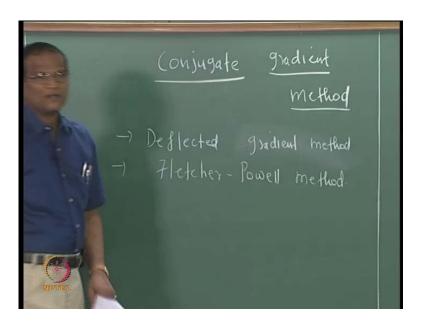
Design and Optimization of Energy Systems Prof. C. Balaji Department of Mechanical Engineering Indian Institute of Technology, Madras

# Lecture No. # 35 The Conjugate Gradient Method Contd...

So, we will revisit the conjugate gradient method. So, in the last class, towards the end, we wrote the algorithm very quickly and without paying much attention. So, the first few minutes, we will discuss the algorithm. Of course, I do not have time either to prove the, prove how you got that beta or to solve problems. It is basically an extension of the steepest descent or steepest ascent method; except that, after first two steps, you will correct; you will deflect the steepest gradient and proceed in a new direction. This is quadratically convergence; it can be proved. The proof is available in many books.

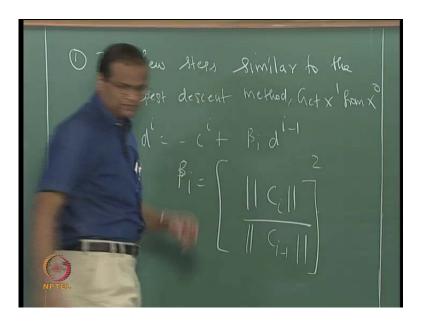
For example, you can look at the book "Engineering Optimization" by Ravindran, Ragsdell and Reklaitis; Reklaitis from Broadwig university; he is one of the stalwarts in optimization; he worked out lot of algorithms; that book is pretty good. Or, "Optimization Engineering Design" by Kalyanmoy Deb, Prentice Hall, that is available; that is a reasonably good book; or, "Optimization Techniques" by S. S. Rao. The conjugate gradient proof must be available in any of these books. We will quickly go through this conjugate gradient.

### (Refer Slide Time: 01:18)



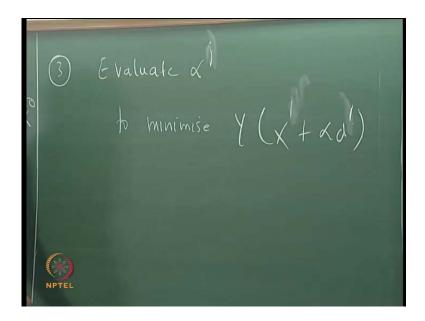
So, it is basically a deflected gradient method, also called the Fletcher Powell. Then, somebody called Davidan, not Davidson, who made some improvements to this, which called Davidan Fletcher Powell - DFP method - which is state of the art, for many of the problems. And, there is no need that your objective function must be quadratic. It is very rarely to encounter a quadratic function. But, what these people have done is, they have shown that the algorithm can work for non-quadratic functions also. So, the DFP method is pretty powerful; you can write papers out of that.

(Refer Slide Time: 02:39)



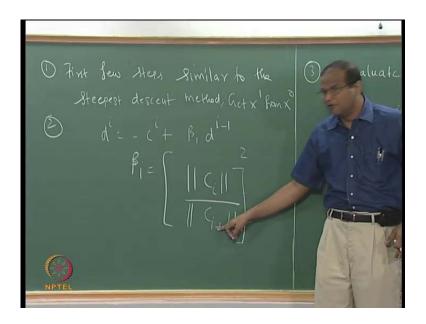
Now, when I discuss this, we said that first few steps, the first few steps are similar; steepest descent or steepest ascent depending upon whether you are looking at the, looking at a maximization or minimization problem. Then, so, you get x 1 from x naught, right. And, then, d i, it should be i minus 1, on the right hand side; that is a correction, you have to make. May be, I wrote it as d i, right; does not make sense; it has to be d i minus 1. So, where beta I; did we put C, subscript or superscript? Does not matter, so long as you understand.

(Refer Slide Time: 04:05)



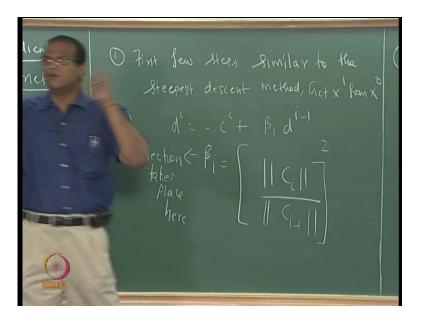
So, now what will be the third step? You get the value of beta, and then proceed to; you get the next value of, evaluate the value of alpha I; or, since, we started with 1 and 0, we can call this as alpha 1. So, the key point is d 1; d 1 is not equal to minus del y; C is equal to del y or minus del y? Student: C is del y. C is del y, d equal to minus C. Therefore, originally in the steepest descent or steepest ascent method, this d was minus del y; but now, this d is the modified d; how was it modified?

### (Refer Slide Time: 05:22)



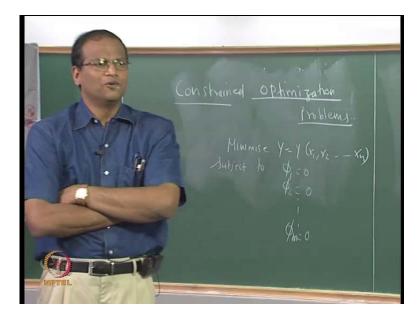
You add the d to the beta i into; beta i, is the deflection parameter. How does it get deflected? Depending on the del y at the current iteration, divided by the del y at the previous iteration; are you getting the point? That is it. So, it seems very logical, it can be proved; it requires 15-20 minutes, I do not have time; if somebody wants, I can take that proof and put it on modal. Let me see, if I have time, I will put it; there is some, I created something for this course; I will try to postulate on that. So, this can be proved.

(Refer Slide Time: 05:58)



So, this is actually where, deflection takes place here. So, first to start with, how many times should we get C i to start working with the conjugate gradient? Student: 2. So, minimum you have to start with 2; it cannot start on its own; it cannot start at initio from 0. So, you have to start with the steepest ascent or steepest descent; evaluate the C, 2 times; and then only, you have an opportunity to get the beta and then deflect the d. Is it alright?

Now, so, though we have these algorithms are impressive, we are not able to handle; so far, we have not been able to handle constraints, right; all these are still unconstraint. How do you handle constraint optimization problem?



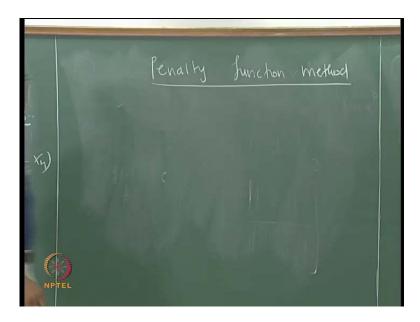
(Refer Slide Time: 07:04)

Needless to say, we are having a multi variable; we are having a multi variable constraint optimization. So, minimize; assume that we are going to handle, we would have only equality constraints. First, we will see how we can develop an algorithm for handling equality constraints; then, we will extend it to handle inequality constraint. Inequality constraints were lot more difficult to handle, Kuhn Tucker conditions and all that, did you remember? It leads to a lot of pain. Equality constraints are easier to handle, except that, the number of equality constraints must be less than or equal to the number of variables.

Now, there is a very interesting technique, where we transform the objective function; we transform the objective function and convert it into an equivalent unconstrained

optimization problem. I come again, we transform the objective function in such a way, that the constrained optimization problem becomes an unconstrained one, and then we solve the unconstrained optimization problem with the state of the, our technically conjugate gradient or genetical algorithm or whatever; what is this concept of penalty?

(Refer Slide Time: 08:51)



So, we introduce, what is called the penalty function method, is one of the most powerful techniques for handling multi variable constraint optimization problem.

(Refer Slide Time: 09:11)

Ophimiza Constraince

Suppose, I want to handle, suppose I want to handle this problem, but I want to use a conjugate gradient or steepest ascent, then I will have to transform my objective functions plus constraints into equivalent unconstrained optimization problem. Therefore, I introduce, what is called a composite objective function.

(Refer Slide Time: 09:27)

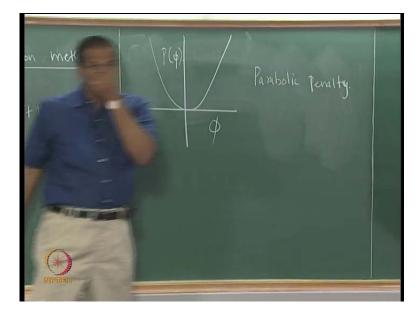
hunchon MINIMIZADO - Pm-) dealars NPTE

I introduce a composite objective function called V; V is nothing but Y plus P1 to P m are scalars, where positive.

What is the funda here? Each of these constraints is expected to be 0, because it is an equality constraint; if each of these phi takes on a value which is different from 0, I square it. Therefore, whether it deviates from the left side of 0, right side of 0, this phi square will always be positive; if you multiply by a positive quantity, P1 phi 1 square, P2 phi 2 square, all these things, will be positive quantities. These are all added to the original cost, because I am talking about a minimization problem.

So, these are all adding to the original cost. Therefore, V will be substantially higher compared to Y; if the phi(s) are not equal to 0. Therefore, the objective function is finalized for violation of constraints; this is the penalty. If your violation is too much, then phi 1, phi 2, phi 3, everything will be more, then if you multiplied by P- P is a scalar, which can be decided - it can be 5, 10, 100, 200, 300; the analyst will decide it; the person who was optimizer will decide it; we will come to little bit later. Therefore,

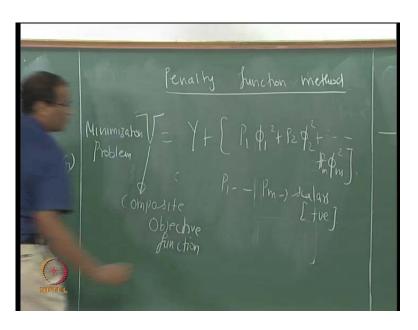
you finalize the objective function for violation of constraint. If all the constraints are satisfied, there will be no penalty.



(Refer Slide Time: 11:31)

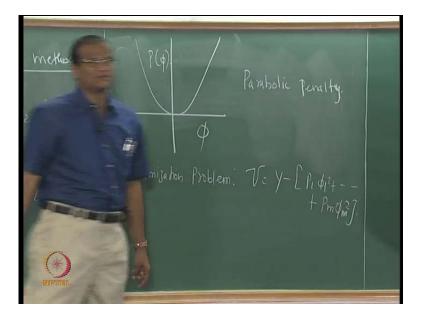
So, this is called a parabolic penalty, because a penalty is like this. Agreed? Now, you minimize V, instead of minimizing Y; that is, if you minimize V, by assigning different values of P 1 to P m, and then keep changing these values of P 1 to P m, and find out, and reach a stage where regardless of the values of P 1 to P m, you are getting more or less the same value of V, that means, it attains stationarity, with respect to this scalars also; are you getting the point?

(Refer Slide Time: 12:22)



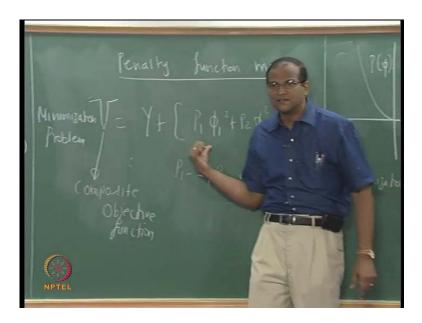
Now, we will write. So, this is called the composite objective function.

(Refer Slide Time: 12:41)



For a maximization problem, though, it may appear, though the minus sign and then positive sign appear to be incongruous, with the maximization-minimization problem; because for a minimization problem, intuitively you feel you have to put a minus, but actually we are finalizing, the cost has to increase.

## (Refer Slide Time: 13:32)



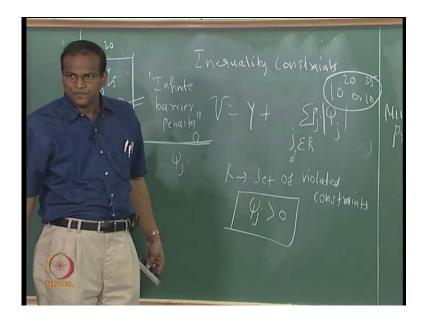
Therefore, for a watch out, for a minimization problem, it is Y plus; for a maximization problem, it is Y minus; because there is a penalty on the profit, therefore, it is minus; there is a penalty on the cost, therefore, it is plus; because it is minimization, your mind somewhat tells you that it should be Y minus; is it not so? If you argue out, it will become clear to you. Upon first glance, it seems incongruous; it seems that we have made a mistake; not so; are you getting the point? Did you confuse, or did you clarify?

(Refer Slide Time: 13:57)

Pasabolic Penalty 0

So, there are other penalty parameters, which are available; penalty functions, which are available for handling inequality constraint. For example, phi of x must be greater than 0. Suppose, there is a constraint like that, for inequality constraints. Now, we are going to solve problems with inequality constraint.

(Refer Slide Time: 14:23)



Inequality constraints. So, V is equal to Y plus, let us say, P, sigma. So, j is a part of R, where R is a set of violated constraints, for example. You can put a, P j, or you can put universal P, it does not matter. If you want to be this thing and put a P j also, which means, you take the modulus of this, and if it is getting violated, then you sum up all the violations, multiplied by; this will be very huge quantity, like, 10 to the power of 20 or 10 to the power of 25.

So, this constraint could be psi j greater than 0. So, if it is like this, the penalty P; the penalty will go like this; the penalty will be 0 here and it becomes negative; it will be 10 to the power of 20, 10 to the power of 25. So, the penalty will be selectively applied, it will always look at the value of psi, whenever it goes; whenever it becomes negative, it will be applied, it will be added to the cost. So, this is called an infinite barrier penalty.

Of course, you can immediately conjure up with new ideas, where you can say that, p instead of mod log of psi x, log of mod of psi x, and so on; you can come up with your own; so, your imagination is the limit; so, you can come up with your own concept of penalty or barrier, which will prevent. See, basically, all this is, basically, you are adding

to the cost, you are ensuring that the psi never become a negative; if the psi becomes negative, this fellow will, 10 to the power of 20, it will, it will become huge, and you will automatically correct it, such that psi is satisfied; are you getting the point?

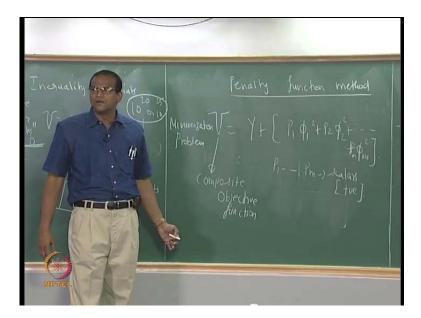
Student: You know, 1 P for each psi, right.

Yeah, yeah so,

Student: Here, we have only 1 P.

Yeah, because I am giving a very huge value of; if you do not like it, you can take the P j inside; I do not have any objection, but generally it is very high, what you are saying is all right, fine? So, but this could be; so, this represents the set of violated constraints; you ensure that, it is not violated; instead of checking each time, whether, it is violated or not, you put it as a part of the objective function, and then put, set up an infinite barrier; infinite barrier penalty, so that, it is not violated.

(Refer Slide Time: 18:02)



Now, who tells us the value of P? How do we choose the value? Without, already, we do not know many things; we do not know what are values of x 1 to x m, which optimize; now, you made the problem more complicated; we introduced additional scalars like P 1 to; for each of these constraints, there may be a Lagrange multiplier also. Is the solution worse than the problem?

Student: When the solution, which will behave, P equal to 0.

What is that?

Student: Sir, even for small evaluations, the penalty will be very huge.

Yeah, yeah, that is; that is, that is the intention; that is a intention, in the case of a, in the case of a inequality, because, I mean, it is non-negotiable, we do not want it to go less than this, some pressure limit or temperature limit; that is why, it is an infinite barrier is set. Here, if there is an equality constraint; equality constraint is also used. In fact, your c f d software, they use this; they use this penalty function method. For example, there is a penalty on not satisfying the continuity equation, how does your fluid work? Dou U by dou x plus dou V by dou Y equal to 0; dou U by dou x plus dou V by dou Y equal to 0, is a two-dimensional equation, continuity equation, for incompressible flow. This will never be equal to 0.

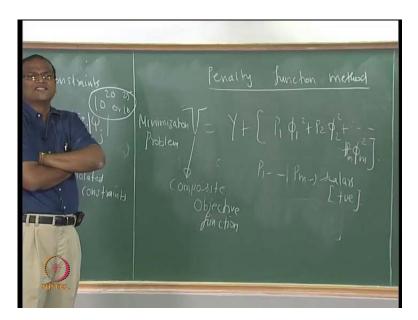
So, you set the limit, and then dou U by dou x plus dou V by dou Y, set it as psi, and thus, this psi whole square multiplied by P. So, this formulation is routinely applied in the petro of Galerkin finite element formulation, for solving Navier Stoke's equation. So, the continuity equation itself is used to introduce a concept, by way of introducing penalty, or putting a penalty, assigning a penalty to the continuity equation, CFD is, you can develop your CFD algorithm.

retted P(p) Pasabolic Peralty. Pasabolic Peralty. Pasabolic Peralty. Pasabolic Peralty. P(p) P(p) P(p) P(p) Pasabolic Peralty. P(p) P(

(Refer Slide Time: 19:41)

Now, who will tell me the value of P? Now, P very high; P very high means, we give lot of respect to the, we give lot of respect to the constraint; so, it will be very slow convergence; too much importance, slow convergence; this is fast convergence, but less importance to constraints. So, what you do is, in order to accelerate the convergence, I will start with small values of P. I will get some solution, then I will keep on changing the value of p; I will increase the value of P. And then, the solution will proceed in such a way, that the phi(s) are not violated too much; a stage will come when the phi(s) are satisfied more or less exactly, and regardless of the value of P, you will get the same value of V; that means, you have hit convergence.

So, it means, it is lot more painful to solve it, using a penalty function method, because you are going to try with different values of P. But, if it is a computer, which is going to do it, then it is very eminently and elegantly done on the computer. So, penalty function method can be ideally used for solving practical optimization problems, instead of, solving problem; I mean, for solving problems in the class, you can use any technique you want, but if there are many variables, you are looking at a real practical example, the penalty function method is a God send for us.



(Refer Slide Time: 21:40)

Now, we will revisit that cylindrical solar water storage heater problem. But now, I am not going to solve it with hand. So, we will try to demonstrate it with MS Excel. What we will do is, we will assign, there is only one constraint; it is a two variable, one

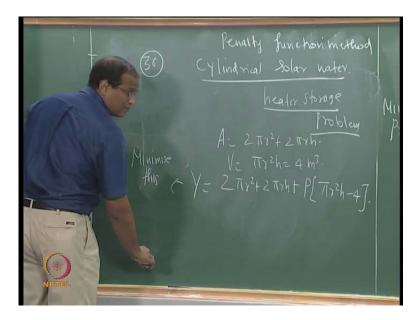
constraint problem - radius and height two variables; one constraint is, volume is 3 meter or 4 meter cube.

Student: 4 meter cube.

4 meter cube. So, we will do that. What is the problem number? 37 or 38?

Student: 38, Sir.

(Refer Slide Time: 22:00)



This already, it is volume, right. So, I will not use V, I will use Y; minimize this. I want a volunteer now. You will come on NPTEL, man; or, you will be volunteered. Do not worry, if we get a wrong answer, we will correct it, the class will correct it; somebody you can copy, paste and this thing and you can do it fast, somebody.

Student: Blade.

Who is blade? Come, you are affectionately called blade.

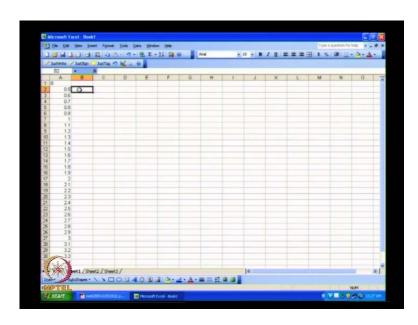
Student: Nickname.

Why, you are such a blade or what? Sit down-sit down, take the seat of honor. So, what was our range 0.5 to?

Student: 3.5.

So, Abishek, 0.5 to 3.5; you go in steps of 0.1 and then, repeat it 10 times.

(Refer Slide Time: 24:08)



Student: 0.5 to 3.5.

0.5 to 3.5, yeah; repeat it, up to 3.5 you have to go.

(Refer Slide Time: 24:49)

Now, copy paste this, another 8 or 9 times, because this is, here, you can write it as r; a 1 you write it as r. We are trying to search out the solution. You know what I am doing? I am just putting various values of r and h; third column, we will put P equal to 1; fourth

column, we will put P equal to 5; fifth column, we will put P equal to 20, and we will look at Y and, in 10 minutes quickly, we can; r; second one is h; no, no; now, the problem is, for each value of r, we should get each value of, you should get, you do that.

Student: For each value of r.

For r of 0.5, I want h varying from 0.5 to 3.5. So, how do you do that?

Student: Voice not clear.

Not 4 by, I do not want the constraint; I want it to, you first, Abhishek did you get it?

Student: Yeah, I get it.

Do not get h from that volume constraint that...

Student: You want, for each r, you want different kind of h.

Yes. So, I want at least;

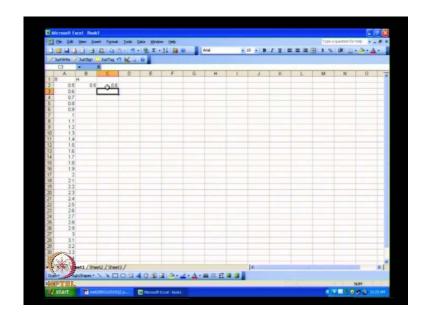
Will have some...

The best way is,0.5 to 3.5, you cut and paste, some 10 times; right side you vary it from 0.1 to 0.1, 0.2; is there any other way of doing it?

Student: Voice not clear.

All values of h.

# (Refer Slide Time: 25:54)



You go ahead.

Student: h from

H is also 0.5 to 3.5.

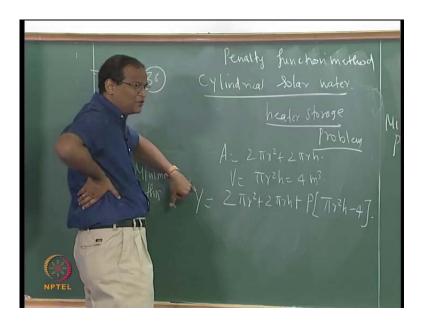
Student: Voice not clear.

No matrix, then it is difficult to do.

Student: No, I can do.

Then, how will you calculate the function?

(Refer Slide Time: 26:23)



You should be able to calculate the value of y.

Student: We can, we can generate, and that is not a problem.

(Refer Slide Time: 26:28)

EL M	icrosoft Facel												unitan for he		
	Can Day Name	Poert Fyrnat	Date De		and the second second						-				
10		3 4 4 1		(S. E - )	24 HA M	A	6 <b>8</b> .	<u>=</u> 10	1.000	( U =	==3	3.5	(年) 日十		
1	Autime 🖉 Auti	kgn 🥃 XastTag 🗸		9											
	82 .	5													
	A 8	C	0	E	F	G	H	1	1	K	L	M	N	0	
1 1		н													
2	0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
3		0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
4	0.7	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
5	0.8	0.5	0.6	0.7	0.0	0.9	- 1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
0	0.9	0.5	06	0.7	80	0.9	-	1.1	1.2	13	1.4	1.5	1.6	17	
6	11	0.5	0.6	0.7	0.8	0.9	-	11	1.2	13	14	16	16	17	
8	12	0.5	0.6	0.7	0.8	0.9		1.1	12	13	14	1.5	1.6	17	
10	12	0.5	0.6	0.7	0.8	0.9	-	11	1.2	13	1.4	1.5	1.6	17	
11	1.4	0.5	0.6	0.7	0.0	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
12	1.5	0.5	0.6	0.7	0.0	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
12	1.6	0.5	0.6	0.7	0.0	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	17	
14	1.7	0.5	0.6	0.7	0.0	0.9	1	1.1	1.2 1.2 1.2 1.2 1.2 1.2 1.2	1.3	1.4	1.6	1.6	1.7	
15	1.8	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	13	1.4	1.5	1.6	17	
16	19	0.5	0.6	0.7	0.8	0.9	1	1.1	12	1.3	1.4	1.5	1.6	17	
17	2	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
18	2.1	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
19	2.2	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
20	2.3	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
21	2.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
22	2.6	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
23	2.6	0.5	0.6	0.7	0.8	0.9	1	1.1	12	1.3	1.4	1.5	1.6	1.7	
24	27	0.5	0.6	0.7	0.8	0.9	1	1.1	12	1.3	14	1.5	1.6	1.7	
25	2.8	0.5	0.6	0.7	8.0	0.9	1	1.1	12	1.3	1.4	1.5	1.6	17	
26	29	0.5	0.6	0.7	0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
27	3	0.5	8.0 8.0	0.7	80	0.9		1.1	1.2	13	1.4	1.5	16	17	
28	3.1 3.2	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
	32	0.5	0.6	0.7	0.0	0.9			1.2	1.3	1.4	1.5	1.6	1.7	
30		0.5	0.6	0.7	0.0	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
	SUP S	0.5	0.0	0.7	0.0	0.9	-		1 3	1.3	1.4	1.0	16	4.7	
-	Provet1	(Sheet2 / Sheet	2/					1	¢	-					2
INH	PTEL		0 4 4	0.4.4		-	1221					-		H I	
1	start T	me62800.125100	22	Herenoft Ex	our - Brock 1							0.70		5 (D) 10 10	

No time.

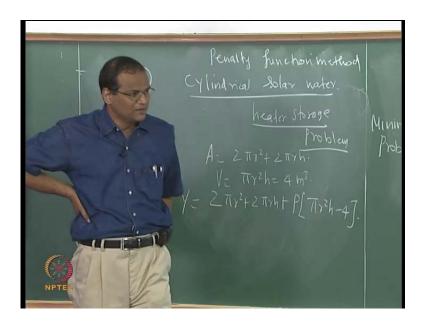
Student: No, it would not take time.

You got it, know? Yeah, now, this has to be completed. You have to calculate the function, Abhishek.

By the matrix, the function will be there? Then how will you search for the?

Student: You want each value for this.

(Refer Slide Time: 27:42)



(Refer Slide Time: 27:50)

E M	crosoft Facel B	losent Figmat	Tools De	ta Wester	1940							Tree	sention for he		
	and and and	10100			112 0	E A		in 10	- 8.7		==3		de mai-	A . A .	
-				14. X - 1			-							<u></u> .	
1	B2 •	A =6.28*A2		P		-	_								
-	A 8	0 46.35°A	0	F F	F	G	H		4	K		M	N	0	-
1	2 0	н	0	R	1			10	4					9	-
2	0.5 -0.46		0.6	0.7	0.8	0.9		1.1	1.2	13	1.4	1.5	1.6	17	
3	0.6	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
4	0.7	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	13	1.4	1.5	1.6	1.7	
5	0.0	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	17	
6	0.9	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
1	1	0.5	0.6	0.7	0.8	0.9	1	1.1	12 12 12 12 12 12	13	1.4	1.5	1.6	17	
8	1.1	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	13	1.4	1.5	1.6	17	
9	12	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
10	1.3	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
11	1.4	0.5	0.6	0.7	0.8	0.9	1	1.1	12	1.3	1.4	1.5	1.6	1.7	
12	1.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
13	1.6	0.5	0.6	0.7	0.0	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
14	1.7	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	17	
15	1.8	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.6	1.6	1.7	
16	19	0.5	0.6	0.7	0.8	0.9	1	1.1	12 12 12 12 12 12	13	1.4	1.5	1.6	17	
17	2	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
18	21	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
19	2.2	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
20	2.3	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
21	2.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
22	2.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
23	2.6	0.6	0.6	0.7	0.8	0.9	1	1.1	12 12 12 12 12 12	1.3	1.4	1.5	1.6	1.7	
24	27	0.5	0.6	0.7	0.8	0.9	1	1.1	12	1.3	1.4	15	1.6	17	
25	28	0.5	0.6	0.7	0.8	0.9		1.1	12	1.3	1.4	1.5	1.6	17	
26	2.9	0.5	0.6	0.7	0.9	0.9	1	1.1	12	1.3	1.4	15	16	1.7	
	3			0.7		0.9		1.1		13	14	1.5	1.6	1.7	
28 29	3.1 3.2	0.5	0.6	0.7	0.0	0.9	-	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
30	32	0.5	0.6	0.7	0.0	0.9		1.1	1.2	1.3	1.4	1.5	1.6	1.7	
30		0.5	0.6	0.7	0.0	0.9	- 1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	
1	SUP S	40	0.6	0.7	0.0	0.9				+ 9	12	1.0	1.6	17	-
16 14	Fieldert1/5	heat2 / Sheet3	1						€	_		1.00		ij,	6
Dee	a sugar	·>> =	0.014	0.8.3	the st	· A · =									
				100 Car 100		-									
	PTEL		_							_	-	_	M		
	start 🔁		- E	<b>Herosoft</b> Ex	cel - Nookt							1 C 1		B 10.33	

I want you to do this, now; 6.28 a 1 square plus, 6.28 a 1 into whatever, plus P equal to 1, first. How do you do C1, d 1? How will you do that?

Do it, let us see, equal to...

6.28 times a 2 square, a 2 that thing also, plus 6.28 into a 2 plus, how will you do that h?

Student: That can be

You have to put that into C, what is it into? Not plus.

No, you did not do that.

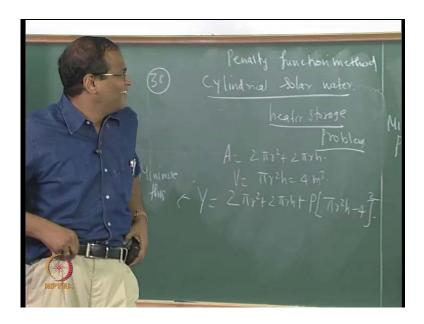
H is there.

Plus, one times, star open bracket; 3.14 star a 2 square, then you have to put brackets, it will work?

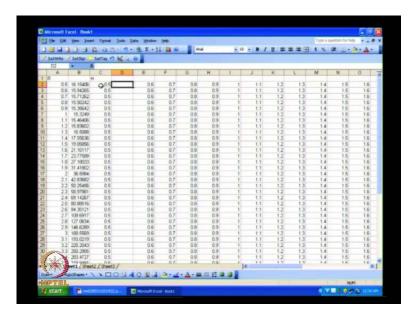
Into C2 minus 4.

Student: Minus 4.

(Refer Slide Time: 29:29)



#### (Refer Slide Time: 29:56)



Minus; into square; finally into; hang on; last term, here; I argued you; t into square of; that is why, it takes a long time to solve the quiz problems. Raised to the power. No, that raise is there, this one. Now, how do you find the minimum or maximum? No, you are copying it in all the columns? It is not a very efficient way of doing.

Student: Only one.

What happened there?

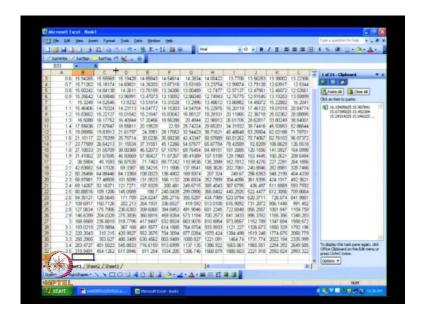
Student: No, just again, the values are same right, so the matrix will have that value.

B dollar 1 is correct.

Student: Yes, sir.

I have very basic technique, which works; so take 0.1, 0.5 to 3.5; keep on copying it, then right side, you just 0.1, 0.2, 0.3 then...

## (Refer Slide Time: 32:15)



It is coming know; that is good; now, we again; now, tell me where is the minimum?

(Refer Slide Time: 32:56)

Penalty function method Minimi

Student: You can put a

You can show, close up.

### Minimum

Student: Voice not clear.

# (Refer Slide Time: 32:03)

11	Autwite		Justing *	1 10	_						/ 1 =				
	833		5 =MN(B	2:AF32)											
	A	8	C	D	E	F	G	H	1	4	K	L	2 of 24 - Cla	board	
4	0.7		15.18174	14.89821	14 26203	13.87318	13 53169	13.23754	12 99074	12 79128	12 63917	12 5344			_
0	0.9		14.04138	14.2011	13 /6199	13.34,98	13.00489	1274941	12.57127	12 91545	12 48072	13 69899		-	
7	0.9	15.30642		13.96991	13.4/0/3		13,2996	13,49612	13/06/022	14.45072	16.22082	16.2041	Paste Al	Cear Al	
10	11	15.46406		14,23113		14 15303		15.22976	16.20119	17.46133	19.01018	20.04774	Cikik an item to	pade:	
9	1.2	15.83602		15.01542				18,28101	20.11965	22.36718	25 02362	20.00095	(8) 15-3249		
10	13	16.5088		16.40944	17.20456	18.56288	20.4844			29.62817	33 80249	38.54001	-		
11	1.4	17 55636	17 67847	18.55811	20 19529	22.59	25 74224	29.65201	34.31932	39.74416	45.9,7653	52,86644			
.12	1.5	19.05856	19.83912	21.61797	24.3961	28.17052	32 94423	38 71621	45.49648	53 25504	62.02188	71 78701	16.15405	425 15.907045	
13		21.10117		25.75714		35 58238				70.74067	82.76103			825 15.452304 825 15.046225 -	
14	1.7			31.15534	37.31551	45.12266	54.57677	65.67784	78.42589	92.8209	108.0629	126.6518	15-24014	625 IS (96225)	
15		27 10033		38.00309		57.10761			101.2005	120.1556	141.0927	164.0998			
16	1.9	31.41002	37.67695	46.50569	57.90427	71.87267	00.41009	107.5189	129.1968	153.4445	100.2621	209.6494			
17	2	36.5964		56.87536		09.77242			162,7812	193.4276		264.1056			
18	21		64 17026 64 88446	69.3367 84.13369	88.34215	111.1806	137 8541 169 5974	168.3626 207.324	202.7061	240.8846 296.6353	282.8981 348.2199				
19	22		77 49505	101 5095	108.0023		206 6904			361 5395					
21	24	69 14267		121 7271	157 8329	200.481		305.4043			511.8589	492.3621			
22	25			145.0599	108.7	240 0439					612 3056	709.0664			
23	2.6	94.35121	128 5645	171.789	224.0247	205.2716	355.5297		523.0794		726.674	841.9081			
28	2.7	109.6917	150.7126	202 213	264,1931	336.6527	419 592	513.0100	616.9092	731.2572	055.1443	991.402			
25	2.8	127.0634	175.7906	236.6383	309.6065	394.6963	491.9045	601.2345	722.6040	056.2557	1001.947	1159.759			
26	2.9	146.6289	204.0328	275.3836		459.9264					1165.356	1348.283			
27	3	168 5569				532 8924				1152,789	1347 694				
28	31	193.0219		367 168	48 5577					1326.673	1550.329	1792 196			
29	32			420.9027		704.3094		1070.424	1284-498	1519.248	1774,675				
30			353.627	490 3499					1454.74	1731 774 1965 551	2022 194 2294 392	2335 999			
31			454.1262			1034.205							To dealer the	task pane again,	44
I		12.46072	404.1404	017.0040	011.254	10.04.200	1,200.740	1966.019	1000-002	2221.010	2772.024	2773-344		d on the Edit men	
39	2802												Options *		
		eet1/9m	et2 / Sheet	3/				10				- 21	Colore -		
10 40	THE	eet1/90	et2/Sheet					]c	-			3.			

Anyway, we generated all the values.

B 2 to...

Student: B 32.

32?

You can do it only column wise.

Student: Voice not clear.

L 32, no.

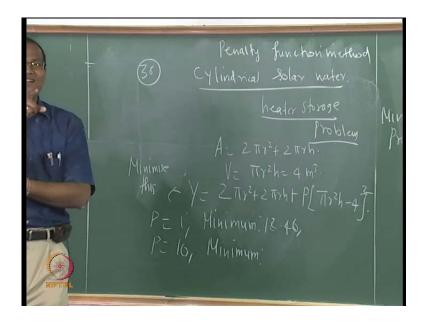
A of 32 name.

12, where?

There is the minimum.

Where?

(Refer Slide Time: 34:21)



12.46, how is it possible? What are the solution to the problem?

What a...?

That is not possible. I think...

(Refer Slide Time: 34:34)

	Hicrosoft E	scel Book	đ										31	
181	-	-	et Fyrnat	Jush D	da minin	- 1940							Type a question for help	# ×
10		E OLA	10.710		8 T -			and.			/	==3	5 % GE	
	Jatwite /		_						-					-
1	82	Ascap	Justing *		Street a Contract	- 1000010		2*8\$1-4)*2)						
-	A	-	0.00	0	21 \$AU U\$1	FILLOU (L)	6	H 14		1	. P. 1	1 -	in the second	
1		0.5	0.6	0.7	0.8	0.9	1	11	12	13	14	15	4 of 24 - Clipboard	• ×
2	0.5		15.90784	15.67395	15.45238		15.04623	14.86163	14,69936		14 3818	14.24651	9 9 9 1	
3	0.6	15 94395	15.55569	15 19420	14 (95043	14.54014		14.00422	13,7706	13 56253	13 39002	13.22306	Paste Al Cest Al	
4	0.7	15.71262	15.10174	14.69821	14,26203	13.87310		13.23754		12,79128	12.63917	12:5344	Conservation and and and and and and and and and an	
5	0.8	15.50242	14.84138	14.2611	13.76159	13.34296	13.00489	12.7477	12.67127	12,47561	12.46072	12 52661	Club an item to paster	_
6	0.9	15.36642	14.59848	13.96991	13.47073	13.10092	12.86048	12,74943	12,76775	12,91546	13.19253	13.59899	18 13017.19625	
7	1	15.3249	14.52546	13.9232	13.51814	13.31028	13.2996	13.48612	13.86982	14.45072	16.22882	18.2041		
8	1.1	15.46406	14 70324	14.23113	14.04772	14 15303	14.54704	15.22976	16.20119	17 46133	19 01018	20.84774		
9	1.2	15 83602	15.22127	15.01542	15,21847	15.83042	16.85127	18.28101	20 11965	22.36718	25.02362	28.08895	······································	
10	1.3	16.5098	16.17752	16.40944	17.20456	18.56298	20.4844	22,96912	26.01705	29 62817	33 80249	38.54001		
11	1.4	17,45636	17.67847	18.55811	20.19529	22.59	25.74224	29.65201	34.31932	39.74416	45.92653	52.06644		
12	1.5	19 05856	19.83912	21.61797	24.3951	28.17052	32.94423	30.71621	45.49640	53.25504	62.02100	71,78701	(K) 15.3249	
13	1.6	21.10117	22.70299	25.75714	30.0236	35.50230	42.43347	50.57689	60.01262	70.74067	62.76103	96.07372	all to any	
14	1.7	23.77589	26.64213	31.15534	37.31661	45.12266	54.57677	65.67784	78.42589	92.0209	108.8629	126.5518		
15	1.8	27.18033	31.55709	38.00399	46.52073	67.10761	69.76454	84.49151	101.2885	120.1556	141.0927	164.0998	1	
16	1.9	31.41802	37.67696	46.50569	67 90427	71.87267	88.41089	107 5189	129 1968	153.4445	180.2621	209.6494	14.15405625 15.907041 15.67395025 15.452304	
17	2	36 5984	45 1593	56.87526	71.7483	89 77242	110.9536	135.2899	162,7812	193.4276	227 2291	264 1956	15.24314225 15.046225	
18	2.1	42.83682	54.17026	69.3387	88.34215	111.1806	137.8541	168.3626	202.7061	240.8846	282.8981	328.7466		
19	2.2	50.25456	64.98446	84,13369	108.0023	136.4902	169.5974	207 324	249.67	296.6353	348,2199	404.4239		
20	2.3	58.97981	77.40505	101.5095	131.0523	166.1132	206.6924	252.7099	304.4055	361.5395	424.1917	492.3621		
21	2.4	69.14267	92.16371	121.7271	157.8329	200.481	249.6715	305.4043	367.6795	436.497	511.0569	593.7692		
22	2.5	00.00516	109 1206	145.0589	108.7	240.0439	299.0906	365.8402	440.2925	522.4477	612.3056	709.8664		
23	2.6	94.35121	128.5645	171,789	224.0247	285.2716	365.5297	434.7989	523.0794	620.3711	726.674	841.9081		
24	27	109 6917	150.7126	202.213	264 1931	336.6527	419.592	513 0108	616 9092	731 2872	856.1448	991.482		
25	2.8	127.0634	175.7906	236 6383	309 6065	394 6953	491.9046	601.2345	722.6848	858.2557	1001.947	1159 759		
26	2.9	146.6289	204.0328	275.3836	360.6815	453 9264	573 1184	700.2573	841.3433	996 3762	1165.356	1348.283		
27	3	168 5569	235.6819	318 7795	417.8497	532 8924	663.9076	810 2954	973 8557	1152.789	1347 694	1558.572		
29	3.1	193.0219	270.9894	367.168	481.5577	614.1585	764.9704	933.9933	1121.227	1326.673	1990.329	1792.196		
29	3.2	220.2043	310.215	420.9027	552.2675	704.3094	877.0294			1519.240	1774.675	2050.779	To diplay this task pane again, it Office Optioard on the Edit men	
30	Jame 11	250.2905	353.627		630.4962			1221.091	1464.74				office Opboard on the Edit men press Onle-C trace.	eux.
38	allow							1306.922			2294.392		Cations .	
		110 0.401	et2 / Sheet	RAT HOME	R11.564	10114-206	1396.746	1440 A75	19901 6173	7774 @58	36933-934	1001 1777	Oferna .	
												- 1		
Des	and the second second				C 1 4		$(-\Delta)$	== #						
N	PTEL	and press Di	TER or choose	Pala								1.1.1	NP	
1	start			22	Record E	scal - Bookt								

What are the solution? 0.866 and 1.732, right. What is the value of, a minimum, we got?

What are the value of, a minimum?

You cannot get this.

Even otherwise, it is square know...

No, a itself is 14.9, you have to get something more than that.

It is not correct; something is wrong with the formula, just go to the formula.

That is, ok; but, that does not matter, there is something wrong with the; 3.14 a 2 square is;

Even, if penalty is relates always; adding know, it should be more than 12.96; even, if we hit the optimum; optimum was, a was 14.96.

It cannot be less than this.

It is straightaway, it is wrong.

Penalty does not matter man.

Student: Sir, at which are not feasible, I might have a very low value, which is...

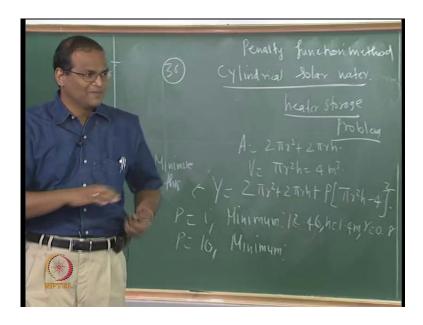
Well taken. What is, you can violate the constraint and still have low, because the penalty is low.

No, tell me the value of r and h?

No, for this solution, before getting confused, for this 12.46, what are the r?

And, this is.

(Refer Slide Time: 36:19)



H is 1.4; r is 1.8.

(Refer Slide Time: 36:19)

	licrosoft Ex	cel Book	1												6	6 🗙
19	(m (m	Sam Sau	et: Fyrnal	Jude De	da Hindow	1940								Type a question	for 748	- # ×
101		(BIB)	12 23 2	5 - 7 -	B. I	21 12 6	- JA	vial.	· · · ·	0 - 8	/ 1 =		81	5 5 18	- A-1	-
1	Jatwite	Jut5g	JustTag V	142.0	6 B		-									
-				A2*\$A2+6.3	38*\$A2*R\$1	+10100.14	"EA2"EA2"	(5:0-128								
	U I	V	W	X	Y	T	AA	AB	AC	AD	AE	AF	-	5 of 24 - Clark		• ×
3	12.96047	13.05908	13.18324	13.33295	13 50822	13.70905	13.93544	4 18738	14.46487	14.76793	15.09654	15.4507	1			
4	13.72207	14.09076	14.5068	14.97018	15.40091	16.03999	16.64441	17.29718	17.99729	1874475	19 53955	20.3017				
5	16,75419	17.62778	10.50213	19.61725	20.73314	21.9298	23.20723		25.0044	27.52414	29.12465	30.00993		R Paste Ad	Cen Al	
6	23.07909	24.77932	26.60993	20.56792	30.65629			37.69765	40.30353	43.03870	45.90341	48.89742		Citik an item to	pate	
7	33.0563	36.8025	39.9469	43.20040	45.82725	50.56324	54.4964		62.9543	67.47904	72.20090	77.1201		(8) 133.2909		
	50.37766 74.07731	55.1023	60.11566	65.41772 96.7%ENO				89.51306		103.293	110.616	118.2277 175.264		- man		
10	106.5217	116 8912	127 8239	139 3199	105.1479	113.9378		132,7443 190,9356			164.0207	251.5624				
	149 4145	163.9298	179 2026	106 2329	212 0207	229.6661	247 969		205 24/5	307 3228	338 6660	350.7454		<b>8</b> ] 13017.19		
12	204 4949	224 3439	245.0902	266.8347		313 3185	338.058		390,5317	418 3659		475.7293		-91 1001/19	06.0	
12	274.0422	300.278	327 8062	356.6267	396,7395	418 1446	450.8421	454 (310	520 1139	555 6883	014 005	633.7141				
14	369 0659	394 0245	429 8301	457 2025	506.3821			633.5625		726.5843		826 1939		100 10 1000		
15	464.316	508.0236	553.0012			703.5543			871.9377	932,2056	994 5436	1058.952		······································	M13	
16	509.7778	644.0634	702 5100	762.7441	825 5392	890.9041			1102.418	1178.062	1256 276	1337.06				
17	738.7727	807.28	678.9423	963 7697	1031.732	1112.86	1197.142	1284.58	1375 173	1468.921	1565.824	1665.882		-		
18	913 9687	998.1573	1086.191	1178.06		1373 302	1476.676	1583.884	1894.928	1809 807	1928.52	2051 089		······································		
19	1118.13	1220.527	1327.544	1439.18	1555 435	1676 31			2066.65	2206.002	2349,974	2498.565				
20	1354.217	1477.589	1606.44	1740.83					2495.55	2663.049	2836.066	3014-601		1		
21	1625-296	1772.611	1926.48	2006.09	2253.843	2427 330		2793.956	2967.078	3186.743	3292.95	3605.699			625 15.907941 025 15.452304	
44	1934.54	2109.129	2291.421	2481.415	2579.113	2984.513			3545.93	3783 141	4027.056	4278.673		15.24314	225 15.046225	
-	2205.319	2490.745	2705.182	2928.631	3161.091	3402.562			4101.042		4745.084	5040.622				
-	2581.090	2921.232 3404.607	3171.844	3432.936	3704.508			4582 102 5333 141				5050.94				
-		3404.507	4290.983		4996.513			6171 909	6691.936			2936.697				
27		4545.843	4932 419				6638.448		7587 299	1085 683	8600.030	9130.368				
38	4798.502	5712.48	4654 669	6115.07		7090 504			(99)01 239	0050 906	9947.784	10453.87				
20	5456 187	5349.063	6452 615	6075.844		8087 334		9290 532	9908 147	10556.44	11225.41	11915.05				
30	6212.59	6760.249	7331.294	7925.724	0543.520	9184.74		10537.3	11248.66	11983.4	12741.53	13523.04				
31	7032.138	7950.845	8295.904	0967.315	9665.077			11916.47	12719.64	12549.16	14405.03	15207.25		To display this t		
12	75.0.300	0625.006	9051.005	10107.4	10092-58	11707 35	12551.72	13425.67	14329.22	15262.36	16225.00	17217.4		Office Opboard press OnleC to		nu or
1.7	0.0.0		100 M 100 M	10000		100000000		and the second of the				and the second second				
	- Here	et1/9w	n2 / Sheet	2/	-			je.		-	-		328	31C **		
	N 8 1 1							-				-				
Dre					0.0	- <u>-</u>	4·Δ·	==#							5 of 24-13	phoard
1	PTEL	and press END	TER or choose	Pate								Sat=1490	2096.4	102 208	tare collected.	
1	start			24. 0	Normalit E	cel-bookt										

But Sampath, tell me the value of volume. Now, you change the penalty to 10; open it, can you do it with the same sheet? Change the penalty to 10; you make it 1000.

Student: 2.81.

Make it 10.

# Recalculate.

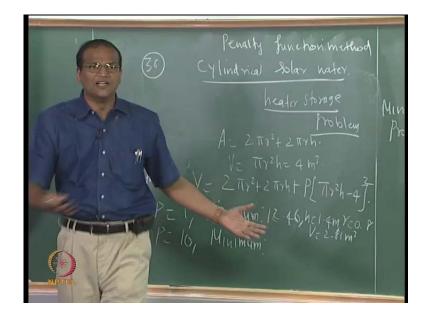
(Refer Slide Time: 36:53)

	dicrosoft f					_										
	CH ON													the scheme		- *
			10	5 - 17 -	H E -	21 12 1		ne .	1	0 - 8	/ 1 =		91.5	5. F	11 · 2 · 4	Δ-
11	Satwite .	AutSun (	Juit Tag. V	1 12 1	ŭ.											
_					6.28"\$A33"	C\$1+10*(()	14"SA33"	A33*C\$1-4	9*2)							
	R	8	T	U	¥	W	x	Y	1	AA	AB	AC	- 1	5 124-0	the second	
13	1896.296		2138.301		2632.009								- 1			_
14	2307 599		2999 206		3536.602					5443.626	5874.422		1		-	
15	3053 365	3424.165	3009.667	4215.869	4542.771			6047.68	6557 383	7007 798 9062 187	7638.092	8210.697		S. Paste Al	Cen 4	1
15	6098.35		4921.59	6890 361		0269.439	7133.456 9006.309	9774.73	0393.579	9062.107		13163.92	- 1	35 an Ann 1	to grantee	
18	6345.223	7060.29		0090.301	9435.69		11210.87	12156.04	13139.66		15221.64			133.29	5625	
19	7861.869	8724.818	9643.97	10609.32		12678.69		14932 63	16128.95	17371.46	10660 15	19995 05				
20	9600 693	10665.67	11765.84	12991 18	14151 71	16427 42	16758 32	18144.39	19585.65	21082.09	22633.72	24240.52				
21	11615.21	12878.63	14207 48	15601.75	17061.44	18586.56	20177.1	21833.06	23554.45	25341.26	27193.5	29111.15		13017.1	19625	
22	13919.92	15420.6	16998.3	19953.03	20384.79	22193.58		26042.24	29082.11	30199.01	32392 93	34662.09				
23	16540.4	10309.63	20169.98	22118.43	24158	26387 67	29907.46	30817.37	33217 38	35707 51	30207.74	40958.09				
24	19503-27	21574.95	23751.43	26032.7	20418.77	30909.64	33505.3	36205.76	39011.02	41921.07	44935.91	48055.55		······································	72463	
2	22536.16 26567.79	25246.09	32295.6	30431.97	33206.31 39561.29	36101.87	39110.62	42256.50 49020.05	45515.75	40096-12	62397.7	56020.40				
37	30727.88	339337 78	37307.41	40836.77	44525.16	48974.66		56551.44		66367 12	70014.56					
28	36347 23		42991.2	46921.35		55547 99			69653.77	74996-59	80301.52	85798.55		(5.324)	•	
29	40457 67	44648.02	49045.14	53649.04	58459.7	63477 14		74132 32	79770.07	85614.59	91665.98	97923.94				
30	46092.05	50848.43	55838.67	61062.75	66520.7	72212.49	78138.14	84297 85	90691	97318.22	104179.3	111274.2	1.1			
31	52284.31			68206.0	75374.66			96459.33	102681 2	110166.7	117915.6	129928.1	4	★C 16.154	25825 15.3029H	t
32	59069.4	65125.97	71478.45	70126.84			99847.48	107679.5	115007.5	124231.3	132961.1	141966.0	31	45,243	4225 15.04622	
	100	160	160	160	160	160	160	160	100	160	160	160		~		
1											124	12	1813	SC.		
14																
37																
38																
30																
40																
41															task party again and up the Edit to	
44	1													press Orl+C		
1	Ste	-												Options +		
10	The second	eet1/9m	et2 / Sheet	2/				Jc.								
De	and the second	Automatical	110	034	0.00	1.0	<li>Δ - 2</li>									
-IN		and press EN			-	-		-				Ser-122	-		No. PE	

What are the values of the volume?

Student: 2.81.

(Refer Slide Time: 37:01)



#### (Refer Slide Time: 37:17)

	Hicrosoft Ex	wei Book	1											2
115	-	Sam San	et Format	Toute Qu	sta Mitaka								Type a question for their	
10		Diat	100	5-1-17-	9. E .	21 10 1		lotel.			/ 1 =	##B	5 % # . · · · · ·	-
	Jutwite 0		Just Tag		÷.					_				_
-	P6			A5*\$45+6.	XI"SAG"PE	+10103-14	*\$45*\$45*	261-61-21						
	1	-	K	L	M	N	0	P	0	p	S	TT	S of 24 - Claboard	-
1	12	13	14	1.5	1.6	17	1.8	19	2	21	22	23		•
2	98.85164	94.4262	90.12401	85 94506	81.09936	77.9569	74.14769	70.46172	66.899	63.45952	60 14329	56 9603	9 6 6	
3	76.66438	71.19249	65.97616	61.01539	96.31019	51.86054	47.66645	43.72793	40.04497	36.61756	33.44572	30 52944	Paste Al Cear Al	
4	54.73578	48.7848	43.30728	38.30322	33.77262	29.71548	26.13179	23.02157	20.3648	18.22149	16.53163	15.31524	Cikit an item to paster	
5	35.20069	29.00252	25.13205	21.26927	10.2142	15.96682	14.52715	13.89517	14.07009	15.0543	16.84542	19.44424		-
6		17 24493			14.17822	15.74353	18.60263	22.7666	38 20214	34.94256	42.97676		[8] 133-2805625	
7		14.51124		20.741	26.81376	34 85844	44 87504	56.86356		86.75636		124.5368		
8		25.40054						124.3346		180.4093				
9	38.41888		73.89378	97 7647	125.7246		193.9113		278 4539			435.935	III 13017.19625	
10	76.48045		139.6397	179.6673		276.6185		396.0977		538 105				
11	137.4604	183.796	237 7069	299.1932		444.0917			720.2544		941 7061	1063.795		
12	225.9588		374.3568	463.5301		671.8251			1059.139	1200.209	1367.262	1536.299	[8] 12.46072463	
13	346.9166	445.1539	556.3143	600.390	017.4048	967 3348	1130.100				1910.032	2138.301	-	
14	505.6153	639.957	790.7684	958.0494		1342.021	1558.711	1791.87	2041.5					
15	707.6772		1085.371	1306.269	1545.868	1807 166	2089.165		2716.265		3424.166	3809.667	18 15.3249	
16		1190.804	1448.24		2040.208							4921.59	-51	
17	1266.084			2246.216				3994 789		5058.35		6248.133 7813.752		
18					3345 141				5668 506		7060.29	9643.97	and the second second second	
20	2073.93	2531.149 3147.409	3034 56		4179 964 5153 523		5510 142		7025.093	7051.059 9600.693	8724.818 10655.67	11765.84	16.15405625 15.907641 15.67395025 15.452304	
	3108.462		AU03.107	5400.055	6279.446	7215.751	8217.48		10417.21	11615.21	12078.63	14207.48	15.24314225 15.046225	
	3090.115		4003.107	9400 965	7571.97			11149.66			15420.6	16990.3		
	4672 377	9630.699	6678 932	7817 378	9045.936		11773 39		14961 29	16540.4		20168.98		
24	46/2.3/7	6702 496		9073 883	10716.79		13916.99				21574.95	23751.43		
X	6593 824		9954 796	10917 09	12600.59	14405.29	16331.2		20546.64		25246.89	27778.83		
36	7741.41	9275 349	10948.76		14713.99		19037 1	21407.96	23918.09		20366.96	32295.6		
27	9026.405	10798 78	12730.88	14822.7	17074.25	19495.53	22056.53		27677 7	30727.88	33037 70	37307.41		
28	10458.06	12495.79	14714.83	17115.99	19699.25	22464.62		20541.71	31953.41	35347 23	39023.16	42991.2		
29	12049.17	14378.59	16914.78	19657.73	22607.46	25763 96			36474.08	40457.67	44649.02	49045.14	To diplay this task pane again, d	
30	1 200 07	16459.77	19345.31					37290.06				55838.67	Office Opboard on the Edit menu	100
35	15346 44	18762.32	22021.52					42320.22			57661.63	63302.46	press CirleC toke.	
		11100.04	DAGER GD	198644-111				\$79×83 00		KONEG A		71.870 AL	Options ·	
10 0		eet1/9m						34				2		
Des	and	Authors *	110	024	0.00	A	L . A . :	<b>二元</b> 二二二二	2.34					
N	PTEL								-			1	NP	
1	start	1 million		20. 0	Normal I	sent - Bookt							CONTRACTOR	

So, the volume was 2.81; so, the penalty is, the FUNDS is, the penalty is so low, that you are deliberately choosing smaller values of r and h, but the volume constraint is not satisfied; this tank will not hold 4000 litres, when you can get lower surface area, right. If, 13.89, ok. So, now, this is, what is the value of r and h?

Student: Move to the right, go to 1.6 roughly.

How do you get?

Go to column n.

It it is row.

No, you no cheating.

You cannot search it out.

13.

P 5 5.

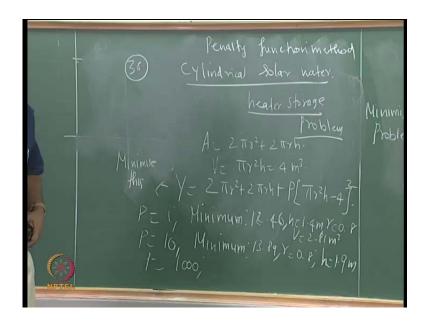
You got this.

No, that is 13.90; it is here, 1.99 and,

Go to the left side.

0.8

(Refer Slide Time: 38:01)

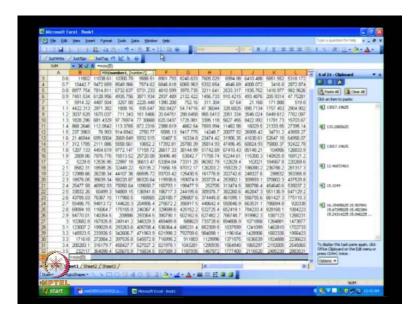


(Refer Slide Time: 38:42)

	licrosoft Ex	cel Rook											201
	-	-	et Fyrnat	Juste De	da minin	- 1940 -							Type a question for help
1.08		COLUMN 1	101241	5-1-17		41.18	<b>B</b> Ta	tui .			/ =	==3	5 % (@
-		_	_		-				-				
1	Autwrite (	Jul Sup		7 14 -									
_	B2	• 1	=6.29*\$	A2*\$A2+6.	28"\$A2"8\$	1+10*([3.14	"IAPIA"						
1	A	8	0.6	0	80.0	.0.9	0	H	1	13	K. 14	15	S of 24 - Claboard •
2	0.5						108.0723	103 4003	90.05164		90.12401		9 9 9 7
1	0.5	122 1231	114.0625	107 0572			00.37484						(Brown) (Brown)
4	0.7	109 6494		91,59252				61.16021	54,73578	48.7840			Peste Al Cons Al
5		96,24343		74,78701			40.66012				25,13205		Cick an item to paster
6							31 96564						(B) 133-2805625
7	1			43.14804			19.956			14 51124		20.741	
8	1.1	65 1654	41.339999	30.40166	22 36042	17 18627	14 9092	15 51923	19.01634	25.40054	34.67183	46.83021	
9	1.2	43.05937	30.12952	21.28965	16.53675	15 87382	19.29987	36.81489	38.41999	54 11184	73.89378	97 7647	(8) 13017.19625
10	1.3	32.83121	22.17081	17.14242	17 74602	23.98163	35.84924	53.34984	76.48045	105 2441	139.6397	179.6673	and the second se
11	1.4	25.2204	18.5287	19.41233			67 51519			183.796	237 7069		
12	1.5	21.02556		29.6637			117.4923						W1 12.46072463
13	1.6	21.10449		49.57779	03.1992		109.2115			445.1539			
14	1.7		45.42811	00.96179			256.3409			639.967	790.7684		
15	1.8	37.81054		125.6989	200.6937		412,7846					1305.269	(A) 15.3249
16	1.9	56 44903		185.8481			672.6837				1448.24	1731.375	
1/	2	83.384	157.689	263.5446		569 9082				1561 244		2246.216	
18	2.1	119.769	221 2342	361 0494			1010 596			2005.293		2960.175	
10	22	166.0168	300.6814	490.7393	706 9907 907 535	979.4355		1662.905 2085.912	2073.93	2531 149 3147 409	3034.56	3584 166	16.15405625 15.007641 15.67395025 (5.452304)
24	23	225.7993 298.0475		625.1074		1245.145 1557.172		2005.912 2579.275		3147.409		4429.636	15.24314225 15.046225
	2.4		514.68U1 653.1763	998.4291	144,255	1920.019		3149.722	3100.462			5400.505	
73			815.3966	1232 948	1740.61		3036,269					7817 378	
34	20	608 5841		1503.277	2107.817		3631 295						
25				1812 487			4317 673			7913 707			
28	29		1466.65				5001 043			9275 349			
27	3	1092 109		2560.423			5960.836					14822.7	
28	31	1299.456	2061.61	3005.874	4132.25			8604.043		12495.79	14714.83	17116.99	
29		1532.846	2414.866	3503.657	4799.219	6301.552	8010.655	9925.529	12049.17	14378.59	16914.78	19657.73	To diplay this task pane again, clock
30	2-11			4057 425			9206.252			16459.77	19345.31	22464.71	Office Opboard on the Edit menu or press Onle-C trace.
35	h A and						10525.82				22021.52	25554.23	
	SK I	7400 711	17m 2	COL INCA	7761.914	Q.471 E37	11077.17	14778.07	17975 19	21202.6.8	14069-01	30424.4 17	Cuttors .
1	No. of the second		et2 / Sheet		-							1.	
Dee	11- 11- 14 M	LuShapes *	1.00	044	0.00		<u>(• A</u> • •	===					
	PTEL	-	_	_		_	_		_	_	_		NP
1	start	1 1 1 m		241.	Plocadt t								化氟甲酮化氨基铵 口收的

So, r equal to 0.8; h is equal to 1.9. Now, P equal to 1000; make it 1000. I hope, everybody able to follow the arguments; so, if we keep changing the value of P, finally the constraint, when the P so high, that pi r square h must be equal to 4; otherwise, you will get a very high value of V.

#### (Refer Slide Time: 39:00)

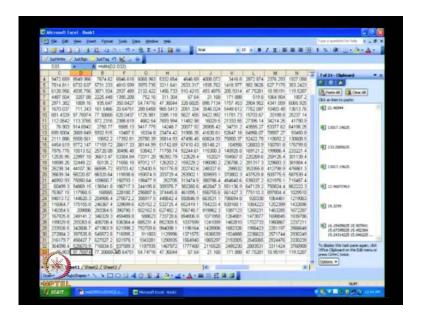


So, did you get the answer now? So, penalty functions is very difficult; obviously, very difficult for me to ask in the exam, you require computer. I can ask only the formulation, that is all, right. I again, just give the papers; we have to discuss the quiz solution, only on Monday, that you have well taken, know; cribs, after the...

Student: I can find it easier if I move it

But, 21 is not correct.

(Refer Slide Time: 39:52)



Student: I am doing it for each column; then I will do it for all the columns; that way I can find it also.

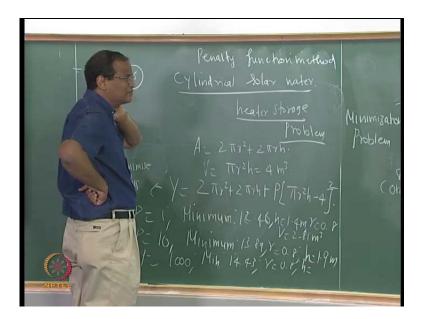
But, last time you are able to do.

No, no. Allow him to, he has some, he has way of doing it, that is good; so, you will find out the, you, will take the minimum of the row now.

Student: Yes.

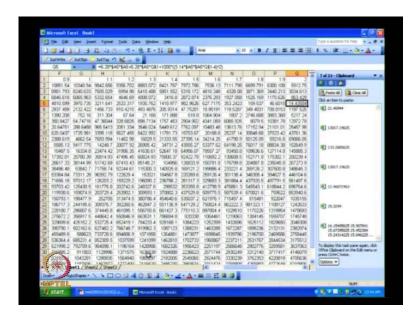
Not bad.

(Refer Slide Time: 40:54)

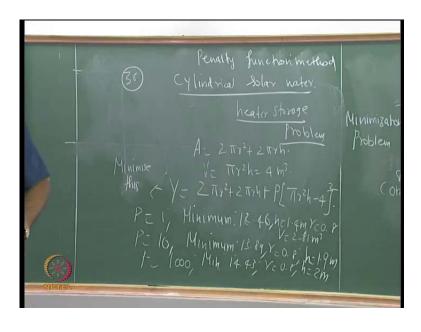


14 points. That is good; r equal to, it will stay at 0.8, because we did not use that 0.86; h is 1.7.

#### (Refer Slide Time: 41:05)



(Refer Slide Time: 41:12)



The actual answer is 0.866 and 1.7.

The problem is not with a penalty function, because we had a least count of 0.1. If you have a least count of 0.01, which may makes a matrix much more bigger, you will, you can resolve it; the problem is not the least count is only 0.1; the answer is 0.866, then the 2 will change to 1.7; this is how the penalty function works.

We did an exhaustive search, after doing this, right; you can do much more efficient searches, or you can solve it as a Lagrange multiplier. Using the Lagrange multiplier method, at each and every iteration, that is, for each value of P, and get the solution. Thank you. You want to do anything more or that is, fine.

So, this how the penalty function method works. So, it is pretty impressive. So, now, finally, we have come to a stage, where we can handle multi variable constraint optimization problem; not only equality constraints; inequality constraints can also be handled using infinite barrier penalty.

So, with that, we are more or less come to an end to conventional optimization techniques. There are small, there are other techniques like dynamic programming, linear programming, which are used basically by management people, operation research people and so on.

In the next class, we will look at the graphical method of linear programming and the dynamic programming. And then, the last 4 or 5 classes, we will just devote to unconventional or nontraditional optimization techniques. Two hours, we look at genetic algorithm. I will give a heat transfer example, so that, you get an idea of what genetic algorithm is all about. I will teach you simulated annealing, yet another powerful technique. And, if time permits, we will do neural network or basin inference and so on.