Advanced Gas Dynamics
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Lecture – 06
Normal Shock in a duct: Throat and Reservoir conditions

So carrying over from last lecture, what we did in the last lecture was basically we developed relationships between the throat conditions, the reservoir of stagnant conditions in a flow field. Let us see if we can do something specifically for a normal shock. Let us go ahead and do that and see if answer some questions in terms of how basically the normal shock is formed.

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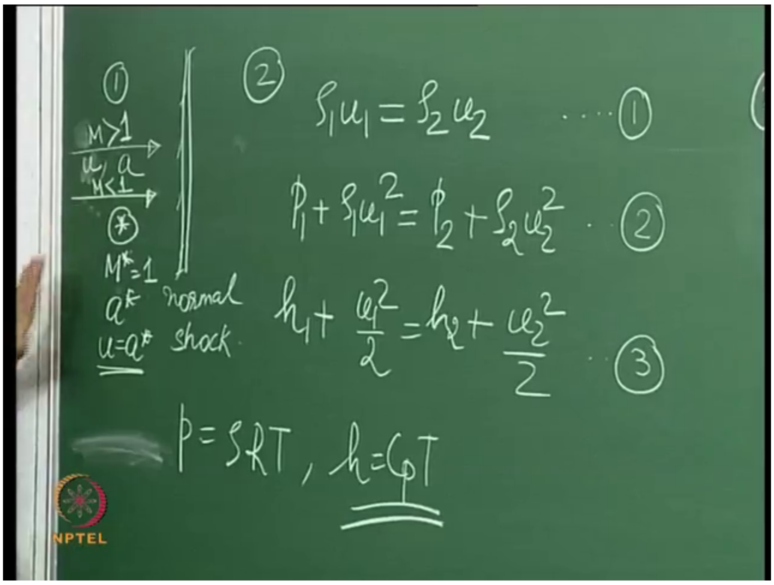

Now say this is a, the again this is my normal shock, this is region 1 and 2. Now I just rewrite the basic governing equations.

This is the continuity equation, this is the conservation of moment equation right and this is the adiabatic energy equation, now one thing I would like you to be careful is the similarity of this relationship this equation with the Bernoulli's equation it looks very similar, but it is very different, be careful when you write this equation when use it.

The flow field when a normal shock is going to be governed by our 3 relationships like this, let us develop a few relationships and see if we can have a little more information.

We will also use these relationships in tandem here, which means that our gas is calorically perfect, now one of the questions to ask especially in a shockwave like we have done before number one is what exactly is a shockwave, try to you know give examples for that or to explain that in a more crude way, the one question to ask is that when exactly is a shockwave formed is it that you know.

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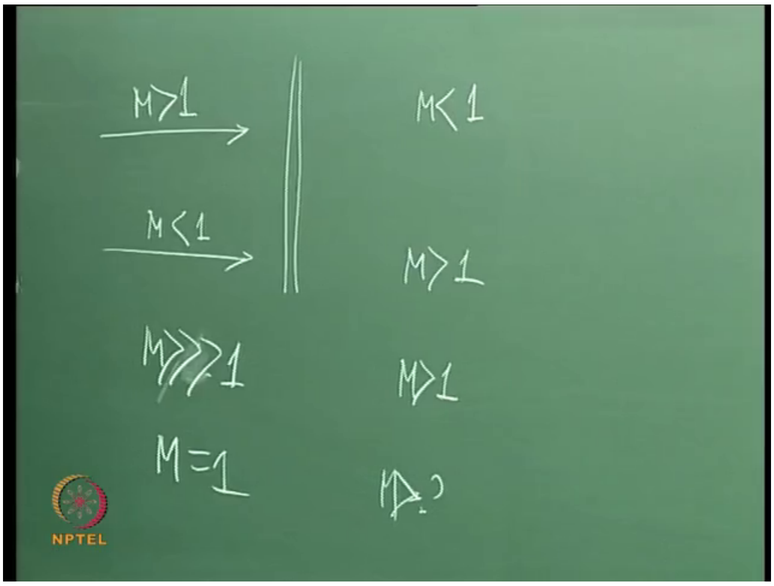

Is it that you have a very high supersonic flow now that really slows down here to subsonic and then we have a shockwave could it also be that I actually have a subsonic flow and it speeds up to very high speed supersonic flow and then we have a shockwave right or is it that you know we have a very very high mach number flow and slightly less mach number flow right both are supersonic by the way and we then have a shockwave and so on and so forth.

If we have a sonic flow and that kind of sonic flow and then that causes you know what happens here does and does that cause a shockwave and what is it that is in our equations or in our relationship that will tell us this one of the questions to ask is that. Let us see if we can use our governing equations to find some of these answers, essentially what we are looking for is a relationship between the flow properties and the incoming mach number right. So, the how the flow properties change across the shockwave depending on the incoming speed of the flow right that is what we are looking for, let us see if we can get some of those answers from our governing equations. So, let us do this somewhat here that is divide the second equation by the first, what we will do.

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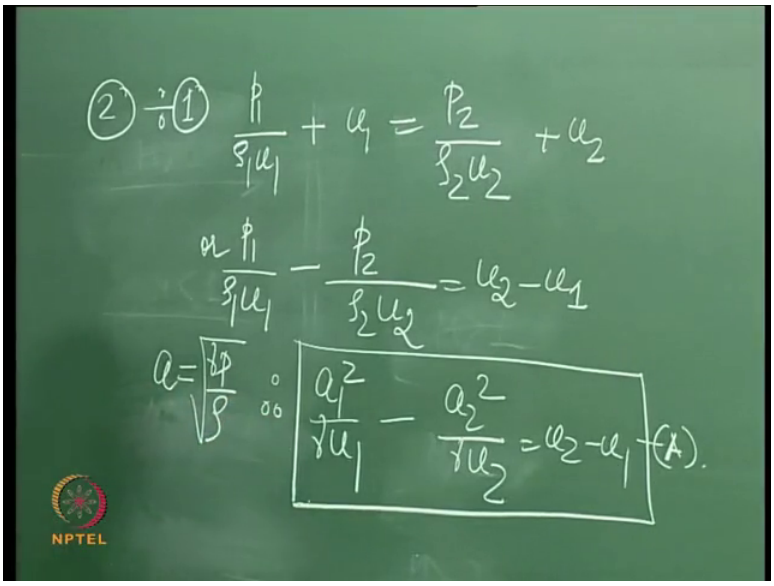

Is take the second equation divide it by the first and what we get is this, right we get this or is this right. Now we can also write let us introduce the speed of sound in here, now, we know that a is equal to rho, what we can write over here right. Therefore, the relationship becomes, we have a relationship like this right, we basically combine the momentum equation and the continuity equations introduce the speed of the sound and this is what we get. Now using the energy equation over here what do we get let us see let us use that so.

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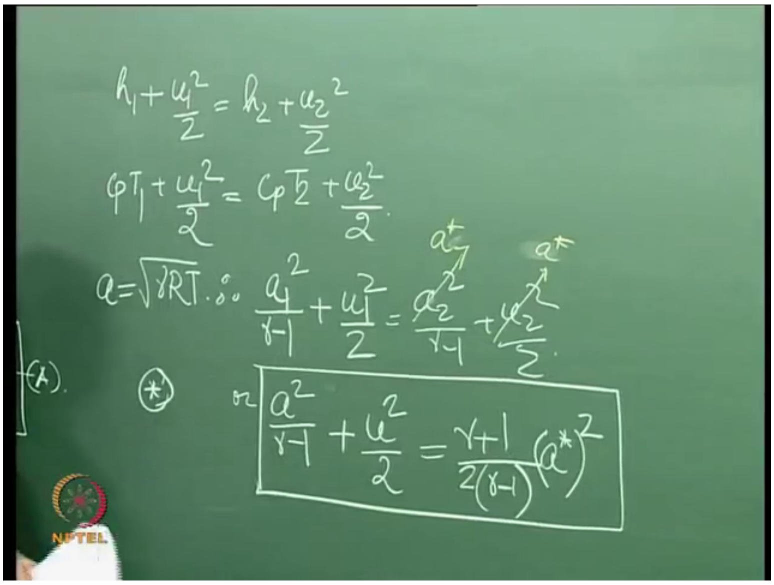

We know that this can be written as and from here again we can introduce this because speed of sound is right this is true as well, if I do that, therefore, what I get from here is also, what essentially I get from here is this is equal to.

Let us do a thing over here let us introduce a region which we will denote by the properties at which we will denote by a subscript star let us just try to understand what we are trying to do over here now say I have in this region in this region which is before the normal shock, let us say we have a supersonic flow we have say a supersonic flow. This is the actual the mach number, we have a speed this is the actual flow speed the velocity and this is the actual speed of sound right.

This is a supersonic flow coming in over here now let us slow this down such that it reaches a mach number 1 it reaches the sonic condition, mach number 1 the corresponding speed of sound here is a star now this is an imaginary case. So, we are just saying that we have a supersonic flow and we are going to slow this down we are going to slow this down adiabatically and bring it down to a sonic condition which is m star which is sonic condition which is m star as 1 and the corresponding speed of sound is a star, which means that the velocity at this point then is a star as well is not it.

If I had to do that now we could do similarly another if we could also do this in this way that says I have speed of sound speed of the flow which is subsonic, say in this region or in this region itself, instead of m the speed being supersonic say this is subsonic, say we have, say we are coming in with the speed which is subsonic and the corresponding velocity and speed of sound are u and a right it could be either supersonic or subsonic. In this case it will again reach this sonic point by speeding up the flow, we speed up the flow, that it becomes sonic and the corresponding mach number here is a star we will see the usefulness of this as we introduce it into our equations it could be either right.

So, what we are not assuming here what we are not able to assume here is what exactly is the incoming mach number right for the speed of the flow which will actually cause a shockwave, we just are assuming that we do not know that, we will do all these studies and then try to get an answer for that. So, coming back to our equation here if we do have that if we introduce this star region over here instead of this taking the second region as the sonic region. So, what we will have there is that this becomes a star and at that point you this is also equal to a star.

If we have that then what we will get this equation therefore, then becomes right, what this relationship is basically telling me that if at a particular zone the actual flow of this of actual velocity of the flow is u the actual speed of sound is a then let us get a region where which is sonic the corresponding flow velocity the speed of sound that is a star and the mach number there is 1.

Then this is the relationship and nothing here is actually telling us whether I am supersonic or whether the you know velocity there is supersonic or subsonic it could be anything from this region because if it is supersonic I can slow it down to the sonic zone and if it is subsonic we can speed it up to the sonic zone. So, nothing here is actually giving me any clues as to the incoming mach number.

If I do this right now let us do that, therefore, what we will do is write this for these 2 zones right say 1 or 2 zones and what we will get is this.

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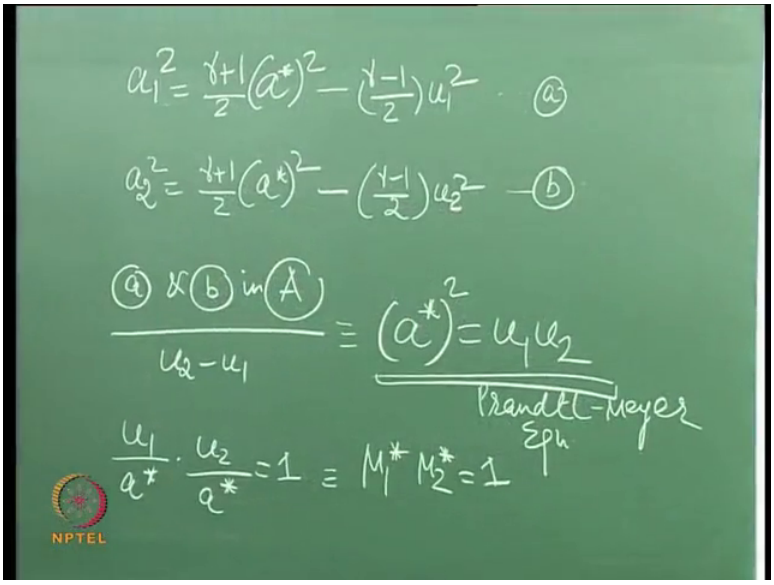

If I write it out, so this is what we get by bringing in 2 regions over here a 1 and a 2 right, either of this a 1, a 1 is in the region 1 and a 2 is in region 2 and nothing here is telling me that the region 1 has supersonic flow and region 2 has subsonic flow or vice or versa we are just saying that the flow is the flow reaches we are able to slow it down or speed it up to the sonic zone. If I do this, what we will do is use these 2 in the relationship over here if you see this is the relationship which we had worked first in general by A, what we will do is use these 2 expressions. We will put input in a and divide that by u 2 by u 1 right we use these expressions in the equation and divide that and what we get is this.

So, what this is telling us is the relationship between the velocities in region 1 and 2 with respect to the sonic zone, then that also I can write as right, what I am doing is now I am basically dividing by the speed of sound in the imaginary sonic zone. If I do that then what I get is what this gives us is M 1 star M 2 star is equal to 1 right, now, this relationship essentially this relationship is the Prandtl Meyer equation this it is square to the (Refer Time:17:40). Then this is the relationship that we get now let us see now if we can make certain inferences from here. We will write it like this and also, I can also write it in this form now.

Let us write that over here, then what I will write is.

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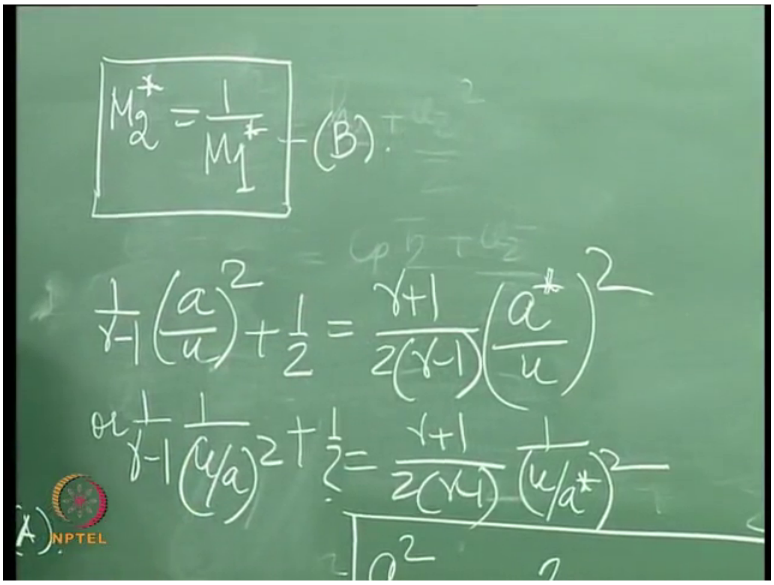

And I am going to call this as the relationship B now if you see over here, you can see it from this relationship as well, what we can do is divide this by u square right if I divide this by u square what I get is this. So, basically I am dividing this by u square, what I get is this right or I can write this as basically or I can write this as right, see if you agree with me on this one, all I am trying to write this as just in terms of because this give you the feel for the mach number, then I can actually write this equation as I can write this equation as.

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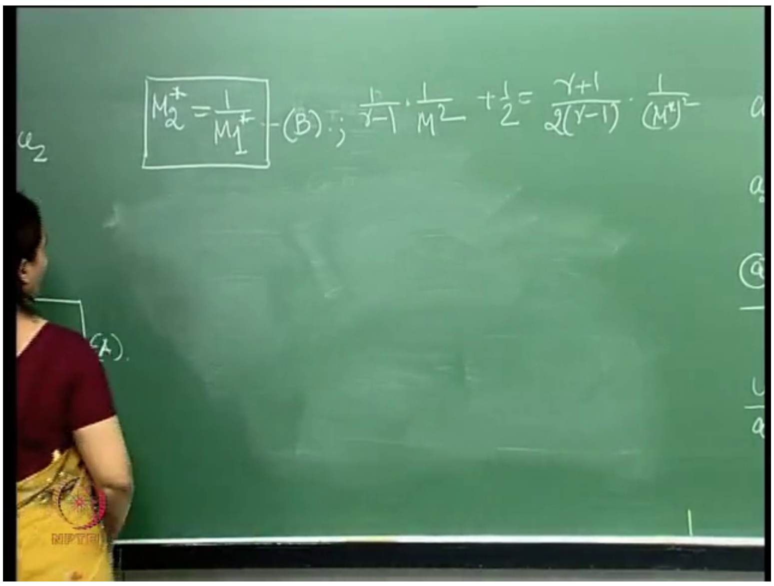

This is the actual mach number right, this is the actual flow velocity and this is the actual speed of sound and this is the flow velocity with the sonic zone speed of sound this is the M star, if I write this, then from there I can basically write that in I will write it in this form.

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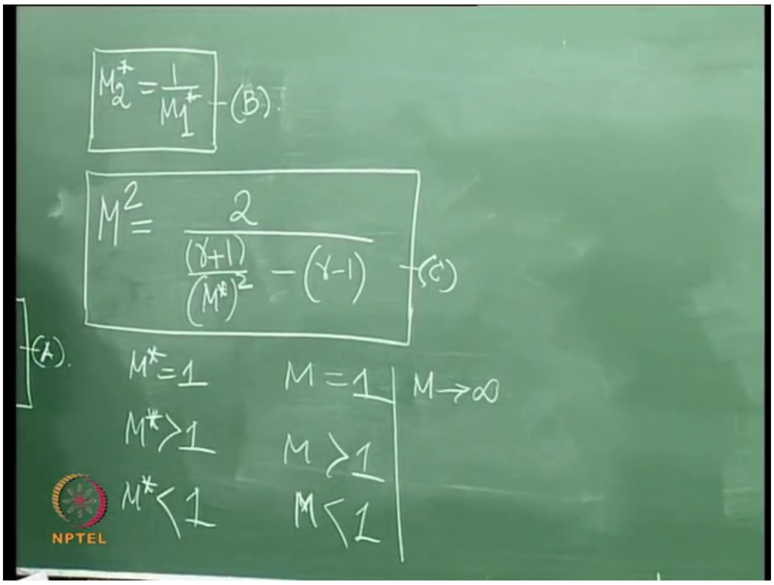

So, M square is equal to, what now we are able to do is basically look at the their relationship between M 1 star and M 2 star right and the actual mach number and the sonic zone mach number. So, we have got these 2 relationships and now let us study from here what are the mathematical characteristics from here and see if we can infer you know something more in the from these relationships.

Although we have said basically said that M star is the M star out here is the sonic zone M star is equal to 1, now just for a minute let us just forget whatever we discussed so far and treat this just as a mathematical entity right if you give this relationship to somebody who has not you know heard the discussion would we just had before then this is just a number to him. If I am just looking at this from the mathematical point of view let us try and decipher what we can from this relationship and we will go back to this physical simplifications after that.

If I do that now let us look at this relationship over here from C and now, if M star is equal to 1 what does this tell us over here if M star is equal to 1 what is the end. So, I will just try putting M is it M star is equal to 1 over here, if I put that what do I get M is also one, if M star is sonic M is also sonic. Let us say M star is supersonic then what do we get over here I think the best way to do that is just take say put a value put a value for M star say 2 out here and then see what will you get for mark, the mark you know M right. Let me just say that mark in this case is also supersonic and similarly if it is subsonic best thing to do is just act put an actual value, take gamma is equal to 1.46 which is the standard value for air and put an actual value of M star if it is subsonic say pre that say you know 0.5 or something and then see what you get for the mach number.

So, verify whatever I am writing over here I am saying that if it is 1 this you can see right now itself M star is 1 now verify this for yourself if I am correct. If it is supersonic then M is also supersonic, if it is subsonic it is also subsonic. This is what we understand from C and there is further more now if say in this case another case is that M tends to infinity, if M tends to infinity then what do we get for M star well just try to pull it over here just put M star. So, what we actually get is that this denominator basically tends to 0 is not, what we get if M star M tends to infinity.

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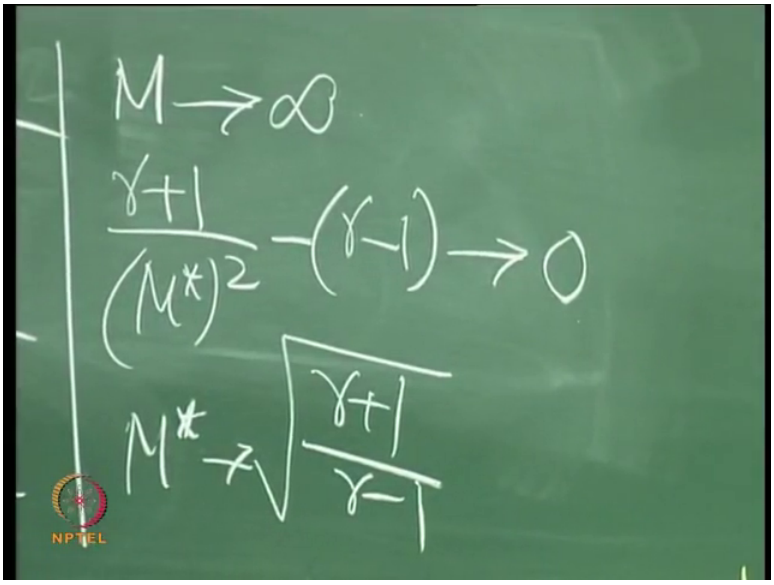

What we get is not it right if that is true then basically from here M star tends to, this is what we get from here, essentially, this is the inferences from the relationship between the actual mach number and this sonic zone mach number.

Let us say this is typically from the book this characteristic mach number, let me say it another characteristic mach number, the actual mach number and a characteristic mach number, this is the inferences from this relationship. So, what is this exactly telling us mach the actual mach number is tending to infinity it is infinitely large and then the characteristic mach number tends to the finite value. Whether we or if we have exceedingly very large mach number. Now this M star has a finite value depending on the type of gas because it depends on the gamma right, it is not tending to infinity or it is not becoming exceedingly small. Now, whether in those conditions also we will have a in a shockwave is something that we still need to see, there is only a finite value of M a characteristic mach number that we will reach even if I increase my incoming mach number to infinity even if I make it infinitely large that is what I am understanding from here.

Now, let us use this information here the between M star and M and try to see get an in inferences from inferences for M 1 and M 2 now if I do that, allow inferring from here now say if.

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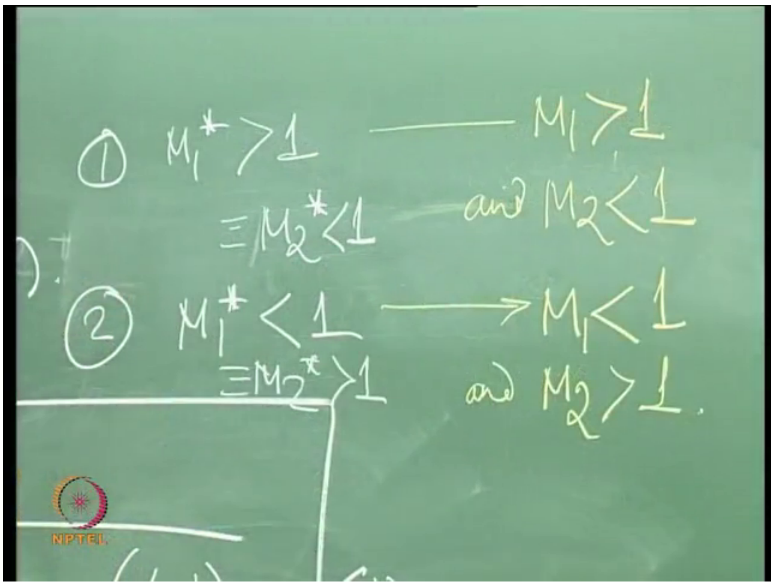

If M 1 star is greater than 1 then what we get from M 2 which means that this inference from here is that M 2 star is less than 1, that we can inform from B, but what can we infer from this again if M 1 star is a supersonic which means the corresponding M 1 is also supersonic. Now M 2 star is subsonic which means the star value is subsonic therefore, the corresponding actual value is also subsonic which means M 2 is subsonic right, therefore what we are trying to say here is that if M 1 is supersonic then M 2 is subsonic.

Another possibility is that, this is say case 1, now case 2 is that M 1 star is less than 1 and 1 star is subsonic, which means that M 2 star is supersonic is not it, M 2 star is supersonic, now M 1 star is subsonic which means the M the characteristic mach number is subsonic the corresponding actual mach number is also subsonic, M 1 is also subsonic, at M 2 star is supersonic the characteristic mach number is supersonic, therefore, the corresponding mach number is also supersonic, here M 2 is also supersonic.

What we understand from here is that if the incoming mach number is supersonic then across the shockwave it will become subsonic if the incoming mach number is subsonic across the shockwave it will become supersonic. Now we still need to answer the question that, they basically therefore, both these possibilities exist, we will see if that is a valid assumption if that is valid or maybe not maybe only one is possible nothing as, far that we have studied or talked about gives us the clue.

Now, if we do that let us see, we develop this relationship over here now let me we wrote that in terms of M square let us write that in terms of say M star square.

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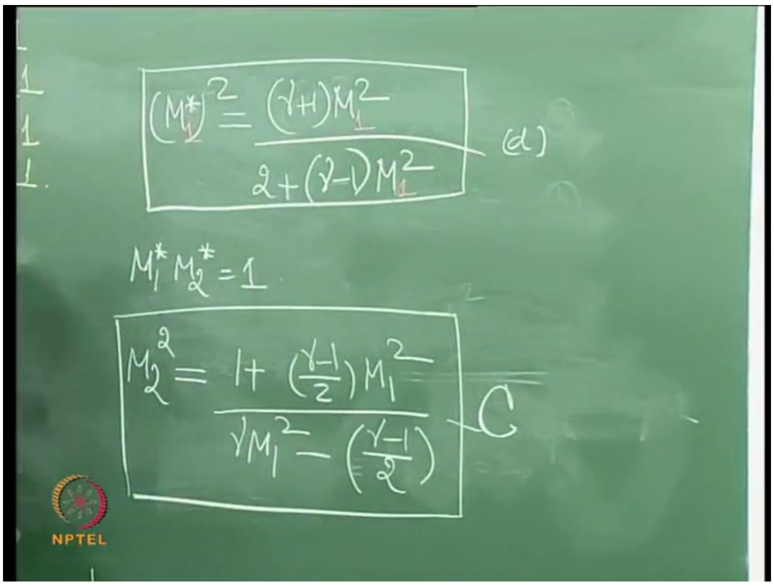

So, what we get from there is this, M star square is basically I am just writing this relationship in terms of M star instead of M square I am just writing M square it is just you know rewriting the equation nothing new over here. So, then this is the characteristic mach number now what I will do is now here we will introduce this into this right this is what, this is the problem our equation. Let us introduce this into this meaning if I say M 1 star, what I get is M 1 and M 1 over here and if this is M 2 star then what I get over here is M 2 and M 2 over here. So, what I am able to do here is introduce the mach numbers introduce 1 and 2, once I do that, let me bring that up into here, what I get is this and I rewrite that, what I get.

So, this is the, and like I said all I did was that if from this equation I bought M 1 and M 2 right, basically if you write M 1 star then you get M 1 right, you introduced this M 1 value over here and we will start value over here and instead of that if you need and then you put.

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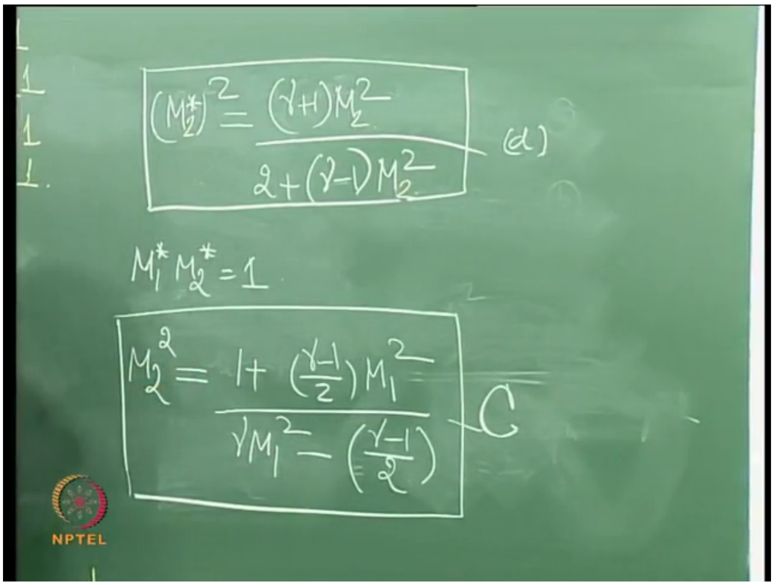

M 2 star you get M 2 here and you introduce that over here, if you do that then I am just rewriting this equation as M 2 and M 1 this is the relationship between the 2 mach numbers in the 2 regions. So, now, that we have done this what we should be looking at now is the relationship between, although we have answered the question right whether like we said the supersonic flow slowing down to subsonic flow will cause a shockwave of or vice versa. Now what we have done here in this equation is that we have written out the mach number in the region after the shock in terms of the mach number in terms of the incoming mach number and let us try and do that for also the changes in the properties in terms of the density change the temperature changes and the pressure changes.

Let us do that and also the entropy change, let us do that in terms of the incoming mach number, let us see how those expressions look like and then we will try to answer the final question if I do that so.

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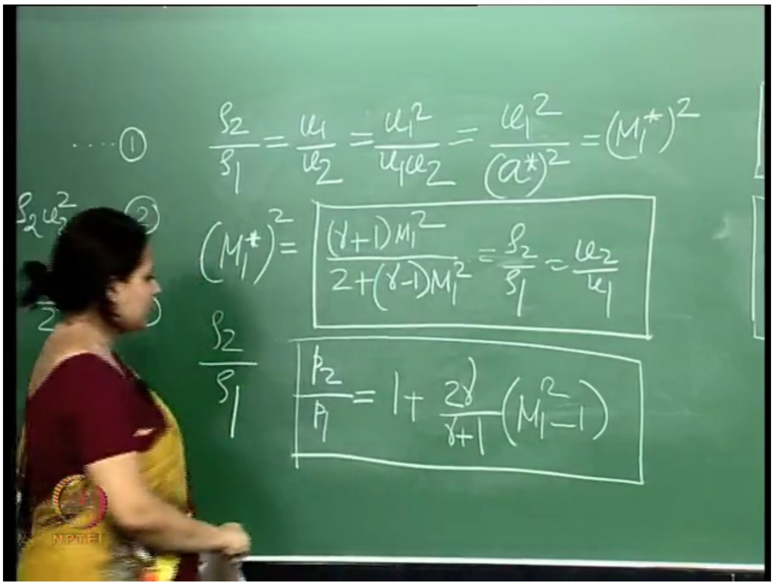

I will just write this as right now this is the Prandtl Meyer expression right u 1 u 2 is equal to a star square this we have done before and right and then what we will write here is basically, this is nothing, but M 1 star square. Now M 1 star square we can now use from expression which we developed over here and this is the M star, if you just write there as M 1 star you get this relationship, what we can write over here is now M 1 star square is see, if that is true then what we can write here is that rho 2 by rho 1, basically this is actually equal to rho 2 by rho 1 actually. So, this is, therefore, this is a relationship between the densities and that we will write in terms of the incoming mach number.

Let us get the expression for pressures and densities now I am not going to bore you with detailed calculations I think by this time you should be able to do this from here, I will just sort of let us say write it out. If I write this now P 2 by P 1 is 1 plus 2 gamma by gamma plus 1, this is the relationship of the change in pressures with the incoming mach number. Now briefly let me just say just use the momentum equation the continuity equation right, this is that the pressures are here, you have the densities here now the densities we have already expressed in terms of the incoming mach number and we have the pressures and velocities and densities just use these 2 equations right together and rewrite the you know values in such a way that you can write it in this form right and then introduce the rho 2 by rho 1 into that and you should be able to get this.

It is just working with those relationships and then we need the temperature change, I have been able to relate the densities then this is the pressure let us do it for the temperatures.

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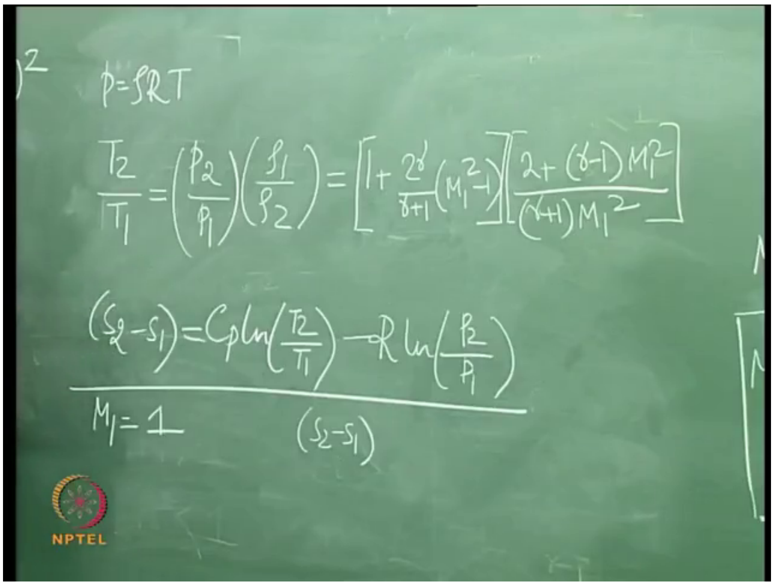

We need the temperatures now you can use this relationship now again I am going to just write this out now T 2 by T 1 is actually right, this is it, so basically we have a relationship for P 2 by P 1 as well as rho 2 by rho 2 by rho 1. If we can use these 2 relationships and get T 2 by T 1 I would still write it out. So, basically I will write this out and this comes out to be, this is what we get now, what is the entropy change and the entropy change is right if I am now let me just cross check right, P 2 by P 1 correct.

So, then this is the entropy change now let us look at this and see what, T 2 by T 1 is something we have calculated we have the expression in terms of the incoming mach number we also have the relationship for P 2 by P 1 right here for the incoming mach number now. Now, let us try to answer the question which we have been discussing throughout this lecture as, what how exactly should be shockwave be formally is it they should have the incoming mach number will be supersonic or subsonic, but if I have to do that, let us say, therefore, let us decide on these 3 cases in terms of say the incoming mach number is 1.

So, what happens to the entropy change the incoming mach number is 1, in that case what happens to T 2 if you put 1 over here what you get is 1 from this bracket and if you put 1 over here and what you get from here is the whole bracket becomes 1. So, basically T 2 by T 1 is 1, which means this term goes to 0 and P 2 by P 1 again is 1 which makes this term also supposed to 0.

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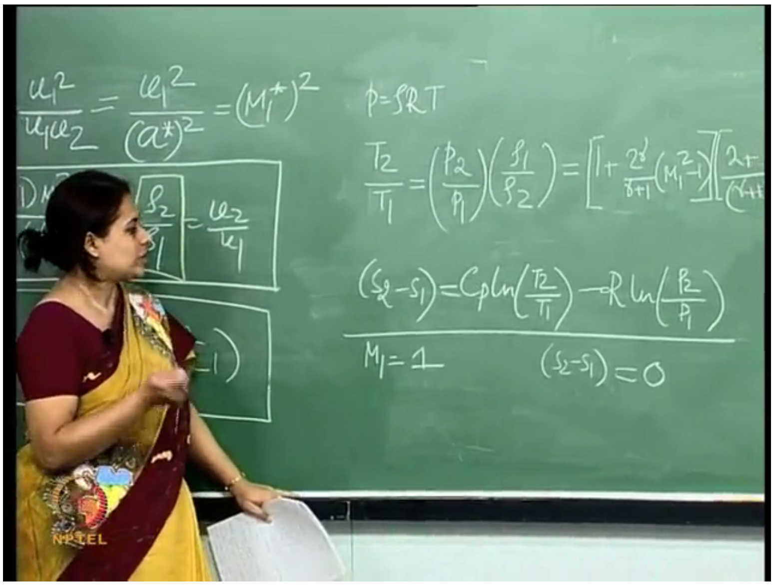

So, which means that, S 2 by S 1 is 0, if the incoming mach number is sonic right if the sonic mach number mach number is equal to 1. So, there is no entropy change, entropy remains constant or if I may write it like this.

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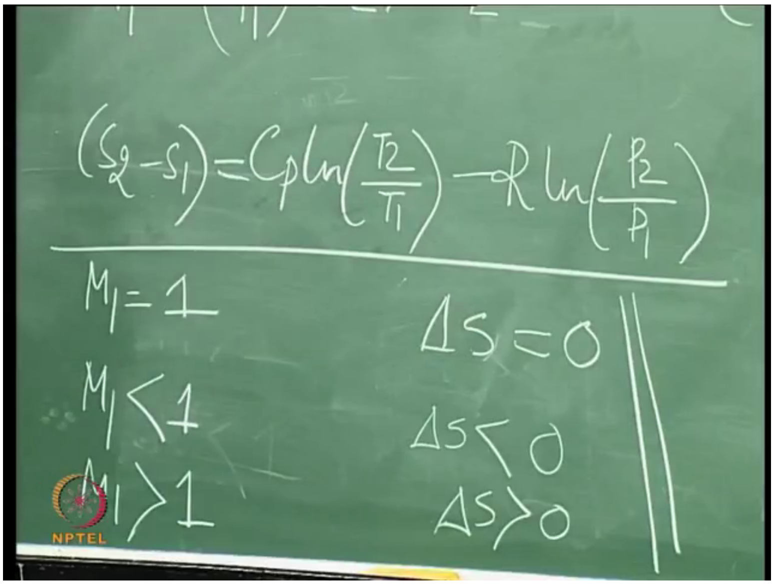

Entropy is constant, now let us look at this if mach number is if the incoming mach number is less than 1 again what I will suggest you is that actually put an actual value into here into the expression and see what you get. So, confirm whatever I am saying over here, if mach numbers are less you know let us just say that if mach number is subsonic.

Then entropy is decreasing and if it is supersonic then entropy is increasing again for these 2 cases just take an actual value of this mach number, and put it there and see what you get for the corresponding entropy change. In this case, if mach number if it is supersonic flow then the entropy is increasing now if that happens now what do you infer from here.

What is your assumption from over here, now in all the discussions that we have done, far right when we also introduced the characteristic mach number we said the changes in all the properties in the flow field are all happening adiabatically right which means that this is a physical process in which the entropy cannot decrease right now that is so much we know from the second law of thermodynamics right that is why it was important to do us a little bit of review of the thermodynamics because again as the second law of what thermodynamic does it is giving us the direction in which the physical process should progress which in here is telling us that it should be at least 0 and increasing, but it cannot decrease, which means that the incoming mach number in a for a normal shockwave should always be supersonic, that is so much we are finally, able to answer in terms once we come in terms of the entropy change.

So, therefore, as we have seen there could be you know from the relations that we developed that mathematically there are more than one possibilities; but if we invoke the conditions for these relationships from the basic laws of thermodynamics we know that the incoming mach number has to be supersonic. So, that the entropy change is at least 0 or constant, the entropy is at least constant or increasing that is so much from thermodynamics. What we have sort of developed today is basical relations of the pressures, temperatures, densities, entropy in terms of the incoming mach number and we have also tried to understand to how a shockwave is going to progress, how I mean it is the income of mach number has to be supersonic and if that is, then the mach number on the other side of the shockwave is subsonic which we have seen. So, what we will try to do is develop a couple more equal relations and look at some typical problems and see if you can understand that all right that should be all.

Thank you.