Advanced Gas Dynamics

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Lecture - 07
Example Problems in Normal Shocks

So, what we did last time was we basically derived relations for the properties of the fluid before and after of a normal shockwave right. So, we will try to understand a little more about that using a problem.

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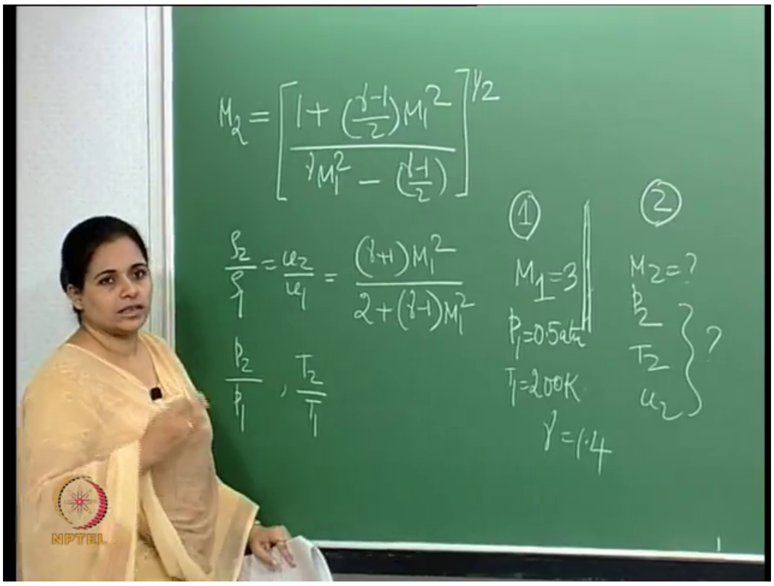

For example, we got relationships like this for example, M 2 right. So, this was one of the relationships then so on and so forth and we also got a relationship for say p 2 by p 1, T 2 by T 1 in terms of a the mach number in front of the shock. So, essentially so if I have this as a normal shock and this region is 1 and this region is 2. So, mach number here is M 1 right. So, we have the properties here listed in terms of a mach number; now let us do a problem where say this M 1 is given right. So, the M 1 is equal to 3. So, mach before a shock is 3, p 1 is 0.5 atmospheres T 1 is 200 Kelvin and what we need to do is find out pressure temperature velocity.

It is as simple as that right. So, the mach number is given press pressures and temperatures in region 1 we need to find out these properties after the behind the shock. So, how do we go about doing this? So, essentially if you now and then gamma is typically 1.4 for a calorically perfect gas. Now, in here you can see that if you put this value of gamma and M 1 you should be able to get them too and then you can do the same thing for these ratios as well right. So, once you get these ratios then p 1 is given and T 1 is given then p 2 and T 2 can be calculated. Now, I did the same thing I did that and then now another thing is if you take a look at any in any standard book right. So, what you can see the basically in the annexes or the behind the book that you know all of this is actually listed as a table right.

So, what you have is for given say mach numbers well.

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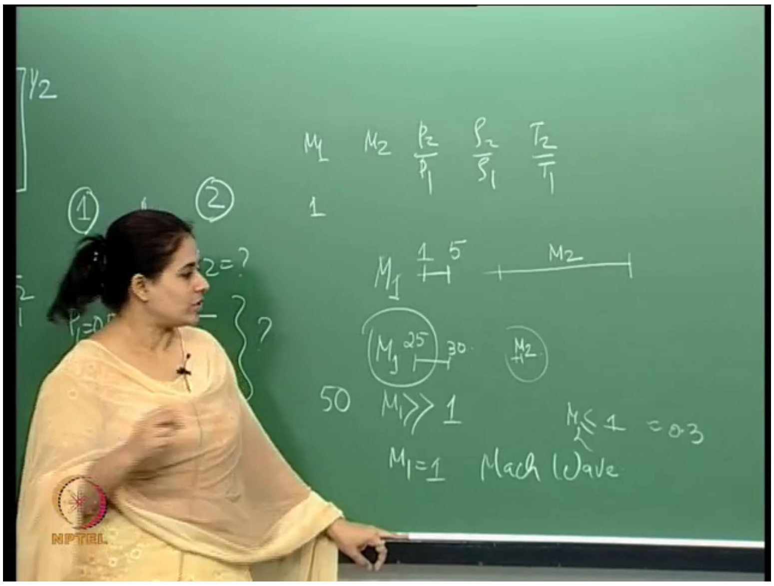

So, for given mach numbers you actually have M 2 and so on and so forth listed, for a series of mach numbers from 1 to 50. So, if you go and take a look at any standard you know book on incompressible flows usually it is pro provided in the appendices at the end of the book. So, you would find these listed in this way. So, if you had to do this problem. So, instead of you know putting your values in here and calculating it yourself you can actually go and look at the, these charts and just get these values.

So, essentially if you go to the chart and look for the mach number 3 and then you get the corresponding values they are all listed. Now, what all it was that I actually put these values in here and I also looked up the values from the chart and I compared them to cross check that you know I calculated right or you know this is actually calculating what it should and it matches the chart values it matched. So, you can try this yourself now nothing that I did is that I wrote up a small little code you know and so that you know whatever human error is there when I put these values up here that can be eliminated. So, just I want to cross check whether I was doing this correctly by hand. So, let us just take a look at what I did.

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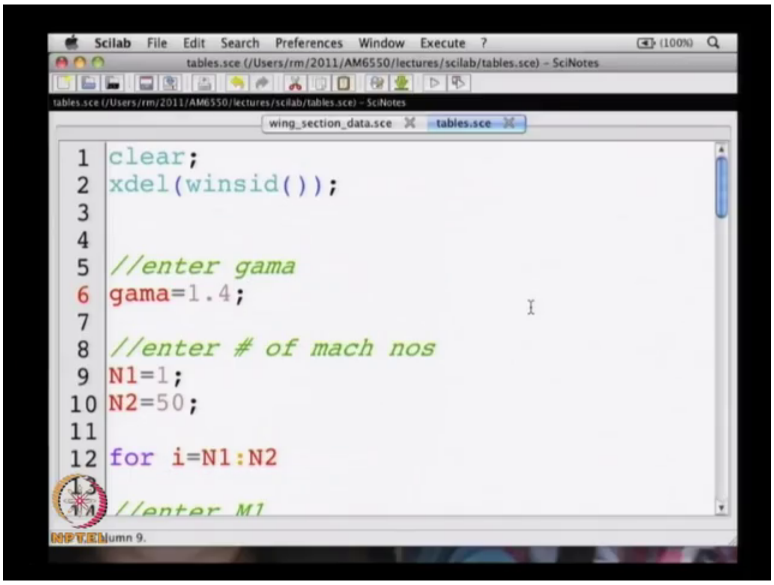

So we will do this in this platform called Scilab. So, if you are not familiar with this, this is basically an open source environment you can just downloaded it and run it is very easy to install it also you could do this in mat lab if that is available, Scilab is free. So, it is helpful it is not too difficult to use. So, if you get you know used to it you should be able to use it this is anyway a very simple sort of a code. So, so let me just sort of work you through this.

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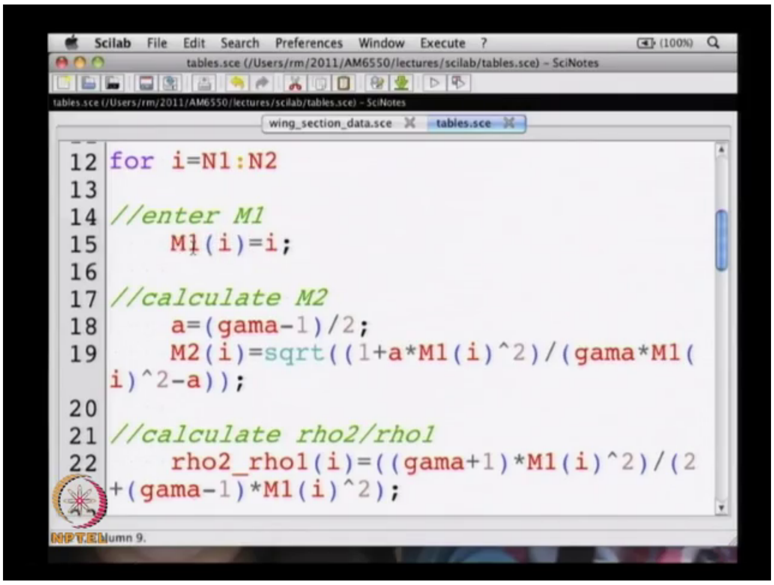

So, this gamma is the ratio of specific heats right 1.4 and what you will see here is that for given say mach number here which is M 1, I calculate this a is not speed of sound I am just using this as a variable for the gamma minus 1 by 2 factor. So, then here you can see that I have put the mach M 2 value in the formula for M 2 over here then I calculate.

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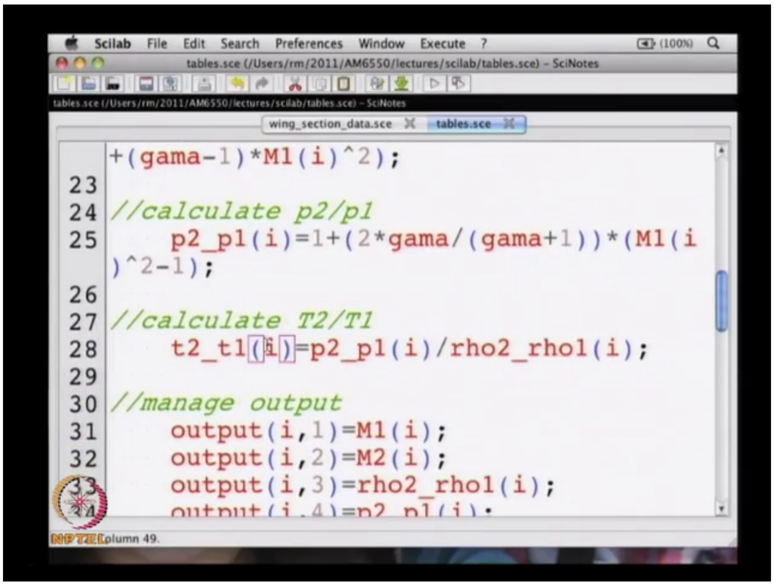

I have put in the formula for rho 2 by rho 1 then I have p 2 by p 1 T 2 by T 1.

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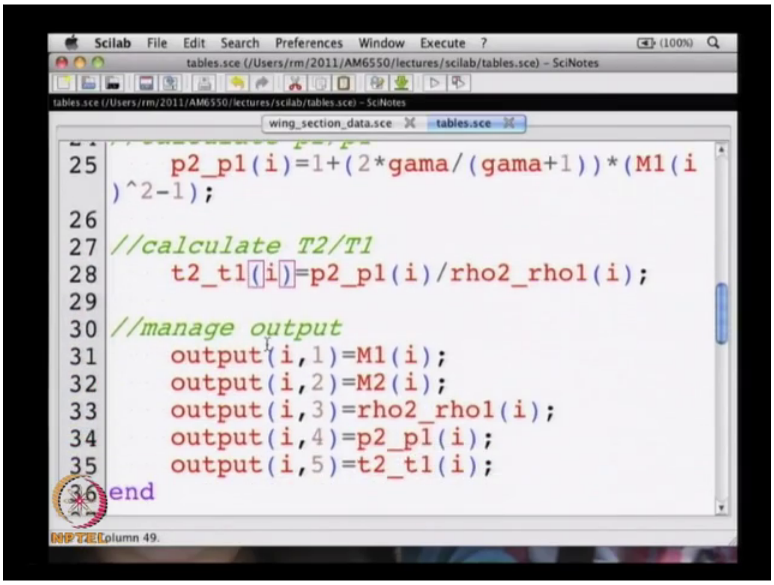

And all of these is you can see are in terms of gamma and M 1 right

So, and then I write this out and I am going to plot this. So, let us see for the way I am going to run this.

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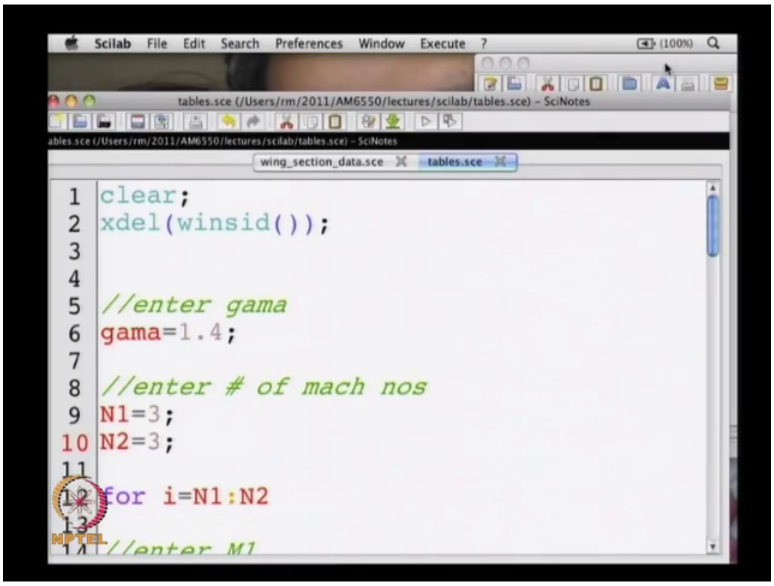

And I put this as 3 and then we are going to run this and see what we will get this is my console.

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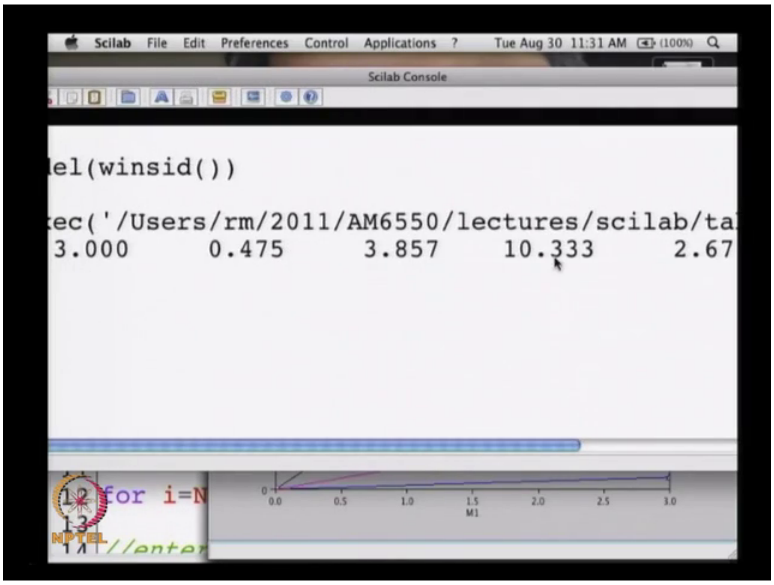

Let me just do this quickly for you to see. So, now, let me run this and see what we get and let me let us go back here. So, what we get over here is this right and this 3 is the mach number right, 3 is the mach number this is M 1 this is M 2 right this is rho 2 by rho 1 this is p 2 by p 1.

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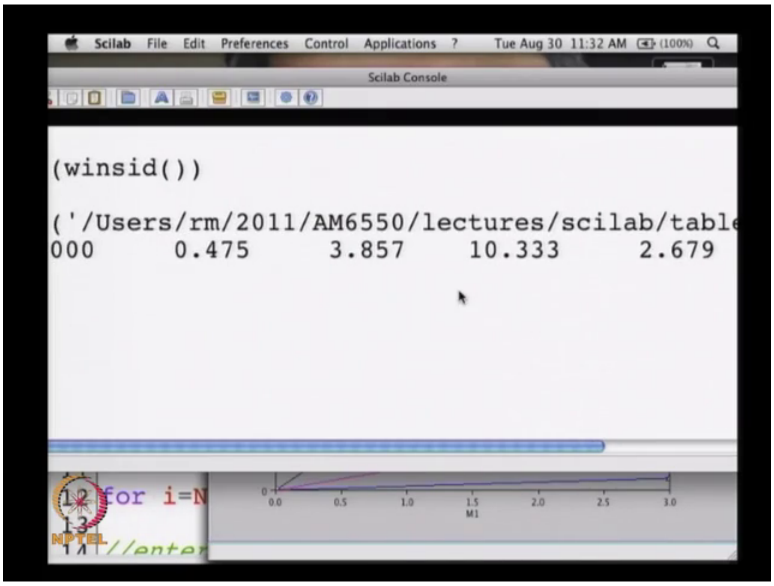

And in the end this is T 2 by T 1. So, this is what we a get as an outcome, now what in this code you can do is basically plot for as many number of mach numbers that you want you know it is just 1 enter button as you saw ok.

So, let us sort of do that, let me see like let me see if I get the same thing as there is in the charts that I talked about.

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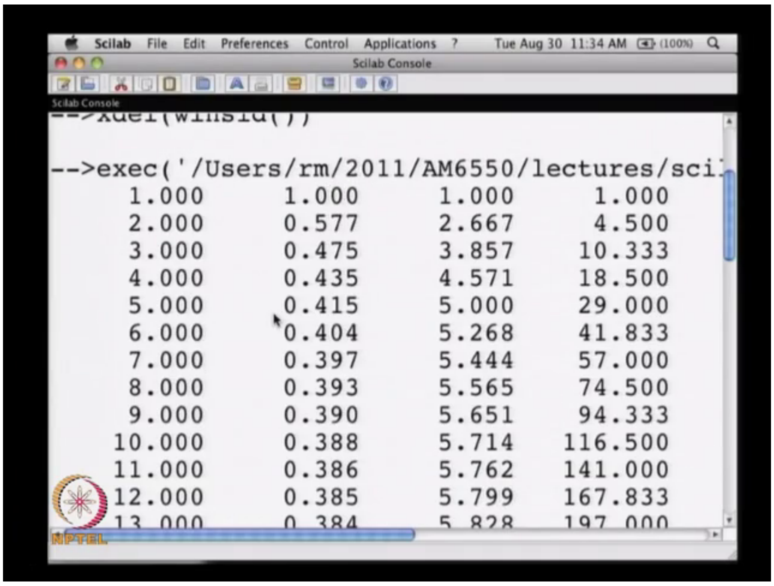

So, if I do this if I do this then. So, what you see over here is basically all this entire chart over here. So, this column the first column is M 1 this is M 2 the third column is rho 2 by rho 1.

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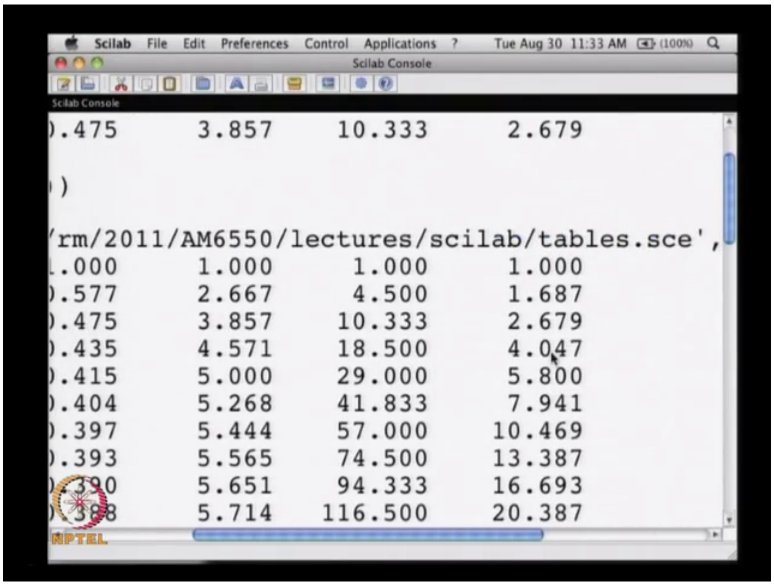

Then it is p 2 by p 1 and T 2 by T 1 right this is what you get and this basically we get for all the 50 values right, we get this for all the 50 values. So, now, when you go and see the chart you will basically see something like this now the chart that have talked about at the back of your books this is what you will see at the end of the book, now let us do something over here. So, let me just go back 1 more time now for example, if I said say mach number is 3 right. So, then the corresponding values of M 2 is 0.475 then you know this is rho 2 by rho 1 and p 2 by p 1 and T 2 by T 1 right.

So, then one way was to come here and calculate it you know putting your values of gamma and M 1 and all these ratios you could get by hand calculations and the other is to go back to the you know to the available charts and use it from that and this is the code then again which gives us you know this basically the chart you know which is at your you can just do it for any number of values. And the usefulness is you know if say you need you know values between say 2 and 3 you know and then you know a code like this comes in handy because it you know you can just you know use that particular value of mach number and get the corresponding values, it reduces you know human error.

Now, let us, for these whole range of mach numbers all these range of mach numbers let us look at these plots which I have brought it over here.

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Now, let us just say look at this one, now what do you see over here what you have on the x axis is M 1 what you have on the y axis is M 2, as you can see in the x axis M 1 goes from 1 to 50 and this is your M 2. Now what do you what do you notice from here right, what do you look when you look at this curve what does this tell you? Now, for example, say if you look at this curve and say you are you are changing your M 1 your incoming mach number say right from say 1 1 to 5 you increase your mach number incoming mach number say from M 1 equal to 1 to M 1 equal to 5 right. That is one way that that is 1 change another change is say you have an incoming mach number of 25 and you increase that to say 30 what is, what do you notice from this curve.

Well I would like to see like what you to see over here that if you are say going from M 1 equal to 1 to M 1 to 5 there is a very sharp change in the mach number M 2 right, that is accompanied by a large change in M 2 whereas, if you go from mach number 25 to 30 the change is not. So, much it is almost n1 right and if you remember that in one of the previous lectures we did say that M 2 always tends to a finite value, even if you increase mach number M 1 to infinity M 2 will tend to a finite value. So, what this is telling us is that say M 1. So, M 1 I have a change right. So, M 1 say I am going from 1 to 5 right and M 1 I am also going from 25 to 30 right, but watch this is going to do I s create a very large change in the M 2.

It is creating a very large change and a very sharp change in M 2 whereas, here the change is not so much right, what this is telling us is that here the change is not so much. Now, if you also sort of look at this if you have say mach number 5 right if you have say mach number 5 then your corres say or say if number 3, then you have a corresponding M 2 around point 5 actually right 0.475 that is what we have calculated. So, around say 0.5, but when you have a mach number of 25 and the corresponding M 2 is around point 3 right.

Now, what does that tell you in terms of the speeds what does that tell you in terms of the speed of the flow right. So, is that giving you some idea about how sharp or how strong the shock is right. So, you have a very high supersonic flow you have a very very high supersonic flow right and that M 2 is really much less than 1 right, is it is it is say around M 2 is around say 0.3. So, does that give you an idea about how strong the shock is right. So, now, the point is when we have an incoming mach number when we are an incoming mach number say M 1 is just about a sonic we have a very weak shock we will talk about this in a little detail in a couple of next classes. So, if you have a sonic sort of mach number nearly sonic then you have very a a weak shock and that is usually the kind of degenerates into up what we call as a mach wave ok.

On the other hand if you increase this mach number right, if you increase this mach number you have you know very large changes in this in the M 2 right. So, the difference between in a very highly supersonic flow and the flow aft of the shockwave is very much. So, then that becomes a very very strong shock, it is a very very strong shock at the same time; however, the more you increase the incoming mach number this M 2; however, tends to a a finite value it does not keep decreasing right for the rest of it. So, that is what we take away from essentially this plot over here now let us say.

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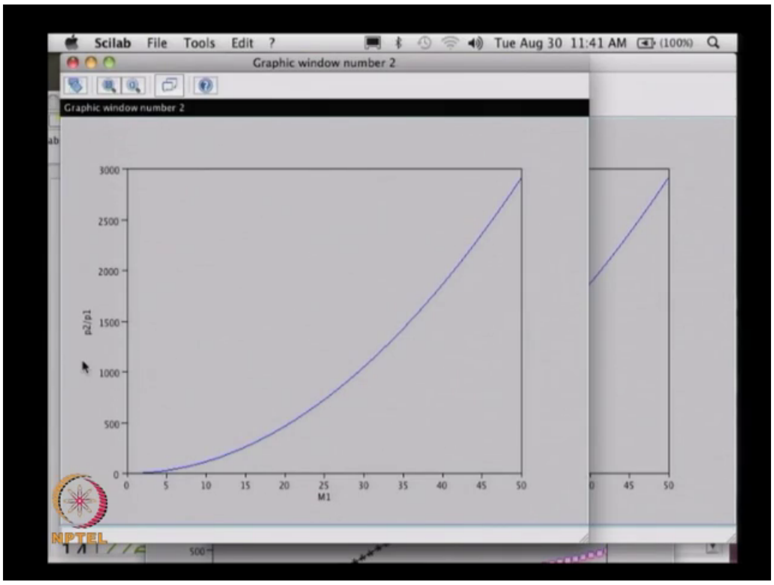

Then this is how the pressures look like this is how the pressures look like. So, you know this is what you can see this is your incoming mach number along the x axis and what you see on the y axis is p 2 by p 1 is not it. So, this is sort of increasing, but more or less smoothly what does this tell us p 2 is more than T 1 ok and then this is the T 2 by T 1 right.

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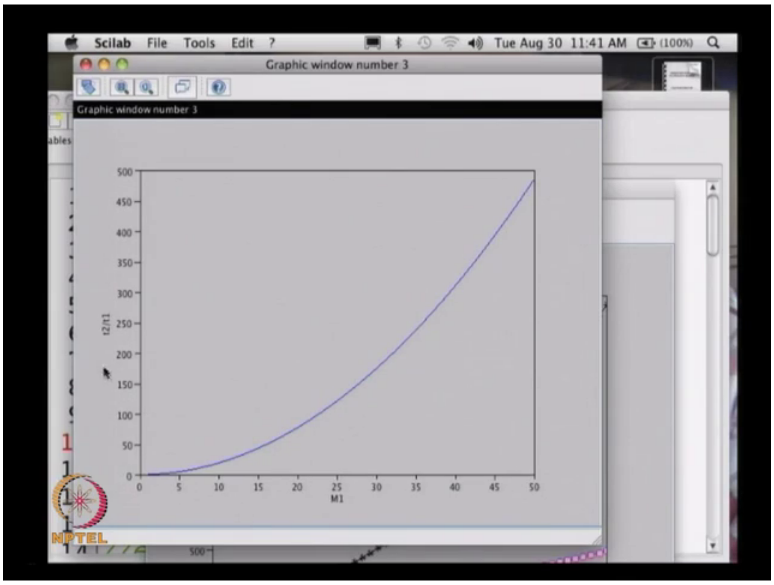

So, x axis is mach number this is going from say 1 to 50 and the y axis is T 2 by T 1 this is also a smooth increase, alright.

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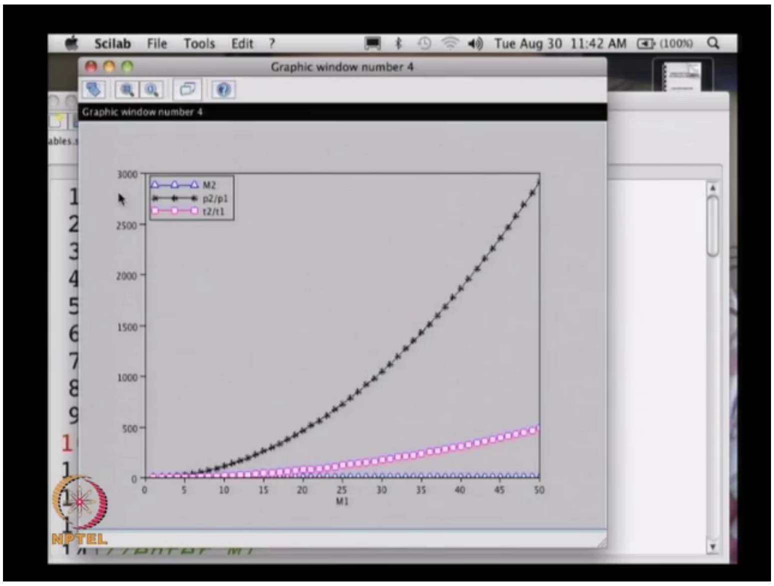

So, I plotted all these things almost at the same you know same plot I do not think this is a (Refer Time: 18:04) one, many different axis actually to do that. So, therefore, in having a code like this is pretty handy it is fun to do for 1 and you can do whatever you want and put as many values here as you want, but if you go look at the charts you will also see something else other than whatever I have listed over here. Which is M 2 rho 2 by rho 1 p 2 by T 1 and T 2 by T 1 you will see something some more values there as well which are the values with a subscript naught which is essentially the stagnation or total conditions. So, let us sort of go and talk about that a little bit and see what those (Refer Time: 18:52) is actually mean.

Now, let us do this here now as we have done this before. So, say we have this shockwave.

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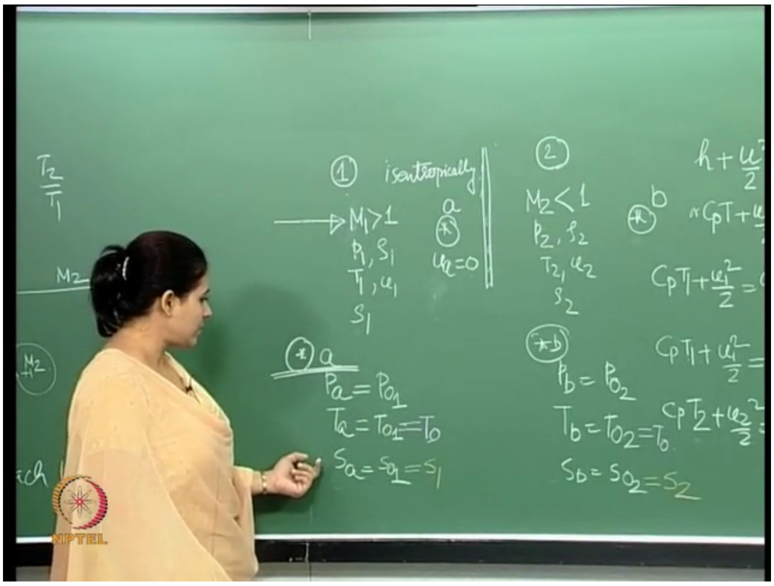

So, we have a shockwave and this is say the region 1 and this is region 2. So, a normal shockwave is happening in this way and say at region one the properties of the fluid are mach number and so on and so forth right. So, these are so this is a supersonic I think this is established now this is the only way we can get a normal shock wave. So, we have an incoming supersonic flow. So, we have M 1 over here and you have a p 1, rho 1, T 1, u 1, S 1 similarly here. So, we have a mach 2 which is subsonic and correspondingly we have E 2, rho 2, T 2, E 2, S 2. So, s is the entropy now these are values which are for each of the actual flow properties ok.

Now, let us divide 2 more regions or 2 more conditions, one is this, this point a and this point b right. Now, these 2 points are the let us define a now this is a point which is essentially I take the fluid at condition 1 right it is moving with these parameters and I i isentropically bring it to 0 the speed of the flow becomes 0 at this point a which is u a is essentially 0 over here, is that right thing to say yes. So, now, therefore, this is a condition. So, where is I isentropically bring it to bring it to a stop. So, therefore, the corresponding values here are let me sort of write it over here. So, this point a so the corresponding flow property is over here at P a, T a, S a so on and so forth now this is the location for a ok.

Now, p a here now these are also. So, since this is a stagnation condition I will also write this as with the subscript 0, this T naught s 1 and this is s naught as 1. So, these are the corresponding properties at this point a. So, again let me repeat that this is a point at which I bring this fluid at 1 isentropically to 0 and the speed of the flow here is 0. So, similarly this is the point b is an imaginary point basically. So, what we do is we take T conditions at 2 here and bring it to a point where this becomes here. So, this here it becomes again 0 right. So, this is from this condition from a subsonic condition I bring this to a point where this is 0. So, again the corresponding property is for b here for b here is p b again this is a total condition of stagnation conditions then T p is T 0 2 let me call this as 0 2 and s b is s 0 2 ok.

Now, having said that do you see we could incorporate anything else, now what I said here was that we get from condition 1 to the condition at a isentropically right and similarly here too we get from condition 2 to be isentropically which means, which means that when I do this process my entropy is constant, my entropy does not change. So, hence it is isentropic right which means that therefore, S a, S a here is equal to S 1 right there is no change in the entropy. So, therefore, I can write this as s 1 and similarly this process 2 is isentropic. So, therefore, s b here this is the entropy at the condition b. So, this is equal to s 2 since there is no change in the entropy, now let us do this now let us take the energy equation. Now, let us take the energy equation which is right.

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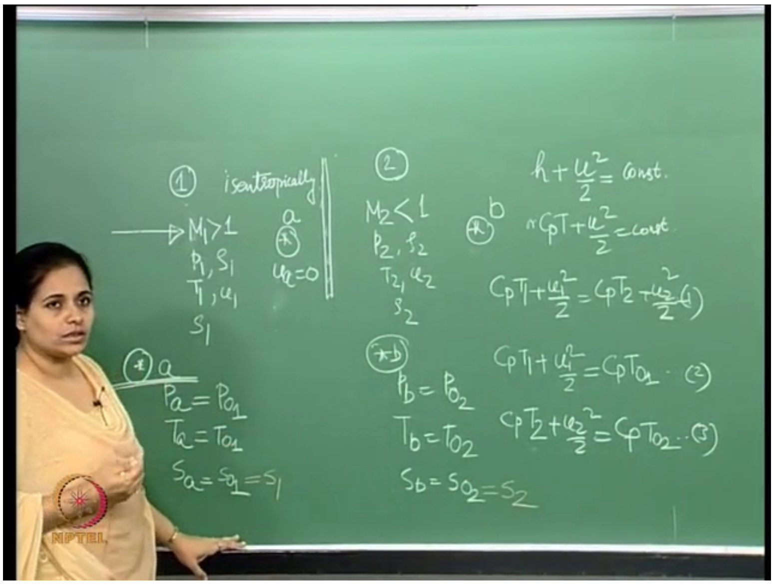

So, this is the energy equation now what we will do is we will take an energy equation and we can also write this as.

Now, we can also now what we will do is we will take that an equation and apply it between different points. Now let us apply that equation between regions 1 and 2 right between the actual flow properties at 1 and 2 if I do that then what I get is this and the next is that I apply this equation the energy equation between this the actual condition 1 and the imaginary position at a. So, if I do that what I get is this c p T 1 plus u 1 square by 2 this is at 1. Now, this is the point a which is C p T naught 1 and u 2 is u 2 is sorry u 1 u a is 0 right. So, this is what we get. So, similarly now I apply the energy equation between the point between the region 2 this is the actual flow conditions and the imaginary position at b right. So, again what we get here is C p T 2 plus is equal to now this is the region 2 and region b here is c p right because u 2 is u b is 0.

Now, once we get these 3 expressions from here what do you infer looking at these 3 equations over here now let us look at this. So, what do we have? So, we have C p T 1 plus 1 square is equal to C p this, now this C p T 1 square T 1 plus u a square by 2 is equal to this is it is not it from this because you can see that from this equation right therefore, what we get what we infer from here is that.

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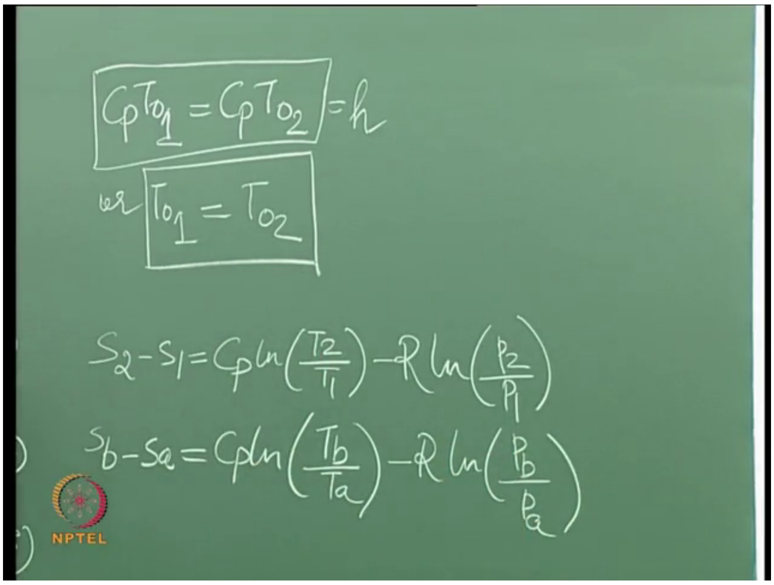

c p T 0 1 is equal to c p T 0 2 or T 0 1 is equal to T 0 2 what does this tell us, what this tells us is that the stagnation temperatures do not change across a shockwave. So, all the total temperatures do not change across a shockwave another thing that it tells us that C p this is nothing, but the enthalpy is not it now this is the total enthalpy. So, the enthalpy does not change across a total enthalpy does not change across a normal shockwave ok.

So, that is what we infer from over here. So, T naught 1 is equal to T naught 2 so if we yeah. So, if we go back over here. So, we have ta T naught 1. So, let us write this as T naught right. So, basically across the shockwave. So, the total temperatures do not change and yes there is an entropy change of course. So, this is what we get from here now if I have to do this now. So, let us therefore, from here let us calculate the entropy change right let us calculate the entropy change across a normal shockwave and how are we going to do that. So, now, let us try and calculate the entropy change between say a and b between a and b now for an isentropic process.

We know that the entropy change is given by, am I writing this correct yeah right now between if I do it between b a between a and b the total entropy change between a and b what is essentially we get is that right you can write it like this right.

Now, this all I am doing is taking the second location as the point b first location is the point a right. So, I am just writing that in terms of this now as we have discussed right.

So, this s b is actually equal to s 2 right, s b is equal to s 2 and s a is equal to s 1 right. So, that let us rewrite that.

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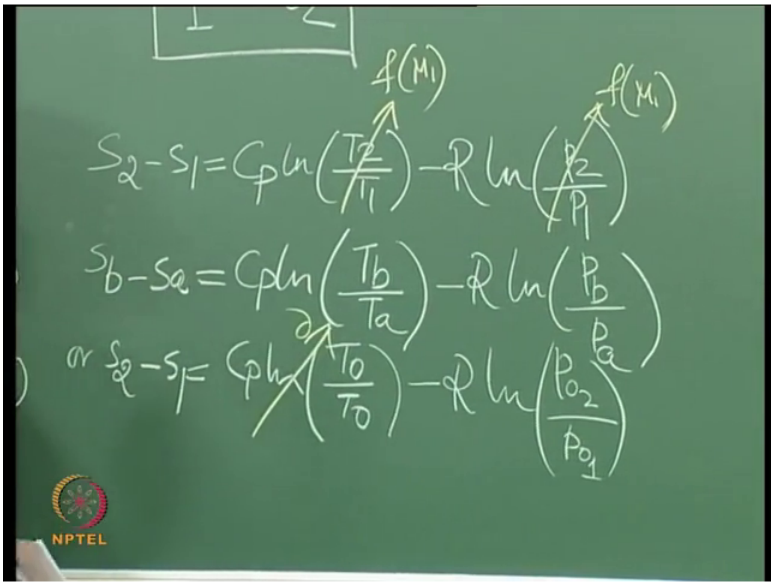

So, therefore, this is s 2 minus s 1 right this is equal to T b by T a right, now we just derived that T b is equal to T naught 2 which is equal to T naught and that is equal to T naught 1. right there is the stagnation temperatures total temperatures do not change across a shockwave. So, essentially this is equal to T naught. So, T b is actually equal to T a which is equal to some stagnation temperature it does not change and minus r and l n and like we have we wrote the expressions since these are total conditions or stagnant conditions we denote this as with subscript naught.

So, therefore, p b we denote as p naught 2. So, what we get over here is p naught 2 by p naught 1, now once we write that here you can see what we get now this turns out to be this term goes to 0 . So, therefore, what we get let us see go back here. So, therefore, what we get is.

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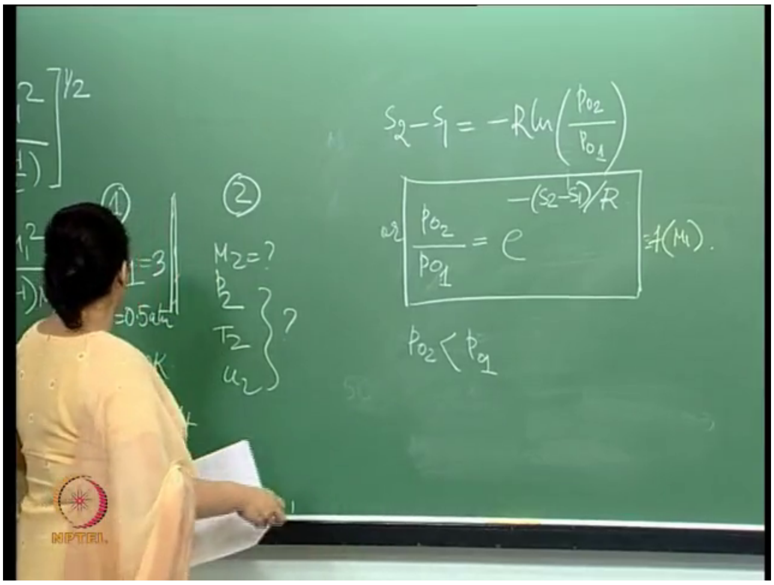

So, we get this or we can write this as. So, this is what we what we get, now what else can we infer from here, now this is a normal shockwave right which is and the process is isentropic is not it now. So, what can we comment about s 2 minus s 1 right, now s 2 minus s 1 is positive is not it and it is increasing across a normal shockwave that comes from the second law of thermodynamics right we discussed this last time and so therefore, what we can see from here is that right which means that the total pressures right the stagnation pressures decrease across a and a normal shockwave right you can see that from over here ok.

So, and also watch if you remember we can write the for example, if you see this expression over here we can write the T 2 by T 1 and p 2 by p 1 in terms of M 1 right we can write these expression, these right and this also is a function of just n 1 therefore, what that makes is that is s 2 minus s 1 is also just a function of M 1 right. So, therefore here also. So, therefore, this is also just a function of n 1 which means that that p naught 2 and p naught (Refer Time: 35:16) or the ratios of the stagnation pressures are also a function of just M 1 and therefore, that also is available in the chart. So, I can just include that in the code that I have written. So, again so therefore, this is also a function of just M 1 this is also available in the in the in the charts. So, having done that. So, I think let me give you some of the numbers that is I got. So, you can get that from the code as well. So, that you can finish this problem you can complete this problem ok.

So, now here in this particular problem what we said was that the initial mach number is given p 1 and T 1 is given gamma is given find out M 2 p 2 T 2 and u 2 this was the problem right.

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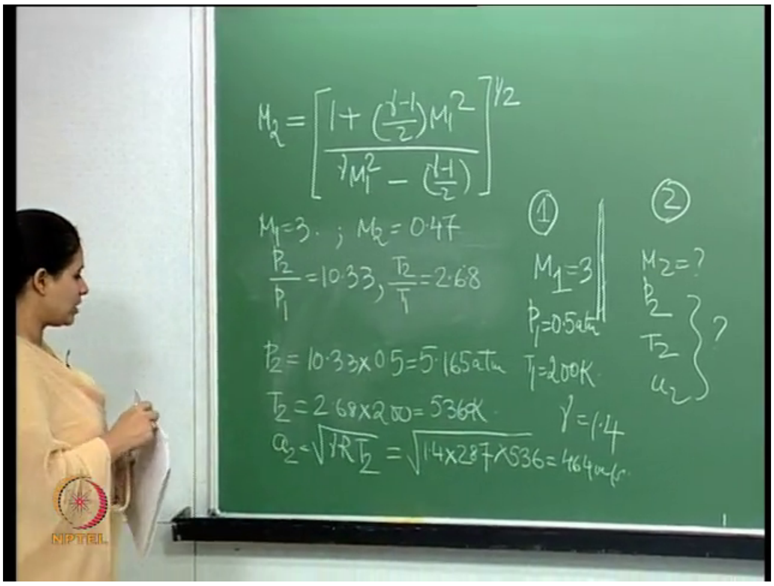

So, for M 1 equal to 3 we got a p 2 by p 1 as 10.33 T 2 by T 1 as 2.679 you can write this as 68 actually and an M 2 is around it is actually 0.47 that is exactly 0.4752. So, I will just write it as 0.47. So, these are the values that you get from your chart you calculate it yourself using the equations or you know write the small code after that how do we get. So, p 2 right how do we get p 2, now p 1 is known right p 1 is 0.5. So, therefore, p 2 is just 10.33 into p 1 right and that gives us ok.

Then similarly we get T 2 which is T 2 by T 1 which is 2 point six 8 into T 1 which is 200 Kelvin. So, this is around 536 Kelvin. Now, then we need to find. So, we found out M 2, p 2 and T 2, M 2 we got straight away from the from the from the charts right and p 2 we just have to sort of take the ratios and calculate depending on the start startup values, how do we get a 2 now well what you need to do here is calculate this, right. So, if I do this. So, this then becomes this is the speed of sound right. So, this is 1.4 I take the standard value of r and T 2 is 536 right. So, what I get over here is 46 right, 464 and from there the way the way we know.

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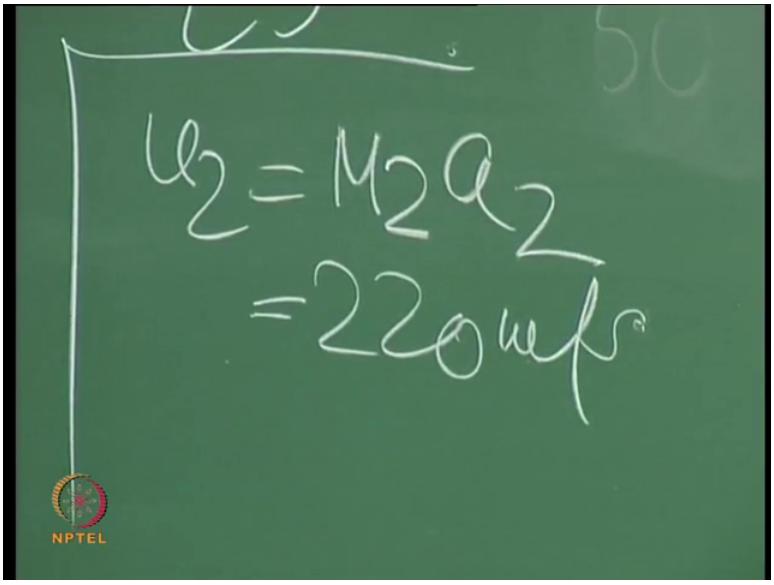

M 2 right. So, therefore, U 2 is equal to M 2 into a 2 and which we get as 220 meters per second. So, this is what is this is what we this is how we sort of finish this problem ok

Now, if I may just sort of do a quick problem here one more time. So, in here actually let us sort of introduce this. So, essentially so basically if you are when you solve problems like this. So, we can get these values for this relationships from the charts and you constantly refer to them and you can look at them. So, there are isentropic charts and there are also charts for normal shocks now for example, let us look at another problem now for example, look at this look at this. So, we have a blunt nosed missile.

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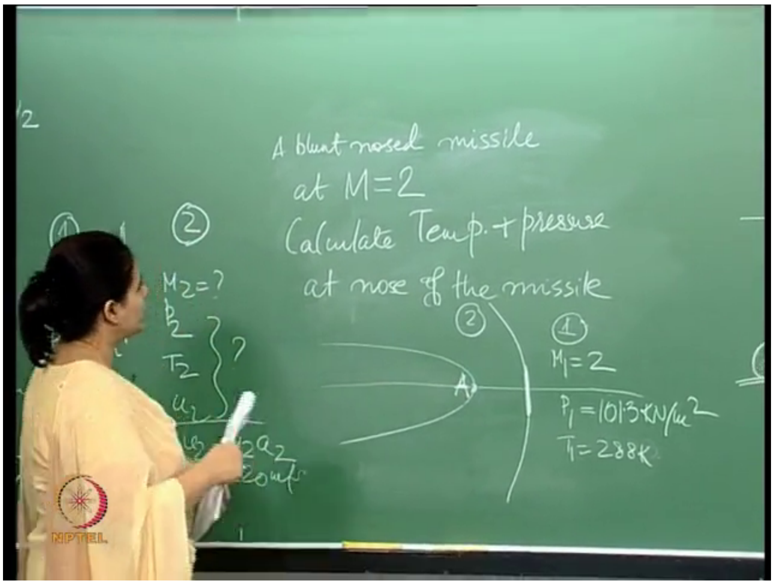

So, we have a blunt nosed right and that is flying it mach at that is flying at mach 2 as standard conditions and so what we need to calculate is the temperature and pressure right at the nose of the missile ok.

So, what we essentially have is we have a blunt nosed missile like this right and we need to find out these says l s located this lets call this is the nose. So, what we need to find out is the temperature, the temperature and pressure at the nose over here and the blunt nosed missile is at mach 2. So, what is given as standard temperature and pressure at standard temperature and pressure? So, what do we do over here now this here is you know this is a supersonic missile right it is a supersonic missile. So, what this will cause is a detached bow shock. So, essentially this part is a normal shock over here is not it. So, basically this is a streamline which is going through. So, if I consider this point over here right if we consider this line. So, it is passing through a normal shock, it passes through a normal shock and then this is my nose. So, if you consider this as say region 1 and a say region 2 right. So, what you have over here is that mach 1 is 2 then standard condition. So, I will just say p 1 is and T 1 is 200 Kelvin and yeah. So, this is a bow shock. So, what we need to calculate is the temperature and pressure.

Here we need to calculate temperature and pressure over here now what the way we will do this the way we will do this. So, we talked about stagnation conditions etcetera. So, far and. So, you know a several times we derived equations etcetera and we saw the imaginary conditions and. So, forth now here is a point and this is at the at the node. So, this is a stagnation point is not it. So, therefore, we will what will go is we will go and look at the charts and look for the ratios of these stagnation points properties are these stagnation conditions. So, from the tables what we will get is this from the tables.

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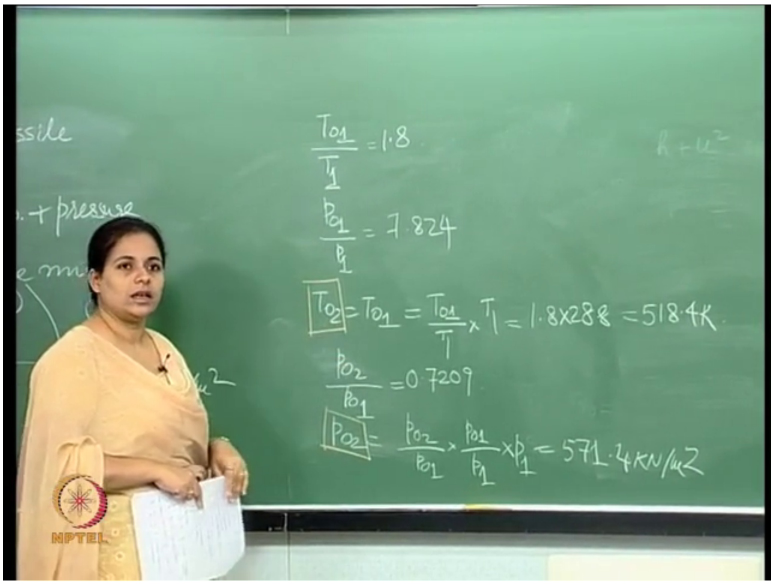

So, we will get it from tables what we get right we get this to be this p naught 1 right. So, what you get this is something that you get from your isentropic tables right. So, if you get this now we know that the stagnation temperatures remain same across a shockwave is it across a normal shock wave. So, you get this. So, the way we will calculate this is again right.

So, T 1 is standard (Refer Time: 44:41) this is known right. So, if I do this then I get something like this. So, you can calculate this yourself. So, what I get over here is it is essentially this is what this is 1.82 88 right this. So, what I get is Kelvin and again. So, this I get from isentropic tables and again from normal shock tables. So, we get from normal shock tables what we get is right and we get that as point this. So, therefore, p naught 2 by p naught 1, which is p naught 1 by p 1. So, this is just writing it in a way. So, that you can use your use the information that is available and just you are doing a doing the mach operation. So, if you put it all the values what you will get is this right. So, what we found out essentially is the temperature here temperature at the nose which is T naught 2 and p naught 2, what we did here is that we noticed or we realized that this is actually a stagnation point. This is stagnation point and we were able to use the the isentropic tables and then we use the normal shock tables and then use the information available and we were able to calculate the temperature and pressure at the nose.

I think what is we will do is see a couple of more problems will calculate the entropy changes etcetera and see how we can do that again is in the tables and then. So, we will stop here today.

Ok thanks.