common sense you can arrive at an initial guess, but since it is a class room environment, I will give you the initial guess. You can either take P is 200 Pascal or Q is 10 meter cube per second.

First draw the information flow diagram, draw two components – fan and duct. Draw two rectangular boxes, something will go in, something will go out, then complete it, this is a, this is a sequential arrangement because output of the fan becomes input to the duct, output of the duct becomes input to the fan.

Two types of information flow diagram are possible, watch out two types of information flow diagrams are possible, please look at the board. You can you can use this equation to determine P, if you know Q. There is, but there is no hard and fast rule about this. I can say, that Q to the power of 1.8 is P minus 79 divided by 10 point7 or rather, Q is equal to P minus 79 divided by 10.7 whole to the power of 1 by 1.8. So, I could try convert this. Do not think that this is on the left hand side, this is on the right hand side, I cannot do anything about it. This equation can be used, to cut a long story short, this equation can be used to calculate P given Q or vice versa. By the same token, this equation need not be necessarily useful to calculate Q, you can also calculate P from Q depending on this.

If you want to calculate P from this equation and Q from this equation, that is shown information flow diagram. If you want to swap, that is another information flow diagram. And since you have got non-linear powers, one is P square, one is Q to the power of 8, one of the two information flow diagrams will, will not, one of the information flow diagram will not, will not work. So, let us see who is unfortunate and, and in tomorrow's class in the, I mean in next Thursday, Monday is holiday, Thursday's class. We will solve this problem and investigate now, why one of the information flow diagram, why is it not working. So, please draw the information flow diagram first, and now any guesses on how you got this Q to the power of 1.8.

Student: (( ))

Correct, regression physics, what is the physics with this 1.8? 4 flv square diameter is fix, you know, I usually in this, I am fixing that dimensions, it could be a diameter of pipe, it got be the, it could be the hydraulic mean diameter. If it is a rectangular duct or

non-circular duct, what is it actually? Actually, that P is the head, right. Shall we erase this?

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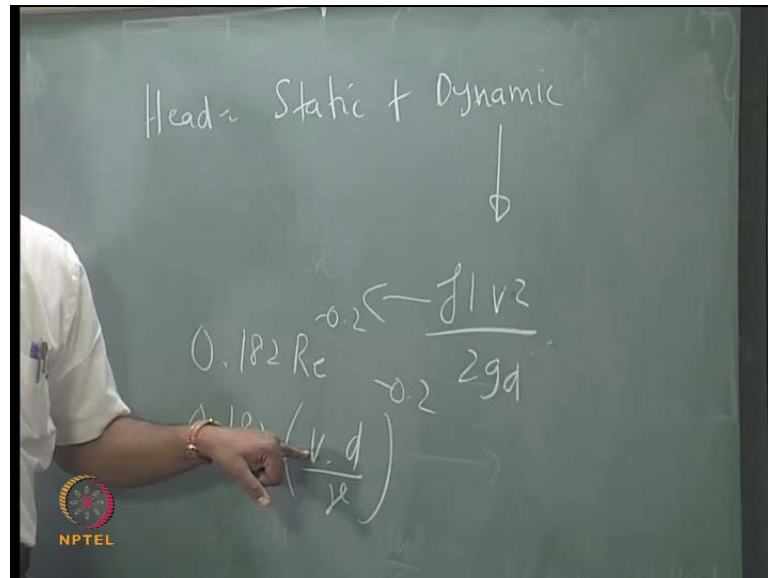
So, head… So, this dynamic, what about its f itself?

V square multiplied by v square multiplied by v to the power of, so what is it given, 1.8. What is this fellow doing? Static. So, using regression you can get that. So, I want it appreciate the physics behind this equation. This equation we cannot get it from physics, somebody, the manufacturer has to do the experiments, you will give it as a manufacture, in the manufacturer's catalogue. It is called the fan characteristics in turbo-machinery. You must have studied, that pump characteristics, blower, axial fan characteristics, blower characteristics, turbine characteristics, they have to give, this is the fundamental experiment they have to do and give, give to the user.

Now, go ahead and first, Vikram do not start solving, information flow diagram started, then proceed. If you finish the information flow diagram, you can start with the solution. I will draw the information flow diagram and stop with that today's class. This problem will take half an hour for you to solve because even if you get the right answer I want you to take other answer, information flow diagram and proceed to a point at which it will cup. So now, I will draw the information flow diagram.

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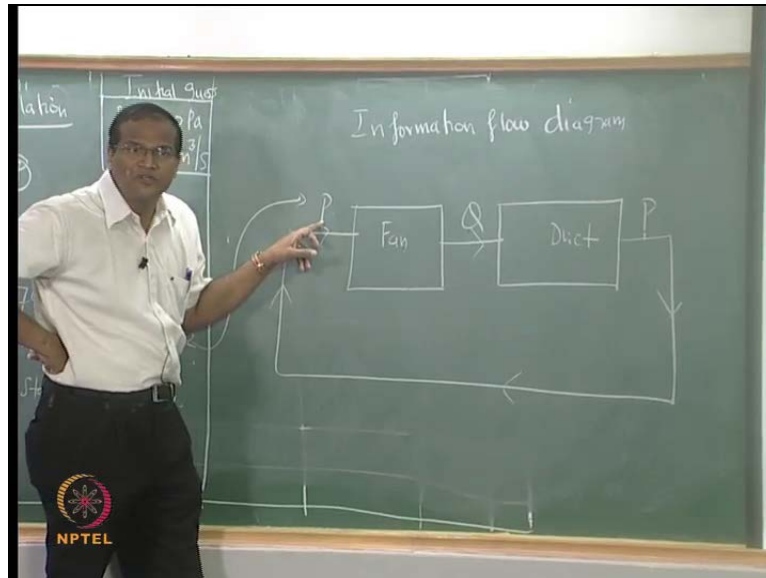
Fan, duct, yeah please tell me. P, Q, I am using exactly in the order, which was given to us in the board on the block board, right, that is information flow diagram. That is, I am using the fan characteristic to determine Q given P; I am using the fan characteristic to determine Q from P and I am using the duct characteristic to get P from this Q. So, whatever at the end of the exercise, whatever P I get, I compare it with whatever P, I assume to start with. If these are equal I can go home. They in general, they will not be the same, so I keep on doing this.

So, what goes through this is the P and the information on P and Q, this. As we proceed to the iterations, there it will stabilize, then whatever coming out will be the same as whatever was sent in, that results in the, you will have a situation where P i plus 1 minus P i whole square is less than equal to something. Is that enough for this problem? What should be the stopping criteria for this problem? Yeah, come on man, it is early in the morning.

Student: (( ))

Yes, Raghav…

Student: (( ))

So, also means how do you incorporate mathematically

Student: (( ))

So, and, or we can do this. So, sigma, that is good, sigma Q i plus 1, we will put some 10 to the power of minus 6, so that you keep on doing or 10 to the power of minus 4, 4 we will easily get. So, this look like, you think that is very, this is not the most important part of problem. You think, that getting the, using the algorithm and getting the solution, the most important part of the problem, but this is also equally important, because in real life nobody will tell you what is the stopping criterion is, it is for you to figure out, it is for you to get convince that you have reached the level at which the solution will not change.

This level of accuracy is ok. I am comfortable with this level of accuracy, because if you know what the true value is, you can compare your result with respect to the true value. And then, if I get 1 percent close to the true value, then I am fine, but when you are doing CFD competition, when you are doing simulation, the true value is elusive. You do not know what is the true value? So, you are always, you are always looking at the difference between successive iterations, but this can mislead, this can mislead you because it is possible for you to finally, beautifully, converge to a totally stupid answer.

So, convergence is necessity. Once you have reached convergence, it does not necessarily mean, that you got an accurate solution. Convergence means, with the present numerical scheme, if you proceed for 100 more iterations, you cannot expect a significant change in the answer, but whether that the answer itself is correct or not, convergence will not tell you. So, you have to do additional measure, you will have to validate your solution.

So, you, you apply your numerical technique to some other solution, some other problem for which either somebody else has done experiments of a, which analytical solutions are possible. Then, for that case, after you run through so many iterations, after you incorporate your stopping criterion, you get a solution, which is exactly the same as what other people have reported or an analytical solution. Then, you say, for this particular problem when I apply this stopping criterion my solution work. Therefore, when I apply to a natural problem for which I do not know what the true solution is, there is no reason why it should misbehave.

So, we will stop here. So, in the next class in the next class, we will solve this problem, and then we will use both, information flow diagram. There is only one information flow

diagram, I can draw the other one and then, will go through some iteration, and solve this. I, as is our practice, I will put it on Excel, and I will also demonstrate it with Ms Excel.