Advanced Gas Dynamics
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Lecture - 05
The relation of physical properties across a normal shock

Let us carry over from the previous lecture. So, what we are going to do today is basically drop a relationship between the properties of the fluid before and after a shock wave. What I mean by that.

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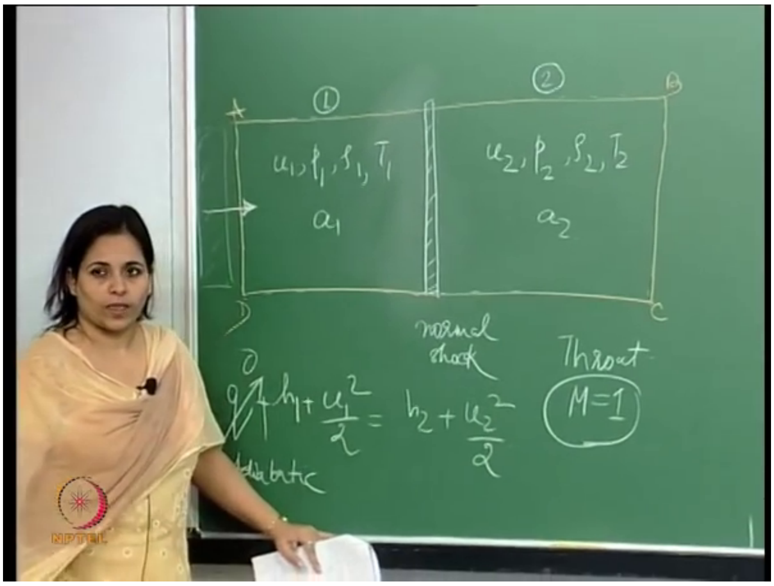

Say this is a shock wave, when I saw this sort of I am calling this say a normal shock wave. Now when I saw something like this for first time saw there is lot of confusion; in the sense that what is this mean that you know the shock wave start from here and stops over this or is this should I draw this just as a straight line or is it how thick should this be. So, when I am going to draw no more shock like this.

Well, there is no really such thing that stops here, starts there is there is really nothing this is basically region where properties are changing it is a region of disturbance. And I choose my domain in such a way that I will be able to capture; the changes in the properties or the basically the properties of the disturbance which in this case is a shock normal shock you know as best possible.

Therefore, I choose you know my domain in such a way that all the disturbances within that region will be captured. And like we define what exactly a normal shock is you know previous times it is basically you have the properties changing in just this direction in one direction really. So, if I do that let us therefore it define a region in which my shock wave is changing and that is what we call as the domain; let us just define that.

So, what this essentially means is that there is some difference or some existence of a disturbance and I sort of put a domain around that region of disturbance and try to study it. So, if I do that. So, let us say this is right; and what I am going to say is that this region is 1 and this region is 2. So, you can say this upstream of the shock this is trans steam of the shock and we typically denote the values here as, right.

So, this is essentially domain that I am going to work with. Now what I am going to do here is now is get a relationship or get a relationship between these properties and these. So what exactly, what is the relationship between these say for example, say the pressures across a shock wave. So, when it is that we have a shock wave and then is this a what is the relationship of the pressure after the shock wave compared to the before the shock wave.

So, those are the kind of questions we are trying to answer. Therefore, let us go and do that. And the way we will starts this; before we start developing this let us just say that clearly so there is a change in properties. Now this change in properties is brought about adiabatically. So, let us go ahead let us do that and start with the energy equation which we develop for a normal shock wave in an earlier case.

Let me just write that. So, what is missing? What is missing is this term the heat flux, but then we consider this as adiabatic and therefore we get rid of this term. So, this term is not there because we consider this as adiabatic. So, what we deal with is just this. So, let us see what information we can get from this relationship.

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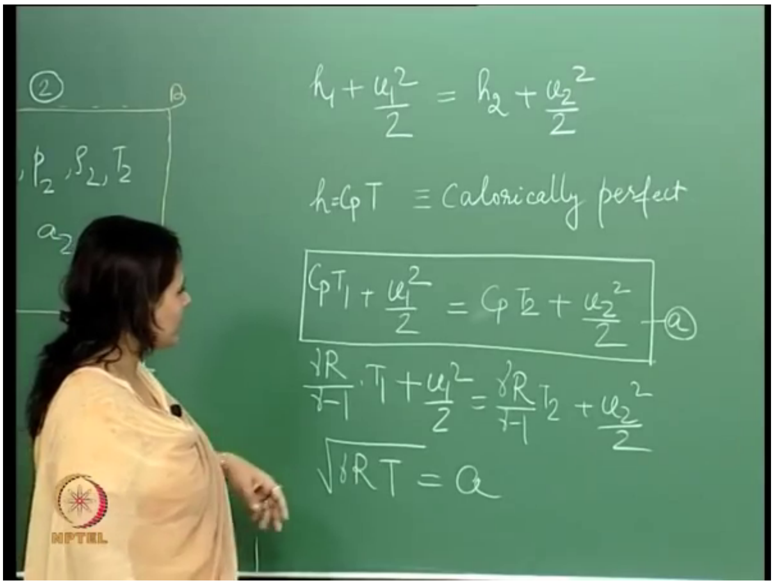

So, basically what we saying is. Now we also know that. So, if I able to write this it gives a some more information about the fluid; the gas that we considering here what is that; it is calorically perfect. Now this is something that we talked about previously- say if I have a calorically perfect gas then we can have a constant C p.

Therefore, I can rewrite this equation as: so then will write this as: now let us write C p in any other form that we know; how can we write C p. So, C p can also be written as, right. Now also from here what we know is that; you see something that is familiar here or we can replace a couple of term with something else.

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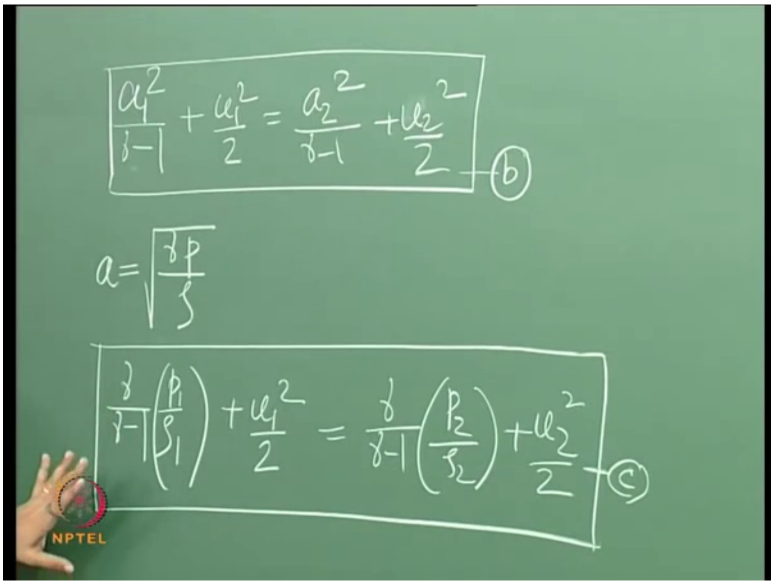

For example: is this term familiar? So, this is nothing but the superior sound. So, there you can see here, so this term up here can be replaced by the square of this superior term. So, if I do that then what I get is this. So, this is what we get. Now, let us sort of label these equations. Now we had this equation out here, so let us call this a and let us call this b. So, this is something that we basically get from the energy equations, you just sort of deal with this.

So, if we do this, so what we have here basically is a relationship between the velocities and the superior sound. Now so, the primary we dealing with here are pressure density temperature etcetera, we still do not know anything about that here. Let us try to get that in into these relationships. So, what to do that; so how else can be write the superior sound because that is the term out here; how else can be write this, can I write it like this.

So, if I p by rho, so then if I replace this term over here by this from the equation b what do we get; what we get is. So, this is the (Refer Time: 10:09) relationship let us call that is c. Now, you can see that this is the relationship where we have the pressure, the density, and the velocity before and after the shockwave. So, when I say 1 and 2 is basically two regions in the flow field and here essentially there is a shock wave in between these two regions, ok

Now, we will take advantage of these relations and go slightly further. Now we talked about a reservoir condition as throat condition in the in the previous lecture. Now, let if I go back to this diagram over here. So, you have a flow which is moving something like this. Now there is a, now before the flow starts say moving now there is a chamber here say for example there is a chamber here where I develop the fuels say. So, we have a fuel which is going to pass you know through a chamber and there is going to be a shock wave.

Now where we are develop the fuel? I have a chamber in which the fuel is developed. Now in that condition, in there we can say that there is a reservoir condition because the velocities are extremely small the fluid is really almost not moving so that we can say that of reservoir. So, that the velocities there is 0. And all the corresponding properties there are the reservoir conditions and we will denote that with a subscript naught. And the other condition that we talked about is that when it is passes you know when it when it moves there is a condition where we have a throat; we have a throat condition. So, let us just use this word here- throat condition which is essentially which means is a sonic sound. So, Mach number is equal to 1.

So now, if I say take say this relationship out here, this relation a if I take this a right and I say I consider this. So, basically 1 and 2 regions on the flow field, so say I consider one of these regions to be a reservoir. So, just say instead of saying u 1 now just use a generic term which is just C p T. Let us sort of do that, let us do this here.

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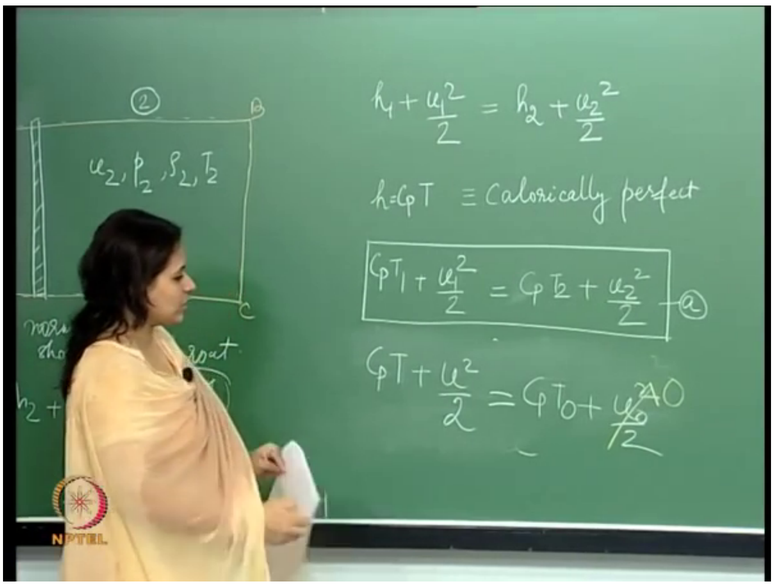

So this is essentially. So, T is the temperature anywhere, anywhere in the flow field. So, this is anywhere in the flow field. And the second location here is essentially the reservoir. So, I can write that as; that as we said in the reservoir the fluid is hardly moving, so this is 0. So, what we get here is essentially a relationship between the temperature velocity and the reservoir temperature over here.

So, if I use this relationship then from here I will try to get relationship or an expression for T by T naught. Let us see what I can do with this.

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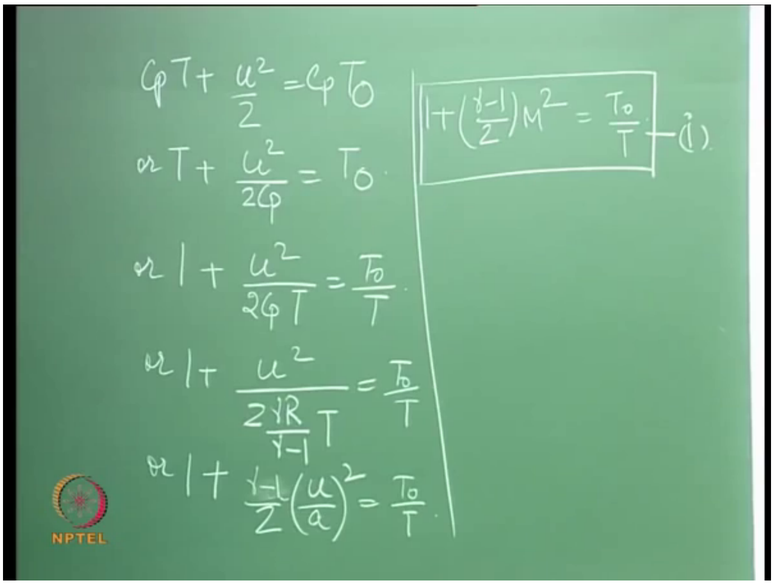

So, essentially what I get is, right if I do this then; so or alright. So, if I again divide by T. So, I get 1 plus u square by 2 C p into T is equal to T naught by T.

Now, this C p here C p again we will write this as in terms of the gamma r by gamma r s 1. So, then I will write this as, right. So, we know this. So, I think by this time you should be able to get what I am trying to well I am going from here so then. So, this then becomes let us do it this way gamma R T. So, it is gamma minus 1 by 2 u by a square is equal to T naught by T.

So, what happens here is this that this gamma R T is a square if the superior sound, so which is a square? So, there I just the values here so this is what I get. And what is this u by a? It is the Mach number and that Mach number is at the point where we are considering this u and T. Therefore, I can actually write this as, right. So, that is an interesting that is still another interesting a result that we get from here. Let us call this is 1.

So, what we are want to do now is that when we bring the reservoir condition, so we are able to connect the temperature at any location or at any location in the flow field of the shock wave with the corresponding Mach number and the reservoir temperature. We consider to do this and then similarly if I have been able to connect the temperature, the temperature with the reservoir condition we should be able to do the same for the pressure and density.

As we said that the process out here or the change in the properties, so the process is that is an isentropic so will use this relationship.

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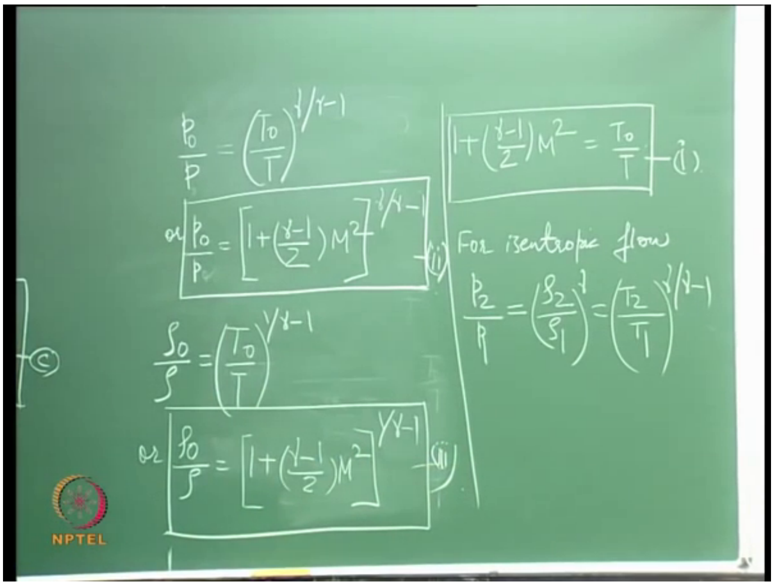

So, this we know. So, for isentropic flow this holds. So, this is the relationship which holds for isentropic flow. So, in one of these if we can just write this in terms of the reservoir conditions. If I do that so essentially; so if I am able to do that so let us write this. So, let us use that and we use it over here to develop the relationship here.

So, if I write that. So, let us consider say- I consider the second location as the reservoir and this p 1 is just this is just any location. So, this is then equal to T naught by T to the power gamma by gamma minus 1, but we already found out an expression for T naught by T given in 1. So, then if I write that then that becomes p naught by p which is 1 plus.

So this is the other, this is the relationship for the pressure. So, we can now do the same thing for the density; we can do the same thing for the density. So, then again T naught by T is available here. So, I will write this as: therefore I get this relationship.

So, what we started out ah doing right trying to relate the properties before and after of the shock wave. So, first things we started with this right we use this. So, this is the relationship between the temperature and the velocities. Then b gives a relationship between the superior sound on the velocity. And c here give us a relationship between pressure density and velocity. And then we said we will use the reservoir conditions. And then we were able to get relationship of the temperature, pressure, and density corresponding to the reservoir condition and we were able to use the Mach number. So, this Mach number is at the point where we are considering the flows. So, this could be really any point in the flow field.

Now obviously, this is a place where we use now the sonic conditions; sorry the reservoir conditions. Now, let us go ahead and try to see if we can introduce sonic condition, because see mathematically what mathematically the usefulness of using this reservoir condition for 1 is that we were able to get rid of the velocity because that is why the velocity is 0. Similarly, mathematically when we use the throat condition the Mach number is going to go to 1. So, let us see if that gives us you know easier relationships through work with.

Now since we are basically trying to study the properties how they vary before and after of the shock wave and the point is there so many properties. So, we have so many relationships, alright. So, if we do that, so let see you know what relationship we going to use over here. So, let us go back and see this over here.

Let us use this relationship over here, let us use this relationship over here. And say that if I use this relationship now where I go back and write this. So, I think I will erase the few things we will just write this up again where we come back to it, ok.

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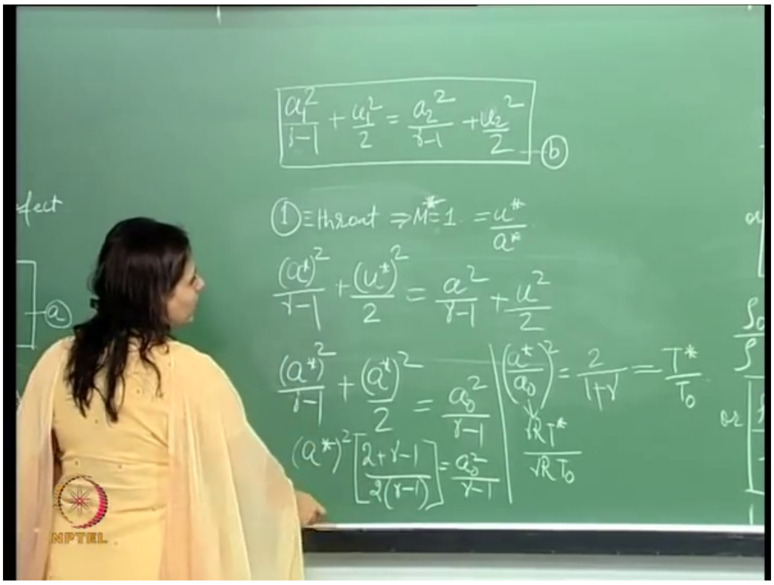

So, we use this relationship over here and we will consider say point 1 location one as the throat, right. And this all the properties here basically denoted by superscript star. So, we denote it this way. So, then if I chose that then what I get essentially is that; if I do this then what else can be put it over here.

So, Mach number is one what does that do to my u star that is also equal to a star right because the Mach number is 1 which implies that this. Mach number is basically the velocity and this superior sound relation the fraction of the u star over a star. Now this star is basically denoting the values at the throat. Therefore, if this is a throat condition then u star is basically equal to a star ok. So, let us do that.

Therefore, what we get over here is this. Now let us do one thing. Now this point here is any point right; so this point is any point in the flow field. Let us do a thing, let us consider this as the reservoir if we consider this as a reservoir then what happens. So, then I will denote that this way right and what happens to this that close to 0, because it is a reservoir. So, that is all we get over here. So, if I get this, so then we will work with this. So, essentially this is our relationship. And what we will get here if I work around this is this should I do this. So, maybe I will do this. So, a star so what we get; let me just take the time and do the math over here. So, 2 plus gamma minus 1 plus 2 into gamma 1 is equal to a naught square by gamma minus 1, ok. So, if I do that what I get over here is a star by a naught square is equal to 2 by 1 plus gamma, correct. So, we get that right.

So, a star by a naught is we get this. So, we get this and let us just remember that how else can I write this. This is this is important here. So, what we got over here is a relationship between the reservoir condition and the throat condition. So, we have basically related these two. Now, this a star square is also equal to right; this is also equal to this, this superior sound. So, if I do that so essentially, if this is it therefore I can actually write this as. So, what we have actually is quite nice and simple relationship between the throat condition and the reservoir condition.

So, let me write that out slightly say let us go here and write it over here, ok. So, what we have is essentially is this.

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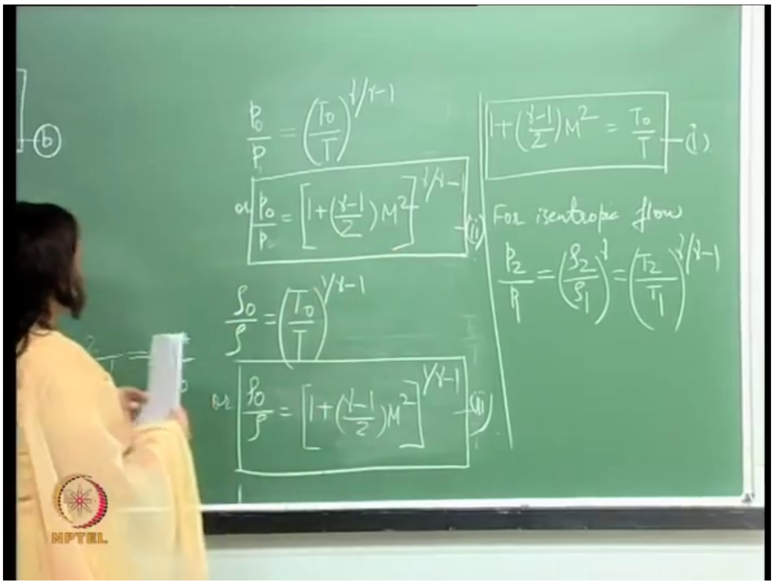

So, basically what we have is, right. And let us call this as the forth relationship. So, we get this, so that is the relationship between the conditions. Now from this relationship over here right, from this relationship over here now the second relationship we developed will at any point and the reservoir.

So now let this any point be is throat condition. Therefore, what happens here this becomes p star and this becomes M star then this M star is equal to 1, because this is the throat. So, let us write that out you see what I mean by that.

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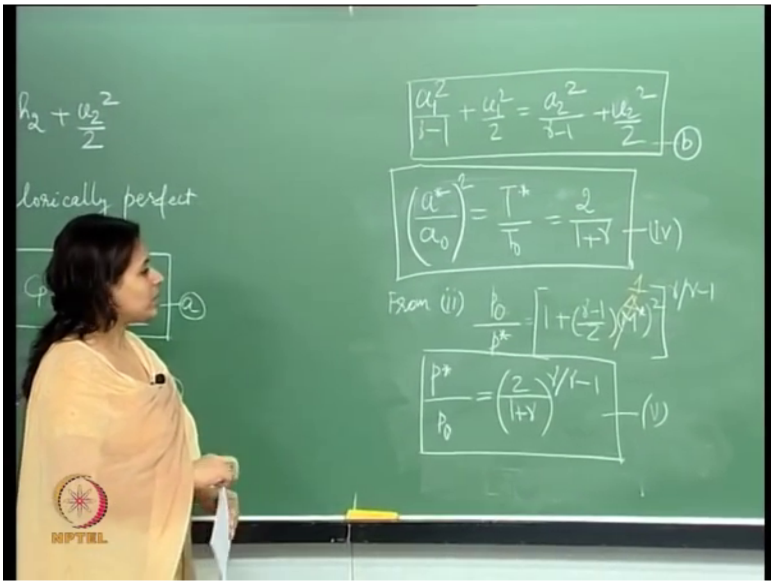

So, what I mean is that, so if you take this from basically the second relationship. So, from two will just take the point the relationship between the throats. So, we have p naught and let this p be the throat. So, we will write that this way, this is equal to 1 plus and this going to be M star and that as. So, this is the point that; this like I said before is this is any point in the flow field and let that any point in this particular case be the float condition. Throat condition means M star; so M star. So, essentially this is the throat. So, that goes to 1, right.

Therefore, what we get over here is say if I just write it like this, say p star by p naught right is equal to 2 by 1 plus gamma to the power gamma by gamma minus 1. So, this is this is the relationship. Similarly we will do the similar exercise for the density and we will found out the relationship between the throat condition and the reservoir condition. So, basically we will use expression three. So, if I use expression three this is what we get. So, essentially this is the relationship. Now if I use, ok.

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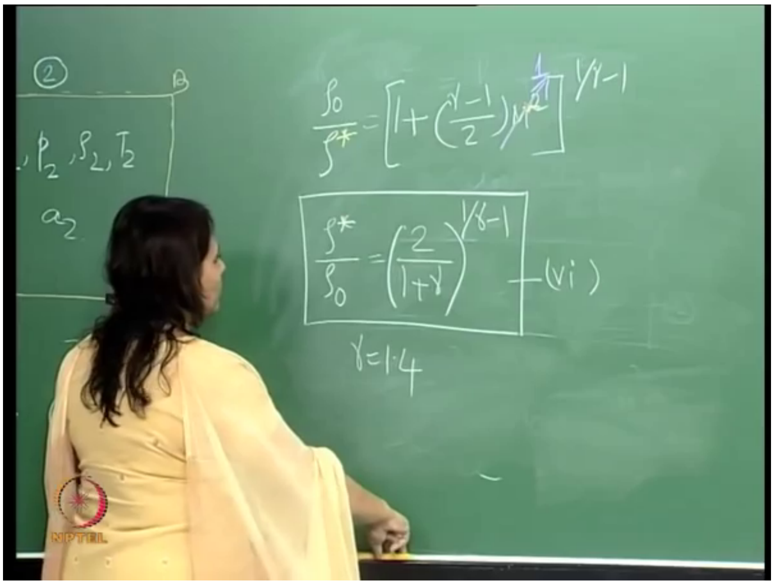

So, the expression three is, ok. So, again let us consider this to be; so let us consider this to be throat conditions then this becomes that and then this M square goes to 1, because it is the throat. Therefore, what we get again is essentially this. Therefore, what in sink here is that all these relationships, right so we got a relationship for the temperature between the throat and the and the reservoir; the temperature the pressure and the density, right. We got a relationship for all of these only in terms of gamma. And for standard conditions for air we all know that gamma is equal to 1.4.

Therefore, if we know this, this is a very easy relationship between these parameters. So, having develop this. So, let us go and sort of see if we will be able to apply this; how you will apply this say to a set problem. So, we have done this before actually. So, we use slightly different analysis there, because we did not know all these analysis, all these relationships at that time now that we know these let see how if you going to able to be a apply this and study this problem.

So, let us see what the problem is all about. So, basically we have a rocket nozzle and there is an isentropic expansion of flow through this nozzle, right.

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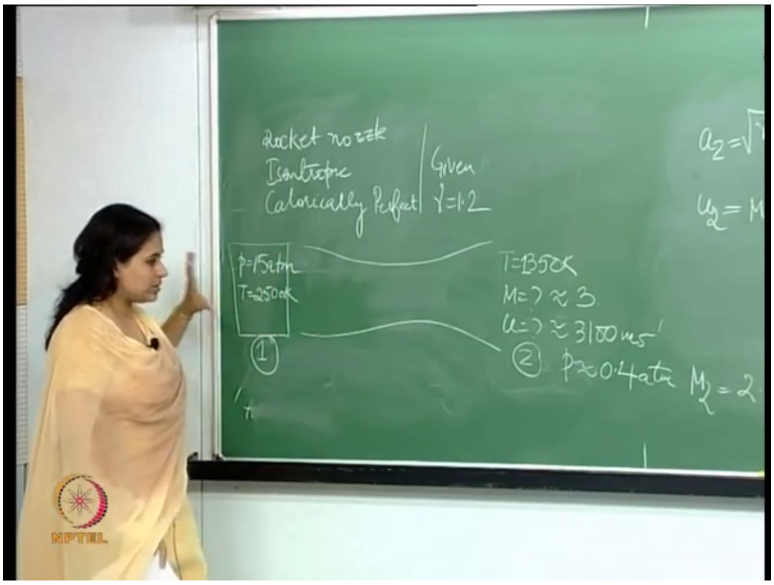

So, basically we have a rocket nozzle that is flow through a rocket nozzle, right. The flow is isentropic in nature and is a calorically perfect gas; which means that you all these equations or relations we developed are really available for use.

Therefore, now we have, so basically now this goes from say here. Now in this say fuel chamber in this fuel chamber the gas is at a pressure of 15 atmospheres and the temperature here is 2500 Kelvin. And the gas expands through the nozzle and the temperature here is its 1350 Kelvin. What we would like to know is that what is the Mach number and the corresponding velocity. What is the Mach number and corresponding velocity in this case?

I think we did last time will we just calculated the drop in pressure. Now in this case what we are asking is what is the Mach number and what is the velocity, and this is you know the flow through a rocket nozzle. So, how do we go about this? Now it is also given that it is given here, right. So, this is the ratio of specific heat is the 1.2

Then what are we going to use over here right, what are we going to use over here, what is the relationships that we are going to going to use over here, how will we calculate Mach number and velocity. I think the way the easier way to tackle a problem to see what information is available and what we can get out of that, right. So, essentially what we know is the temperatures you know; the temperatures at the two places. Here we have pressures we do not know anything about that. All we know is the temperature before and after that is all, ok.

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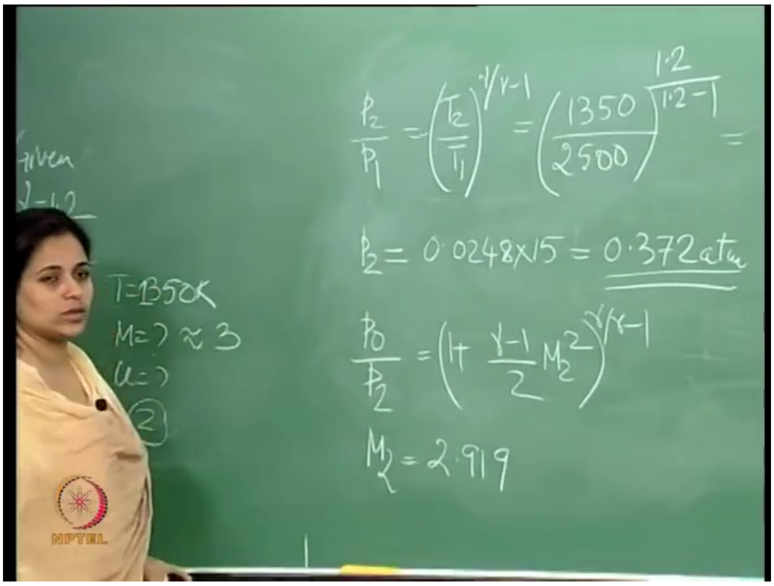

So, if we do that. So, we know that from isentropic relationships. So, p 2 p 1, right. If I do this now let us consider this region as 1; let us consider this region as 1 and this region as 2. If I do that then I can write this as; and gamma is 1.2 we do this. And this comes out to be ok. Now, we need to find out the. So, this is what I know from my isentropic relationships. Now what I can find out from here is my pressure; pressure at the point 2 which is at the exit which is because p 1 is 15 atmospheres, so we get this in and so what we get is point which is this is something that we had done the previous (Refer Time: 38:25) problem. So, this is less than actually 0.4 atm; 0.4 atmospheres.

So, now, the point is we need to find out essentially the Mach number and the velocity and how do we go about this. What are the relationships you see we can use over here. Now first thing is first that if we were going to use the reservoir condition that is where the Mach number information is involved, right. So, we developed all the let say- so we developed this here, so this is 1 2 and 3. So, these were the relationships of the temperature pressure and density which involve the Mach number; the Mach number and the reservoir condition.

Now is there anywhere I can consider a reservoir in this problem over here. So, you can see this is the chamber where the fuel is being generated. So, the fluid velocity hear is nearly 0. So, we can consider this 1, this location 1 as a reservoir and that is what we will do. We will consider that as a reservoir and see what. So, basically if I do that.

So, p naught; so I can write this. Now this is the pressure that I know this is the pressure that I just calculated, so there and gamma is something I know. Therefore, I can actually get M 2; M 2 from this relationship. And this comes out to be from my relation. So, you can take the time and do this. So, p naught is know that is 15 atmospheres, then p 2 is nearly 0.4 atmospheres, gamma is 1.2 which is given. So, you can just calculate M 2 which comes out to be nearly 3. So, the exit Mach number is nearly 3.

Now, when we do this? Now I also need to calculate the exit. So, here what is the exit Mach number which is say nearly 3? We will kind of infer a little bit or may be a lot from these values. So, let us discuss that after we get it. Then we have the velocity, exit velocity; what is the exit velocity, how do we calculate that.

So, to calculate the exit velocity what we need is the exit speed of sound; we need the speed of sound. So, how do we calculate that? So, let us do that.

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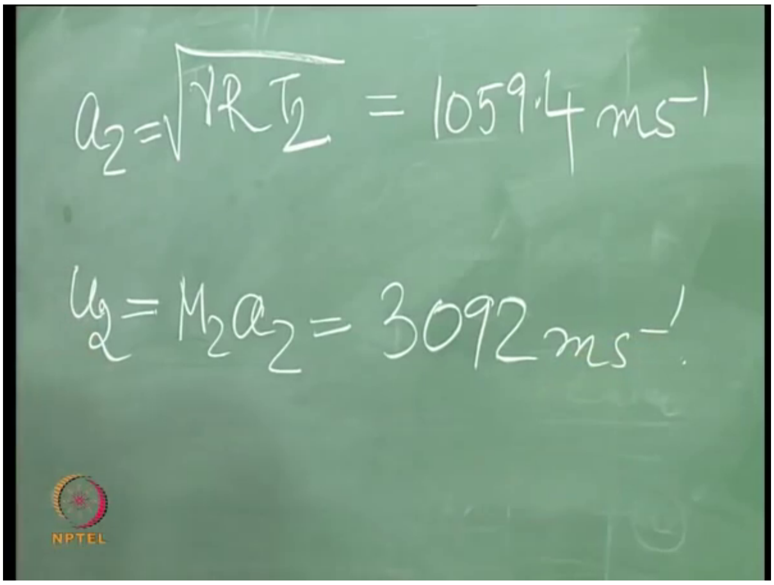

Now, we know the exit temperature, so we can find out the exit Mach number; sorry exit speed of sound and this comes out to be, right. Therefore, the excess speed can now be calculated as, right. So, exit speed is nearly. Now, what is the interstate is c over here is that we have a rocket nozzle here and we have a flow which is going through this convergence diversion nozzle and the exit pressure here is say is actually the say around actually less than 0.4.

So, basically we accelerated the flow, while we accelerate the flow and it went from like 0 to near to near a 3100 per second. It went up with that high speed, you know in normal standard conditions this speed of sound is what around 350, 360, 340 meters per second. So, let us say 340 meters per seconds. So, you can see the marked number is nearly 3 which means it is travelling at nearly 3 times the speed of sound.

So, we basically when from 0 velocity, like from stagnant conditions to the velocity which is highest this. And what is that involve? That involved, therefore a drop in pressure from 15 atmospheres; 15 times the atmospheric pressure to less than 0.4 actually, 0.4 atmospheres. So, that is how large drop in pressure. And there is also a corresponding loss in temperature from 2500 Kelvin to 1350 Kelvin.

Therefore, all these time that we have the discussion that we are right from the invention of Laval that to get such high speeds. So, if I have to get you know such high speed then move it through you know shape like this nozzle you know nozzle like shape. So, if I move it through duct, which is converging diverging here. So, then I get speech which is as high as this. At the same time however, that it involves very high changes; very high changes in pressure and temperature.

So, you cannot have a field for what exactly could be numbers like. So, basically what we can say is that if I have a nozzle and I am trying to speed up my flow to nearly three times the speed of sound then this is the kind of pressure drop and temperature drop that we looking at. So, that is what example like this can sort of tell us.

So, also let us quickly the problem that we did right in the first class. Let us quickly do that before we end today’s class. So, this is about.

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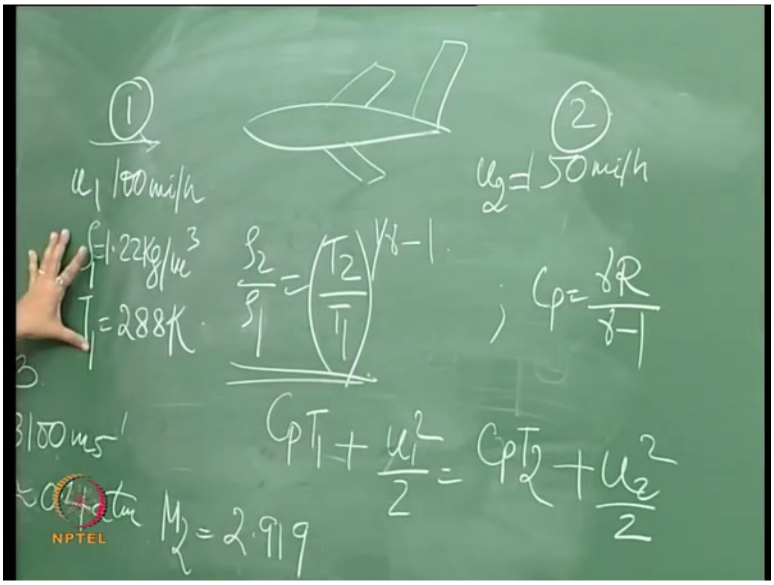

Now if you remember in the first lecture that we did a class where we said that; if we have a up an aero plane and you had a flow. So, which is here say in a free stream, which will accelerating from 100 miles per hour to 150 miles per hour, so then there what is the corresponding pressure drop. And what we did here we said that this is let us consider this incompressible. And then we calculates pressure drop and then we connected that to the compressibility of the compressibility and we said that the pressure drop is small enough and therefore the density changes and not going to be dominant. And therefore, we said we can really consider this as incompressible. That is what we did in the last lecture.

Now, what will do here is let us not consider this as incompressible. Now we know enough, let us consider this as a compressible flow and calculate the actual values of density and temperature in c if we are capable enough to ignore them. Then say it is no the change is small enough, so we can just consider that as incompressible.

So, if you do that, so how do we go about doing that? So, this is like; so this one ok. So, C p will just use basically you know if I use this relationship. So, we will use a isentropic flow relationships. And what we will do here is that we have at the standard sea level conditions. So, our density is 1.22 and temperature is Kelvin. So, these are the standard sea level conditions.

So, now if I do this what I know from here is the velocities; that is all I know from here. So, this is the isentropic relationships. Let us look at here. So, in here we need to calculate the C p and the C p we can calculate. And his gamma is given as 1.2; 1.2 or 1.4 here you can take it as 1.4 is this standard here right. And then this is of velocity which is known and this is of velocity which is often known, right.

So, if you do that then you get a relationship for and relationship between this T 1 and T 2. If you do that T 1 is something that is available over here, you can take this as the this; point you can take as 1 and this point is a 2. So, in here: so T 1 and gamma 1 are known and this is u 1 and this u 2. So, this is all that is known to me. So, in here so therefore we can find out you know T 1 this relationship.

Therefore, I can actually get a relationship for T 2 by T 1. If I do this then what I come up with if I do this, what I come up with is this.

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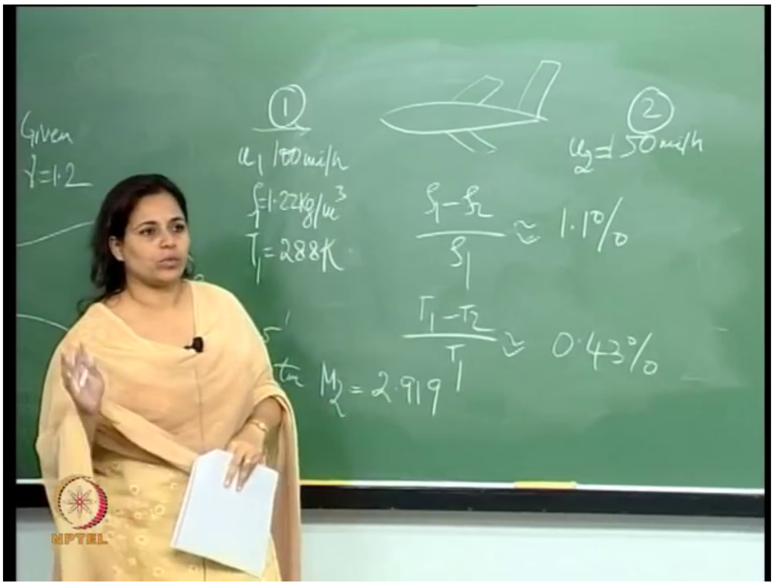

So, basically the density I get; the density change is around this and the temperature change. It is really you have just mentioned off sink whether these changes are you are going to really consider this changes, consider this flow as compressible or you can say this is more enough will disregard the changes and just say that this is in compressible.

Well, so density change the actual density change or here is really just about one percent corresponding temperature changes less than 5; 0.5 percent. So, it is like less than half percent. So, I think it is safe enough to say that the problem that we had at the time said it is incompressible that stays. We do not really need to use the energy equation here or any isentropic relationships over here. We use the Bernoulli's equations to just calculate the pressure change so that should be fine. There is really no dominant density changes in here.

That should be all.

Thanks.