Advanced Gas Dynamics
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Lecture - 09
The relation of physical properties across an oblique shock

So we were on oblique shocks. So, when we wrote down the equations for oblique shocks we saw that basically they are all same as the normal shocks, except that the parallel component of the velocities they do not change across the oblique shock. So, just to get a reminder as in the normal shocks, we wrote out all the changes in properties in terms of the impinging Mach number. So, we had an expression like this. So, this is the Mach number beyond the shock.

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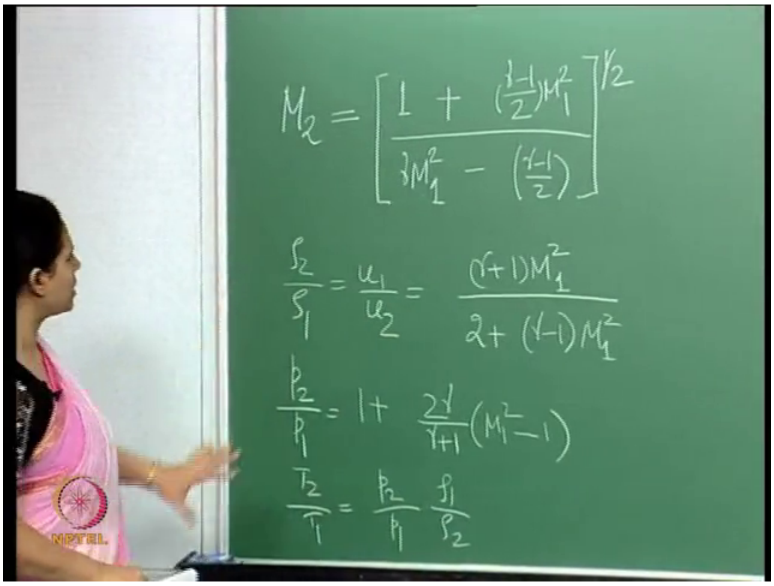

So, this we wrote down as right. So, this is just two kind of remainder themselves, similarly we wrote say this for. Then again this is for density and then this is for the pressure and right. So, these are expressions that we had for a normal shock. So, these are all expressions for a normal shock now like we found out in that for oblique shocks.

Now, the component parallel to the oblique shock does not change, component of the velocity parallel to the oblique shock is does not there is no change in that. So, basically the oblique changes across an oblique shock changes in the flow properties across an oblique shock essentially governed by the normal component or the velocity change across it.

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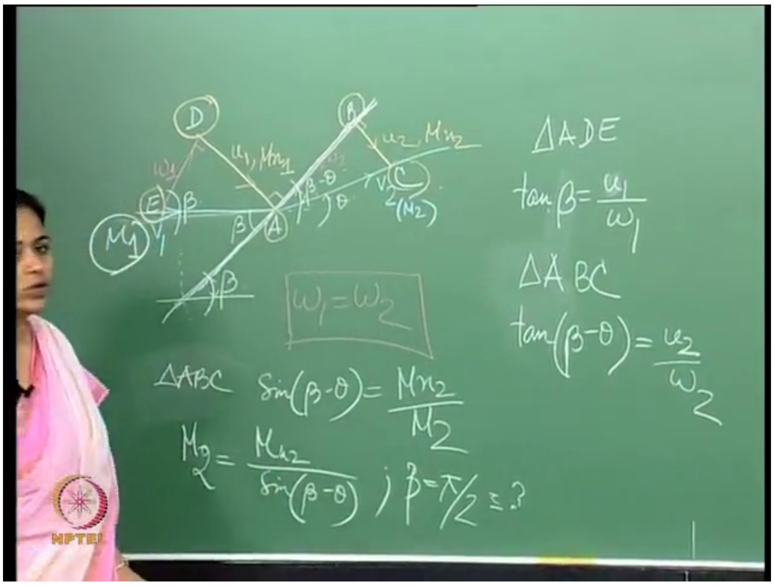

So, if I redraw this, if this is my shock right and say this is my right.

So, this is v 1 and this is v 2. So, then, this is w 1 and similarly we have this on the other side. So, this is u 2 and this is w 2. So, this is from geometry and we said that this angle is going to be beta and this angle is theta this is the thing. Now what I am trying to say here is that there is no change in this component of the velocity which is that across the normal shock w 1 is equal to w 2 this is what we proved from last time ok.

So, therefore, the changes basically or happening because of the changes in the normal component here which is u 1 and u 2. Now if I were to write the corresponding Mach number here as M say n 1 and M n 2. So, the changes basically adjust this velocity component. So, therefore, the change is because of this Mach number. So, if I were to write this the expressions for all the changes in the properties, in this with respect to this Mach number then we should be able to use all the normal shock properties you know from the tables ok.

So, let us do that. So, if I have to do that here, instead of this. So, let us just write out these expressions.

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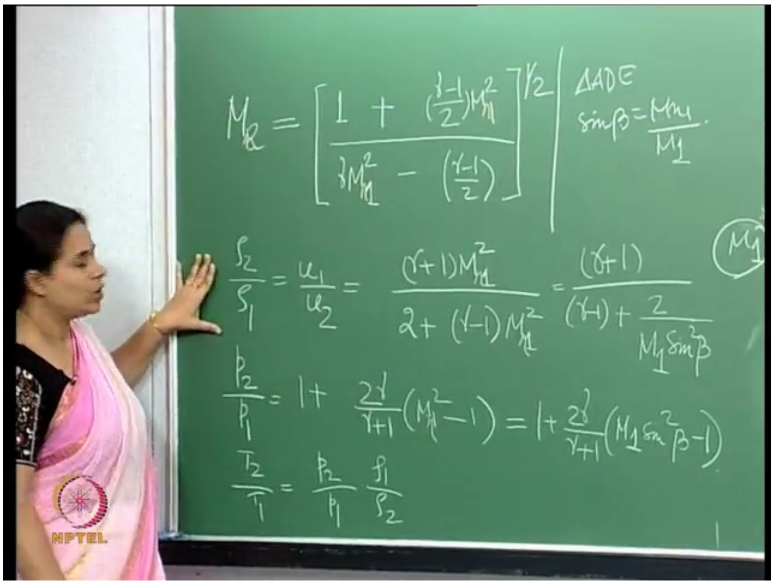

So, instead of this If I were to write n here and I introduce n here the do the trick. So, all I write here is n all I write here is n and so on and so forth right. So, therefore, these are essentially. So, what you can see here that I have written this in terms of.

So, I have written these the these relations now are basically valid for an oblique shock and I write all of these expressions in terms of the normal Mach number over here. Now taking from there now this is our geometry, now we can also write like this now M 2. So, basically if you look at say these triangles over here say for I say A B and C. So, this is say M 2 right the correct Mach number corresponding to this is M 2 right.

So, if I were to write that. So, this angle here beta minus theta right. So, if I use this triangle right. So, what I get is right we get this or basically you can write M 2 equal to. So, this is an expression for M 2. Now why is that important? Because this is the impinging Mach number; so this is my M 1 right this is my M 1 and how do I find out M 2 well we need to go slightly run about way, because this is an oblique shock and like.

If this is impinging Mach number across a normal shock, we would just go look at the tables and find out the corresponding M 2 right. In here though I can do that first of all I need to find out M n 2 which I can do from here right and which I can do from here and no because I know M 1 and I can find out the normal component.

So, once I find M n 2 from here. So, knowing the geometry from here, then I can find out M 2 this is a corresponding Mach number; now looking at this just if you look at this just think about this. So, basically this is beta, if you just look at the geometry over here if you just look at the geometry. So, if beta is equal to pi by 2 then what do you get right; if beta is equal to pi by 2.

So, then what do you get just think about it and if you also put pi by 2 over here then what do we get? Just think about it here as well. So, now, basically I think you can see from here just from the geometry this that if this beta is equal to pi by 2 you actually get a normal shock is not it then the this the shock is no more inclined it is normal to this velocity stream line.

So, having said that, now let us use a couple of things over here. So, this is my beta. So, in that case let this is my beta then what do we get. So, this is my beta right this is my beta. So, then this again becomes my beta right. So, if I do this, then what I can find. So, let us do some geometry here let us call this a b c and ok.

So, this let us just name out triangles. So, A B and C and let us call this say D and E here. So, if I going to do that, then what we get is from say this triangle A D E right looking at this triangle over here. So, then what I get is right this is just from the geometry. So, this is just from the geometry, geometry this is tan beta need to write this right this is.

So, tan beta is equal to u 1 by w 1 and again similarly. So, now, we look at this triangle A B C. So, we are looking at this triangle over here. So, then what we get is tan right tan beta minus theta is equal to u 2 by w 2. So, u 2 by w 2 is not it.

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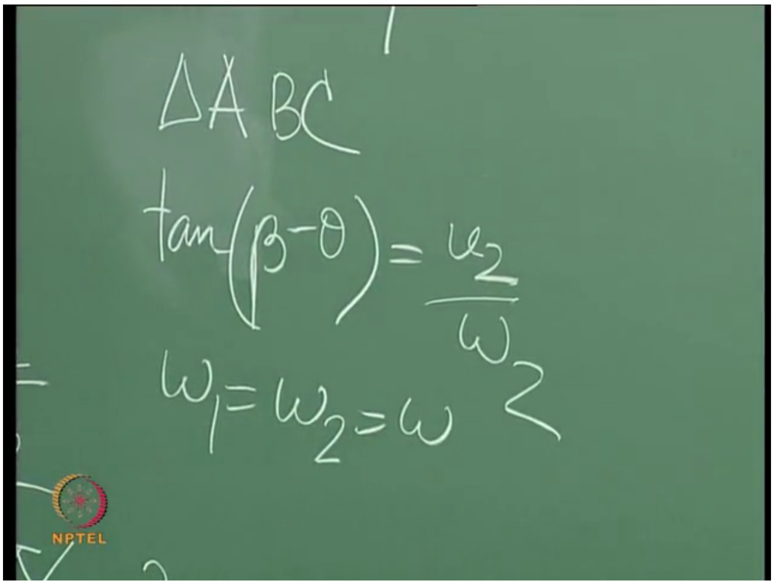

Now like we know that w 1 is equal to w 2 which is equal to w say ok.

So, if that is true, then if we can write in that case we can write.

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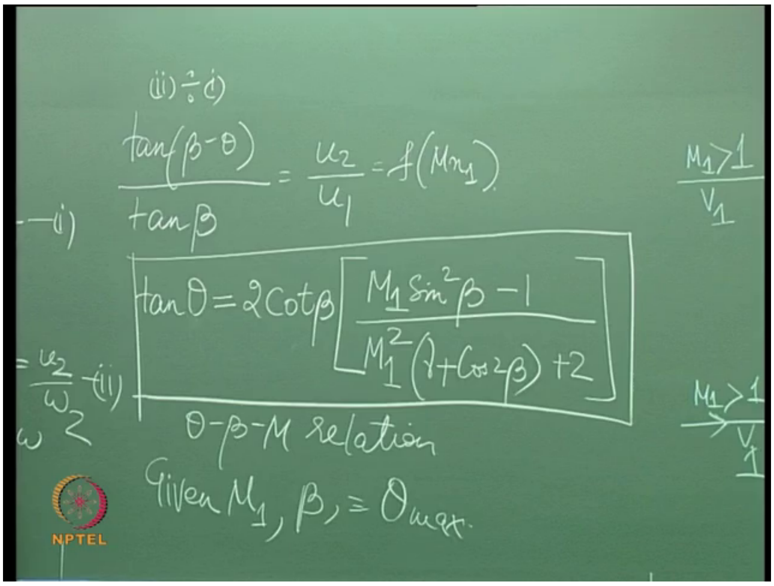

So, what we can write is u 2 by u 1 we can write this as u 2 by u 1 now. So, I hope you understood about what we did over here. So, tan of beta minus theta is this, I just divided basically. So, if I write say this is one right. So, all I did here is this right. So, if I do this. So, basically what I get is tan of beta minus theta by tan of beta is u 2 by u 1 because like we said w 1 is equal to w 2 right you found that before.

Now, if we come over here you can see we have an expression for u 2 by u 1 over here in terms of the normal component of the impinging Mach number right. So, I can actually write that expression, I can relate that expression with this. So, if I do that, so what I am saying is this is also now a function of the impinging Mach number right and if I do the math what I will get is this, tan theta is equal to 2 cot beta ok.

So, what we did over here is that this is an this is what we trying to do over here is that we trying to connect the geometry with the Mach number and what is that purpose behind it. Now let us try and understand this a little bit now for example, now what we were saying earlier on in terms of say we have a supersonic flow coming like that.

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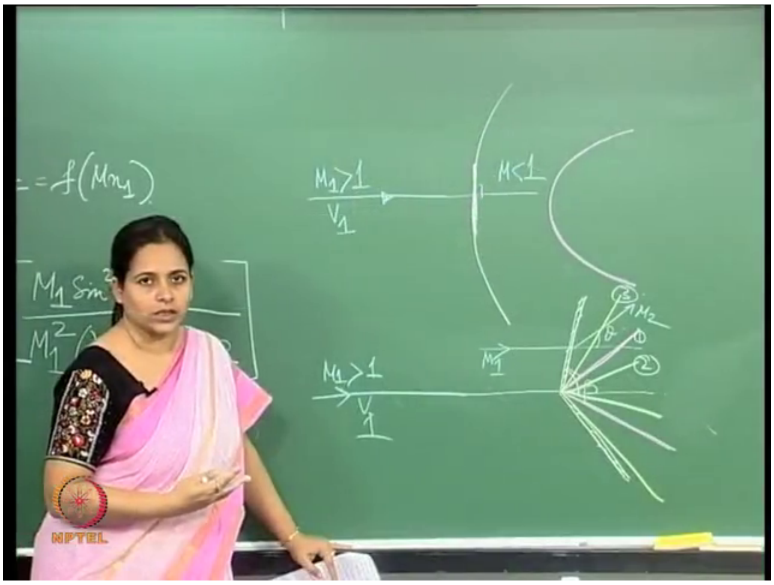

Say we have a supersonic flow coming like this right and it encounters and say it encounters say a more you know an obstruction of this nature. So, what we are must probably going to see is right a shock like that. So, this is essentially this is essentially the normal part of it and this is a detached bow shock right and this will be a normal shock this will be our normal shock in this case. However, let us change the picture a little bit.

So, say we have the same and. So, this gets compressed and so on and so forth. Now we have the same thing say over like that. So, again we have say supersonic flow over here now compare to this structure over here right this is more of a bluff kind of a body right. So, if I have something like this instead if I have say something like this if I have say something like this right then what is the kind of structure that we going to see right what is the kind of structure that we are going to see or we going to encounter this sort of a shock or what it is it that is going to happen.

So, what really happen here is that, we are going to get we are going to encounter basically an attached oblique shock right. So, what we are essentially trying to calculate over here is that this is say my M 1 and this is my M 2 right and in here right now this is an angle, which I am calling as beta and this is an angle I am calling as theta. So, essentially if my shape of the body changes then this is the kind of structure that we going to see. Now the next question to ask is that ok.

Now, what governs this beta right or what governs this theta. So, clearly what we see from here is that whether it will be an oblique shock or a normal shock, there is some connection with the geometry over here right there is some connection with this geometry. Now if say if we have say you know this sort of a you know this is the kind of say which we get say we have a edge like this or a corner like this sharp corner like this.

So, we are encountering this right. Now instead of this what if we have something like this? Let us say instead of this say what if we have say something like this, in that case how will this how will the geometry of this oblique shock change or will it change or will it not change. Now clearly there is some connection of the geometry of the shock with the geometry of the body.

So, therefore, if I have say a shock like this, you know or say I have a shock say like this right. So, if I have say. So, what I have drawn here is basically say three cases right. So, in these three cases how is the geometry of the shock going to change or is it going to change right. Now based on that what we see over here that the theta right which is the angle with which the Mach number behind the shock is deflected right because of the oblique shock.

Now, beta is the angle of the oblique shock and M 1 is the impinging Mach number. So, therefore, what we see is that these are connected. So, the geometry of the shock is connected with the impinging Mach number right. So, this is the connection and. So, therefore, if we do change the theta over here there will be a corresponding change in beta or having said that for a given theta and a given M 1 beta is going to be a particular value. A particular value or more than one value right could it be more than one value for a particular for a given case of theta and M 1 let us see from here.

So, for a given case of say you know theta right and M 1 if you look at this equation you will actually have two values of beta. So now, this is an expression therefore, right again. So, it gives us a connection between the geometry of the shock with the impinging Mach number right and this is the more very common theta beta M relationship which is associated with you know shocks oblique shock right and this again is given in is available in charts in your in any you know normal any regular text book.

This will be available from there and you should be able to you know get your values very easily from there. So, now, let us just sort of look at this a little bit. Now the question is like we said that how are we exactly relating this geometry. So, let us just look at that in say little more little more in detail ok.

So, if say beta is given and M 1 is given, now there is a corresponding maximum theta that this can attain right there is a corresponding maximum theta, which means that for a given Mach number and beta there is a corresponding theta max now what this is this mean what is this exactly mean when I when I say this? Let us take a look at that, let us sort of detail this out.

Now for example: now the point is that every time we have a edge should be have a attached oblique shock or what do we encounter.

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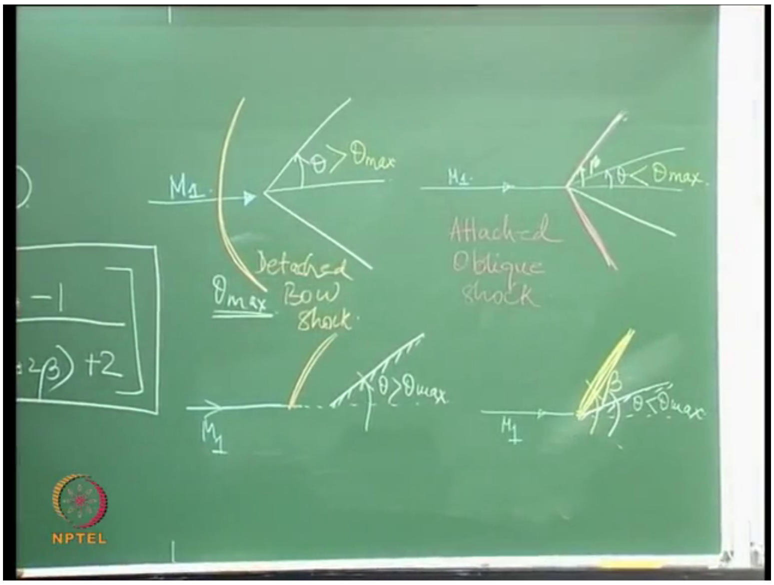

So, say this is the edge essentially this is my theta right when you compare this is the geometry of the body. So, this is the theta, and we have impinging Mach number impinging Mach over here and.

Let us look at these cases this is also the theta and we have the same and I have the similar. So, like if you look at this expression over here, now for a given say for a given Mach number and say a given beta for let us basically just look at the maths part of it over here. So, for a given Mach number and M 1 and beta, there is a corresponding theta max ok.

Now, what is this mean physically? Now as you can see here and like we were asking over here, that if I change this theta right if I change this theta what happens to the geometry of the shock over here now what we can see from here is that in here. So, for both these cases there for this given Mach number and for a given beta, there is a corresponding theta max. Let us consider these two cases over here, now this theta here this theta of the body is greater than the theta max and this theta here is less than the theta max what happens in either of these cases, what is just that mean what kind of a shock or how is the geometry of a shock going to be defined.

So, what we have in this case actually, in this case there we are going to have essentially we will have an attached oblique shock right in this case what we have is an attached oblique shock right and what happens in this case. So, there theta is less than theta max, now just think about this. So, in this case we have an attached oblique shock ok.

So, now this theta is increased and so therefore there will be changes in the geometry of the oblique shock as well. As we increase the theta further more and this theta then becomes larger than theta max in this case what happens? In this case what will happen is this because this is exaggerated; so just to try to make the point. So, in this case what we will see is a detached bow shock ok.

So, in this case when the theta is greater than the theta max, what we will see is a detached bow shock. So, from this what I can sort of see for myself and I hope you can in further say is that, I have a smaller theta and depending on the Mach number we will have an oblique shock which is attached to the body right. You start increasing this theta right you start increasing theta this angle beta of. So, will probably this is my beta right.

So, this body kind of tries to move away the shock from itself right. The more and more increase it this shock the angle the beta, the angle with the oblique shock make also increases and finally when the theta is large enough larger than the maximum value of the theta, there is actually pushes away the shock further from itself. It detaches itself and lands up out here then this becomes a detached bow shock.

So, this is for say edges, we can get the similar picture say for you say if I have a corner; if encounter if the same flow it encounters a corner. So, this is you know typical edge structure. So, if I have say this is my M 1. So, what we could have is a structure like this. So, basically I have a right. So, I have a sharp corner like this, if I have a sharp corner like this and I will pretty much get the same you know if I have something like this ok.

So, if I have a theta say greater than theta max. So, in this case what we will have is a detached shock in front like that. This and similarly over here if I have another corner like this, get that right. So, if I have this write theta you have less than theta max. So, then this is the M 1. So, in this case that will be my attached shock. So, in this case also we will have an attached shock structure and in this case we will have a detached bow shock.

So, this is for basically co ordinates edges and coordinates. So, that is essentially the physical meaning what we take away from here. So, this is the connection between the

Mach number and the geometry of the body and the geometry of the shock has generated. Now again now coming back to this relationship again you can see like two mistakes.

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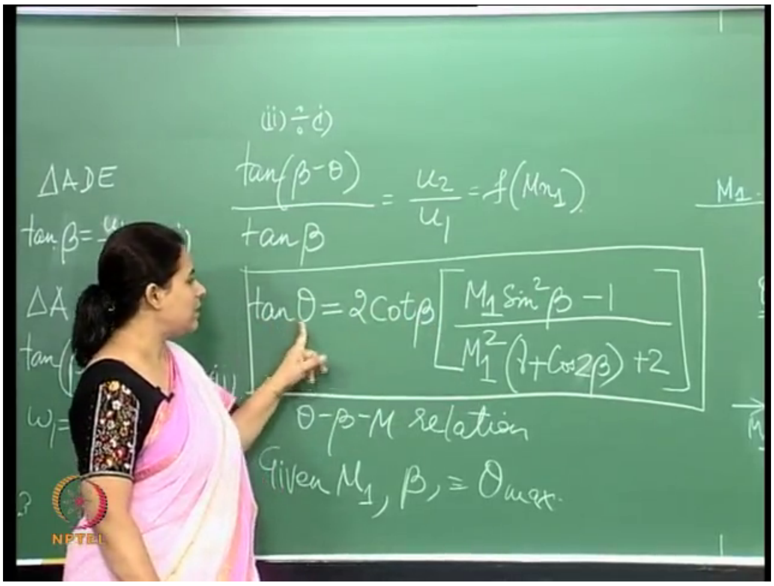

So, this is cos 2 beta, this is cos 2 beta ok.

So, if you look here. So, basically for a given theta and M 1 there are two values of the beta right what is that mean? That for a given Mach number like this right and for the given geometry there are two possibilities of this beta right there may be beta 1 and beta 2 right two possibilities in which this the shock is going to be generated. So, now, let us introduce the beta right which is the shock wave angle in to this equation and see what we get.

See if we can infer something more from this relationships. So, this is the change in density across the oblique shock right. So, I am going to change that. So, what I would get from here is this right. I get that and then again p 2 by p 1 what I get again is. I hope you can see how we are introducing M 1 here instead of the normal component. I think hopefully you can see that from here right I think we did that somewhere over here.

So, if you look at this triangle here A D and E. So, all I am trying to do is can it M n 1 and this is M 1.

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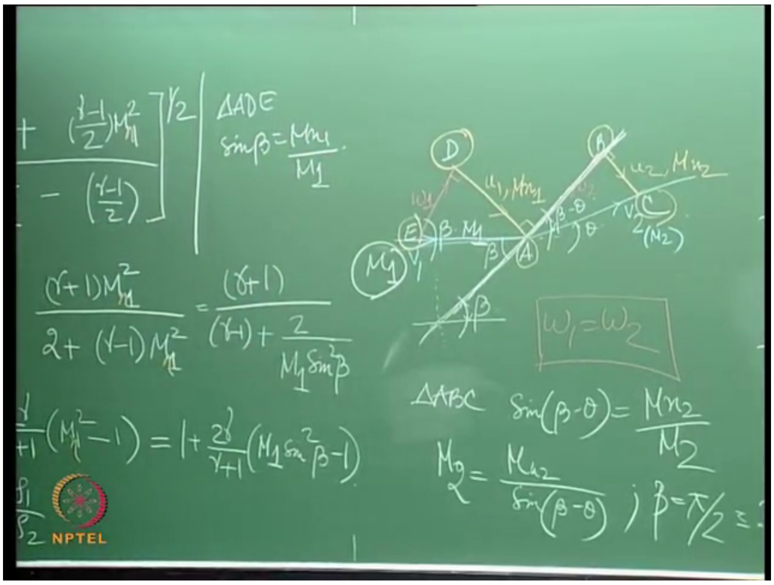

This is M 1. So, if you take this triangle, then sin beta. So, essentially if you check that triangle. So, here just clarify this. So, if you take triangle A D and E. So, you get say sin beta is M n 1 by M 1 right; so M 1. So, M n 1 is essentially M 1 sin beta that is all I have done over here that is all I have done.

So, this is what we get from the expression over here right now. So, what I said over here is that now. So, this is the relationship of. So, what you trying to do over here again is connect the change in geometries change in the flow properties with the geometry of the shock right because what we saying over here is that if for a given Mach number right, if for a given Mach number if I change the geometry of the body the geometry of the shock also changes right. So, beta also changes.

So, the question is that important thus that effects the change of my properties? It is all right if you know this beta is smaller or this beta is larger, as long as my properties are not affected the change the properties are not affected by the geometry or by the size of beta I am fine right. What we see however that, that is that may not be true right; so what we trying to do over here if see is that will actually make any change.

So, what we did is the change in properties over here which is rho 2 by rho 1 we wrote that in terms of the Mach number which of course, is a defining parameter and the shock wave angle which is beta, and we did the same thing with the pressures this is what we it looks like. Now if you look over here, therefore, now if you look at this relationship ok.

Now, if we increase beta what happens to rho 2 by rho 1 if we increase beta? And again if we increase beta over here what happens to p 2 by p 1? I think the best way to understand this is to actually put values of beta in there and probably and then get expression for this. So, what we will do is we will do a couple of problems in see if we understand that. So, we will do that in the bit.

But just think about it. So, if you increase beta over here what happens to rho 2 by rho 1 if we increase beta over here what happens to p 2 over p 1 and what is that tell us about the nature of the shock. In the shells if the shock becoming stronger right or if the shock becoming weaker if we increase the shock wave angle right and the shock wave angle we can increase or decrease by a change in a M 1 and the geometry of the body ok.

So, I think what we shall sort of do is a do a problem and then go ahead and see, how this we can relate this all right. So, to do that let us sort of look at this.

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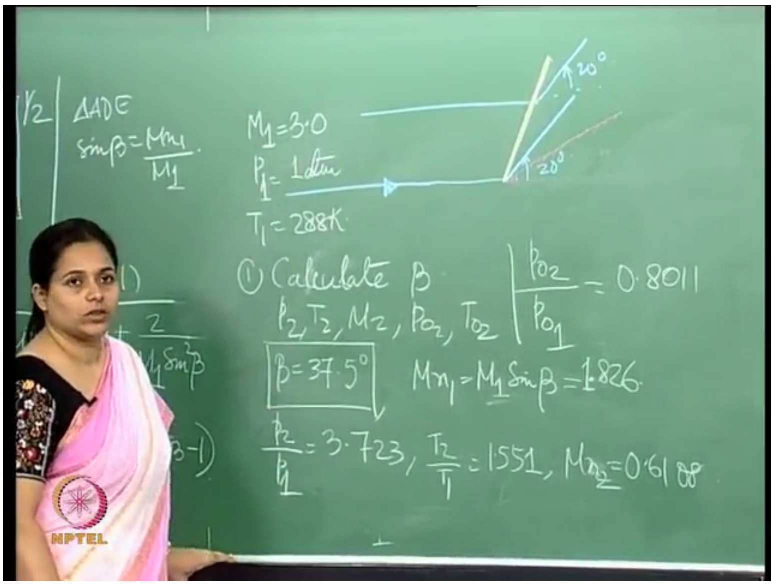

So, we have a uniform supersonic stream and the given values are M 1 p 1 is 1 atmospheres t 1 is 288 Kelvin and this encounters a corner something like that.

So, basically it is encountering and. So, and you know or a or a oblique shock happens. So, right and that deflects it. So, basically what happens is that physically. So, you have a flow like that. So, you have a flow like this right and physically then you have a corner over here and this is a incoming free this is an incoming flow of level of flow and it has these properties, when it encounters a corner like that it is basically deflected by 20 degrees ok.

So, what we are basically. So, and then this is deflected right by 20 degrees. So, this is a supersonic flow would just impinging. So, now, what we are basically seeing is that there is actually a shock wave is not it. So, I think I should draw this right. So, let us say. So, therefore, this is. So, say it is deflected by 20 degrees.

So, essentially I can say that there is attack chock over here right you can also draw this in this ray that you basically have you know a shock this is a field and this its deflecting it in a such way that this is 20 millisecond, this is essentially the geometry it encounters. So, I have a flow like this encounters the corner it deflects it by 20 degrees. So, first things first you know you need to find out is calculate. So, calculate the shock wave angle which is beta calculate beta and then a let us calculate p 2, T 2, M 2, the stagnations behind the shock right.

So, behind the shock we need to calculate all of this we will do that. So, now what we shall do now is a to find out. So, we need to find out the corresponding beta right now we assume this relationship, so theta beta m. So, we know the theta right. So, theta is 20 degrees M 1 is 3. So, from this expression we should be able to find out the beta right.

Now, this is something we can get from the charts which is again available. So, if I use that, what I will get is this in here. So, beta is equal to if we look at that beta is equal to 37.5 degrees. So, this is it now again. So, now, what is M n 1 now how do we calculate this or we need to do is like we have seen before all of these things, we can use the normal shock tables provided we know what M n 1 is. So, we will have to we can find this ratios with respect to M n 1 and not M 1.

So, now that we know M 1 we know beta, we know theta then how do we calculate M n 1 right; so M n 1. Therefore, M n 1 is right M 1 sin beta right and that is equal to 1.826. So, M n 1 now corresponding to M this M n 1 from the charts of the tables, what we will get is from the now we will go back to the no more shock tables right. If I do that then p 2 by p 1 is 3.723 this is what I thought T 2 by T 1 is 1 point say that and M n 2 right. So, this is the M 2 for an normal shock which is M n 2 over here is equal to 0.61 (Refer Time: 43:20) 8 and we also get p let me write it better ok.

So, you write this also I get 0.8011 now what we need to do is basically once you calculate M n 1 which is this. So, just corresponding to that we will get all these values from the normal shock tables, you just discuss just go look it up and then this is all that we get. Now having known this ratios, now already we can find out all these values because p 1 T 1 are all known.

So, therefore, we can calculate p 1 is known. So, we can calculate p 2 from here, and then T 1 is known here we will calculate T 2 from here and what is we need to do calculate; so M 2. So, we can calculate M 2 because we know M n 2 and P02 and T02. So, if I do that. So, let me just sort of ok.

So, if I do that.

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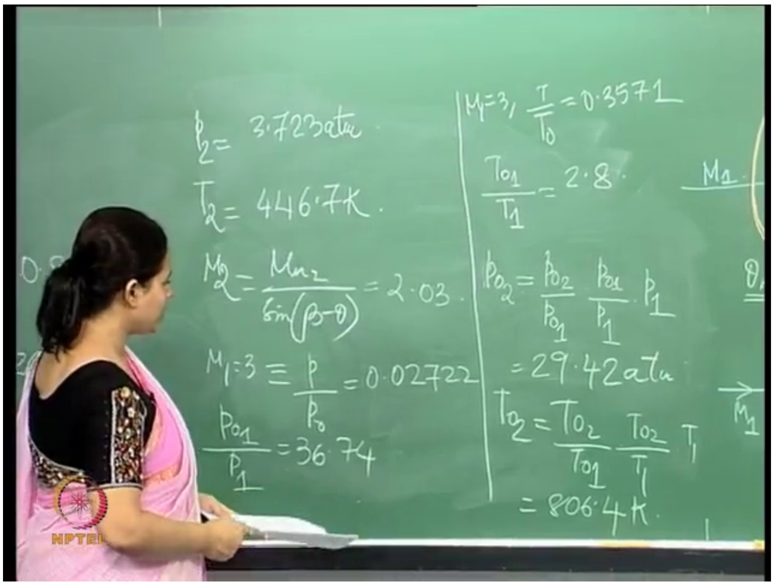

Let me just write what I get. So, p 2 is a essentially 3.723 right atmospheres, then what we get for T 2 is 446.7 Kelvin then M 2 how do we get M n 2 sin of now it is just mass ok.

So, we know better which will be deflection angle and theta is given theta is 20 degrees. So, I am just basically if you look at the geometry, where we design a if we look at the triangle a b c I think then you will get this. So, then what you get here is this ok.

Now,. So, again what the stagnation conditions and how do we find out P02 and p t naught 2. Now corresponding to this M 1 for say M 1 equal to 3 we get values for p by p naught this is also from the charts, we get p by p naught to be equal to 0.027 like that.

So, therefore, from here I can calculate say P01 by P 1 right; so P01 by P 1 which is 36.74. So, corresponding to this M 1 what we get is the stagnation conditions like that. So, this is also from the chart from the tables.

So, then P01 by p 1 is this. So, you can see that p 1 is something that that is known to us what we need; however, is p02 by p01. So, this is it is a similarly we will also get a result. So, again we will again get for let us write it over here. So, similarly again for M 1 equal to 3 we get T by T 0 as right. So, therefore, so, T 0 1 by T, T 1 is basically 2.8 ok.

So, if I do that then the way we will how do we calculate p02 right. So, we need to calculate. So, we need to calculate P02 and T02. So, the way we get p02 is you see you will just be little clever and use this right. So, this is something that we were able to find out corresponding to yes. So, we were able to calculate this with respect to a M n 1 right.

So, we were able to find this out and then using corresponding to M 1, we were able to find these. So, these are the stagnation conditions. So, then we get p02 in this fashion you are just working with this. So, what we get over here is if you just use those values and then what we get over here is again, this is again just playing around; so T02 T01. So, this is just a using right and what we get here is 806.4 Kelvin right. So, this is what we get. So, then this is you know this is what we get. So, let just sort of stop here.

So, we will do a little more with this problem and try. And again address little more in terms of what kind of a shock we get. This is a strong shock or a weak shock how do we decide that, depending on your Mach number depending on the geometry of the body ok.

So, let us do that you know slightly more with this problem and we will discuss this is a little more detail we will stop here.

Thanks.