Advanced Gas Dynamics

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Lecture – 04
The Mach Number and Compressible Flow

So, picking up the threads from last time, we talked about what exactly is a shock wave right and how the disturbances travel through the medium and we also use the governing equations and applied it for a normal shock and saw how the governing equations look like.

Now, the next question is that we every time you talk about shock waves or compressible flows or gas dynamics we constantly talk about the Mach number right, high Mach number flows and things like that. So, I think I said last time that we will see why right, what is the connection of Mach number now with a compressible flows. So, let us see what exactly is the relationship between the speed of sound right and why are we talking about that in context of shockwaves.

Now let us see; what is the relationship of the speed of sound.

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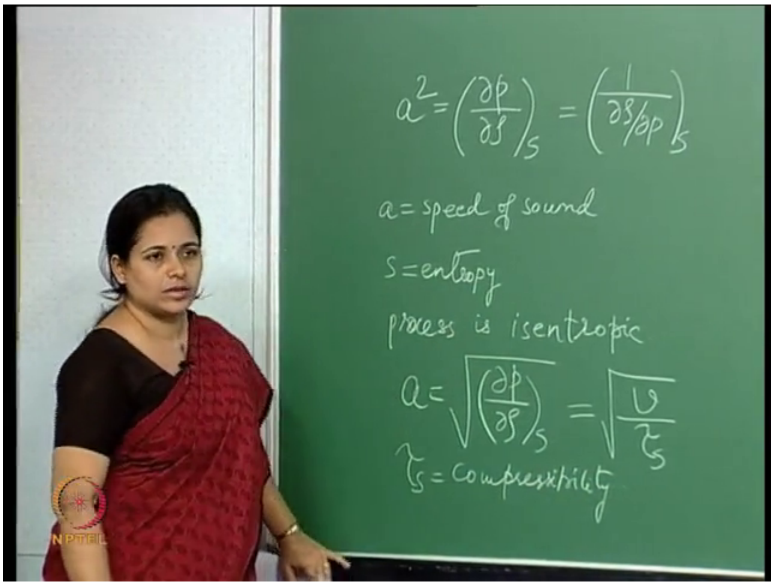

So, now let us just now without going into the derivation of this, we have a relationship like this right now a here is the speed of sound, a here is the speed of sound, p is pressure and rho is density and if you remember correctly this is entropy right. And if I write something like this then del p del rho at constant entropy which also implies that I am talking about an isentropic process right. What this essentially means is that or if I may this is kind of easy for me to visualize right. So, this is a unit change in density per unit change, this is a change in density per unit change in pressure right, and that this change is happening over constant entropy. So, that processes change is isentropic. So, that s is basically entropy and the process here is isentropic.

So, that is essentially the relationship, a square is equal to this or if I may write this. Now, if you can recall some of the derivations or the definitions that we did last time could we write this in some in another form could we write del p by del rho in another way, well think about this.

Now, now v here is the specific volume and tau s if you remember is the compressibility or isentropic compressibility. So, s denotes, tau s basically says isentropic compressibility right. So, therefore, now this kind of gives us some idea, some relationship between the compressibility of a gas and the speed of sound right and if you remember of what the definition of the compressibility was it was the fractional change in volume per unit change in pressure right.

So, now, essentially what we are trying to say here if I were to take this definition right and ask the couple of questions you know. Like we said yesterday that you know there is we are looking at say this point, so you are looking at this point right and you are say traveling from here.

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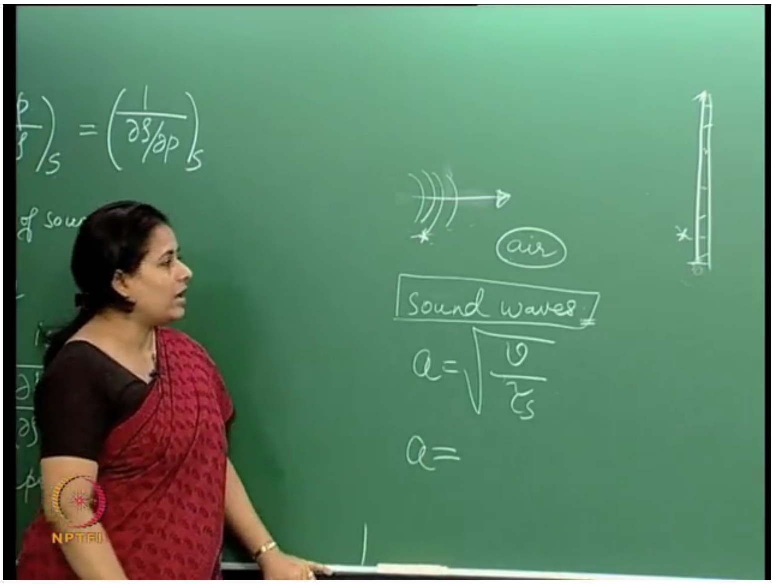

So, you are traveling from here right you have a certain speed right the example that I gave last class was there you know there a group of people standing over here you are riding a bike right and then you ring your bell now say you have a certain disturbance right because of you whizzing past is a certain disturbance. Depending on the speed at which you are traveling and the speed at this at which these disturbances are traveling through the medium over here and the medium of travel over here is air right, we will have a shock wave or not have a shock wave right.

Now, if these disturbances are traveling faster than you right, they reach this area yeah they reach the area and then this point here is able to then it almost gets notified that there is this person coming so I need to adjust myself right. So, then you have then it adjusts itself in, so there is a certain change in pressure temperature etcetera right; however, if the you are traveling so much quicker then these disturbances that you reach way quicker then these disturbances can reach over here then this has to suddenly adjust itself and that is when you get a shock wave right.

Now, the question is that so therefore, now just let us just look at the disturbance is traveling right, these disturbances traveling through the medium over here right and let us say these are causing a shock wave. Now, the question is that now if these disturbances are traveling through air they travel at a certain speed right now what if I change this medium what if I change this medium. If I change this medium to another gas or for example, you know make this a slightly heavier gas or if I make this slightly lighter gas then what happens you know, does the do the disturbances get affected or do not they get affected they will travel anywhere the same speed right.

Now, the crux of to answer that question the crux of the matter is that now sound waves right sound waves our material waves unlike electromagnetic waves like light. So, they need a medium to travel hence they definitely depend on the type of medium that we are traveling in right and the quantification that we just did and the speed of sound is, so therefore, this is the compressibility. So, this affects the speed of sound the compressibility, which means if the compressibility is more if it is more compressible the speed of sound will be less. So, that is essentially the relationship between the sound waves.

Now, again still we still need to answer the question that what is the connection of then sound waves of sound waves with these disturbances in the atmosphere, in any medium for that matter. Now they are kind of related to the mean molecular velocity you will not go into the exact relationship of that, but let us just say that these sound waves are somewhat related to the mean molecular velocity right. And when these disturbances travel there basically they are basically traveling say from this point to this point by passing on that information from one molecule to the other right. So, essentially these molecules are being disturbed right and therefore, these disturbances hence these disturbances perturbations whatever they travel from this point to the other.

So, and therefore, we use sound waves sound waves as a measure of finding out how fast these disturbances will travel right and hence the Mach number is such an important factor when we talk about compressible flows all right. Now there is yet another way that I can represent the speed of sound.

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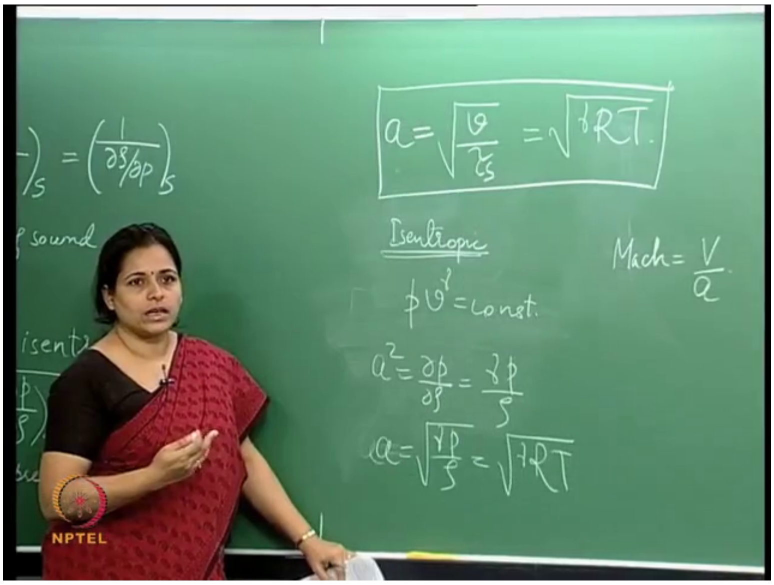

If I consider now, if I consider having say written this, we have one quantification right we have one quantification and that is the relationship of speed of sound with the compressibility right. If I consider isentropic which is adiabatic and reversible, if I consider isentropic then we can get this sort of this is a relationship if I consider isentropic. If I do that now I have also defined as isentropic relationship this way.

Now, instead it, now, in this relationship over here del p del rho if I can use this expression over here what I get is something like this right and therefore, I can actually therefore, again write. So, I can actually also write this as and we will see how we will use this relationship we will do a couple of problems and then we will see how we will use this. So, what we have done now is basically relate the speed of sound with the compressibility and you can see that is also related to the temperature right.

So, now having said that, so then of course, you know that the Mach number is speed by velocity by speed of sound right. So, Mach number is V by a and then we say subsonic sonic and supersonic depending on whether you the Mach number is less than one equal to one or greater than one I think that is something you know already. I think what we will do now is do a couple of problems and see if we can put these relations into practice and some other relations which we derived couple of classes back I think. So, let us start doing that.

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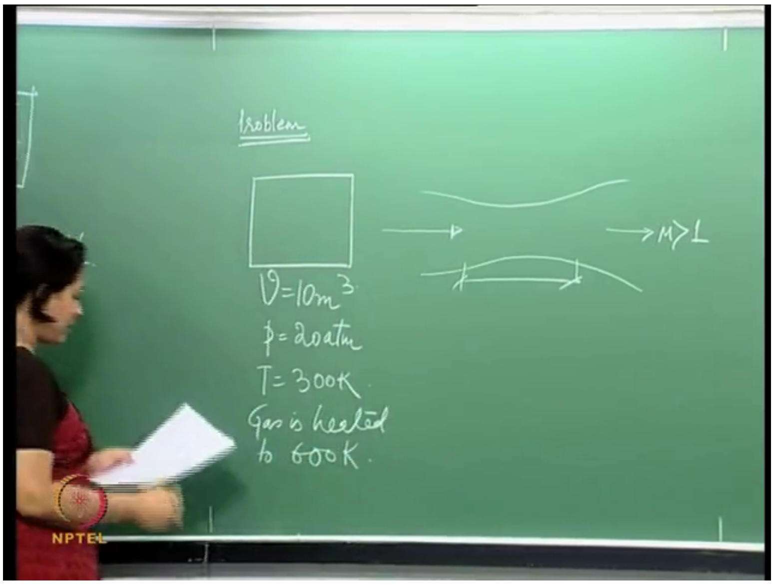

Now, say we have a pressure vessel which is restores high pressure air for operating a supersonic wind tunnel right. So, what we have is a vessel like that now what we do is we store air over here at a, this is the volume, what we have here is that we have we are storing air in this vessel at a very high pressure which is 20 times 20 times the normal atmospheric pressure. So, 20 atmospheres it has a volume of 10 meter cube and it is has a temperature of 300 Kelvin and we use this to operate a supersonic wind tunnel. So, what does that mean so basically what I am going to do is make it run through, if you remember this, basically we are going to make it run through here right past this gas through here make it undergo some changes right this that there would be some changes here and that is what were going to find out.

Now, what is what sort of changes is required to get this air in here which is stagnant right here to go through this to pass it through this tunnel so that it comes out with a Mach number which is greater than 1? So, what changes in its properties do we need to you know make this air to now become to true to be traveling at supersonic speeds. So, what we do now is we heat this gas right. Now this gas is heated to 600 Kelvin's, it is heated to 600 Kelvin's.

So, what we the question to be answered is what is the entropy change right, what is the entropy change. So, essentially this is what we are looking at, so what is the change in the entropy.

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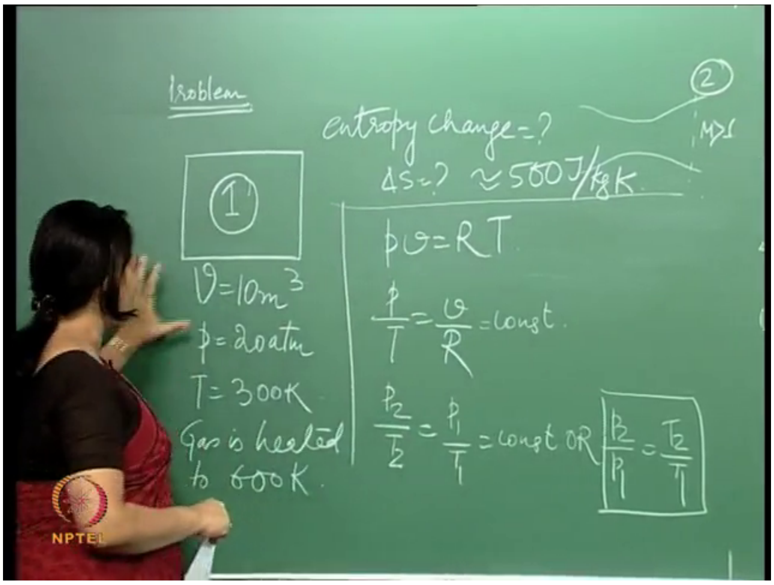

So, again, basically we have stored gas here were going to run it through a through a wind tunnel and the and this gas is going to then travel at speeds higher than Mach number. And during that process there will be an entropy change and we want to find that so, that is the problem here. Now, how do we go about doing this?

So, let us start let us start with this relationship. So, pv is equal to RT that is the basic gas equation of state right now what happens to the volume over here. This is a chamber this is a vessel now this will most you know always will have very strong steel walls right. So, this volume is not going to change right the volume remains constant is not it. So, therefore, if I can write this or basically what I am saying this is the universal specific gas constant. So, essentially what I am saying is this is constant right. So, this particular process p by t is constant right. So, now, let us say that this these conditions is condition one let us say that these conditions which is the stagnant conditions is one and when it become supersonic finally, at the exit let us call those conditions as 2.

So, say this is how it is traveling say; here same Mach number is greater than one is in the exit of the tunnel. So, let us call this as let us call the properties over that as 2 right. So, what this essentially means is that is this right is not it, or we can write this as or we can say is not it. Now p 2 is something that we do not know right, p 2 is the pressure at the exit and p 1 is given, p 1 is 20 atmospheres, now T 2 is known which is 600 Kelvin T

1 is also known which is 300 Kelvin. So, therefore, we can we get this relationship. So, let us mark that all right.

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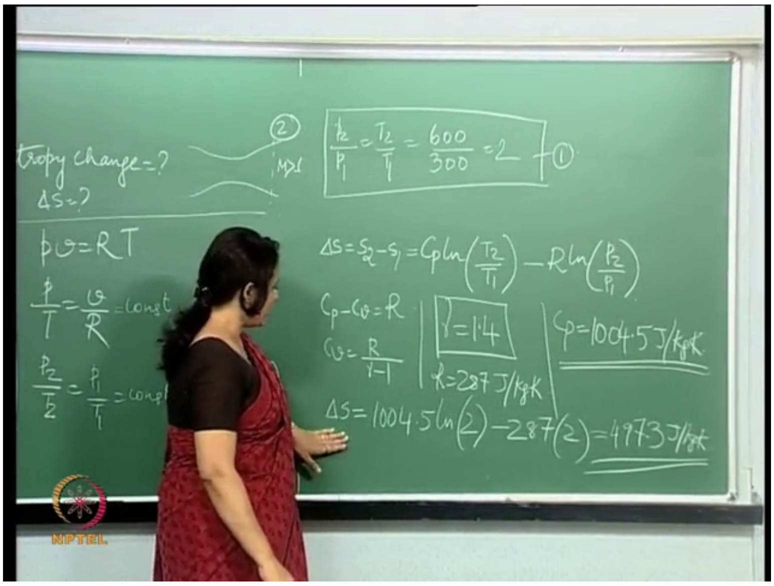

The next thing is we need to find the entropy change. Now if I have to find the entropy change what is my relationship what is the. So, again S 2 is here and S 1 is at the stagnant conditions in the vessel right. So, this is equal to right. So, in this relationship we see that we know T 2 by T 1 which is 2 and p 2 by p 1 is also 2 right and we know the specific gas constant this is 287 this is a standard constant right.

Now, we need to find out C p right we need to find out C p and how do we go about doing that. If you remember the relationships for example, C p minus C p is R right. So, C v is R by gamma minus 1 and so therefore, you can you know combine these 2. So, in there if you take if nothing is specifically given you can take the gammas 1.4, if you do that then you can find C p from here and R is 287 right yeah, 287. Now if I do this, I know the value of R and I know gamma, I can find C p from this right and if I do that then I get a value of C p which is you C p can check that. So, essentially I am just you know putting these values in here and I get C p. Now once I do this I should be able to get my change in entropy all right, once I have done this I should be able to get the change in entropy which now means that.

So, let me just write this out. So, this becomes C p is 1004.5 the natural log write that, we get this. Now this comes up to be what I get here is 497.3 Joules per kg Kelvin all right. So, what we get here is the entropy change for which is around 500 you know Joules per kg Kelvin. So, we got some answer for this and, so in here, this is say around 500 Joules per kg Kelvin. So, as you can see this is basically specific entropy right, we defining that by over the unit mass.

Now so, we would really like to know what is the total entropy change is not it because we have a total volume of gas to be 10 meter cube right. So, for that we get a specific entropy as 500 use per kg Kelvin, so what is the total mass here mass of gas. So, that we can calculate the total entropy change right and why is the entropy change important and why should we be concerned about the entropy change. Let us answer that question, let us find out first the total entropy change.

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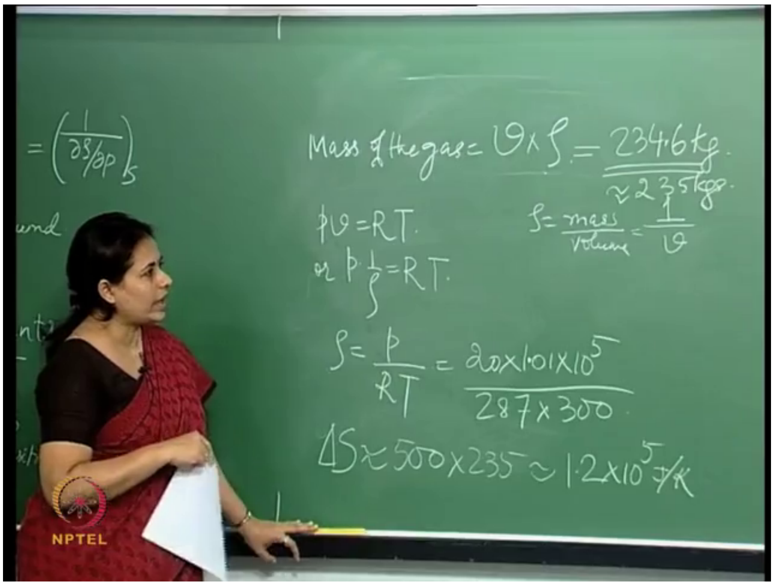

So, to do that to get the total mass of the gas we need the volume which is given 10 meter cube into what into the density and how do I get this density from right this is what I need the density right. Now, we will again use this right, now this is for this is the specific volume is not it this is for unit mass. So, I can also write this as is not it, I think you should be able to figure out why because see density is mass by volume right.

So, when you say volume as this v this is specific volume; that means, the mass is 1 unity right because this is volume per unit mass. So, therefore, this if this is v then this is 1. So, that is why I can write v as one by rho if I write that from here then I get rho to be this is my density right this is my density of the gas and this we can find because pressure is given to be 20 atmospheres right, 20 atmospheres and temperature is 300 Kelvin right.

So, this is this pressure is 20 into this, this is Newton per meter square right. So, this is the total pressure this is specific gas constant and T is the temperature. So, these are the conditions under which the gas is stored in the vessel right. So, the density of the gas in the vessel which is not explicitly given can we calculate it. So, this is your rho and I did not calculate that explicitly you can get it get it separately if you want. So, therefore, the mass of the gas therefore, is this rho into the volume which is into 10 right and that I get as 234.6 kgs, which is say around 235 kgs.

So, we get 235 kgs of gas right that is being stored in the vessel therefore, the total entropy change. So, what you see if I write that as kept s would be say around, so I had around 500 Joules per kg Kelvin in 235 kgs. So, I kind of get you know this is I get around say 1.2 into 10 to the power 5 Joules per Kelvin right. So, this is the total entropy change for the given mass of gas right.

So, therefore, like I asked the question then why is this important? Why is the entropy change important? So, basically what we are seeing is that we have a gas which is stored in a vessel right at a certain volume temperature and pressure and as a result of which it has a certain density right and then we heat the gas I make it travel through the tunnel right and there is an entropy change there is an entropy change of nearly 500 Joules per kg Kelvin and total is around 1.2 Joules per kg for a given mass and I get supersonic speeds right.

Now, why however, I need to know this, is this information important and why are we sitting here in calculating all this entropy change and so on and so forth.

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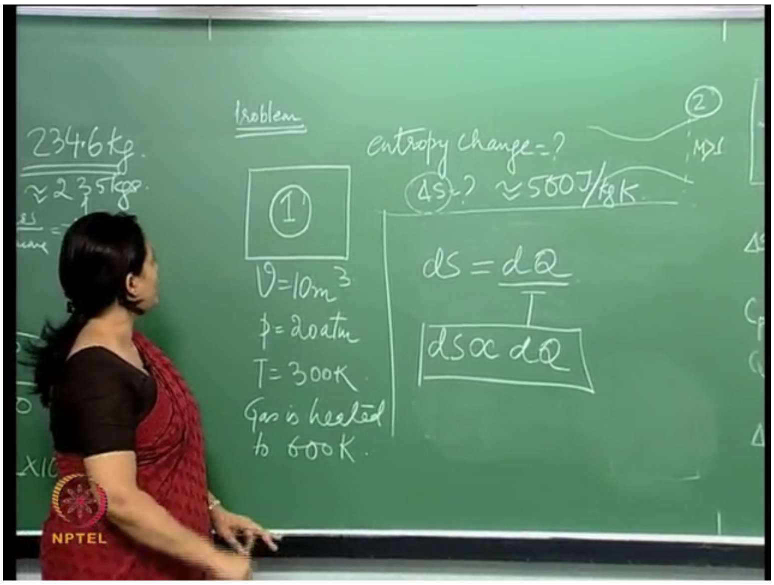

Now, if you remember the definition of entropy. So, the entropy change gives us a measure of the heat generated right the heat or the heat generated in this case heat is generated or heat required right. So, it is directly proportional to the heat generated. So, therefore, we will have some the total entropy change will give us an idea of how much heat the how much heat is going to be generated when such a process takes place right and this information is important right. So, that was one example right where we were able to use you know basically a simple relationships of you know these relationships of you know for an isentropic process and we were able to find out the you know total entropy change when we were trying to run a supersonic wind tunnel. Let us take another problem and then try to understand a little more.

So, now, the next problem would be and let us consider the flow I think you will be doing this quite a lot now, you try to understand flows through nozzles etcetera right. So, we will try to concentrate and consider the flow through a nozzle.

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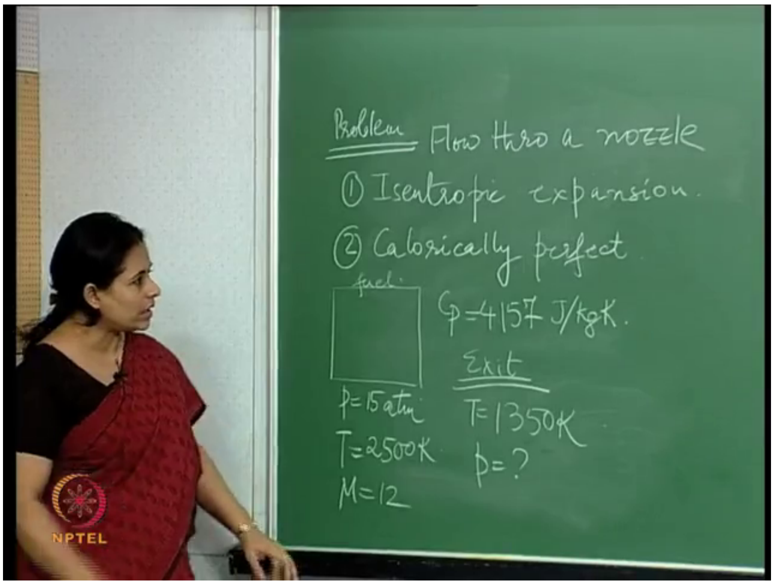

So, we will consider the flow through a nozzle and there are two considerations given right. So, basically when we say a nozzle right the gas is going to expand through the nozzle like we did in the previous example. So, we will consider that process to be isentropic, isentropic expansion of the gas and. Secondly, we will consider the gas to be. So, what this essentially means in terms of solving a problem is that we have a handy equation, the moment I say isentropic expansion I have a handy equation moment I say calorically perfect we have a handy relationship right. But time and again I think we should remind ourselves what exactly we mean by isentropic or calorically perfect I think once we keep doing that that it kind of justifies the relationships and the equations that we use right.

So, therefore, now we in, so therefore, we have the chamber right we have a chamber where we have fuel and oxidizer right. So, they burn so basically the gas and the oxidizer they burn and you know create the fuel now the fuel is generated in this chamber right. So, this is the few chamber right and this here again the pressure is 15 atmospheres and the temperature is then the molecular weight is 12 and the C p is also given to be. So, this is the fuel which is being stored at these conditions right. And this now expands again, again you are going to run the nozzle. So, at the fuel here just before entering the nozzle is at these conditions and at the exit of the nozzle, at the exit nozzle the temperature is 1350 Kelvin. So, what you see is that the fuel is being generated at 2500 Kelvin right and the at the exit there is a drop in temperature to 1350 Kelvin. So, there is you know quite a large drop in temperature right.

So, what we are going to basically ask is that what is the pressure? What is the pressure as the exit? You know for this change this the change in the temperature what is the corresponding drop in the pressure right. So, how do we go about solving this?

So, what do we need for this process? Now, one thing is that let us see now we said it is isentropic right. So, now, the change is isentropic.

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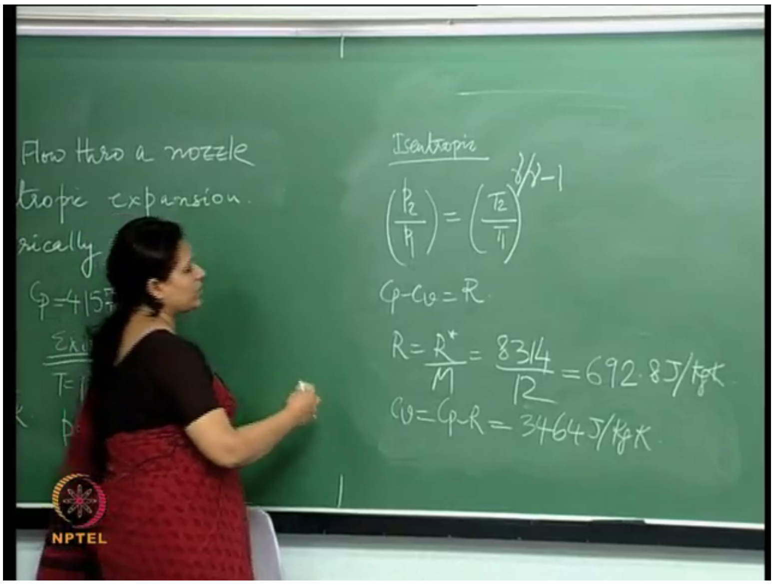

So, let us we will use these relationships. So, the relationship is, this is the relationship. So, basically all we need to do is you know plug in our values into this relationship and we should be able to get p 2 right. The subscripts again say let us say the you know the fuel chamber is 1, so these are ones and at the exit is 2 right.

Now the exit temperature is known the inlet temperatures temperature is known the pressure is known we all are asking is what is p 2 right. So, we just need to plug in our values into this and we should be able to get the exit pressure. Now, before we do that what we need is the value of gamma right. Now, unlike in the previous case when we took that as just as one point four here C p is given C p is given over here. So, what we need to find out is C v and then we will take a fracture of that like C p by C v and that is what will give our gamma right.

And how do we do that? Again we will use these relationships right this is a specific gas constant. So, this is the specific gas constant right. So, that this specific gas constant is this is the universal gas constant by some mass right. Now, what mass is this? I think we talked about this a little bit in detail in the first class right. So, this is, this is the specific gas constant which we need over here right and. So, this is the universal gas constant this is the mass. So, in this case this is given, this is the molecular mass which is given and this is the standard value which we will use 8314 for I think right, 8314 and this is the molecular mass. So, therefore, we will get the specific gas constant.

So, I hope you are beginning to see why we use these you know these expressions for a specific gas constant or specific volume etcetera you know what is the significance of that because especially when we are considering different gases with different molecular rates that is when we talk about you know these we talk about in a specific gas constant specific volume. Because the same number of molecules of different gases will weigh differently. The mass of the same number of molecules the Avogadro number of molecules will weigh differently for different gases.

So, if you see if you change the gas like we saw in the previous example right so the density is not going to remain constant because that will depend on the pressure temperature etcetera. At the same time if you change the gas itself right now these values also start changing because the molecular mass is changing. So, in this case the molecular mass is given as 12. So, therefore, now this value here becomes this R is Joules per kg Kelvin which is unlike it is nearly 693, this was 280 in the previous case you know just to bring out the differences. So, once you get this C p is known.

So, therefore, we can get C v right, we can get C v and that C v is basically C p minus you know just write that and that what we get is 3464 Joules per kg Kelvin. Once we do that then we should be able to find out our gamma right. So, in this case the ratio of specific heats therefore, become ok, so gamma is C p by C v right, which is, so and that is 1.2 that is 1.2. So, once we get that then for the eighth for this particular isentropic process we will again use this, so we just need to find out we will just plug in the values now.

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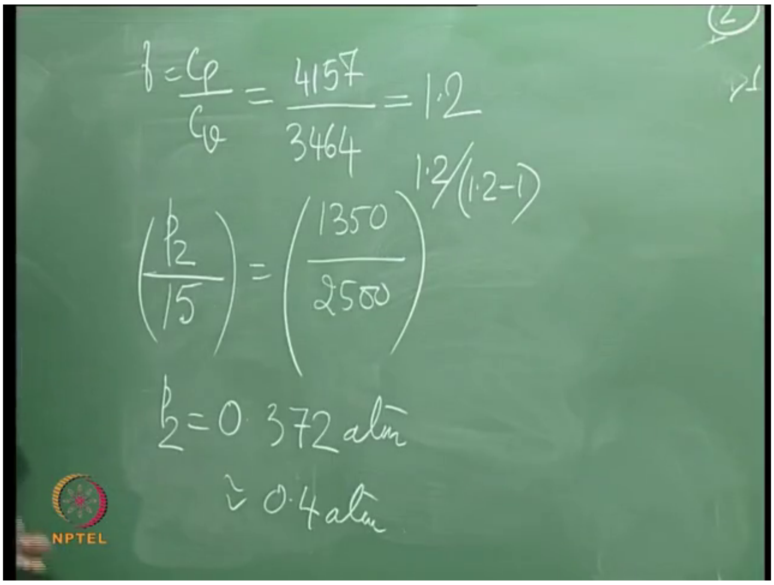

So, p 2 is what we need, p 1 is 15 atmospheres right that is equal to T 2 which is a 1350 and T 1 is 2500 and this is gamma by 1.2 that right. Now, if I do this what I essentially get from here is that p 2 is a quarter 0.372 atmospheres which is an out point for atmospheres. So, this is essentially my p 2.

Now, what is interesting is now let us just sort of you know take a step back and look at all these properties. So, what did we do? Now we have we have a nozzle right and then we are burning fuel making that fuel run through that nozzle right. Now, when the fuel is generated it has these conditions you know it has a molecular mass of 12; it is a pressure of 15 atmospheres temperature is 2500 Kelvin C p is given. Now, when we run this through the nozzle at the exit there is a temperature drop which is 1350 right and see the corresponding pressure changes pressure change is less than half atmospheres right. So, you see the large pressure change from 15 times the atmospheric pressure we actually come down to 0.5, less than 0.5 atmospheres right.

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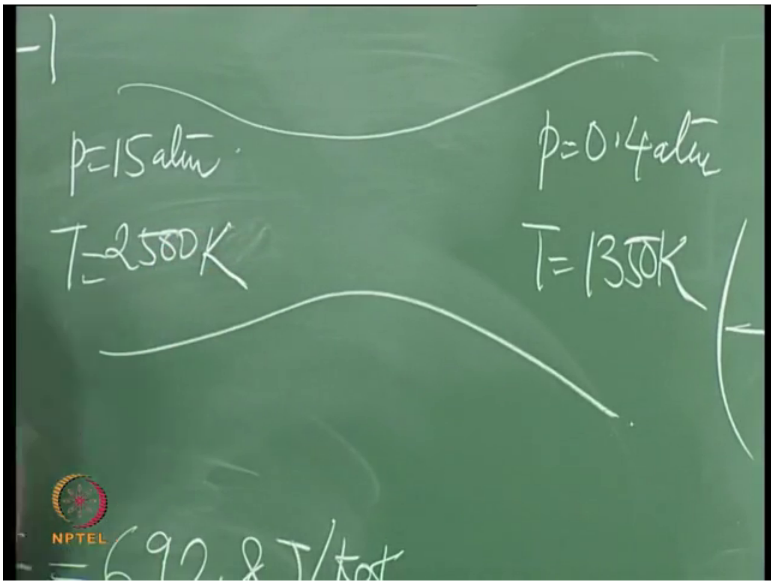

So, therefore, when I run this, if you right. So, pressure here is 15 atm the temperature is 2500 and the pressure here is 0.4, actually it is less than 0.4 and t is 1350 Kelvin sorry Kelvin. So, basically, in the nozzle, therefore, you can see the physical change here. So, you can see the physical change here that to move from, that once it moves from here right, we have a temperature drop of quite which is quite significant which is 2500 to 1350 and look at the corresponding pressure drop right.

So, like we spoke in the previous lectures etcetera what does not nozzle do that they it brings about a correct. So, now, think about this that if I bring about you know a pressure change which is say you know 15 atmospheres to 0.5 atmospheres say, then I am going to get you know there is a corresponding temperature drop as well, but I am also going to get supersonic flow through it right. So, I mean to the two ways to sort of look around this.

I think we kind of stop here and where we will pick up the next time I think what we will start looking at is the way we looked at the relationships using the governing equations right for a normal shock wave. Now we will see if we can you know use those and for a nozzle especially for a nozzle and how best we can use those you know equations to you know study flow through nozzles right. Now, let me just sort of just you know briefly start that process and we will pick it up in the next class as well right. Since we have talked about nozzles over here I think it is you can just sort of introduce that over here. Let me just go back over here.

Now, therefore, in the nozzle, as you can see here the fuel is stagnant right the fuel is the stagnant in this case. So, this is you know we can call this chamber or this condition of the gas to be reservoir conditions. So, if I do that, so say this is my nozzle right, so this is my nozzle. Now, here you know before just before the flow it actually starts happening now I am going to call these as reservoir conditions right and these are going to be denoted by a subscript naught.

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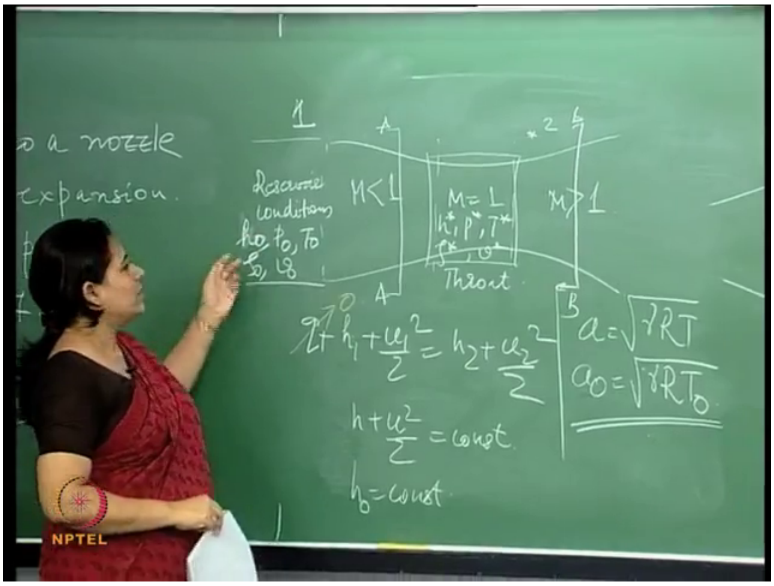

Now, if you remember the relationship that we derived from the energy equation for a you know here, for a normal shock wave was this yeah. So, we said basically right. So, what this again 1 and 2 basically, in here so this is the relationship that if the flow is moving from say 0.1 to 0.2 right, 0.1 to 0.2. So, it could be anywhere this could 0.1 and this could be just 0.2 right. If it is, so the relationship between the enthalpy the velocities and the heat is this right.

Now, let us consider this as isentropic. So, we are left with this. So, therefore, I can bring this down and say that this is constant right. So, if I do that this is constant, now let us define this constant now let us say this one or say this one this location one is the reservoir. So, if it is a reservoir u there is 0 right. So, therefore, we have h is equal to constant if I have the reservoir conditioned and like we said we will denote this specially.

So, we will denote this as h naught right. So, therefore, in the reservoir, so in the reservoir therefore so now, my, this is my 1. So, my conditions therefore, this is h naught then the corresponding t naught, rho naught, v naught and so on and so forth right. So, this is essentially h naught. So, these are the, this is the notation.

So, every time you see something like this right, every time you see something like this you will know that we are considering basically the reservoir where there is the flow is stagnant; there is essentially no stagnant condition. So, this is and this is a special kind of a dilution right. And the corresponding again so if I had to calculate the speed of sound then now the speed of sound is again given by right. So, if I consider, if I consider the reservoir; then I again I am going to call this as right, you are going to call this as this and yeah, this is d, this is the condition. So, therefore, this is the reservoir conditions or you know total conditions actually because after this when you are going to expand this and accelerate this through the nozzle there is going to be a temperature change, pressure change etcetera. So, these are the reservoir conditions and special dilution with naught you know corresponding is this.

So, now in the nozzle usually right the way we would kind of define this is here it is say subsonic, it is kind of you know more or less kind of defined like this. So, now, this area, this area this more like sonic zone is actually kind of you can say that this is kind of the throat right. Now, in this case we can denote these values, these values whatever we define here with a subscript naught we will define those values on the throat with a superscript star. Meaning I will denote that as say h star, p star, t star, rho star etcetera.

So, I guess we will just stop over there right and we will develop this relationship again for the nozzle and see how we will basically what we will do is develop the relationships in terms of and see if we can introduce these values somewhere into the relationship. In the sense that if I have you know values like this, so basically if I have any condition in any point over here say, if I have a point or say section over here a all right or I have a section over here right say b. So, I will try to find out these values at the properties at a and b with respect to say the reservoir conditions, and the throat condition. And we will see what is the usefulness of that why do we need to do that, it is all right.

We will stop there and we will continue this in the next lectures.

Thanks.