Advanced Gas Dynamics
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Lecture – 40
Supersonic Flow past a 3D Bluff Body at an angle of attack

So, now, that we sort of comes in yielded, but try to figure out or try to understand 3D flow of past cone of angle of attack and we saw that unlike the axisymmetric case right where we will basically dealing with just one independent variable right which was the theta then we came to the cone at an angle of attack and we saw that the flow was still conical and the meaning that the properties we to the spherical coordinate system, where the properties would still be constant along the radial direction, but would vary on the a prediction of that which is n theta and phi directions right.

And we try to see if you understand what; that means, you know in a 3D space geometry vertex; so on and so forth and I tried also give you a brief overview of a numerical procedure in which the way which we bunker in to implement in order to study the properties across such a flow now. So, essentially the first one where we had the axisymmetric case; you can you know we had just one you know independent variable and this was also; this is also a 3D flow; the next one the angle of the cone at an angle of attack is also a 3D flow, here the number of independent variables is 2 theta and phi right now. So, let when we say 3 D. So, when we say we are going to graduate from 2 d to 3D we basically looking at 3 variables numerically that is what we would say right.

So, let us look at something like that we would have a you know a real 3D flow with 3 independent variables, which we basically if the body is a blunt nosed body at an angle of attack and let us see how the governing equations would look like solve with the small figure and yes, I am not going to the detail geometry the way we try to understand it for the cone at an angle of an attack, but hopefully you know you will be able to visualize that little more now and what we are going to use here cylindrical coordinates. So, let see what I mean by that.

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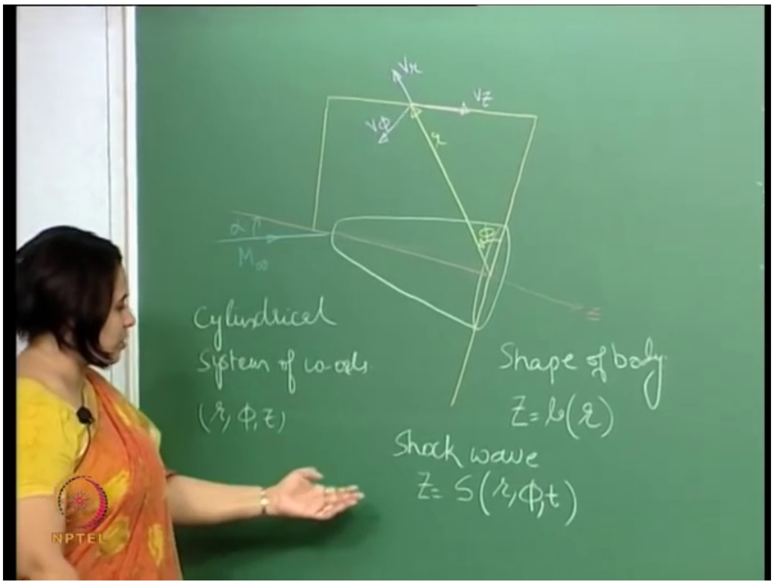

So, let us say I have something like this .

Then we go and somewhere over here yes it, is a small icon. So, and this would be more or like the free stream which makes an angle of attack alpha right like this and this is again c I am going to call that as c now I am basically looking at a plane like that.

So, I am looking at a plane like that looking at a plane like this and I draw and I take basically radial vector like this I take a radial vector like that, this is r. So, this is r. So, clearly now this angle is my phi. So, hopefully this makes will be this is also sort of you know take this radial vector and rotate it; it is just that the you can see that here the radius is going to sort of differ as we go along. So, this is the phi geometry device and let us draw some velocity vectors ok.

Let us draw some velocity vectors here. So, if I virtually look at this point this point is special. So, this would be V r this would be V r in this direction would be V c and so, this is what we are looking at a cylindrical system of coordinates this is a basically r phi and V. So, we in this particular case. So, r the phi and the c right. So, this is. So, let us see therefore, if I were to write out the governing equations in this system of coordinates what that would look like. So, like we did in the previous case ok.

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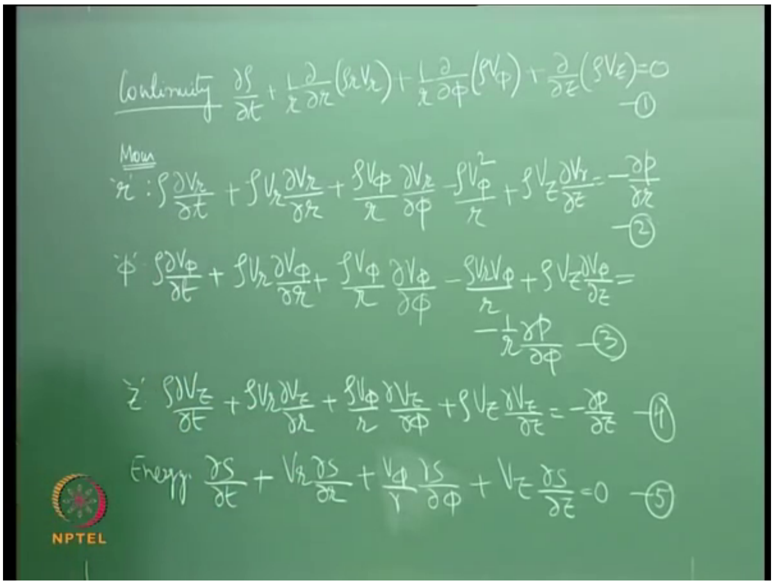

So, let us let me write that down. So, again. So, this will look like this continuity .

Let us call this is one that is what the continuity looks like then we will take momentum. So, momentum first of in r direction momentum in the r direction right now that looks like that is a rho. So, that is the momentum in the r direction then in the phi direction yeah that is the in the phi direction and finally, in this Z direction that is.

So, these are the momentum in the r phi and z direction and the energy finally, the energy. So, then we have V phi by r. So, these are the. So, these are our basically governing equations right in the cylindrical coordinates now numerically how do we go about solving this. Now first things to do is you have to transform these equations onto a rectangular coordinate system what does that mean. So, what is that mean in order to do and in order to do that we will have to use some kind of transmissions we have done a little bit of that when we did linearization right and we transformed into the zeta eta space if you remember.

So, we were going to do it do something like that. So, transform this space into more of a rectangular system of coordinates in order to able to implement this numerically and we would take and the shape here shape of the body does not have to be you know in this case the way I have drawn it is like symmetric it does not have to be it can be any arbitrary shape. So, let us say that you know we will we will you know. So, hope you understand this here. So, we have a blunt nose body over here we have a blunt nose body here and we will have a shockwave which will come somewhere which will be generating from here ok.

So, in. So, therefore, here let us you know gate transformation for here you know for this sort of in order to do that let us say let us say the shape let me write this here let us say the shape of the body the shape of the body we will define as right. So, shape of the body is equal to a function V and b basically for the body.

Now, this in this particular case I have taken it axisymmetric which is why I write it like this it does not have to be you know it can be anything. So, let us just say. So, in here let me write the shape of the body as say some function you know which calling it is b in this particular case and then the shape of the shockwave shape of the shockwave.

So, let us call that. So, here of course, the shock wave is the function of you know r phi and you know standard parameter. So, the here clearly you know the shape of the body being axisymmetric is just a function of you know r Z is equal to r the shape of the body does not change with time hope that that is clear right. So, then the shockwave; however, now the when I define the surface of the shockwave. So, will call that as some function s for shockwave right. So, this is arbitrary shape we do not know what. So, the function of r and phi and also changes over the time right. So, this is how if we define.

This if you define this then we will now let us go into calculate this.

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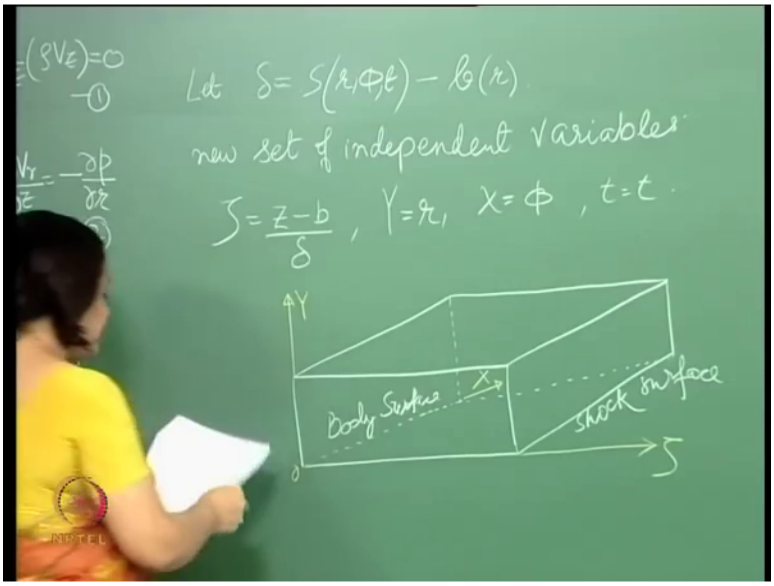

Now, let we will just say you know let us consider just this let us consider the s to be some delta delta to be equal to this. So, this is essentially the space between the shock and the body right. So, this is the or say if I want to write this little more. So, say delta. So, if I want to write it like this if I want to define like this I am defining you can see here just geometry ok.

If I want to define this then we will now set some independent variables. So, let us now define a new set of independent variables how do we define that delta. So, essentially this is the spatial coordinates time domain time is time. So, now, the independent variables we have a tau I think right. So, this tau is Z minus b by tau this delta out here Y is equal to r, X is equal to phi. So, let me draw this you know how would this look like. So, we basically going to draw a cylindrical coordinates right. So, which means this then let us call this as the origin and this is the tau direction.

this is the Y direction I hope my art work is more or less all right. So, this is it and this is my X. So, if you look at. So, this is basically the transform coordinate. So, the way I transform this over here. So, this is the tau. So, this is my tau over here. So, basically now the body on the shock I am I am observing that sort of a transformer we need to this you know 3 dimensional coordinates which is this tau X and Y and this tau out here is Z minus b.

By delta Y is equal to r and X is equal to phi. So, if you want to essentially look at this. So, what we are looking at is that this is the body surface on this phase and this is a shock surface on this face right this is the shock surface on this phase and so, this is my geometry and they somewhat transformations the dependent variables ok.

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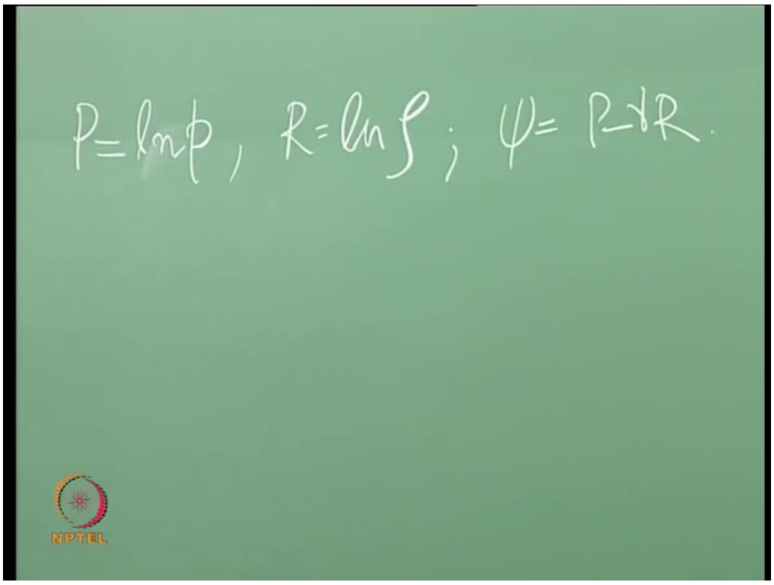

We will transform the dependent variables as let us call these.

So, which essentially meaning that you know when we had pressure density etcetera in this coordinate system what will that be in this coordinate system. So, we transform the dependent variables in this fashion now if we do that then the governing equations right the governing equations which we wrote in terms of say density V r V phi V Z etcetera over here right. So, we wrote all these terms V r rho V phi V Z etcetera see all these terms what happens to these because these are no more there in this particular coordinate system. So, now, with our transformations the governing equations also change right governing equations also change and. So, let us just see how what they look like now ok.

How they look like now. So, let me write those down let me write those down and. So, what I am going to do is write it right next to the equations in this cylindrical coordinate system. So, we will write it there that you can make a quick comparison. So, the continuity ok.

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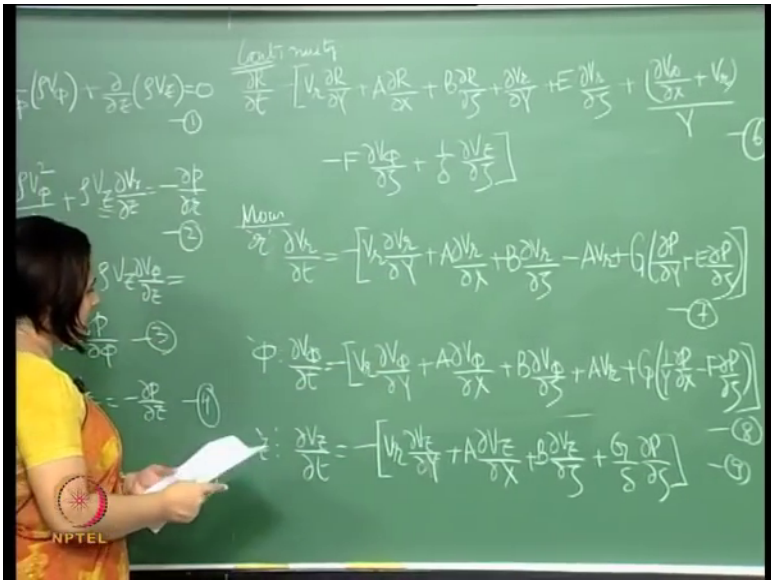

So, in the continuity you can write as and I will write all the time dependent values time dependent derivatives on the time derivatives on the left hand side that is how I am writing the equations.

And let us call this as say 6. So, this is our new continuity equation this is our new continuity equation again now momentum in r direction momentum missile this is the continuity actually then momentum in r direction. So, if I write that there is in r direction then in the phi direction in the phi direction.

and in Z direction I think it is important now. So, that you for you to compare the 2 sets of equations and see that you know how the derivatives of you know all the velocity components V r V phi V Z, etcetera, they look like in these set of equations in these cylindrical coordinate system and what are we doing with those in the corresponding equations in the corresponding you know rectangular system of coordinates into which we have transformed our space. So, I think just sort of you know make a quick comparison between the 2. So, finally, the Z. So, these are essentially the equations that we got one more the energy equation right.

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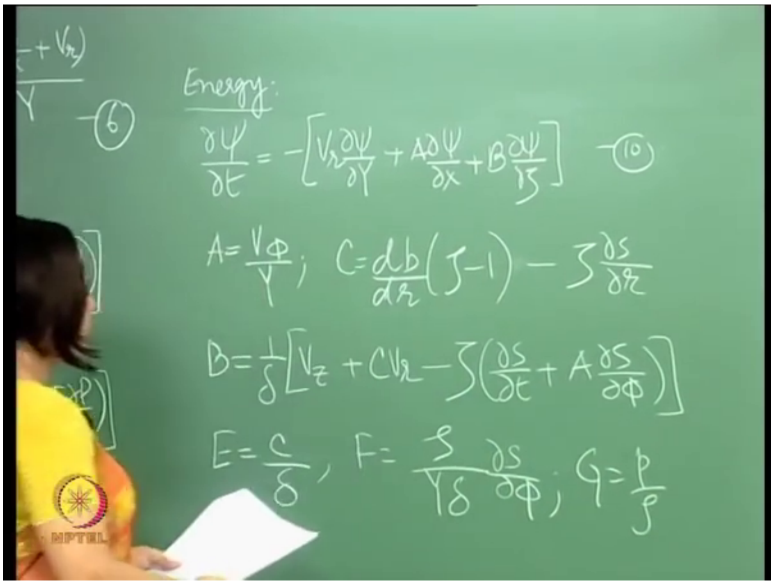

So, the let me write the energy equation again. So, here. So, where you can see we have this constants A, B, C; let me write quickly write the other one; that means,. So, that is essentially our equip the governing equations in the transformed coordinate system.

So, they look like this and as you can see I think this is a time where we tell how little more respect for a computers right to solve these equations you need to solve these equations on this rectangular computational grid and solve for your properties for this blunt nose body you know son sonic flow. So, let us again the go ahead and see how we will do that we illustrate the number the steps of doing that which is pretty similar to what we did for the you know cone at an angle of an attack. So, let us sort of take a re look at that.

So, essentially what we do what we say now is that when we implement this when we implement this problem when we implement this problem. So, that that is we have a blunt nosed body and we have a supersonic flow in here. So, then the equations the governing equations look like this it is as simple as that the governing equations look like this in a. So, numerically what this is what this means now to numerically solve this problem is that we have these set this is my equations to solve and a physically what this means is that now these equations result after transforming you know our governing equations from the cylindrical coordinates to a rectangular system of coordinates ok.

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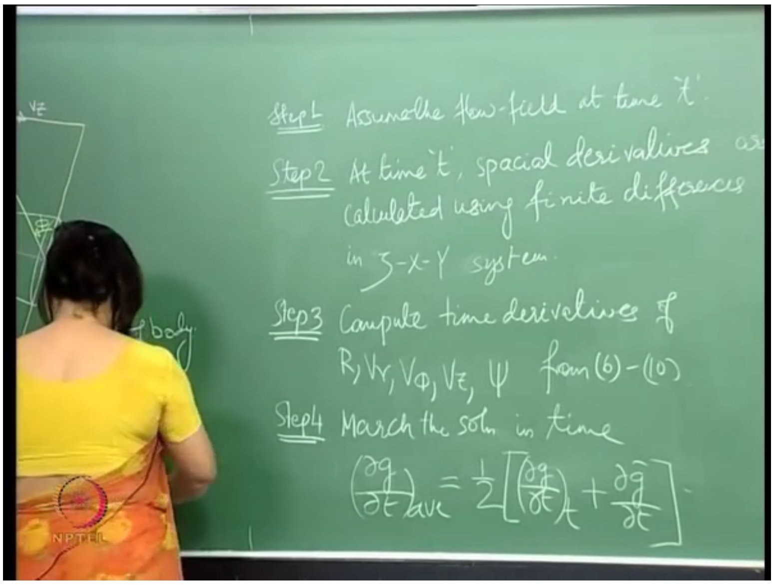

So, all right; so, step one. So, how do we go about this now if you look here. So, essentially. So, all these the time variance are all on the left hand side time variance are all on the left hand side well I think I can write the first one I miss the equal to sign over here this is equal to all the time variance are on the left hand side. So, if. So, the first thing we will do is assume the flow at time t. So, we will start by.

So, we assume the flow at time t. So, now, if you look at each equation on the right hand side we have the derivatives we have the derivative of the flow field with respect to X Y tau, etcetera is it not that each and every equation here on the right hand side basically consists of the derivatives of the flow properties in X, Y and tau directions. So, we have the derivatives of the flow properties in space the what we will do here in you know step 2 is having assumed the flow at once we assume the flow at time t, we will get the derivatives of the flow we will get the derivatives of the flow which is which are these del r del t del V r del etcetera, etcetera, we will get the derivatives of this using finite differences in space on the right hand side let me clarify.

So, let. So, let me first write this down. So, at time t. So, spatial derivatives calculated using finite differences right finite differences finite differences in in zeta X and Y coordinates is it not; I just call this this here. So, let us say will I am calling this as say step two. So, at this particular time the moment you assume the flow field you calculate the spatial derivatives spatial derivatives here in the in this coordinate system using finite differences once you do that then calculate the time derivatives using the equation 6 to 10 so.

So, then we calculate the time derivatives of basically say off r V r V phi V Z and psi. So, which is what is given by. So, then we compute the time derivatives you know of this which is what we what we get is del r del t del V r del t V phi del t etcetera from 6 to 10 makes sense and then the next step is. So, therefore, now having done that we will march the solution.

In time we will march the solution in time using the product decorator of predicted corrected approach which I briefly talked about yesterday. So, now, next thing is. So, what do you see over here. So, essentially by this time what I have done is calculated I have just assume the flow and I have calculated the derivatives of the flow at you know at time t. So, clearly that is not a you know that is not the correct values or it is not the exact values that we are looking for because you just assumed it. So, what we will do is march the solution in time.

Yeah; so, march the solution in time which means that now we will find out the flow properties at time t plus delta t right. So, using the predictor corrector approach. So, let us just go and talk about that little bit little bit. So, let me erase just this because I want to use these equations is important for you just see. So, these are the equations that we are using; we are equating using 6 to 10 using these equations out here ok.

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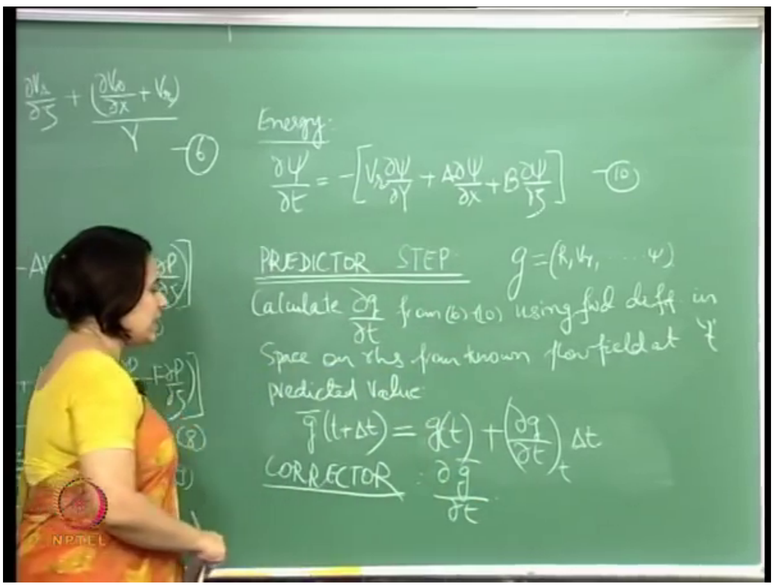

So, how do we do this. So, now, this. So, let me call you know any of these parameter these value parameters here or properties are V r, V phi, V c and psi let me call that I say let me just give that name as g I am just call that is g. So, g could be anything it could be r V r anything I am just giving it a random name. So, in here. So, what we will calculate is here what I will calculate using 6 to 10 right 6 to 10 we will yeah you know calculate this del g del t using the fire using four differences in in space on the right hand side; right hand side.

So, so how do let me write that term. So, calculate what this means is that we calculate del g del t from right 6 to 10 using forward differences in space actually in space on right hand side from known flow field. at time t; this is what we have done actually. So, we calculate the you know the derivatives the time derivatives of the flow properties using finite differences in space on the right hand side for the known flow field at time t then we calculate the predicted value then we predict the value at t plus delta t as follows. So, predicted value. So, predicted value and we are going to call that say g bar I am going to just call it that. So, this g bar I will write. So, that is that that I can write as.

Now this is the known value this is something that we have assumed right at the beginning which was step one. So, we assumed this what we are doing here is that first we need to calculate del g del t. So, and this that del g del t is given by 6 to 10 these equations in which on the right hand side we have the derivatives of the flow properties in space. So, we use forward differences on the right hand side and calculate the derivatives; derivatives at time t from the known values we do that. So, then we predict the value at the advanced time which is t plus delta t as. So, now in the corrector step basically what we do is we calculate the derivatives using a backward difference backward difference ok.

You using backward difference relations using the predictive values you know say what I am saying. So, in the corrector; corrector step in the corrector step basically. So, we again we basically calculate now we calculate the derivatives using backward differences you know backward differences using the predicted values you see the predicted values is at an advanced time. So, therefore, and we are going to call that the derivative as let me call that derivatives as del g del t. So, then then. So, in here. So, therefore, if I were to write over here. So, the average value here.

This average value is going to be. So, this is the predictor and this is the corrector step right. So, this is the if you if you remember right. So, we did this for the we did this for the we advanced the time right we advanced the we advance the solution in the r direction right for the cone at an angle of attack in this case we advancing it in time. So, then how do we V 1 when we march solution in time. So, similar to what we did in the last case last time. So, in this particular case. So, the solution will be let me write it over here in this particular case the solution is going to look like this.

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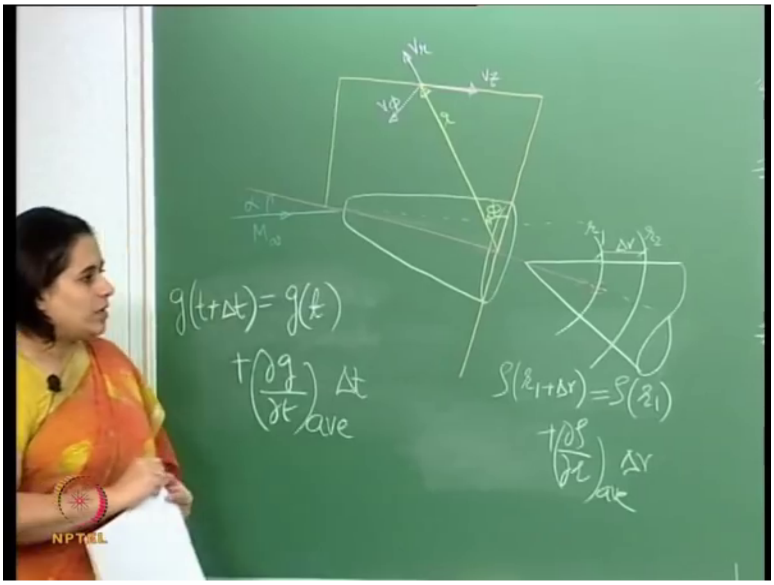

So, we are going to base for example, if you have say r you know the value r or let me call say g. So, g now g at an advanced time is going to be this right. So, so this. So, now, what we have done is this this del g del t, we have calculated that using a predictor corrector approach and this is how we advanced it in a time now again.

So, once we when we do that of course, there will be you know we will have there will be errors and you have to you know deal with the errors and iterate over because we came up with all this solution assuming the flow start at time t. So, clearly that is not the correct flow. So, we will not you know come up with the correct values in here. So, we will need to iterate and then the once the once the errors are within a certain tolerance you know a limit which you are comfortable with; then your program is your you know numerical code that you written has converged right. So, if I were to sort of you know look back a little bit just to give a comparison.

So, this was this is the time marching. So, I just want to notice the difference this is the time difference here we have we have transformed the coordinates what we are looking at here is the we are marching at in time right we have marching this in time. So, if you are looking at this particular picture looking at this picture particular picture at different instants of time you understand you are looking at this picture at different instances of time you just marching from this time is equal to the 0 to 1 to 2, etcetera, etcetera; what we did in the previous case we march the solution in space itself in the r direction. So, I will just want to sort of you know draw a little bit of you know connection there if I if I can. So, in in that particular case what we had was the instead of this.

So, we actually let me let me draw it here. So, we actually had say this is the free stream right this is the. So, let me draw a cone out here instead of this let me draw this cone at an angle of attack . So, this is what we had earlier right and. So, therefore, I said the we are going to march the solutions of for example, we started off with say r naught right. So, we went from r one to r r r 2 here and you know the way we said for example, the let me call say properties in in this particular case say density.

So, density at say r 2 and the distance between these r 2 is say delta r. So, I can say r one plus delta r is equal to you see the difference now in this particular if you see look at this and if you look at this a hopefully you can see the difference in this particular case like I said we are marching the solution in time. So, if you this is a picture you are looking at every you know at regular intervals of time regular instances of time here I am actually moving you know I am moving in this space I am moving in space from one radial direction into the other.

So, I am marching the solution in the r direction over here. So, well I think that concludes the entire you know this course which is advanced gas dynamics. So, hopefully you know if taken away something from this you know class the last bit I tried to give you an overview of the numerics which is involved in this the only way we are really going to get a whole picture or understanding of how to do this is implement this in the computer program. So, that calls from an assignment that calls for you know regress also experienced in in in coding right so; however, I hopefully you have been able to pick up something or try to understand a little bit about the physics or the geometry the 3D geometry the 3D flow field which is happening in this 3 cases right.

So, and we have done basically 3 problems in the 3D case, one is the axisymmetric one is a cone at an angle of attack and the other is this blunt nosed body at an angle of an attack right and what we have basically seen is that the there is a change there is of course, a change in the physical property of the shock and the body, but numerically is still boils down to you know one independent variable 2 independent variable in this case we have a 3; 3D coordinate system. So, you can say this is like a truly 3D flow although the flow is still 3D where whether it is an angle of attack the cone is an angle of attack like this or it is axisymmetric.

So, I think will call it quits there. So, you should be able to contact me or ask any questions if you have any suggestions criticisms or are all welcome more than welcome actually and. So, it should help me to improve next time I do this. So, you know please feel free to contact me not a not a problem at all ok.

Thanks.