Advanced Gas Dynamics
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Lecture - 08
An introduction to Oblique Shocks

So we were still on normal shocks. So, let us do one more problem and we will sort of slowly move towards oblique shocks today. So, let us look at this problem. So, we have a point in a supersonic flow where the static pressure is 0.4 atmospheres.

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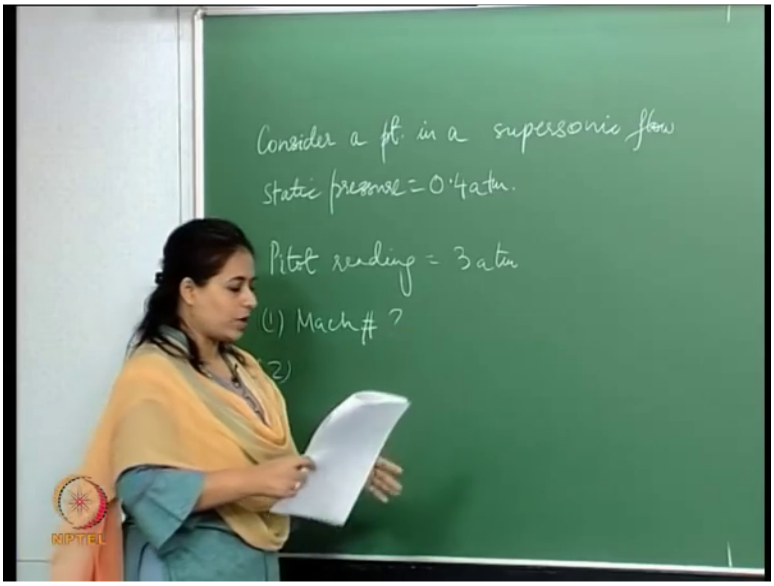

So, let us say you consider, right. So, you consider a point in a supersonic flow where the static pressure 0.4 atmospheres and then when a Pitot tube is inserted right at this point we get a pressure of 3 atmospheres. So, then we have a Pitot tube which reads 3 atmospheres, we need to answer what is the Mach number. And secondly what is the; let us first a answer this question then what is the Mach number. So, we have a supersonic flow static pressure is this Pitot tube is this. So, what is happening is that the Pitot tube is actually reading the total pressure at this point.

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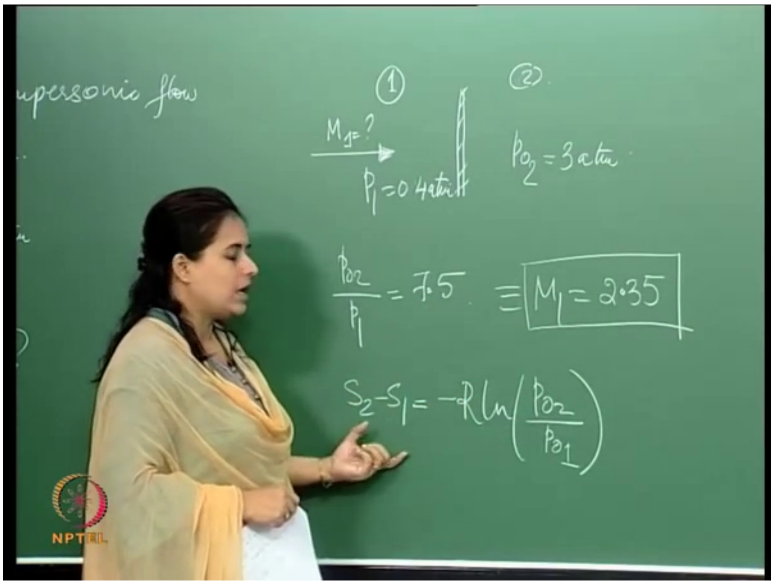

So, what we can say essentially is that if you know it is basically it is a supersonic wind tunnel. So, it is going through a normal shock right. So, what we can see over here is that if I say this is 1 and this is say 2. So, basically we have a flow coming in here. What we require is this, right. What we require is this. And what we know is, right and this is what is known to us. So, this is the static reading at a particular point and when we insert the Pitot tube, we get a reading for the total pressures which comes out to be 3 atmospheres.

So, what we can do from here is get this and this is equal to say 7.5. Now, again if you go back to the tables as we spoke in the last lecture; if you go back to the tables you will find a corresponding Mach number right; corresponding Mach number which is 2.35. So, therefore, we have a supersonic flow Mach number 2.35 moving in a supersonic wind tunnel which has pressures given like this. Therefore, what we know now is that; there therefore we have this normal shock over here. So, now, in here what we would like to therefore calculate is the entropy change, right. So, essentially what we see here is that there is a pressure change like this and based on that we were able to calculate the incoming Mach number, so from the tables.

Now, what we would like to see is; what is the entropy change. So, this is the first; this is from the tables, right. Now, from what we have done previously. So, S 2 minus S 1, right. So, this is something we derived in the last lecture. So, the ratio of the total pressures after and before the shocks and this is known to us. Now how do we get this ratio? Well, we can get this ratio again from the tables corresponding to this Mach number. I would leave you to do that. So, just look up for Mach number 1 for M 1 equal to 2.35 go look at the tables and find the corresponding ratio here and that should give you the entropy change.

With that we will kind of put a little bit of, we will take off from this normal shocks and look at few things and then probably even come back and connect to this one more time. So, this is part of the problem. Now what we have done so far is we have connected the temperature changes, pressure changes, density changes, etcetera; as well as the we have looked at total pressures, temperatures, densities, etcetera across a shock wave in terms of the incoming Mach number that is what we did right now.

So, we got these ratios, we calculated the corresponding Mach number and we therefore got the corresponding entropy change. We did exactly that right. Therefore, everything that we have done is in terms of the incoming velocity then Mach number, etcetera ok.

Now, what we could do one more time is basically connect all these properties just by themselves. In the sense that if there is a pressure change what is the corresponding density change or specific volume change etcetera, without any reference to this Mach number at all. So, let us see if we can do that. Now if we go back. So, we have a continuity equation, right.

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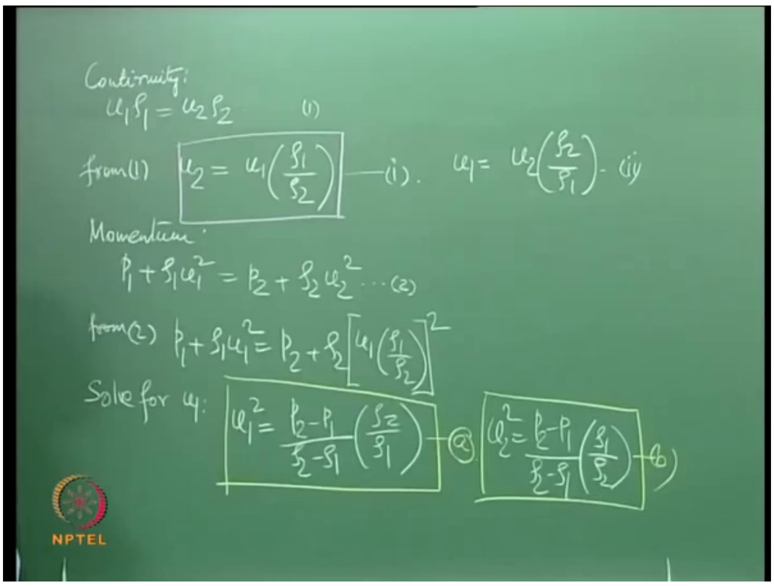

So, this is from the continuity, right this is from continuity. So, then from here so let us say call this as 1. So, what we will do here is we will write; so we will write this from the continuity. Now again from the momentum equation; now from the momentum equation which is; so what we will do is basically let us say this relationship is right. So, what we will do is we will incorporate this relationship here; we incorporate this in into this. So, in this equation 2 what we will do is we will replace this u 2 by this. So, which makes this equation therefore, u 2 is u 1; this is what we get.

So, you can see that we do not have any u 2 term over here. And then we will solve for u 1 and what we get is this, we will solve for u 1 so what we get is this. So, we get this and say let us call this as a. So, we get an expression for u 1 in terms of just the pressures and densities, right.

Now we will do a similar thing for; we will solve similarly for u 2. If we do that, so basically for that what we will do is we will write this equation in terms of u 1. So, we can also write this as, right this is u 1 and then into this equation we will replace u 1 square by this expression; by this expression. And what we will come up with is then we will solve for u 2 and what we get for u 2 is this. So, this is what we get for u 1 and u 2 square.

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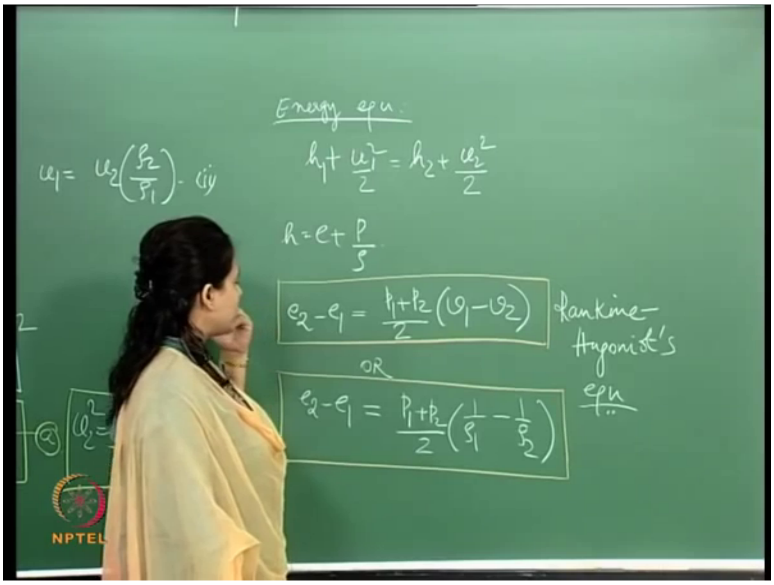

Now, what we will do now is let us go to the energy equation. Now energy equation: now here so we basically have I hope you can see where I am getting at. So, we have this u 1 square term and u 2 square term, so we have velocity terms there. And what we can do if we use these two expressions for u 1 square and u 2 square is totally get rid of any velocity term and just replace it in terms of just the variables; this thermodynamic variables here. So, we have pressures and densities. So, we can actually do that there.

What about the enthalpy, can we write that in any other way. For example, h as you know is, right. Therefore, if I use all of that, if I replace this by all the h 1 h 2 by an expression for enthalpy and we incorporate a and b what we end up is something like this or so, what we get over here is a set of these expressions. All we have done is worked with the continuity the momentum and the energy equation is simple you know playing around with the properties and all we did was we took away any sort of reference to velocities. And we were able to generate this equation with just the thermodynamic variables, ok.

So, we have the specific energy change in terms of the pressures and specific volumes and corresponding go for this in terms of densities. Now these two basically forms are known as basically the Rankine-Hugoniot's equation. So, the these two energy you can see like way of coming repeating this that basically this has an advantage because there is really no reference to the Mach number and velocities, and is specifically relationship just between a thermodynamic variable. Also we do not have any specific reference to a gas, in terms of we do not have a gamma over here. So, it is independent. So, this is valid for any sort of fluid.

Now having said that let us say if we can use this a little bit more. So, in order to do that let us say. Basically you know if we can use that more functionally. So, if I have to do that so let us do this, ok.

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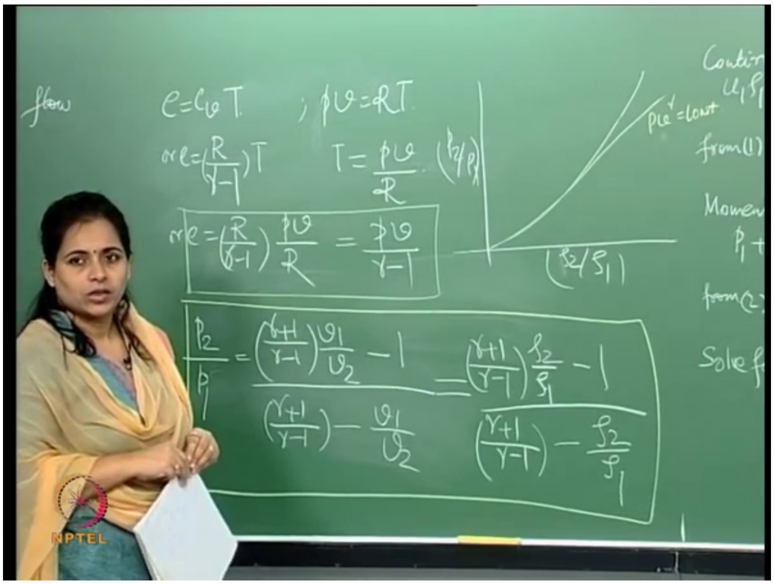

So, we have let us look at this. So, we know this right; see if we can sort of use this in any way. Now see v again can be written as; and from here, right. So, what we can write e as is, right. If I do that and then incorporate this relationship, right if I incorporate this relationship into the Rankine-Hugoniot's equation we will get something like this. And we can also of course write this as. So, this is.

So what we have done is; first we wrote out our basic equations of flow governing a normal shock, then we got rid of any velocity terms using the three basic governing equations and we came up with the Rankine-Hugoniot's equation. Now in the Rankine-Hugoniot's equation we did some more manipulation and we came up with this sort of a relationship. Now what this? This is a very interesting relationship, because if you see what this is telling us is the way the specific volumes in with a particular value of specific volume change the pressure will change. This particular range of density changes how the pressure will change.

So, essentially if I were to plot this right, for various values of v 1 by v 2 or rho 2 by rho 1 then I will basically get a curve which is telling me how the corresponding pressure are changing. Now this basically expression is called the Rankine-Hugoniot's relationship. And I would suggest you that you go and look in any standard book, so you will get the Rankine-Hugoniot's curve and it will look you just see.

So, if you will get curves like that and for say corresponding say right you will get curves like this. And what it will show is how that differs from the isentropic relationship. In the sense that you will have an isentropic curve, right. Now this is an isentropic curve, whereas this is the curve which is resulting from this relationship and this is basically telling us the difference for across a relations between across a normal shock. So, this is something you can look up in any standard book. This is available in just sort of get familiarized with it.

So, that kind of more or less takes care of what I want to talk about normal shocks etcetera. Now the next thing to do immediately is oblique shocks. Now at the outset let me just say that normal shocks are a special case of oblique shocks. We will see what that means, ok.

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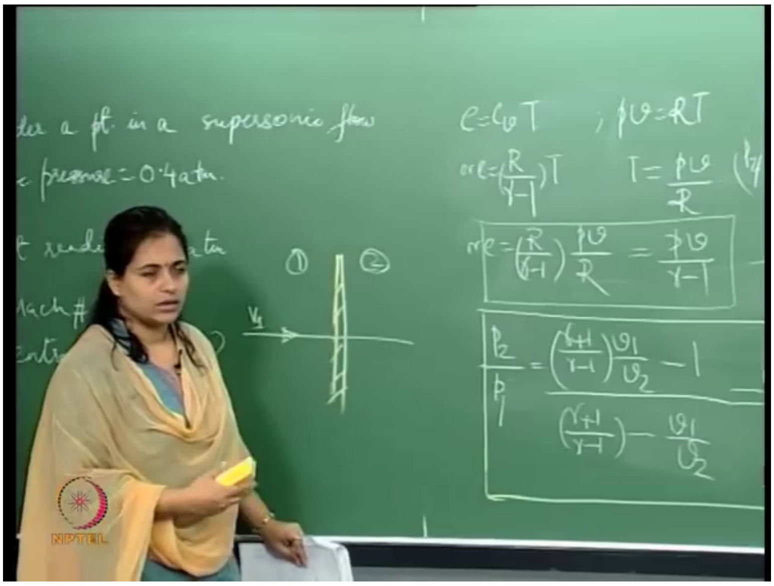

So, so far what we have done is that say we have a normal shock like that, we have here flow coming in, so we calculate velocities. We calculate basically the properties you know regions you know in front and behind the shock.

Therefore, this is like a normal shock. Now what happens if there is no oblique shock? So, we look at some pictures of oblique shock, but I think we did that in some of the earlier classes. So, let us see what we are looking at and see if how basically our properties will change across an oblique shock, and if it is, how different it is if at all from the normal shock relationships that we have developed so far. So, let us get some geometry in place.

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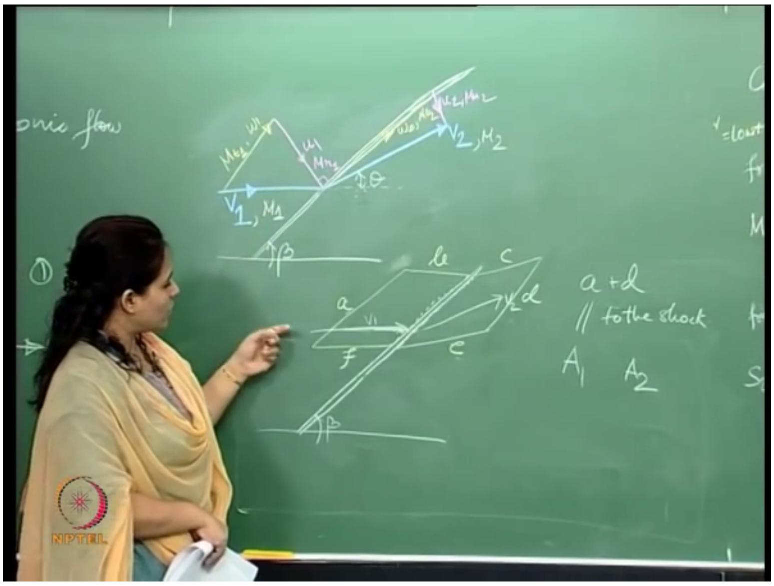

Now, say this is basically an oblique shock. Again this is basically a very thin region across which there is a large gradient of the flow properties. So, what we have seen in the corresponding normal shock is; this is the small region across which properties are changing. Now what we will consider is this is not perpendicular, but this is inclined to the horizontal at some angles at beta.

Now, what we will say here is; so therefore here to so we have an incoming velocity like that. And let us say that when it hits the oblique shock here it moves with v 2. And, so the angle that it makes with the horizontal is theta. So, angle of the shock is theta and angle of this velocity vector behind the shock is theta. If we do this now we will do a couple of things over here.

So now, my corresponding say Mach number here is M 1 and my corresponding Mach number is M 2. Now let us do a couple of things over here. Now let us do this I probably need to this at the bottom. Now we will take the same shock here. On this shock let us take basically what I want to do is right this is my. Now, what I will do is consider; so this is my v 1 and this is v 2; this is v 2. Now let us do this over here. Now what we will do is; we will draw a control volume like this. I need to get this right, now this. Therefore, this is actually v 2.

Now if I take this over here. So, if I call this A or let us say this side is a b c d e and f. Now what we will do is essentially. So, this is my control volume. Now, this f and c these are parallel to the oblique shock. So, this sides here ok; so let us sort of it is rubs me do, this let us call it this and let us say this is a b c d e and f.

So, now here this a and d these are parallel to the shock. I hope I have drawn it right. So, basically a and d these sides are parallel to the shock and let us say the corresponding areas are A 1 and A 2. So, you can see that these are actually equal here. And then as we know that this is essentially the velocity vector, this is the incoming velocity vector.

Now what we will do here is if I do this over here that I will draw components of this velocity vector v 1 like this. One is, this is a velocity vector which we will call as u 1 and the other one; u 1 and w 1 and the corresponding say Mach number here is, right. Because, the reason we write n here is because this is normal to the shock. And the w 1 here is it is tangential so we call this as this.

So similarly, on the right hand side we will do the same thing. So, we will drop a perpendicular and this is my u 2 and the corresponding mn 2. So, this is u 2 and this is w 2 and mt 2. So, this is essentially the geometry over here. And I am drawing this separately just to give some clarity. So, all I am doing is drawing out a control volume which is encompassing this. So, I have this velocity vector over here and I draw control volume so that I can apply the integral forms of the governing equations to it. So, what I have done here; now the question is why did we do this. Now you will see also the usefulness of this in a bit I hope; is if you just look at this part or this u 1 here this is just like a normal shock, is not it.

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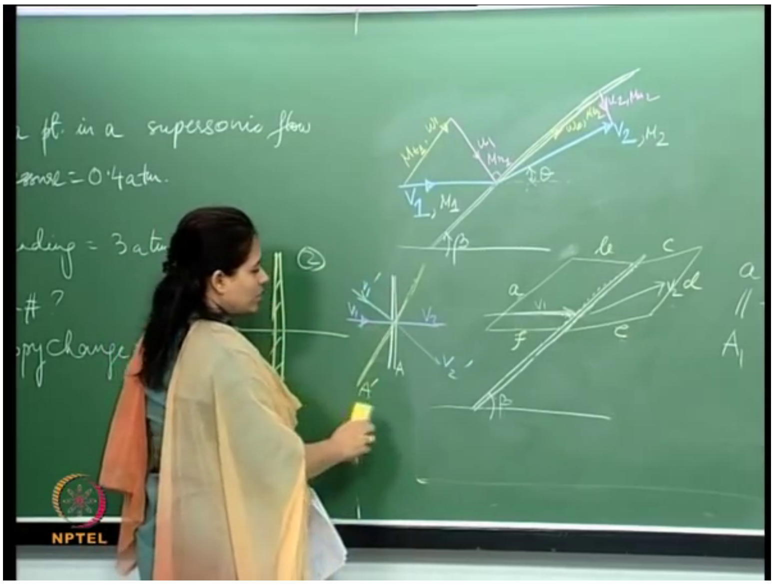

So, what we had earlier was the shock was like this, and you had a velocity vector coming in this way say v 1 and v 2. Now just think of this. Just take this and rotate it in this way. So, say I rotate this. So, let us say I rotate this whole thing. So this is the cor; this normal shock, so say I call this as A. So, therefore, this is say A dash I have just rotated that. Therefore, this velocity component how does that change; now that becomes, that is also rotating this way. So, then this is my say v 1 dash or and v 2 dash.

Now if you compare this that is exactly what I have done over here. So, this is my shock now. Therefore, the corresponding velocity component is normal. So, one component is normal and the other is tangential to it. And we will see the usefulness of this, and if at all this we will have any special implications on another properties across a an oblique shock we will see that. This was just to bring the sort of relevance to the normal shock.

Now, if we do that what we will do simply is what we have done before apply the governing equations to this o oblique shock. Now, what we will use is the steady form of the integral form of the governing equations. So, I am not going to write out the entire equation.

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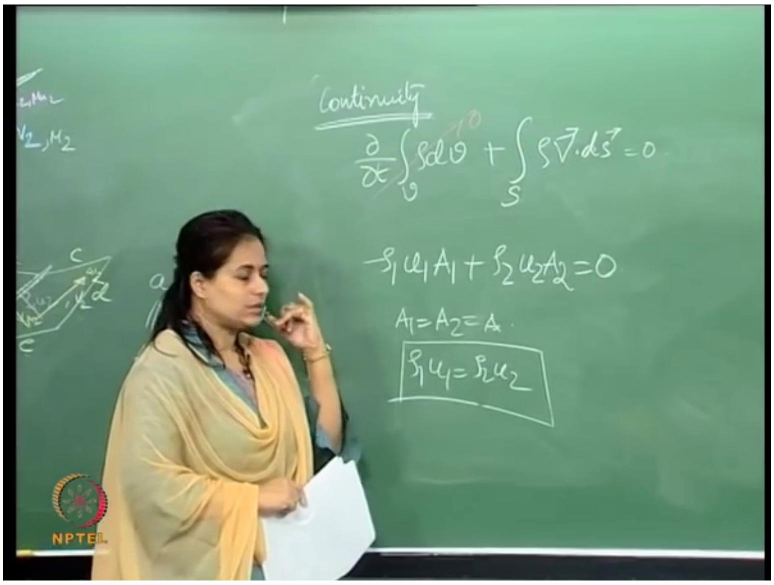

So for example, let me write this because this is small. So, continuity is essentially. So we will consider the; this is basically the integral form of the equation. Now we can. So, we will consider the steady form of the equation, so what we get is this. Now what we have seen if you recall what we did in the earlier case.

So, essentially we have the velocities this is the v 1 impinging on this face and this is my total control volume. Now if you consider this in this case v dot d s, so areas like I said areas is A 1 and A 2. So, I will just write it for this face and this all this faces and see what we get. So we have two components one is u 1 and one is w 1; you know u 1 is perpendicular to the face and w 1 is parallel to the face to the shock here.

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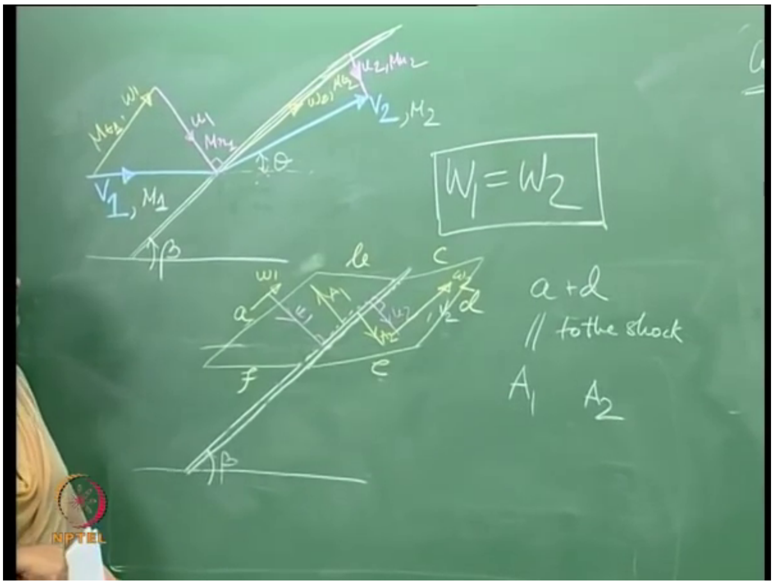

So, basically I have this, so this is my u 1. And, if you remember correctly my area vector is essentially this; right this is my area vector.

Therefore, if I had to write this integral for this component of the velocity, so what we will get here is. And similarly, now for the normal components; so for the normal component is this, so this is the w 1. So, this does not contribute anything over here, because these two are perpendicular so dot prior does not give us anything. So, in that case we get this. And on the up side again if we do this right, so here essentially we have this, we have another perpendicular and this is the u 2 and again this is the A 2 if you remember correctly.

So, what you have to notice essentially is that the area vector and u 1 vector are opposite to each other; here they are in the same directions. That is what just brings in the sin over here; because the angle between u 1 and A 1 is pi is not it and the angle between u 2 and A 2 is 0. So, that is the reason we get this. So, plus rho 2 u 2 A 2 is equals to 0. So, that is what we get. And what we had to notice here is that again here. So, then this is the parallel component and this again the angle between this and A 2 is 90, therefore that does not contribute to the integral over here. So, what we get from here.

Therefore, inhere we know that; sorry. So, A 1 is equal to A 2 right. So, therefore, what we get from here is; so this is what we get from the continuity equation rho 1 u 1 is equal to rho 2 u 2. Now how is this different from the normal shock? It is not, is not it; but what you should notice over here is that u 1 and u 2 are the normal components of the velocity which is impinging at the oblique shock. So, this is a similar to; the continuity equation if you applied for an oblique shock is same as the normal shock provided we take the normal component of the velocity. So, what we wrote there is not rho 1 v 1 instead what we wrote was rho 1 u 1. So, that is important to notice here. So, what we are using over here is the normal component of the velocity across the oblique shock, and then that this is a relationship we get from there.

So now, having done continuity let us go ahead and see what we can do with the momentum equation, like we have done earlier. So, I am not going to write out the entire momentum equation.

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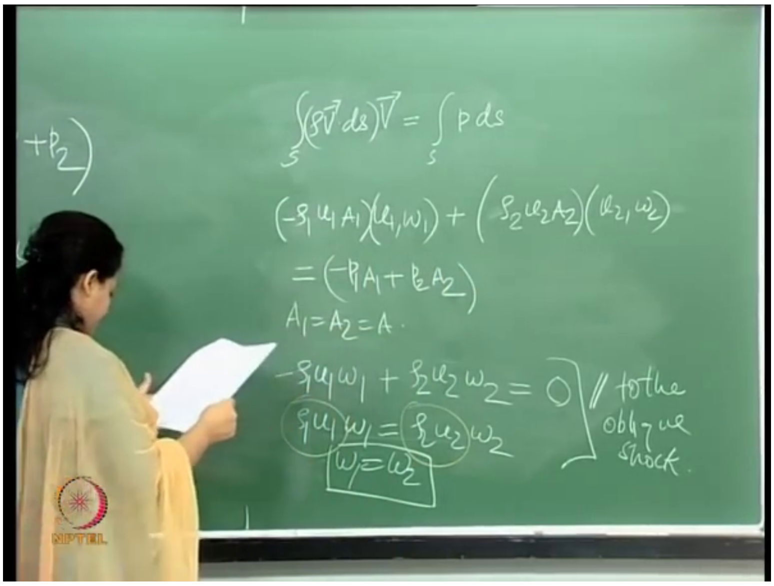

So, for steady case this is what we have done before. Now, we will apply the same thing to the control volume that we have derived over here. So now if I do that what I should get is this. Again let us do this. Therefore, here now this v what this can be here is. So, basically we have again two components; so we have u and w is not it. So what, let us do this over here.

Now, basically we have two components and then this; so for the right hand side what we get is what clearly we do not have a pressure component parallel to the shock I think that we know. But if we do this, now if we do this let us just take; before we do that, so A 1 is equal to A 2 right that we know so far, so that sort of that all the area terms sort of disappear from here. Now let us take the parallel component first that is w 1 let us take that from here and see what we get. So what we get is this: rho 1 u 1 w 1 plus rho 2 u 2 w 2, because from here the pressure there is no component in the parallel direction; there is no component of the pressure in this direction the pressure is only acting normal to the oblique shock. So, if that is, so when I take this velocity as w 1 I get this is not it. So, there is no pressure term in this direction.

If I do that now what we have in; so essentially what we have seen here is that rho 1 u 1 w 1 is equal to rho 2 u 2 w 2. So, this is what we get right, this is what we get if we are looking in the parallel direction. So, this is the parallel to the oblique shock. Now if I do this right, if I do this now from the continuity we have just obtained the rho 1 u 1 is equal to rho 2 u 2, is not it, we have just obtained it. If you look here, rho 1 u 1 is equal to rho 2 u 2 so these two are same which means that w 1 is equal to w 2.

Now this is a very interesting result for an oblique shock that in an oblique shock there is no change in the parallel component right. Although the u 1 is changing right, the u 1 is changing there is no change in the parallel component. So, the change in the velocity is act; so we basically have an impinging velocity v 1 which becomes v 2, but this v 2 is because of the change in the normal component and there is no change what so ever in the parallel component. So, for an oblique shock w 1 is equal to w 2 and that is a very interesting result.

So, this is what we get from the momentum equation. So, let us just sort of complete this and use the energy equation. Before we do that, so we use the parallel component let us also do the normal component and see what we will get.

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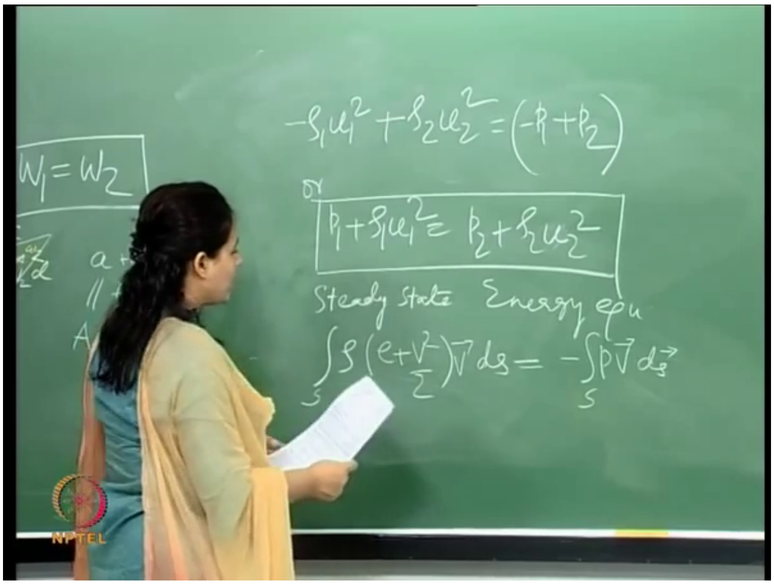

So, if I do the normal component then what we get is; right or. So, if we work up with this what we get is. Now I think that we missed a over here this, right. So, if we get this. So, then we get this relationship which is exactly the same as for a normal shock. This is exactly the same as that and again you can see that what we are using here is the normal component of the velocity across the oblique shock.

So, therefore, what we got from continuity and from momentum with the normal component of the velocity is the same as that we got for a normal shock. So, let us complete this and write out the energy equation. Now the energy equation again it is; I am going to not write the whole equation what I will write is for the steady state case, right.

So, this is equal to; so this is my energy equation. And what we should do over here is that here we basically we will put v 1, so this will be v 1 and v 2 across the right. And again we will write out this equation. Now if we write this out; right if I say right this out so what we shall get without sort of you know I am writing out the details of this.

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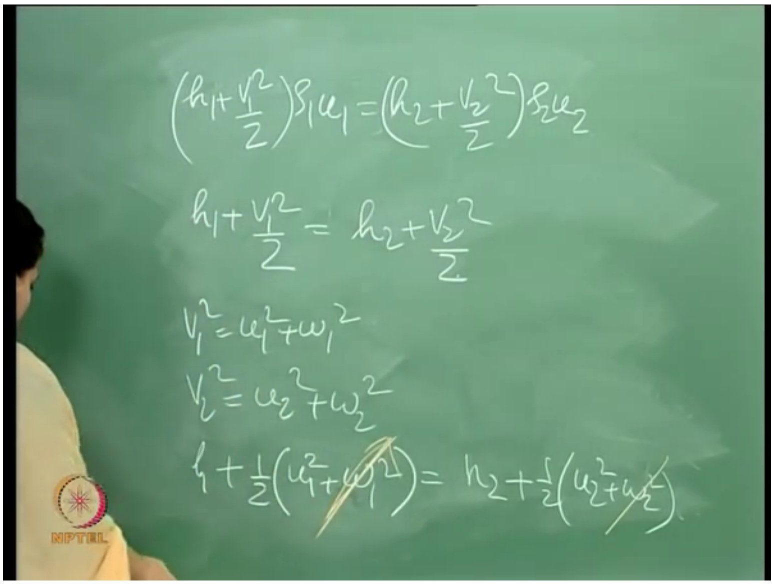

What we get is this; right, but rho 1 u 1 is equal to rho 2 u 2 therefore what we get is. Now here of course what you can see is that we are actually using the total velocity, we are not using any component we are actually using the total velocity before and after the shock. Now v 1 square of course is equal to this and v 2 square is also equal to right. But, what we have seen now is that w 1 is equal to w 2.

Therefore, I can actually write this; I can write here now this equation so this equation as; so it becomes right, but w 1 is equal w 2 we have just shown that. So, which means that we can sort of, we can cancel it out on either side.

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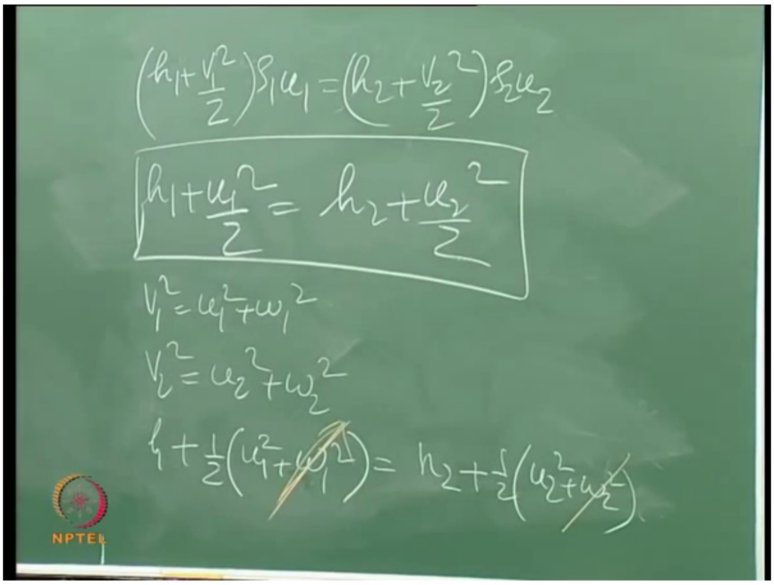

So, then what we get for the energy equation is. Finally, what we see is that that if we are going to say I was going to summarize this, ok.

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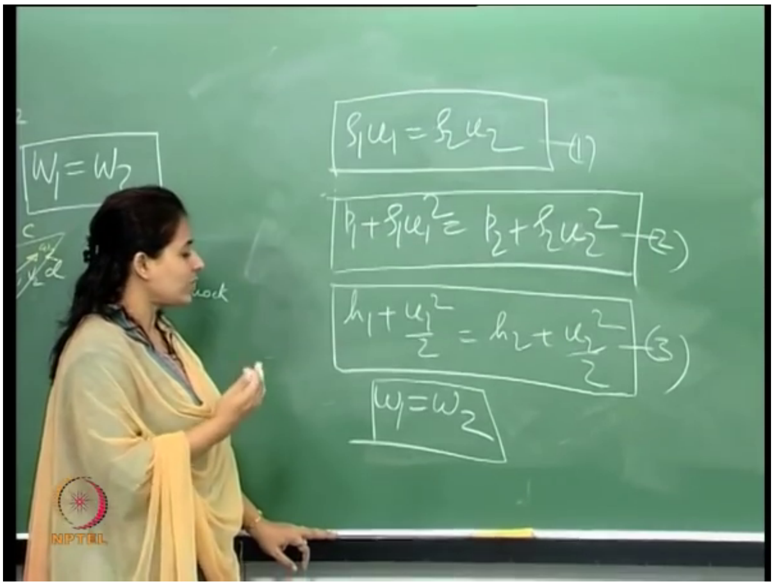

This is my continuity; this is my momentum and this. So, this is my continuity, this is my momentum and this is my energy equation which is exact for an oblique shock and what we see here is that we are using the normal component of the velocity before and after of the shock. And these relations are exactly the same as we have derived for the normal shock. And what we know this from here one more inference that we got from here was that w 1 is equal to w 2 which is that the parallel component of the velocity does not change across the oblique shock.

Therefore, basically that is the connection with the normal shocks. And therefore what we can do here is we can use the same tables that we used for normal shock; only the only thing would be that we will use the normal component of the velocity and the corresponding Mach numbers. So, we will see how we will do that in the next few classes. That should be all.

Thank you.