Design and Optimization of Energy Systems

Prof. C. Balaji

Department of Mechanical Engineering

Indian Institute of Science, Madras

Lecture No. # 23

Unconstrained optimization

Okay, I think we will complete the solution to this.

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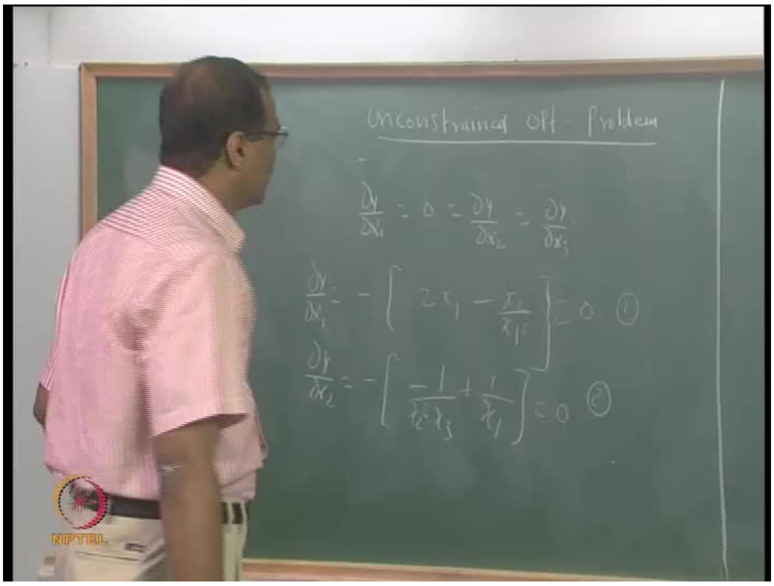

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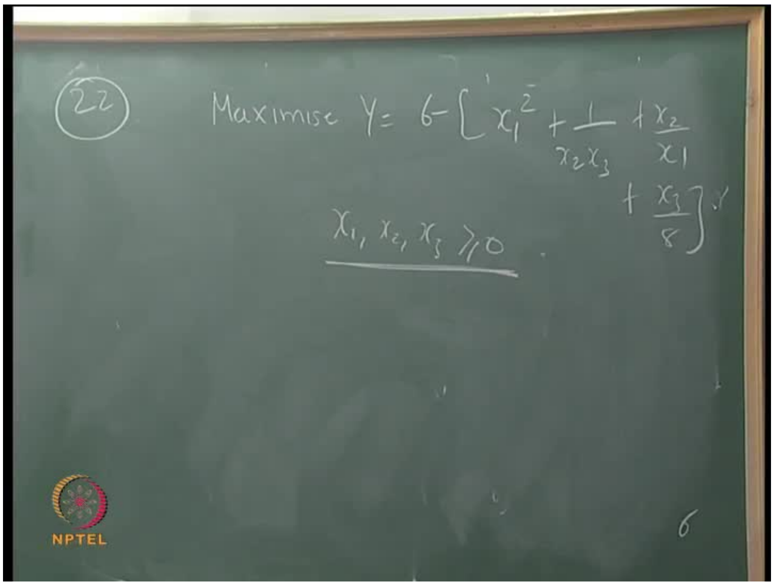

So, it is a basically unconstraint problem. So no, no, no, what is this? Yeah, so this is minus plus 1 by x 1, fine.

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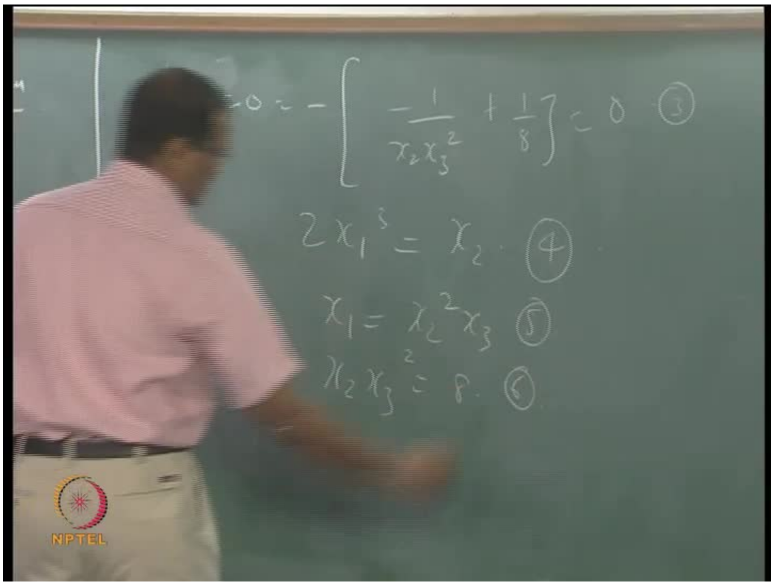

So, this is a Lagrange multiplier equation, is this correct? So, what we do now?

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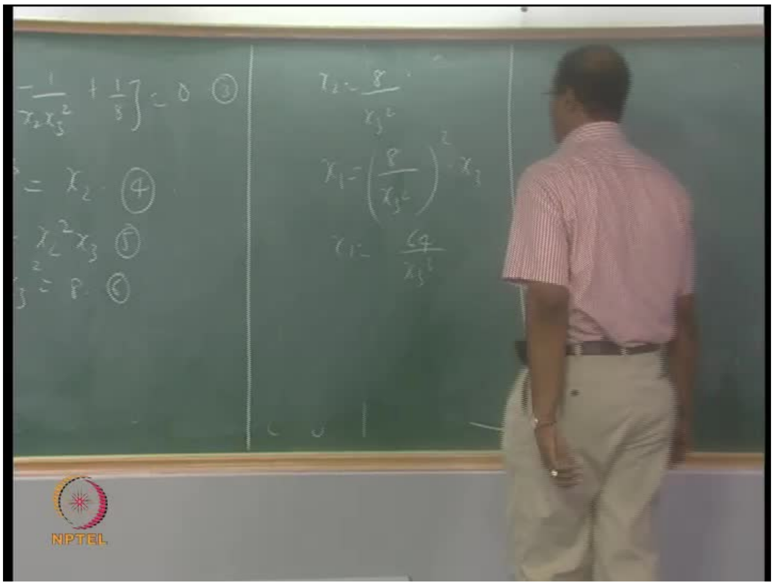

So, what do we do now?

Student: Substitute x 2 is equal to 8 x 3 square.

So, is it right, correct?

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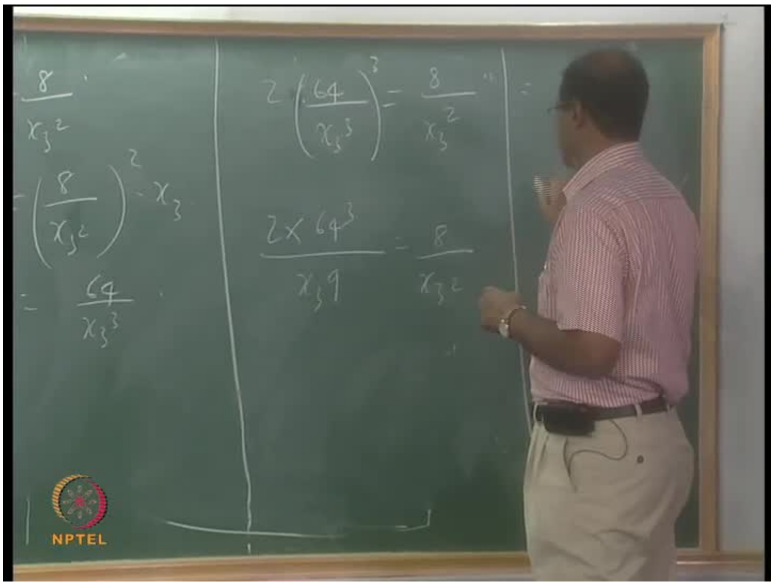

You can substitute in, it is cube, okay. How much is this? 64 into which, it is a large number; are you getting all this? What is this?

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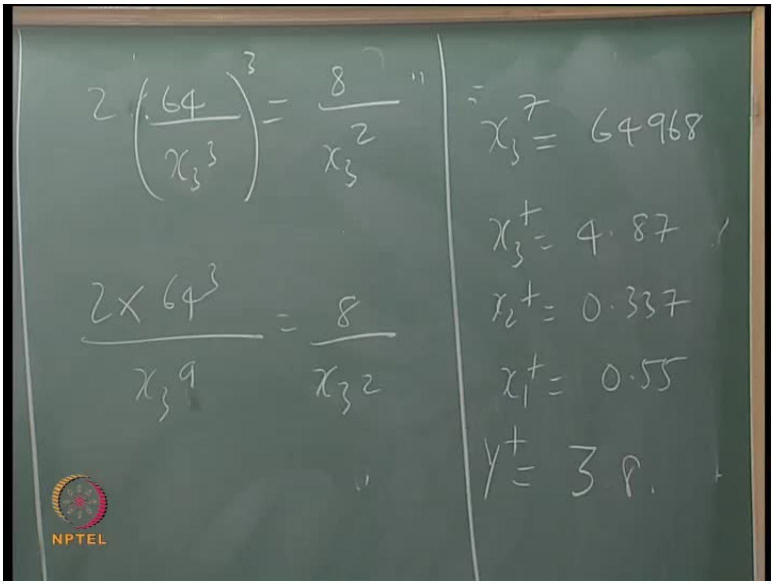

So x 3 to the power of 7, correct. How much is it?

Student: 4.865.

No, no x 3 to the power of 7 is?

Student: 64960.

So, x 3 is?

Student: 4.87.

Okay, x 2 plus?

Student: 0.337.

Student: 0.55.

Y plus?

Student: 3.866.

That is it, okay. We are not checking for the second order and all that, right. So, basically it is, okay anyway it has to be maximum, right, y minus some 6 minus. So, let us take more practical problem, is this clear to everybody?

Yeah, please take down this problem, problem number 23. Consider a solar thermal application where the hot water produced by a solar collector is kept in a cylindrical storage tank and its use regulated so that it is also available during night time. The storage tank has a capacity of 4000 liters and convective losses from the tank have to be minimized. Radiative losses can be neglected, ambient temperature t infinity and convection coefficients h are constant. The hot water temperature may be assumed constant in the analysis. Solve this as an unconstraint optimization problem in r and h where r is the radius of the tank and h is the height, using the Lagrange multiplier method.

Watch out I am asking you to solve it as an unconstraint optimization problem. There is a constraint in this problem. I want you to substitute the constraint into the objective function and convert it into an unconstraint problem. This is for starters; after you finish this exercise we will solve the same problem as a constraint optimization problem. There we will get an additional optimization on lambda, then we will discuss what lambda is all about and by the time it will be 2.50, okay. Is the question clear, yeah?

Student: The top and bottom also, will not be there sides.

Top and bottom also, process will be from top and bottom also. I will take attendance; you can this close for 5 minutes. You have to just minimize A that is all, correct.

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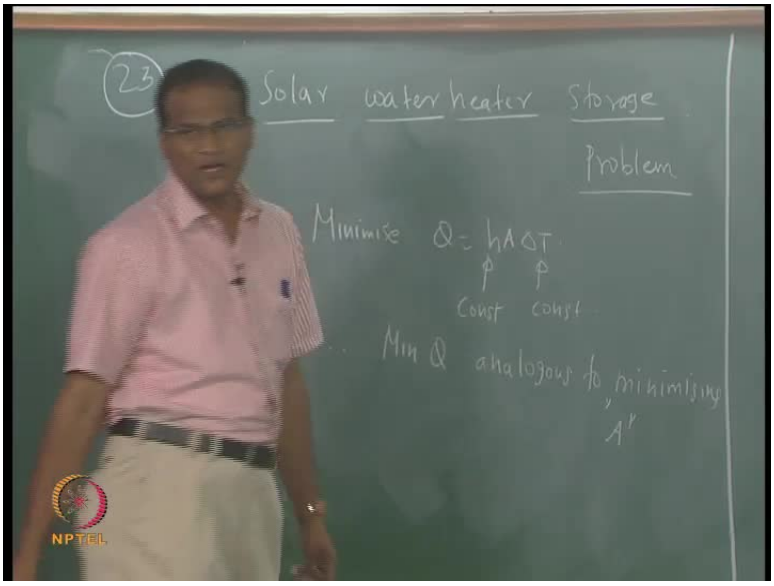

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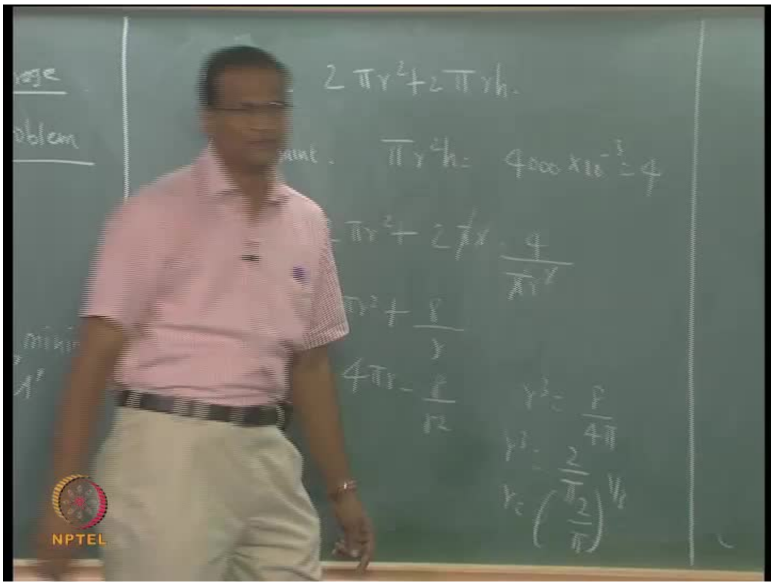

Okay, lateral surface area plus the top area and the bottom area, 4000 liters 1 liter is 10 to the power of minus 3 meter cube. So 4 meter cube, fine is it correct.

Student: 0.86.

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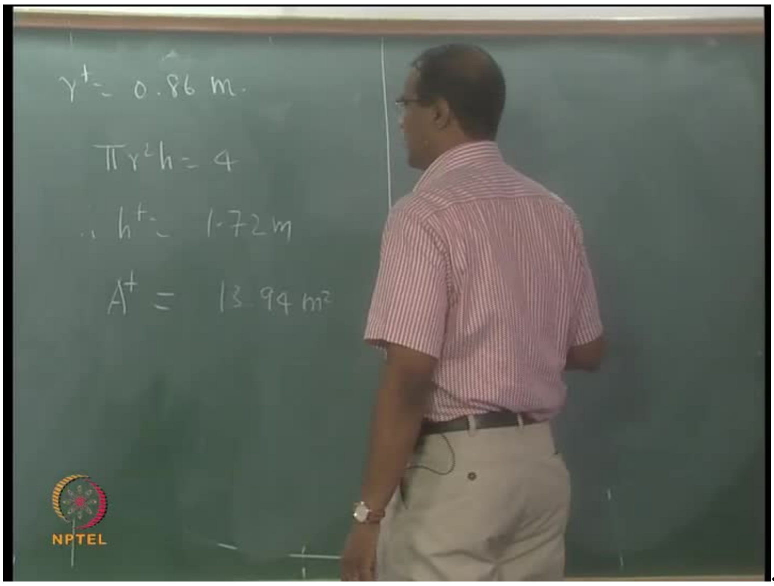

Fine; so r optimum, A optimum; please work out A plus.

Student: 13.94.

13. 94, yeah correct; Anand you got it, okay?

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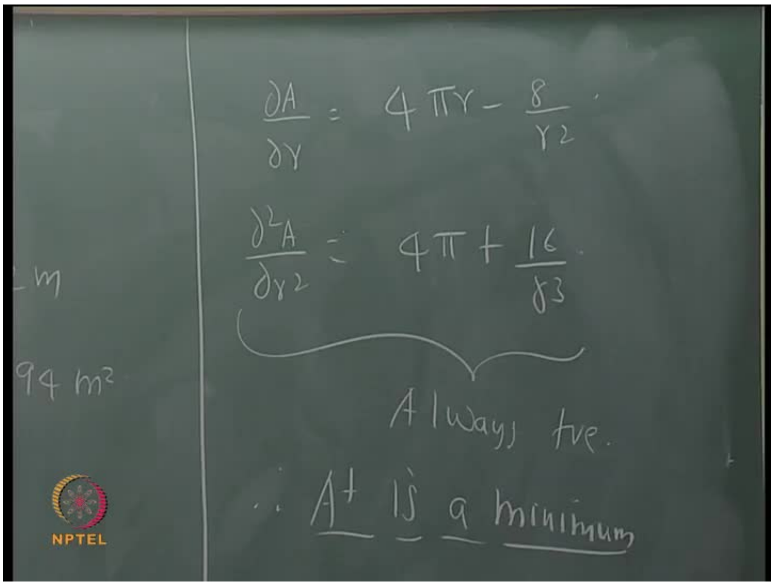

Now dou A by dou r always positive, right; therefore A plus is whatever is a minimum. So, if the temperature is constant the convection coefficient is constant, the height of a cylindrical tank must be twice its?

Student: Radius.

Radius, so it will be 4 times its no, no it will be equal to the diameter, okay alright.

So, application of Lagrange multiplier method in for a very practical problem; is this clear, any doubts? Now problem number 24, revisit problem number 23 but now solve it as a two variable 1 constraint optimization problem; needless to say you will get the same solution, you have got to get the same solution. Apart from that you will get something you will get a new value new parameter called lambda, what is the story about that lambda we will see shortly; yeah work this out. I will close for 5 minutes let them work. Okay, I think we will solve this.

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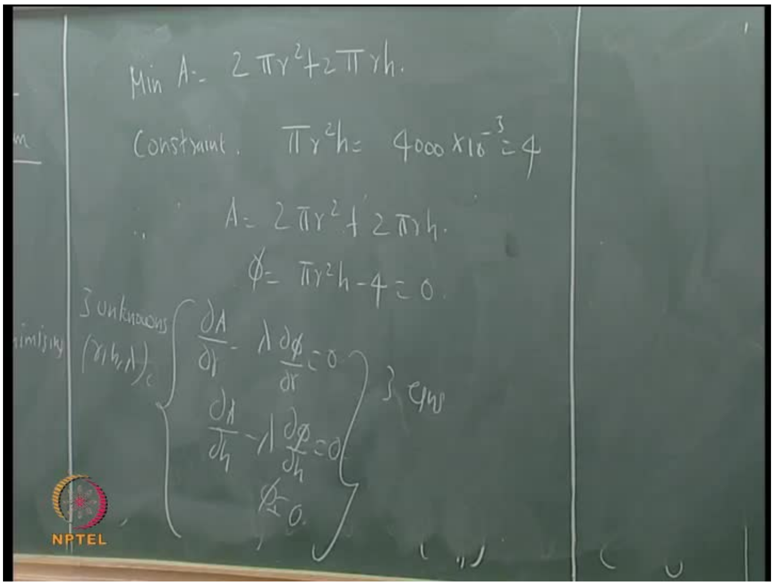

So correct, so I will say this is our first Lagrange multiplier equation. So, three equations and three unknowns; okay, do not ask me sir what happened to A? A finally we will get A; do not worry, A is a global unknown, we do not worry about that. The unknowns are r, h and lambda and then we can start doing that.

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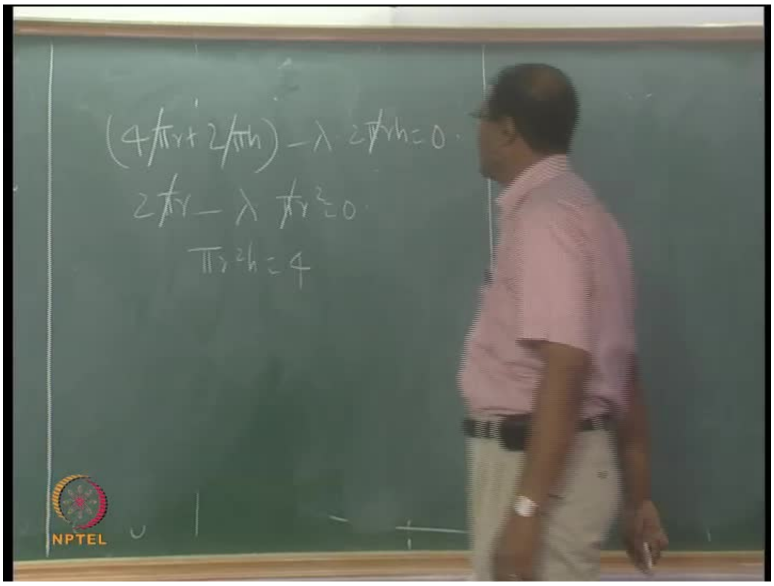

Dou A by dou r is what? 4 pi r 2 pi h, is it right. So, what is the simplification now? This fellow can be removed, correct.

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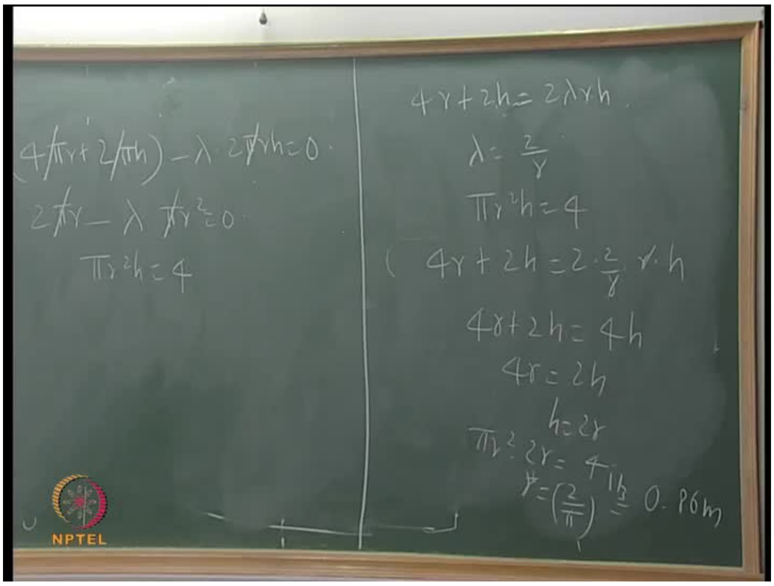

So, 4 r is equal to 2 lambda r h, correct. The second one you can cancel? Lambda equal to lambda, that is very good. I like something like that so simple. Third one is so pie r so 4 r, what? h equal to 2 r, not bad; we are almost there h equal to 2 r, then r is equal to? So, not bad at all 0.86 meters, I know the answer.

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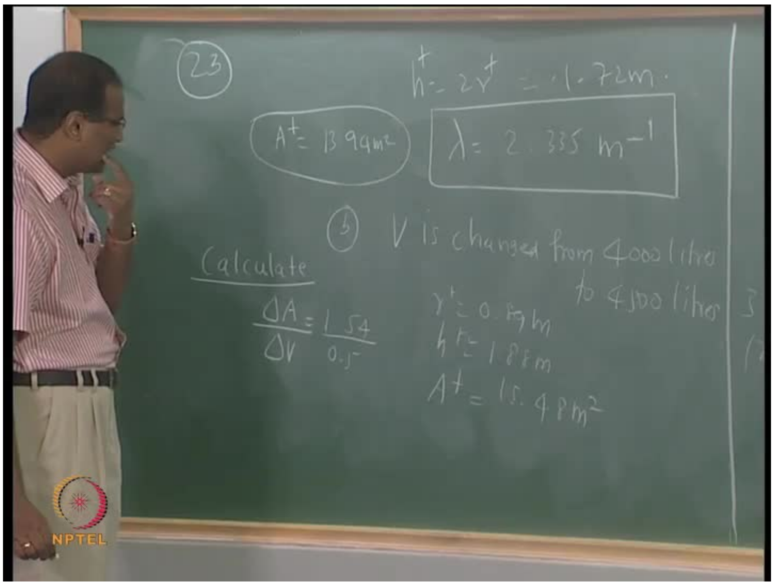

Proceed; h equal to 2r, oh please put the plus lambda.

Student: 2.335.

2.335, what are the units of lambda?

Student: Meter inverse.

Very good, what are the units of lambda? Meter inverse, did everybody figure out this? Meter inverse, what meter square? Area by volume man, okay. So, area by volume meter inverse. So, where is that 4 r coming? It is because of that, okay. So, now we got the optimum A, A plus, r plus, h plus. We got one more value lambda; we do not know what to do with that, we got it. But now we have to interpret what this lambda is all about. So we will do one more exercise; I am not giving problem number 25, 24 b.

Suppose V changes from 4000 liters to 4500 liters what happens to the solution? Can you work it out; can you get me r plus, h plus, A plus? That is why I thought it would be better if you had written everything in terms of the volume and other thing here. So, we are trying to interpret lambda; in order to do this it will be better if we rework the problem with the new volume of 4500 liters you need a 4000 liters.

Student: 0.98 is r.

r is 0.89, okay r plus. What is A plus here? 13 point the same thing as we got before, 13 point meter square, alright. Now r plus is?

Student: 0.89 m.

Good, h plus is 1.88 meter. A plus; Abhishek, A plus? Please do not feel lazy, this 5 minutes will be very critical; this is very very insightful.

Student: 15.48.

15.48, yeah I hope everybody has got this. So, what we have done is we reworked the Lagrange multiplier, we reworked this solar water heater storage problem using the Lagrange multiplier method for a two variable 1 constraint problem; by changing the volume from 4000 liters to 4500 liters we are trying to see what is happening to the r plus, the h plus and the A plus. So, we got the new values of A plus, okay is it right.

Now please calculate the change in area divided by the change in volume 15.48 minus 13.94 divided by 0.5 meter cube; are you getting the point. From 4000 liters if you change to 4000 liters you are getting a new optimal area; for 4000 liters you got an optimal area. I am asking you to take the difference between the two optimal areas divided by the change in the volume. This is 1.54 divided by 0.5; is it correct, is it 15.48? No, it is not correct.

Student: 15 sir.

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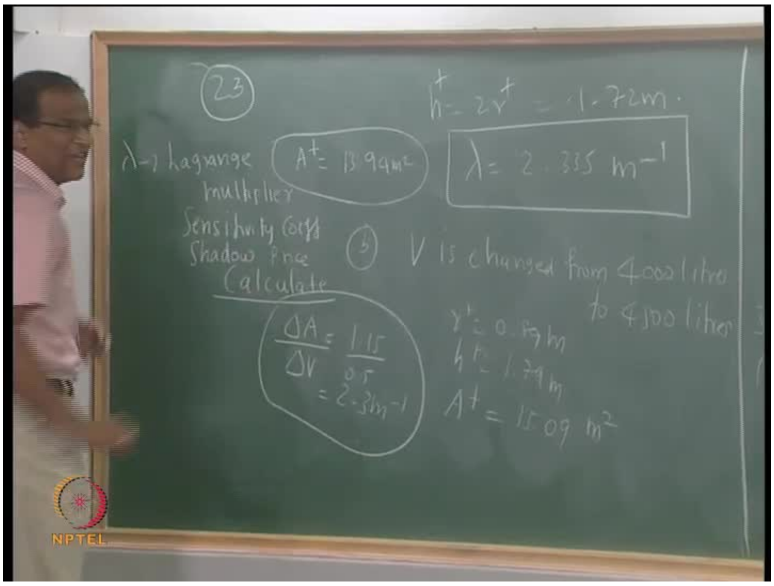

It is 15 eh?

Student: 1.79.

Sorry. This is how much?

Student: 15.09.

So, delta A will be 1.09 1.15, correct, divided by 0.5. How much is this?

Student: 2.3 meter inverse.

2.3 meter inverse, what a nice result? What is the value of lambda? What is that delta A by delta V?

Student: 2.3.

So, lambda is nothing but the change in the objective function with respect to the change in the constraints. Lambda is called the shadow price in operations research. So, lambda is the Lagrange multiplier; it is also called the sensitivity coefficient; it is also called the shadow price. If I relax the constraint from 4 meter cube to 4.5 meter cube how much additional area I can get, what is the sensitivity?

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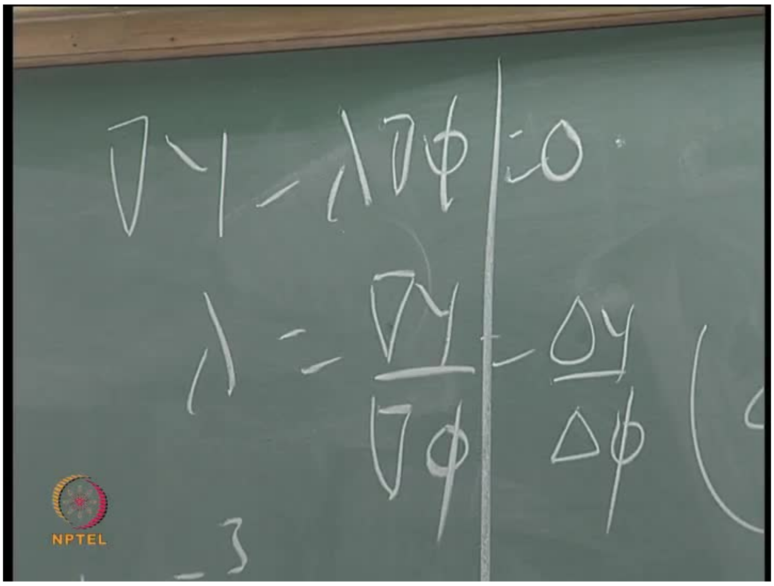

In fact it comes from the governing equation itself; so, delta Y minus lambda delta phi equal to 0. This is nothing but lambda is del Y by del phi but finally we have made it as del Y by del phi. How this inverted delta becomes delta we will see in one of the later classes. Conceptually we can figure out that it is basically del Y by del phi; the change in the objective function with respect to the change in constraint. If there are m constraints each of this lambdas represents the sensitivity of the objective function to that particular constraint. In this case we had only one constraint, so the sensitivity of the area to the constraint. So, lambda is a very important physical parameter; lambda has its physical interpretation.

So, when you work using the constraint optimization approach with the Lagrange multiplier method, apart from getting the optimal values you also get to calculate this sensitivity coefficients which is helpful to evaluate whether you can relax this constraint, whether you can get you can put up the optimal solution to get better results. I think we have covered fairly good areas today, we will stop here. If any doubts I will answer, we will go to some more problems. In Lagrange multiplier method we will have the graphical interpretation of Lagrange multiplier, the mathematical proof of the Lagrange multiplier and one or two problems. Okay, so there are no doubts, I will stop here.