Design and Optimization of Energy Systems
Prof. C. Balaji
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture No. # 14

Example for Lagrange Interpolation

A long heat generating wall, a long heat generating wall, a long heat generating wall is bathed not baked, a long heat generating wall is bathed by a cold fluid; a long heat generating wall is bathed by a cold fluid on both sides. The thickness of the wall is 10 centimeter. A long heat generating wall is bathed by a cold fluid on both sides; the thickness of the wall is 10 centimeter and the thermal conductivity of the wall material, the thermal conductivity of the wall material k is 45 watts per meter per kelvin; k is equal to 45; thickness is 10 centimeter; the temperature of the cold fluid is 30 degree Celsius; the temperature of the cold fluid is 30 degree Celsius. Thermo couples located inside the wall, thermo couples located inside the wall, thermo couples located inside the wall show the following temperatures.

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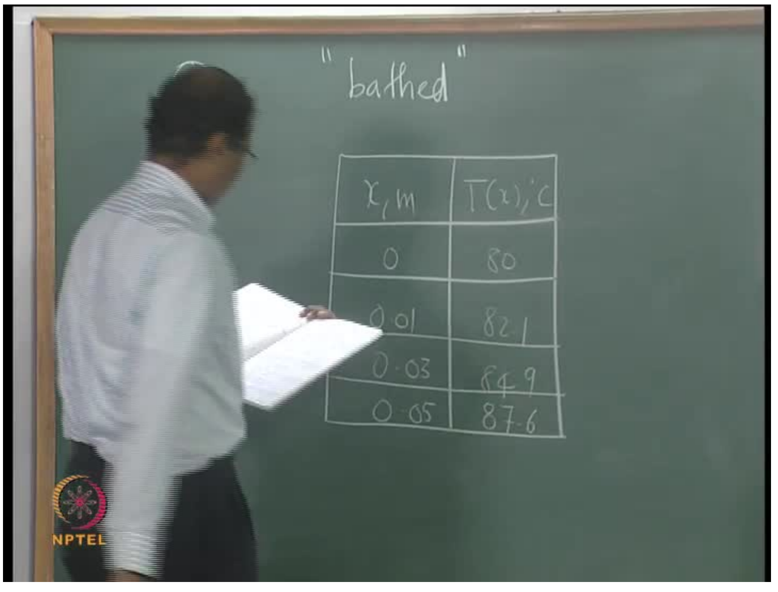

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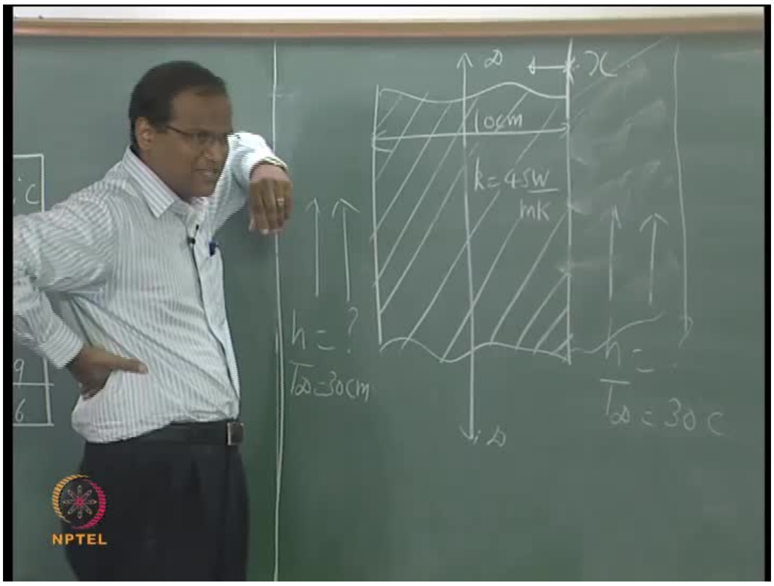

This is the wall. So, the thickness is 10 centimeter; k is 45 watt per meter per kelvin; on both sides bathed by a cold fluid; I do not know what is h; it is infinity; infinity is just a depiction to show that it is infinitely long. You have taken down all the data. The problem continues. The temperature distribution is symmetric about the mid plane. The temperature distribution is symmetric about the mid plane and the origin is indicated in the figure; that the temperature distribution is symmetric is consequent upon the fact that there is uniform volumetric heat generation; will come to that little later.

Using a second order Lagrange interpolation formula, find h at the surface; using a second order Lagrange interpolation formula find h at the surface if the cross sectional area of the wall is 1 meter square, if the cross sectional area of the wall is 1 meter square; that means I am talking about this area; if the cross sectional area for the wall is 1 meter square, determine the volumetric heat generation rate, determine the volumetric heat generation rate, determine the volumetric heat generation rate in the wall, for steady state conditions in the wall, for steady state conditions; just think about this problem.

No, the temperature distribution is like this; cross sectional area is like this right.

No, but this, no no that infinity is basically in order to make it tractable is 1 meter square, what you are saying is correct; when it is infinity, I am saying with respect to 10 centimeter, it is alright.

No. He has a point, but without 1 meter square, I cannot proceed. If you want I can remove. If you want me to remove this, I can do that. Now, which is the cross sectional area, we will find. Just think about the problem. What is it we are trying to do here? You have to get T as the function of x and then do what?

q is equal to k times dT by dx. This q which is coming from the wall is the conduction; that which is going from the wall to the fluid is convection. I already told you it is steady state. So, whatever heat is coming from left to right through the wall has to go out. Therefore, this q conduction is equal to q convection.

Now, you can say, sir, straight away, I can; why go through this pain? Cannot I keep thermocouples here and all that; it will disturb the flow and all that; it may make a laminar flow turbulent and all that. We do that using hot wire anemometer. We can directly get some but the boundary layer is so thin; it is very difficult to do those measurements. Therefore, you would much rather prefer do the measurements on the wall where it is easy to put thermocouples and then using the conduction convection coupling, we just make use of the fact that q conduction here will be equal to…, here it is q conduction, it is q convection, and determine the heat transfer coefficient. So, this one way of determining the heat transfer coefficient;

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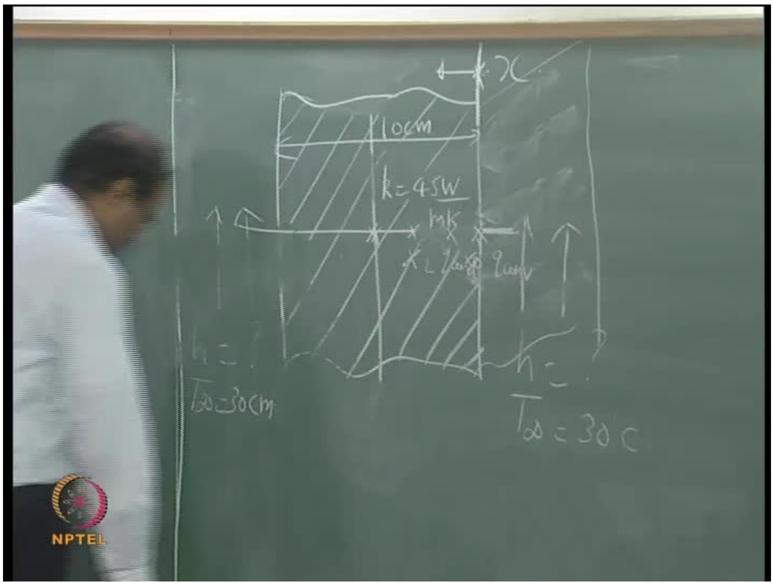

So, I pose it; this is typically what is called as inverse problem; I have a mathematical model for this; the mathematical model for this is d square T by dx square plus q v by k equal to 0. I have mathematical model for this; I have some measurements of temperature; that is I have x naught, x 1, x 2.

So, I have a mathematical model; I have some measurements of temperature; so, if I manage a mathematical model with these measurements, I am able to get much more information about the system. What is the information I am getting? I am getting two more parameters; I am able to get the heat transfer coefficient; at the same time, I am able to get the volumetric heat generation rate. This straight problem would be very simple, what you learn in ME 3 7, that is a basic heat transfer course will be. There is a curve there is a wall which is conducting and there is a convection on the boundary. T transfer course is given to be 250 watts per meter square kelvin it generates; there is the volumetric heat generation of 10 to the power of 5 watts per meter per kelvin; what is the surface temperature? What is the temperature? 1 millimeter away from this surface, 1 centimeter away from surface, whatever; so, this will be the, so that will be the straight problem.

Now, we are making some measurements and we are inferring some properties of the system. So, this is what is typically called a inverse problem. Fine, now, I gave it as 10 centimeter,. So, I hope 1 is here, 0? What is this? (Refer Slide Time: 09:46). So, 1 is already at the center right? But I gave you 4. Now, we were having a problem. You have to choose 3 for the Lagrange right? Which 3 did you decide?

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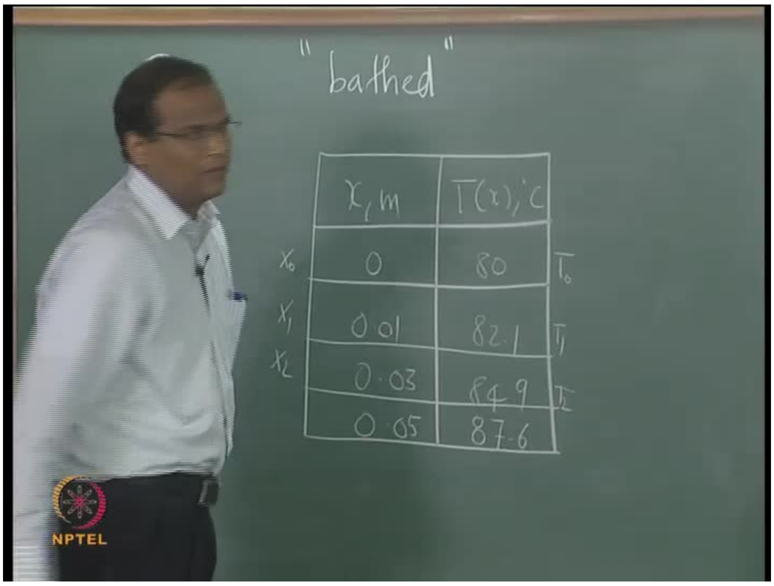

We would choose something which is closest to the wall. So, now, I have no problem. You can choose another one also, right.

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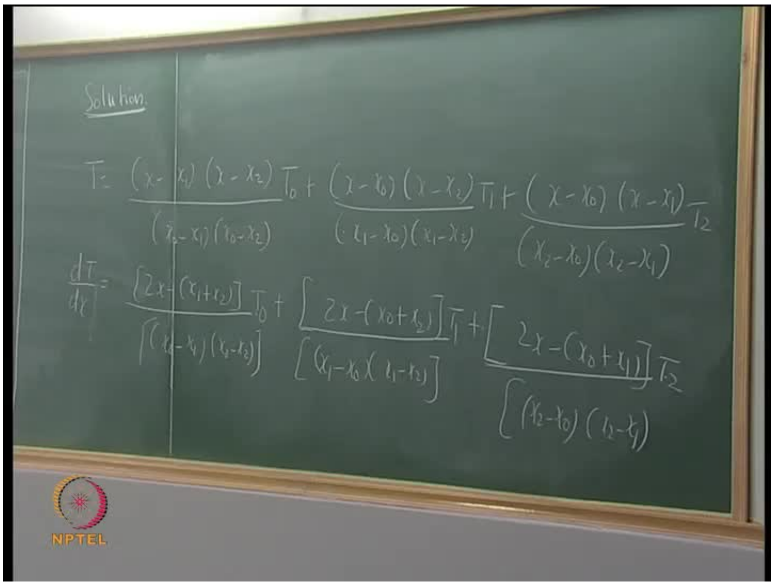

So, do not look at what I am writing on the board; you do parallel processing; then, you at the end, you can exchange notes; you can look at the board and see whether what I am doing is alright; you just proceed. Now, I want dT by dx where? at x naught; so, first, we will put dT by dx. Let us not try to combine too many steps. So, just do the dT by dx. You can substitute for all the values of x.

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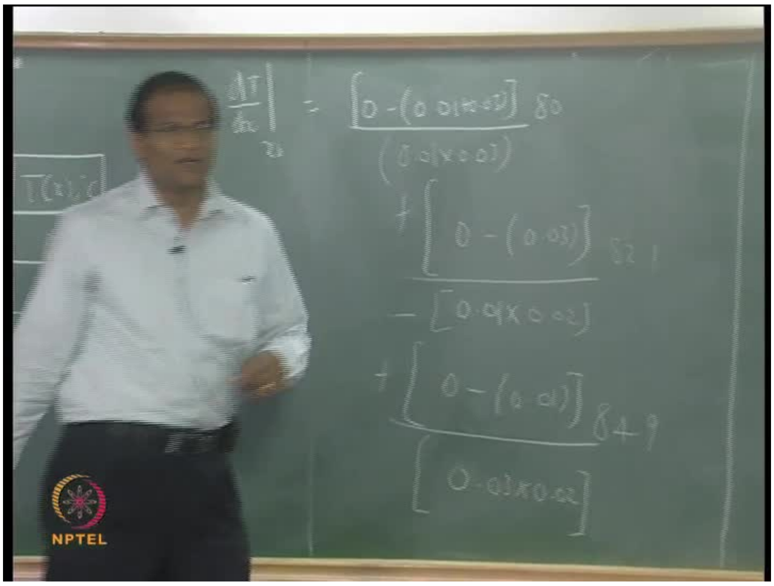

So, dT by dx; both are plus right? Minus into minus is plus right? It is multiplied by x 2 minus x naught; is it right? Did I make any mistake? It is alright? Akshay, is it okay?

So, Sampath, what is that dT by dx at x naught?

What is the dT by dx at x naught?

Student: 23.34.

23? It will be very large.

It will be very large.

Student: 21,500

Is it?

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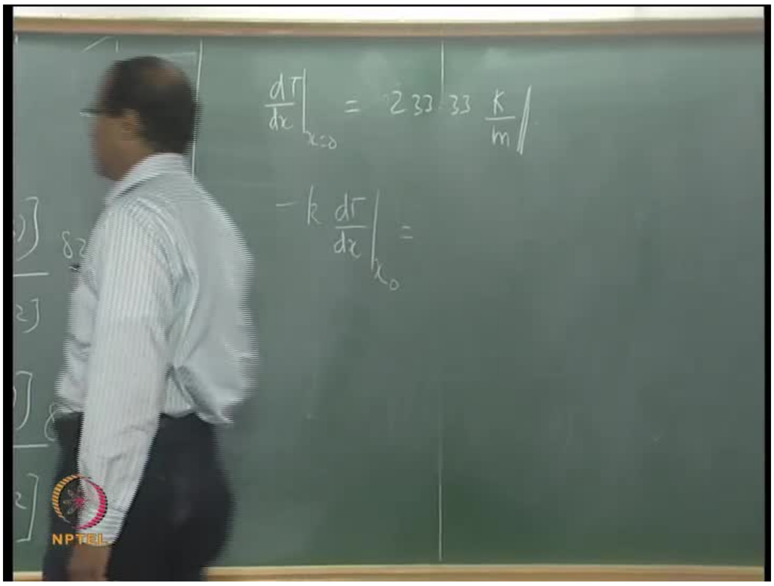

No, the first is minus right? The first is minus; the second is plus; the third is minus 233 correct, 233.33. Yes, what are the units? Kelvin per meter.

What degree Celsius?

Please, what is bothering you? 1 degree Celsius is equal to 1 kelvin if you talking about differences in temperature; correct? All right.

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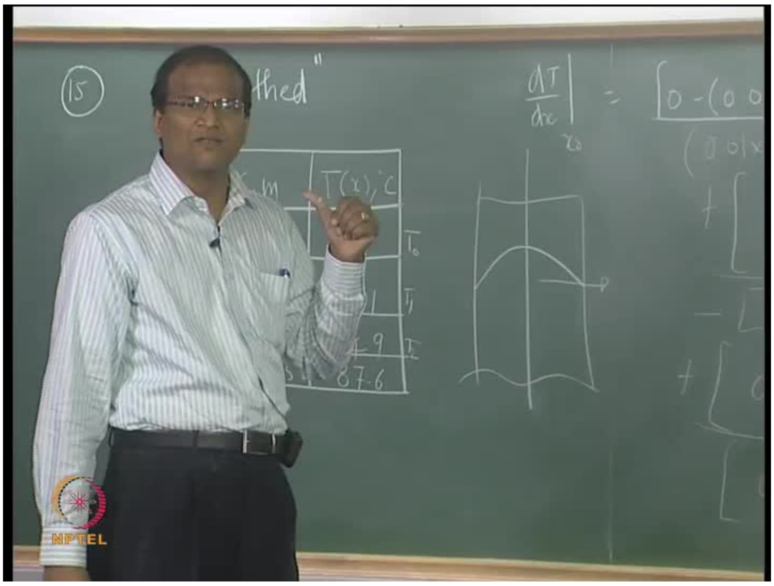

So, now, so, how is it going, the temperature? It going like this; so, in which way is the heat is flowing? Like this. So, it is in the x is in which direction for this? Therefore, we have to adjust this.

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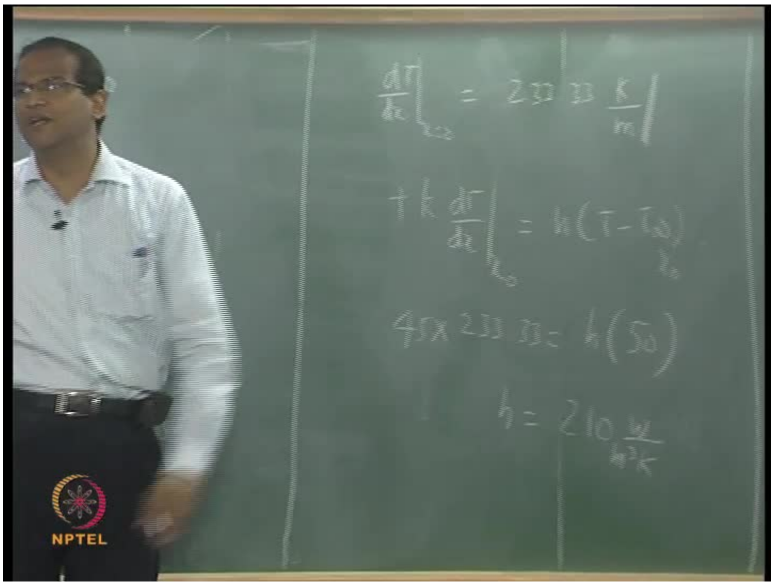

80 minus 30 is 50. So, h equal to what did you get? You got something different.

But is it can be so, it cannot be the way of…

There is no not the value there is 209.

Student: 209.2.

Other one?

So, what was the value of the gradient? You got it 233.3; see, when it was all these complicated, tough, you are taking .01, .03, minus, plus, and all these and there is a possibility that you can get lost, but back to basics is very very important. If you took a very desi approach, you will take only two temperatures and divide it by delta x. So, 2.1 divided by 0.1, it is around what is the gradient? 210; so, this is the ball park value. So, you should get of the same order. If here you get 200, here you get 900, something has come here; some minus you put as plus or if suppose you get 23600 or something, then you should…; then you know that it is always in Lagrange, one will be minus, one will be plus, you have to be extremely careful; you have to try it with caution.

Now, what is the last part of the story? What do you get as the volumetric heat generation rate? What is the problem? You can go to the equation, but there is several ways of doing it. What is the heat transfer from the right side? h a delta T. What is heat transfer from the left side? h a delta T. Total heat transfer is 2 h a delta T. Where is all the heat coming from?

Heat generated per ohm; so ,q v, so many watts per meter cube into 1 meter square into 0.1; so, the q v into volume must be equal to we get 2 h a delta T; there you are. q v equal to then get the value. So, it should be the order of 10 to the power of 5 watts per meter cube. This is without taking recourse of the equation. The equation is d square by dT dx square plus q v by k is equal to 0; it is very threateningly formal way of stating the energy balance. Is it Nishanth? Anish you got it?

Student: Yes sir.

Temperature.

While taking the wall temperature, conduction convection use this temperature.

Can I use this or should again find out.

No, but if you do this, you are not getting the gradient of the wall. You have to get dT by dx; so, you should use 3 temperatures; you should use at least 0; so, see whenever taking three temperatures, you could have taken any three out of the four, but 0, 80 degree centigrade is very crucial because you are evaluating the gradient there. For example, what Sampath has done is, anyway Lagrange interpolation second order has three points; so, he has taken .01, .03, .05, but with these three, you cannot get the gradient at 0. Gradient at 0 is what matters because k dT by dx at x 0 the heat transferred by convection, you could have taking this, this, this, or this, this, this, or this, this this but as you go away and away from the wall, because of non-linear effects the effect is more error prone.

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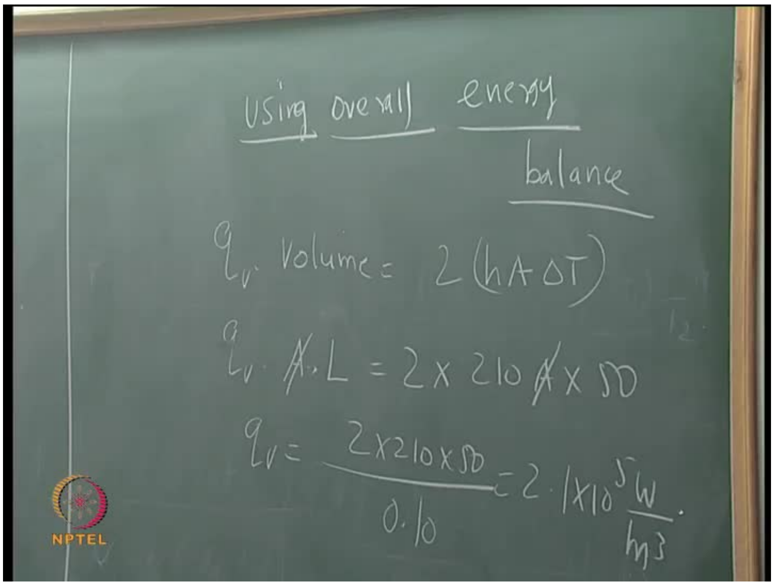

So, using overall energy balance: Thickness, we will call it as L. is it okay?

8.5 into…, that is all?

Student: 2.1 into 2.5.

Did I make a mistake here? Alright; so, there is a heat generating wall; so there is a heat generating wall which has the thickness of 10 centimeter whose thermal conductivity is given by 45 watts per meter per kelvin, and heat generation rate is estimated to be 2.1 into 10 the 5 watts the meter cube; now, it is possible for you to apply the knowledge of heat transfer and get the maximum temperature and you can turn around and see whether the temperature is same as what is measured at 0.05.

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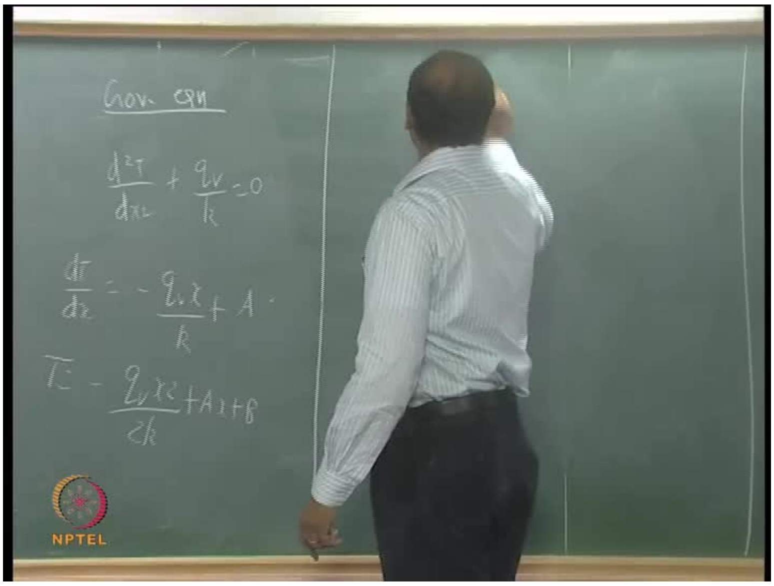

So, what is the governing equation?

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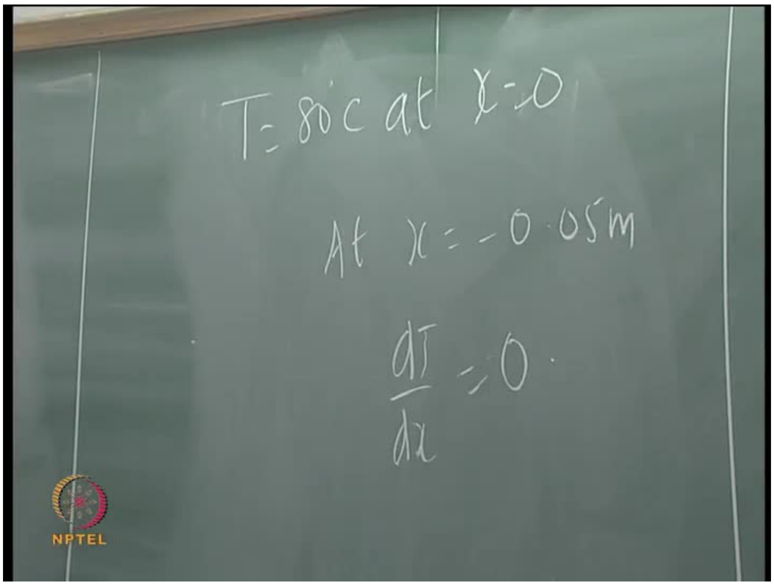

So, this is symmetric at the center. With these two boundary conditions, you can solve for the a and b; then you can get the temperature. This is an additional check; so, this is very simple presentation of inverse heat transfer. so, I surprisingly taught you inverse heat transfer though the original intention is to teach you Lagrange interpolation formula; this inverse heat transfer can be very powerful; for example, last year when people were…, at the international terminal when people were arriving, they we are taking thermal infrared image; it is basically they were working at an inverse problem.

So, when they find that the radiation from the nose and the certain parts exceeds certain value, then you are having fever; then it has to be swine flu. And for example, thermal infrared, for example, for breast cancer detection, you can use infrared camera and get the image. Now, if there is a tumor inside the breast, the metabolism will be high; metabolism is high, that means, watts per meter cube, that volumetric heat generation will be high corresponding to the tissue which is non-cancerous.

Therefore, this will show as a signature on the surface temperature of the breast. So, This called a Breast Thermogram. Now, from the temperature on the breast, if on the surface on the breast, if you can solve these equations; that is, you can assume the location of the tumor and the so many watts per meter cube; for different scenarios we can recreate what will be the temperature on the surface. Then, find out which matches with that which has measured; then you can get exact location and size; like that u can do for several, you can use for additional thing.

So, the Computerized Axial Thermography, the CAT or the CT scan, basically works like this. It tries to create three dimensional views by taking repeatedly to 2D views in several angles and so what is running behind? What is what engine is behind all these CT scans? There is a powerful optimization program which is working; so, medicine is more or less orthopedics is mostly mechanical engineering. They put plate, exercise things, nuts, bolts, everything they do lots of… Orthopedics is engineering now, almost; so, there is a journal of engineering in medicine. Lot of orthopedic doctor’s journal of engineering in medicine; orthopedics doctors have also got PhD in engineering now a days that there are some in Madras Apollo hospital, who have the super specialisation orthopedics surgery, they also have PhD in engineering.

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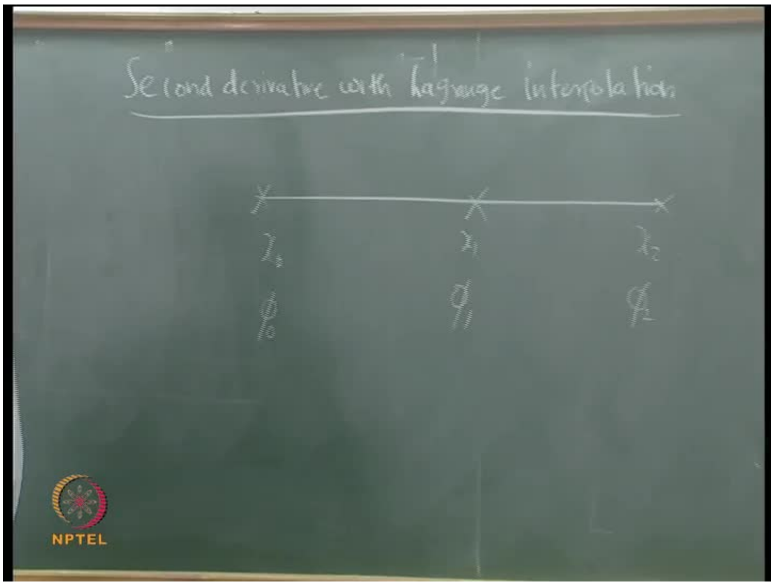

So, now, was there someone who got the value of T at the center?

What you get as the second derivative with Lagrange interpolation? For example, I denote a general variable phi or phi instead of T; it could be any variable; you can write the Lagrange formula. You can take dT by dx. Now, you get dT by dx; like that you have to define d phi by dx; can you get d square by phi by dx square?

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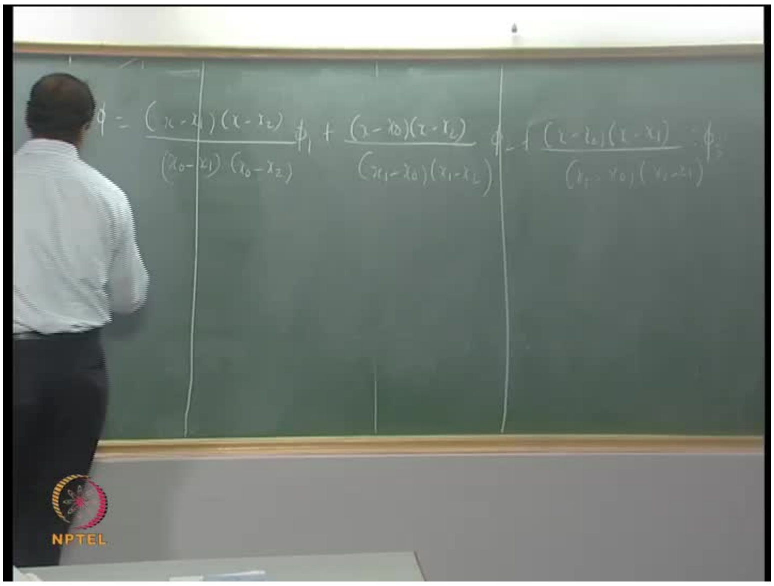

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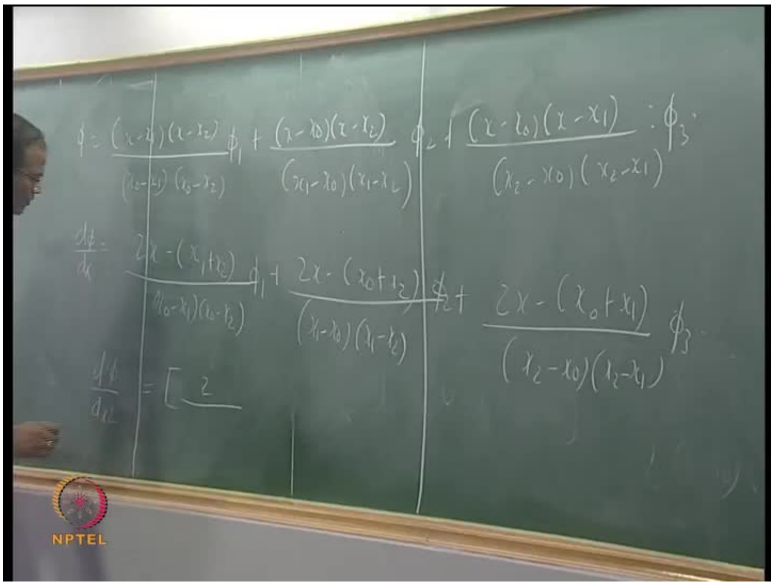

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So, first derivative, 2 by…. right; so, what is the big deal?

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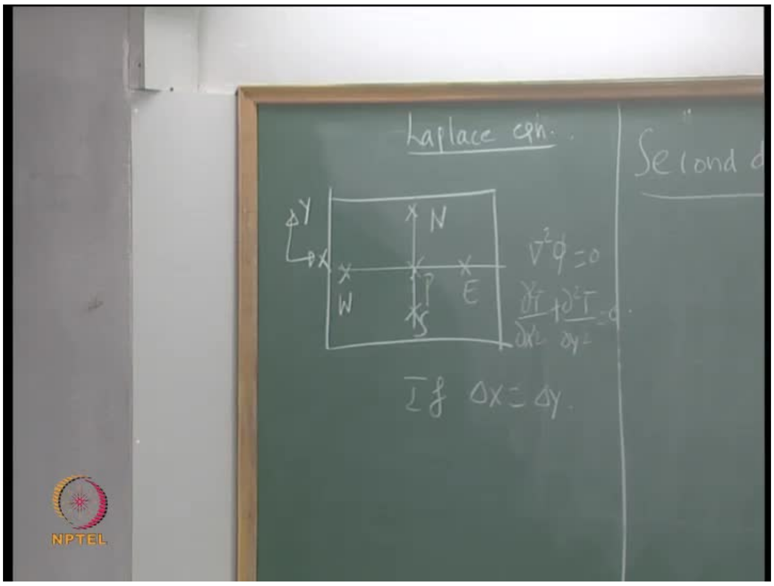

Let us consider the Laplace equation. The Laplace equation del square phi, where the del square phi is the Laplacing operator, d square T by dx square plus d square by T by dy square is equal to 0. Let us consider, I consider this as point p; I consider this as point east; this as west; this as north; this as south. If for this is x and this is y, if delta x equal to delta y, using Lagrange interpolating formula, it is possible for you to get second degree derivative and write the second derivative at P in terms of the values at east and west if you are considering d square phi by d square phi by dx square; by the same token, it is possible for you to write the second derivative at p, that is d square phi by d y square in terms of north and south; we simplify and we will get an amazing result; please do that; you will get an amazing result if delta x is equal to delta y.

See, this cannot be straight away; this d square phi by dx square is the same at all the three points; it is not a function of the x, is it not? Correct? So, what does it mean?

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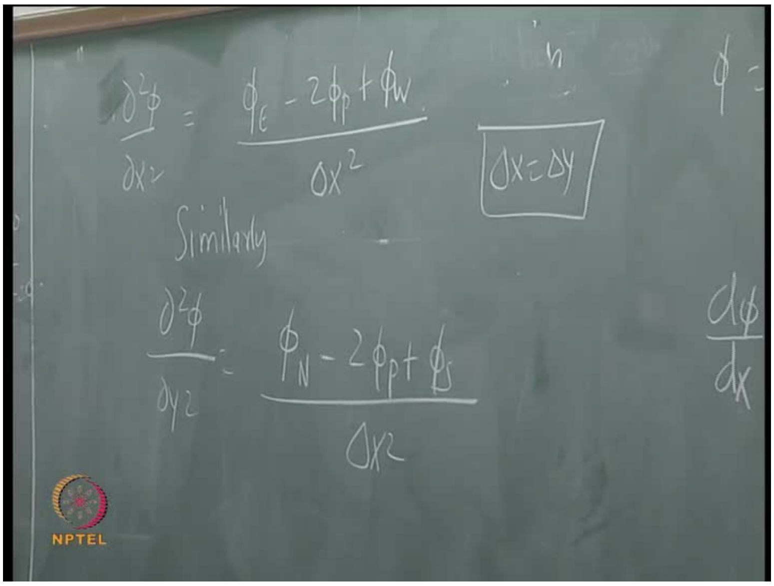

For example, I should have… you got reduce it to that form; will you do that? So, this will be delta x, what is this? So, this will be delta x. What is this? This will be, this will not be delta x; it should be, please correct me, minus delta x. So, this will be minus delta x; this will be minus 2 delta x; this will be delta x minus delta x, but this will already has a 2 here; this fellow already has a 2 here; this fellows have a 2 in the denominator; therefore, this fellow will get multiplied twice. So, that is the node under consideration. Similarly, is it too fast? Is it going beyond our head? Or, above your head? Fine, right? Difficult?

Why did not you get d square phi by dx square at the center using the Lagrange; can you write the Lag range polynomial in terms of x and then can you get the first derivative and second derivative at P? That will be this; the second derivative, anyway, is not a function of x. So, whatever you have written is applicable. Except that, this is 1, 2 right? Why did you write 2, 3?

I wrote 1 2 3 no, but this not consistent with this.

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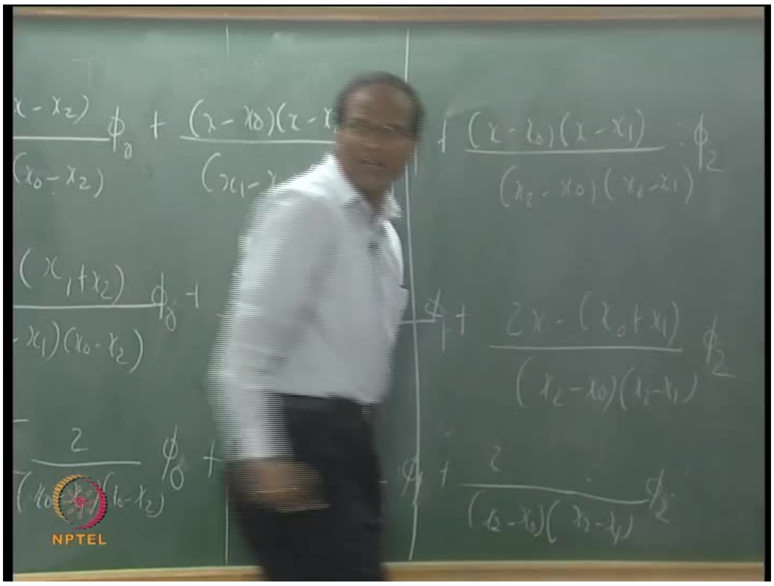

No no, we will change. Deepak you got it now? What is bothering you?

What is the… I am trying to evaluate the partial derivative using the Lagrange method.

Which one?

No, no. We are with the understanding that they are looking at the derivative in the x direction, it is alright, it is perfectly okay; there is no problem. Now, if you do this, so what I am trying to do is, I am trying to write out this second derivative using the Lagrange interpolation formula. Now, if I do this…

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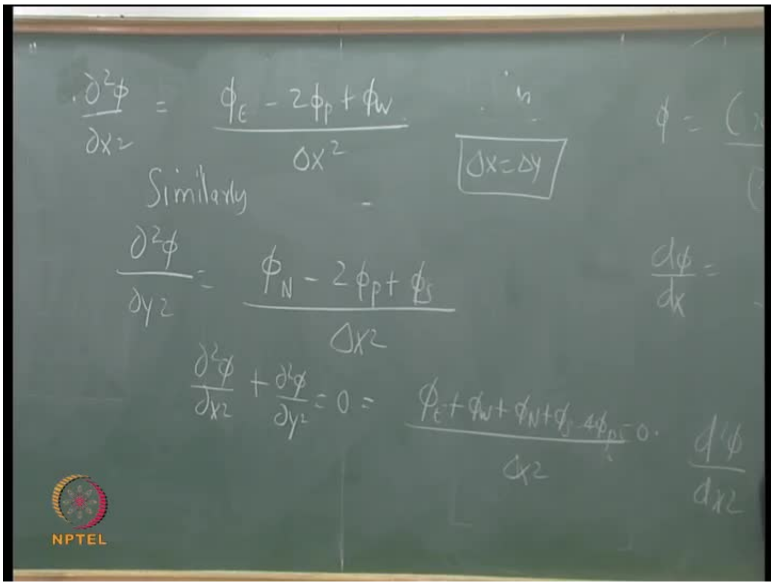

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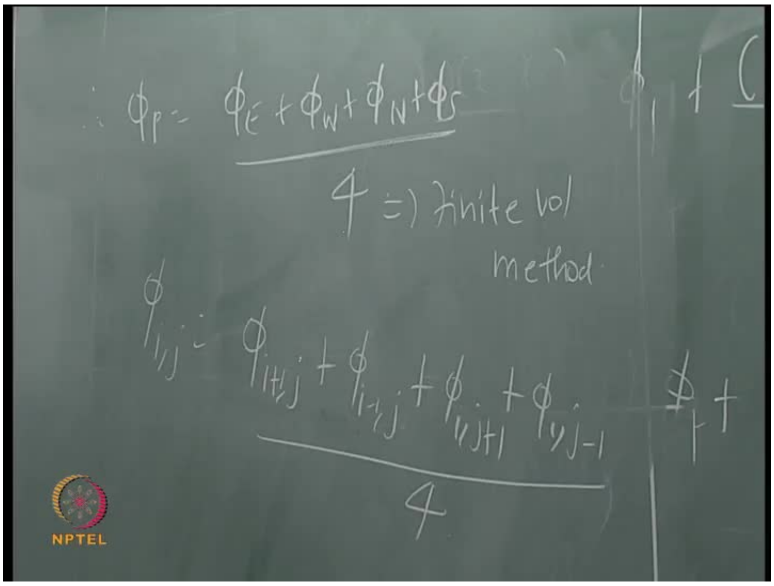

What is this? When you write like this, what it is known as? This comes from where? Finite volume, this is called finite volume. Of course, you can say, no, East is what? What is this? This is finite differential. So, the Lagrange interpolation formula can be used to get the finite difference and finite volume representation of partial derivatives also. More importantly, when you solve it for a, have you taken this down?

Very good question.

So, this basically the finite volume, though the end result looks to be the same, the process by which we do this is completely different. In a finite volume method, we actually integrate the equation and you conserve; you conserve the mass momentum and it turns. It finally turns out like this, but this fellow we break that derivative; dT by dx is t i plus 1 minus t t t i plus 1 j minus t i j divided by delta x. So, eventually it may lead to same this thing; but it leads to the same result eventually because it has so many things, delta x is equal to delta y and this thing.

So, finite volume is basically a conservative scheme; it conserves mass, this thing. Finite difference, basically finite difference works on Tailor series approximation. You cut it off after a certain number of terms. This basically integrates.

Finite volume method you can actually work further and develop what is called the finite element method. Even there you can integrate the governing equation. Once when you integrate the governing equations, all the partial differential equation will be conservation of fluxes; correct? What is this u dou u by dou x after all? dou by dou x of e square into half, and you can apply the continuity equation; you can write the whole thing in term; you have not learnt bound late theory and Navier-Stokes equation; so, you can rewrite and rewrite all that and you can you integrate the governing equation once and then you will get the finite volume at the finite element of (()).

So, to cut a long story short, from the Lagrange interpolation formula, it is possible for you to proceed the solution of partial differential equations. So, even this, you can setup as scheme where I can where I ask have east, west, north, south, they either for a next fellow. When I ask you the set of four equations using the Gauss Seidel method, you can keep on iterating. You start with some initial guess value and you keep on iterating. This fellow depends on these four fellows; this fellow depends on these four fellows, and so on, but the beauty of this is, this is mathematically from the, from the method of characteristics, we know that this is what known as a elliptic equation in mathematics. An elliptic equation requires information from all sides for it to be mathematically close so that we can solve it.

A parabolic equation is one in which the information does not flow downstream; it is typically like this; a parabolic like this; so, the information does not flow downstream. So, an initial value problem, you start you start with an initial concentration and initial temperature; how this marches in time? You do not worry about, you do not go backwards in time; that is a parabolic equation; so, you do not require any information from that. So, you march ahead in time. Similarly, in an elliptic equation, you require information from all the boundaries. So, information, so, here it is to symbolize why that the nodal value at P is dependent on east, west, north and south.

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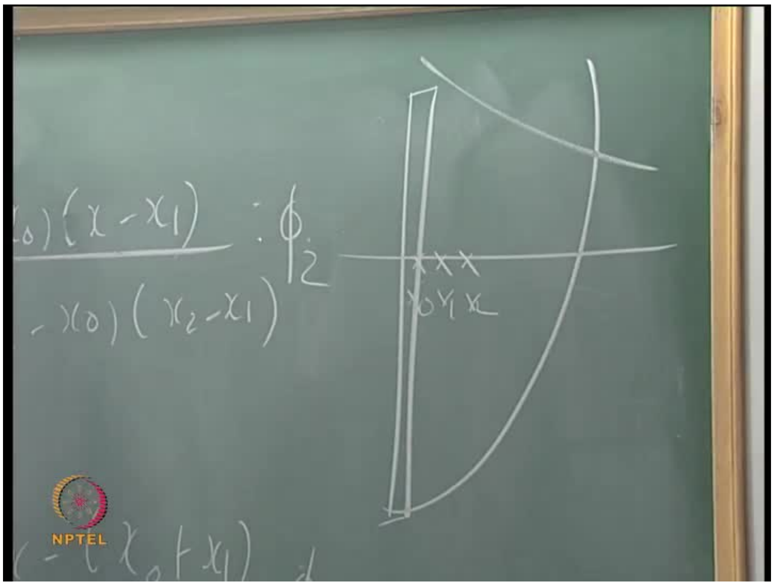

So, to round out the discussion, for example, if you have boundary layer which is developing like this and you want to measure the temperature or something, so you can take here anemometer and get points, and get the temperature three points: x naught, x 1, and x 2. Using second order Lagrange interpolation formula, you can get a better estimate of the gradient rather than taking x naught and x 1 and taking dT by dx 1 by taking t 2 t 1 minus t naught divided by x 1 minus x naught.

So, this can be used for post processing or you are doing fluent calculations. From the temperature and the velocity fields obtained, you do not have to believe the values of heat transfer which is reported by fluent. You can take the raw temperature data, you can use a Lagrange interpolation formula and use a second order of a cubic polynomial, and get a revised estimate of the gradients and then the heat transfer.