Advanced Gas Dynamics
Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 39
Supersonic Flow past a 3D Cone at an angle of attack: Numerical Procedure

So let us begin from where we left off last lecture. So, we looked at the governing equations right and what do we are going to do is sort of briefly or kind of outline the method the numerical method with which we are going to go and solve something like this. In method is primarily what amoretti outlined. So, give you the reference for that. So, I will this basically outline the procedure and which this can be done and let me re to write that the best way to learn this or you know understand the numerical method is to really implemented right write a computer program and implement it.

So, what I will do is basically go over the method, and then like I said I have pulled up more recent paper right and we will you know has some it is a CFD works it has some nice you know colorful pictures they always make for you know you know good reading. So, I will just show you some of the results you know which you might get in numerical analysis ok.

So, now let us. So, go ahead. So, what we have here is our cone of an angle of attack right.

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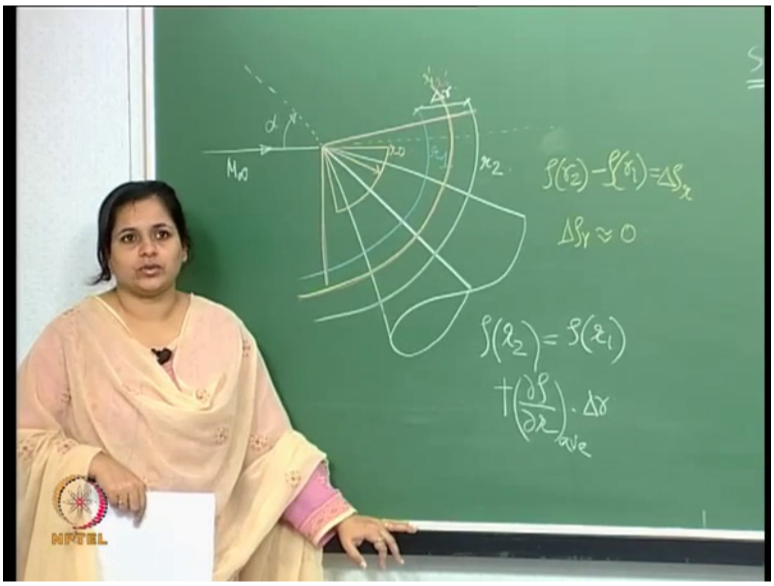

So, this is our free stream this is our free stream and right. So, this is our cone at an angle of attack this and this is our alpha right. So, and we will have a shock structure somewhere like this right. So, this is our shock structure.

So, let us how do we you know sort of. So, this is our problem. So, we have a cone at an angle of which is inclined, at an angle of attack of say alpha with the on coming supersonic free stream we need to calculate the properties. And what are the flow properties are we are looking at the 7 properties that we talked about yesterday 7 why because we are looking at spherical coordinates right.

So, the velocity from components has it is thermodynamic properties right. So, that is what we looking at. So, how do we go about this.

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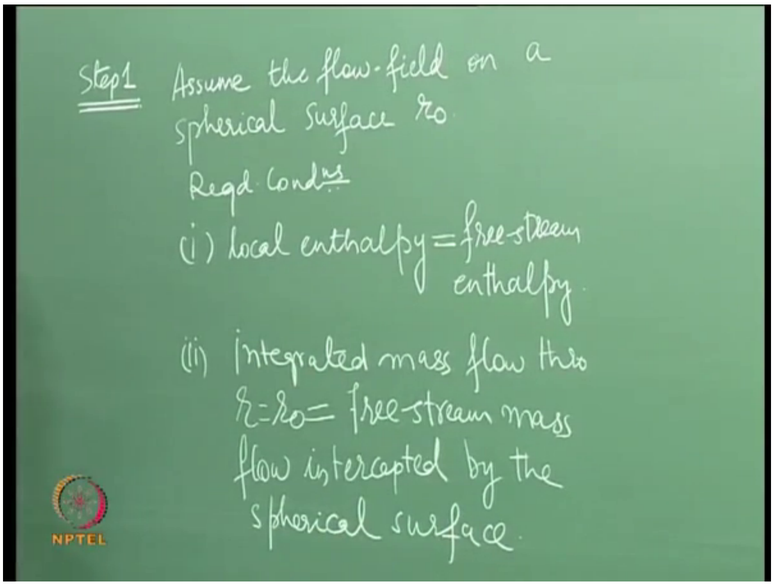

Now, I am going to sort of based on this, you can actually develop a skeleton for a nice computer program you know and then slowly it will try to implement it. This would call for an assignment which will for first timer programmer as we will take little more time that is as you get used to it you kind of get hang of the whole thing right. So, the way we are going to do this you know you have seen we have we wrote on the equations in the spherical coordinates ok.

So, the way we are going to start this is by assuming a flow properties you know and then merge forward from there. So, first step is. So, assume the flow field on a spherical surface r naught what is that mean let us come back here. So, what this means is that. So, you know these. So, this is the vertex of the whole thing.

So, we have various radii extended from that vertex, some of this radii from the surface of the cone the other radii from this surface of this shock structure. So, let us consider a surface which is within the shock structure which is between rather the shock and the body right what is that mean? So, you have two concentric cones you remember when we did the green and the blue. So, they were basically embedded cones within the red one right.

So, let us take a shock let us consider a shock structure say which is inside here like that. So, this we will draw it like this and this radius. So, this radius here this is r naught. So, what we are going to do is assume the flow flied on a spherical surface r naught we going to. So, if if you have a if you can think of a cone right which is embedded within this shock structure which is between the shock and the you know body right the radius of that is r naught then we will assume the flow on that right as we start we are going to when we start this is this is the assumed flow is non canonical the assumed flow is non conical, but and you know you can really be arbitrary when you assume.

If the flow out here all though there are two conditions that have to be there to which is the conditions which are like required conditions, say required conditions local enthalpy should be equal to the free stream. So, local enthalpy should be equal to the you know free stream enthalpy and the mass flow rate through r is equal to r naught. So, total or you know integrate mass flow. So, total mass flow to let me write it this way integrated mass flow through is equal to the free stream mass flow by a spherical surface.

So, the this is non conical the flow the flow field that you assume on this r is equal to r 0 and it can be arbitrary, but number one local enthalpy should be equal to the free streaming enthalpy. And secondly, the total mass flow through this cone right should be equal to the free stream mass flow.

Which is intercepted by this conical surface right now next is step 2. So, having done that this is the first part where we assume the flow right.

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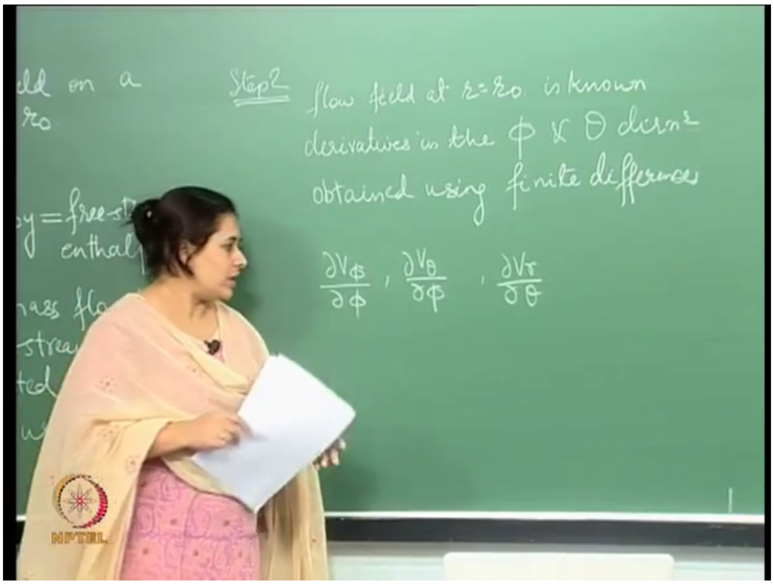

So, step 1 basically consists of begin by assuming a conical surface within the shock and the body and assume the flow on it, but which can be arbitrary, but these two conditions should hold. So, that cares of your step 1. If you look at it governing equations it has all the derivatives, it consists of derivatives for example, del v phi del v, del phi right del v r, del r then del v r del phi so on and so forth. So, how a now we know the flow field at r is equal to r naught. So, that is known.

So, now, we can calculate the derivatives with respect to the phi and theta right. So, phi and theta directions using finite differences.

So, now, the derivative is just like I said. So, del phi, del v r, del phi, del v theta, del r and so on and so, del phi del theta. So, all the derivatives if you see your governing equations to solve this governing equations which is equations basically 4 to 10 that you have. So, to solve this equations the continuity equations so on and so forth we need for example, the continuity equation just give you an example the continuity equation has this term, it has this term then it also has this term. Then the momentum in r direction for example, it has. So, all these terms all these. So, phi and theta.

So, the dderivatives in the phi and theta directions are obtained by using finite differences right now. So, when I say finite differences so obviously, you will have to need you will need to have a basic idea of what I mean by that right. So, it could be you know you could basically go over what you have learn through before in case you are not familiar with what I have talking about. So, take a basic book and very basic finite the differences what; that means, it is something that you should be able to figure out. So, having done that in here. Now, we know the derivatives ok.

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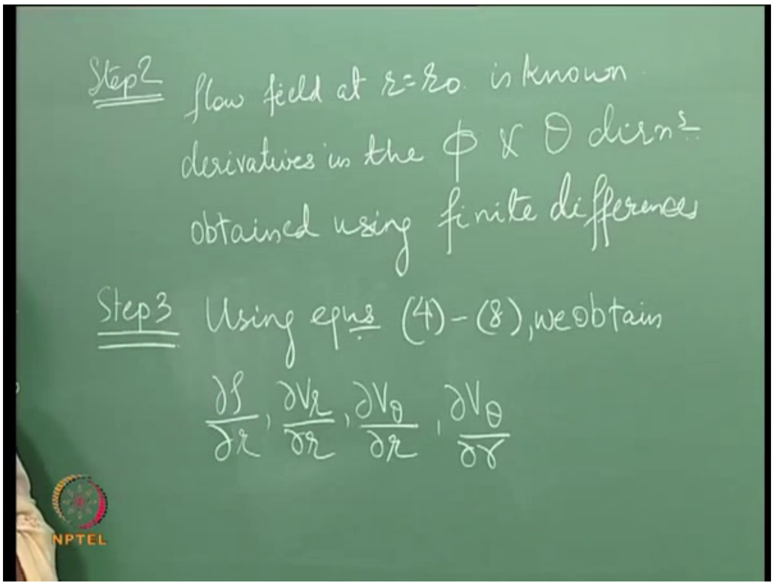

So, then if I do that. So, clearly if I am able to find out the you know the derivatives that if we use the equations the governing equations the continuity.

The momentum in r theta and phi directions, the energy equation, use all of that which is 4 to 8 it is 4 to 8 or 4 to 10 I do not remember how to name it is 4 to 8 actually. So, 4 to 8. So, which contains all our equations the energy momentum and continuity right. So, using the equations which we derived yesterday for 4 to 8 we therefore, if you look at those equations they give us these. So, the continuity for example,. So, we obtain what we obtain is, if you see the continuity gives us this oh sorry right.

The continuity gives us this momentum in r direction gives us this, then momentum in theta direction right and what else and we get h and we get. So, these are the stuff that we guess. So, these are the derivatives that we are able to get. So, having calculated the derivatives in the phi and theta how do we get this? We calculate this using finite differences using the values which we have assumed and once having done that you put those you put those derivatives into these equations 4 to 8 and we obtained these derivatives you can clearly see now these derivatives are in the r direction.

So, these were in the phi and theta plane which was defined by the regulator r naught which we assumed. So, therefore, now what we will do is since if you remember right. So, we came up here this was an arbitrary assumption, we did not know about this. So, we just assumed we took an r and we assumed a set of values over here. Now we need to merge these values further in these directions in the r direction because this r is not something that we know for sure which we just assumed that.

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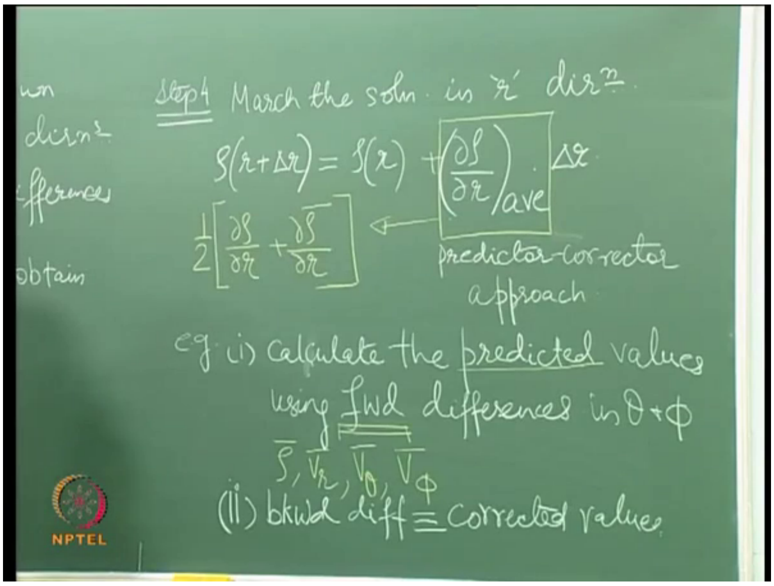

So, the next step of course, is that we merge the solution in the r direction. So, if I say come here. So, let us call this.

So, essentially say if I have you know if I go say delta r in the direction of r how will my properties be . So, now, the way we would sort of do that for example, let us merge the say density, the density which we assumed that r is equal to r naught let us march it in to r like the next r that we would assume. So, which is say density.

So, let us just try and understand what this means out here. So, if you look at this, if I march the solution in the r direction. So, what I am saying here is that see the density here I mean this could be you know any property here which could be out of the 7 you know the density temperature velocities etcetera now if I have the density here in the at r. So, the density at r plus delta r that the next radial position is equal to rho r plus del rho del r average into delta r which is the variation the variation of this density as we go from r to r plus delta r into the.

total change in delta r this is the whole idea. So, let me sort of try and clarify that here. So, we are going from r naught to another r. So, let us say we are going. So, let us just you know forget about this, let us forget about this r naught for a while. So, let us say we have. So, let us call this as say r 1 and let us call this as r 2. So, what we have done here is that.

See if you look at r 1 if you look at just this particular you know radius of curvature if you look at that and if you project it right is a particular circle right on which theta and phi are will vary on which theta and phi will vary is not it. The r remains the same. So, that theta and phi phi variation is what we calculated using finite differences and how because we assume the flow at r is equal to r 1 that is how we started. Now having done that what we have found out is the variation of properties is the derivative of the properties in the r direction right we found out.

So, we using r governing equations. So, now, we want to move from say if I have to predicted it if I have catalytic properties in say and radial you know. (Refer Time: 20:03) distance r 1 I want to move to the say if I want to look at properties which is further down then what would I do? Now what we saying here is that if you look at this particular picture is that here if you look at this that the say density, density at r 2 is equal to the density at r 1 plus the average change of density over r into delta r and that delta r is nothing, but this delta r. I think this is easy to visualize is not it. So, we have properties here.

So what I am saying is the property here is equal to the value of the property here plus if I take you know on an average change in the the rate of change of the property in the r direction that into delta r, because that is how that is the difference between r 2 and r 1. So, that is what is basically my step 4. Now in here this delta rho delta r now what we have calculated in step 3 is basically these values, you have calculated delta r del rho and del r here what we are looking to do here though is dell rho del r and average value what is that mean; now this here actually this term.

So, this term out here now this is calculated. So, this term out here is calculated using a predict operator approach now let us just see what; that means, this average value. Now using say the continuity equation, we calculate the predicted value using forward differences in theta and phi. So, for example.

So, number one we calculate what is important here is number 1 we do 2 6. So, calculate the predicted values, calculate the you or rather predict the values using these forward difference; forward differences in theta and phi. So, we will predict the value at a particular point. So, how will we do that.

You see we have this for example, del rho del r here, now this is available to us this is available to us. So, now, using a forward difference in a theta and phi, we calculate the predicted values we just use these predicted we predict the values using forward differences theta and phi and let us call those values as let us call those just call those values using a bar like this. We are going to call this you just you know we use calculate the predicted values using the forward difference theta and phi and we call this is this and again now using these you know differences again ok.

We calculate the corrected values we use corrected values using backward difference in theta and phi. So, again the second thing you will be. So, let us say here let me just right here. So, using backward differences we use backward difference to calculate the corrected values. So, when we again we come here. So, we use now backward differences and that gives us the corrected values. So, we will calculate the corrected values right in this fashion.

So, once we do that using this then we will calculate this for example, del rho del r average as an average of this predicted and corrected values which means what, which means this which means that this is basically equal to. So, the way again let us just sort of drill on this little more. So, we need to calculate the rate of change of this property as an average using a predictor corrected approach.

So, what we. So, here is we use a forward difference to predict the values and forward differences in theta and phi and we calculate the predicted value let us call that as this then we use a us on the same property here we use a backward difference and calculate the corrected values. So, it is you can you know sort of crudely saying it is like using two different approaches you get two different values for the same property and hence you get an average an average for this change in rate of change of that property as this. So, this is the predicted predicted corrected approach this is also again in the standard book or explanation this is sort of something that you know you can just look up in case you are not familiar with this, if I have time I am probably going to go ahead and explain on that a little more.

So once that is the protector corrector approaches right. So, once we. So, that is it. So, in this. So, then we march the solution in the r direction. So, how long do we do that. So, how long do we do that. So, we have come here and we can we are now calculating. So, what I did just now here is this average value. So, this average value we said we will use correct I mean calculate this using a predictor corrector approach we will do that. So, essentially now what I will get my the whole objective of all of that is to get an estimate for this rho of r 2 for the density at r 2.

Now, I started out by assuming this which is in no way the correct value, which is which is not the correct value. So, therefore, I am trying to get a value which is on which I will be able to you know converge. So, therefore, slowly what we will see is that the difference between say rho r 2 minus say rho r 1. So, say if I would call this as say difference or say let me call this as say this is equal to delta right. So, let me call it this delta rho r. Now slowly what we was see is that our approach here will basically be the this delta rho r approaches 0 of course, when you writing a computer program you going to say it is within a certain tolerance, on that tolerance is something that you design ok.

Say it is 10 to the power minus 10 or whatever so; however, you want your things to progress. So, slowly what we will see that we will be able to correct the error that we made in assuming it to be r naught. So, as we proceed from r naught to the next one. So, we see how close we are to what we had assumed. So, which would mean that this the distance. So, finally, what we will have is that we will converge to a r. So, say this r was in correct this r was incorrect. So, we assume another r. So, say we come over here. So, this was say we this r 1 actually say moves to say r 1 dash, but then r 2 also moves say somewhere right here r 2 also moves say somewhere over here.

So, this is your say r 2 dash. So, if you think if for you if r 1 dash and r 2 dash are close enough, if the if you do not see much of a difference here then your solution is converged. So, this is a solution that you will take. So, that is the basically you know the numerical procedure as we know it. So like I said this method is basically illustrated you know first started out by amoretti. So, I have given you the reference for this for details of it you will have to go through that number one.

And secondly, I think the only way to really understand this is through you know implement a computer program on your own catholic in that is really necessary as it was necessary for the conical flow which we did in the first day axis ssymmetric case also unless and until you do it is difficult to understand. What I will do now is pull like I said I pulled out of paper which is more recent 2009 I think and we will look at some of it is interesting some of the results that are there are sort of pretty interesting. So, let us say look at this ok.

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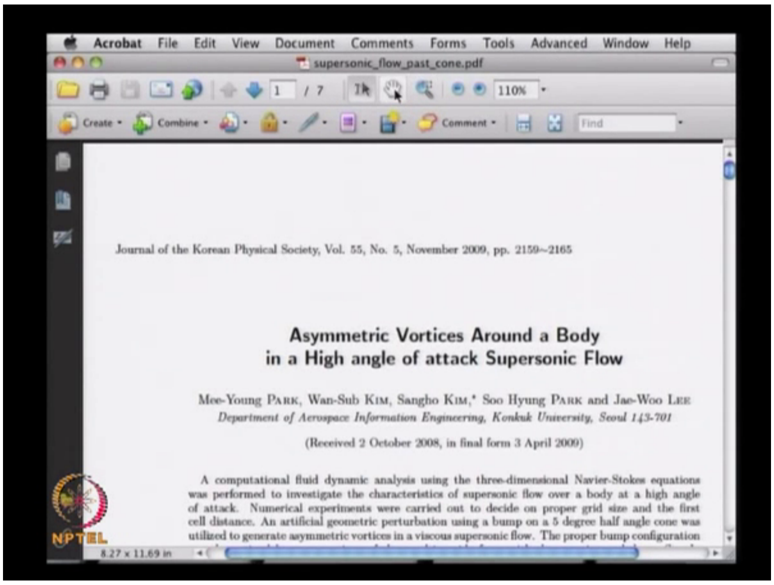

So, this is the paper that I am looking at, this is the paper that I am looking at.

So, let us look at this. So, this is the name of the paper, should this is the name of the paper if you have to look at this a asymmetric vortices around the body in a high angle of attack supersonic flow right. So, I need to send this particular journal this is the journal of.

So, it is in 2009. So, what I found interesting here is some of the results because they make for nice pictures.

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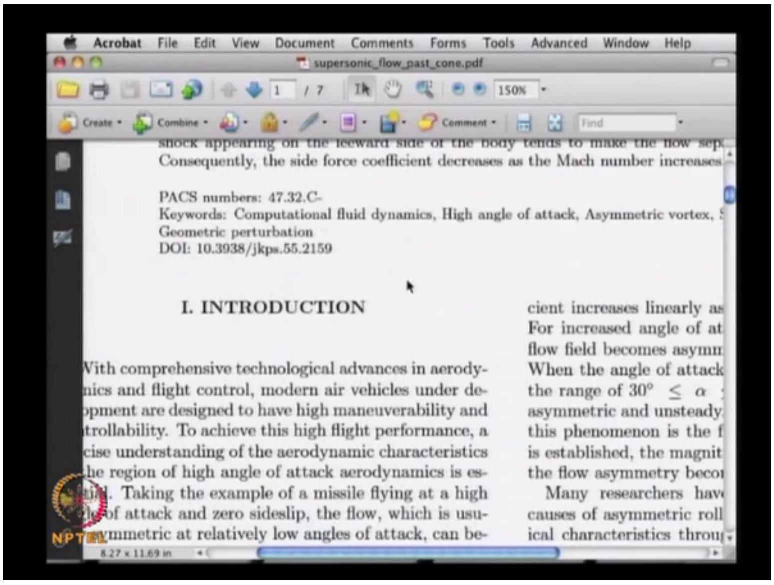

And I think you know pictures are nice way to see things. So, you can you know in down of this newspaper for yourself and I think this should be enough for you to get high and get a hold of this particular paper, if you already do not have it. So, this should be enough for you to know. So, let us just look at this thing let us just look at this. So, I am not going to detail is to how they have calculated and things like that.

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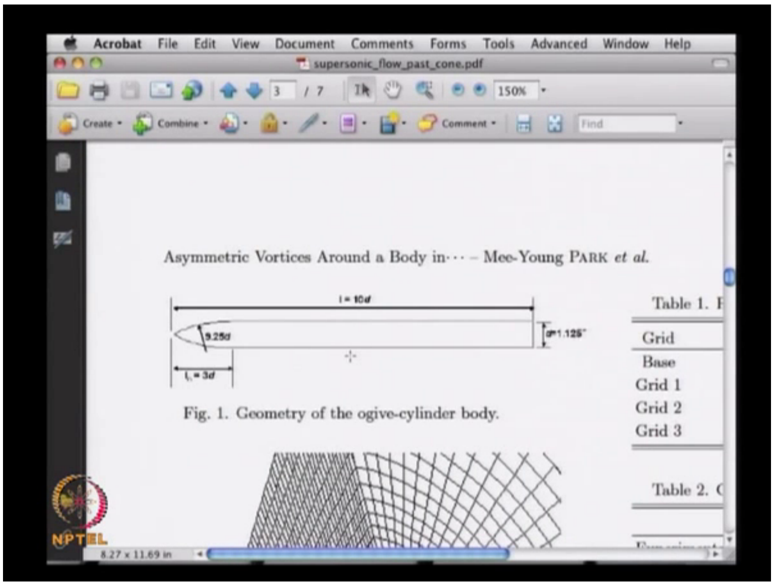

I am just going to show you the picture as to just to give an idea what sort of you know numerical results are we looking at how will this results look like once you get because it is very difficult to get a bunch of numbers is I mean how do you interpret those I think that is the most important question.

Right. So, for example, look at this picture here, let us look at this picture. So, this is essentially there. So, if you look at this, this is for the cylinder that they are considering which is like this give cylinder that we are talking about. So, you can see here right you have a pointed you know nose or cones it similar to what we have been starting so far is not it. So, if you look at this. So, this is l is 10 d. So, this d is basically this whole this links out here is not it. So, if you look at the this is the entire length. So, this d is given and this is l n I think which is 3 times the d, 3 times or from the cone vertex to the bottom I guess to the base is 3 times the d and the entire length is 10 times this diameter fuel look after the cylinder and. So, this is it alright.

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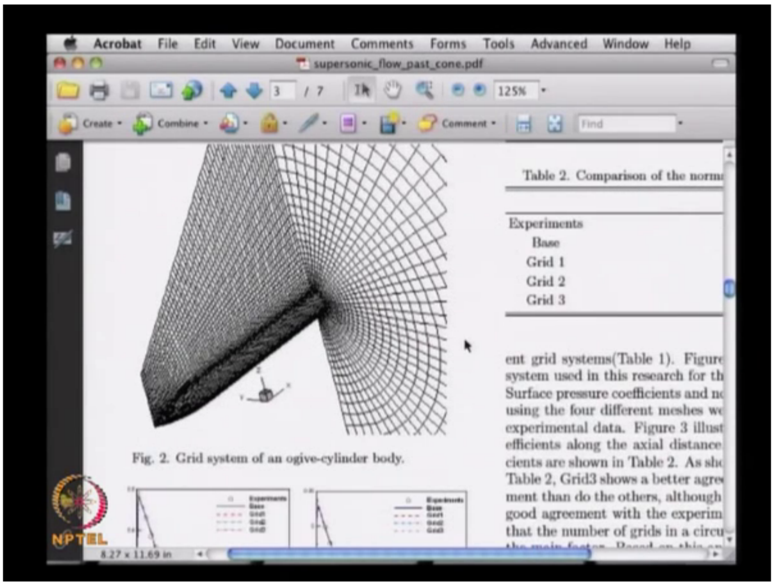

So, this is the this is the geometry that they are considering and this is the picture of how their mesh looks like, this is the picture of how the mesh looks like. So, the grid system. So, this is the. So, that is the a grid system and if you look at this, this is what I was talking about.

So, right. So, if you look at this. So, this is how the grid looks like right. So, this is how the grid looks like. Now here I would like to point out actually all though you know might be knowing, this is typically done I mean for all those of few who are new to you know see if decomposition work and so on and so forth. You can see that you know do you have lorry like dark regions around here you see these dark regions out here and then you seem to have you know these this is actually called mesh or a grid whatever right. So, now, you can see that these sort of rectangles I have found out quite a lot it is I mean you know this is quite large once you get away from it, but here it is pretty congested.

Now, this is a very typical thing to do is not it and it makes sense to us it is not it, because you know if you remember the analysis that we did that as we go further away from the you know very well you remember and use using the linearize theory what happens if it is a supersonic flow, does it do the disturbance die out. So, here too I think the reason we have you know a more and more you know congested you know mesh around the body or near these edges especially the chap one is because we do not want to miss out any information. So, there is a lost happening in a small little place or you know these looks in corners we do not want to miss out on any information.

So, we this is the greatest what is going to give us the information. So, something for analysis on a grid base method is about that. So, we have to really positional grid in such a way that we get the information that we looking for right. So, having done that, so let us go and look at some of the results I thought ok.

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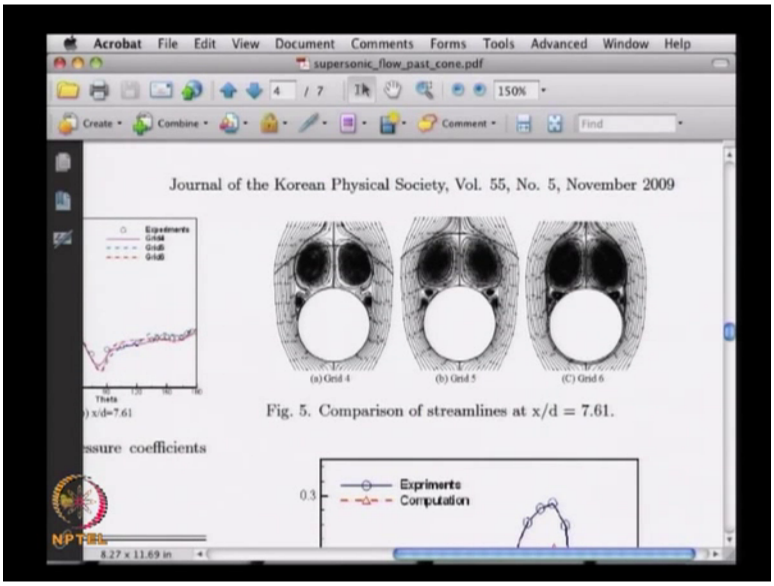

So, this is interesting I think its. So, if you look at say figure 5 here; figure 5 is what I am looking at here, let us see if I can get this up so cannot do it. So, if I am say looking at this right. So, for example. So, let us say right. So, we are looking at this, now this is you know looking at 3 types of grids basically. So, grid 4 grid 5 and grid 6 over here. So, comparison of streamlined the x by d equal to 7.61 what is this mean? X by d equal to 7.61. So, let us go back to the first diagram here.

So, if you consider x along the length of the cylinder a length along the cylinder then. So, at some position x. So, if x is 7.61 times the this d is the diameter, that is what x is equal to x by d is equal to 7.61 means if you look at this. So, x is equal to 7.61 d. So, if you start from the vertex of the cone then you travel basically 7.61 d, that is the section of the conical structure that we are looking at. So, if you look at that. So, what this is showing is 3 types of grids in the showing the solution here and you can clearly see.

You know the difference between the 3 cases here we just point out of few things. So, you can see that we have if we have these two vertices out here you know large circulation here to here we have the same thing, we have so, if this is much you know closer together we have a large condition which means that we have you know like really dark areas out here and again if you look at this here. So, we have two of these vertices which are predict pronounced and which is pronounced mostly for this grid 6, and we can see that in grid 5 is well, but this is not that pronoun that second one does not pronounced. This is how basically you change your grid (Refer Time: 42:50) to see what sort of solution we are looking at and I think what these sort of you know lines these two sort of lines are looking at.

I have not read the paper and whole, but I would our think what they are looking at is to comparing the sizes of these two vertices over here, which seems to be growing which seems to be growing, but I think this has a more congestion the grid 6 to see is has a more it has a lot more congestion of streamlines over here and that is expected you have a larger grid system, that is what you would have.

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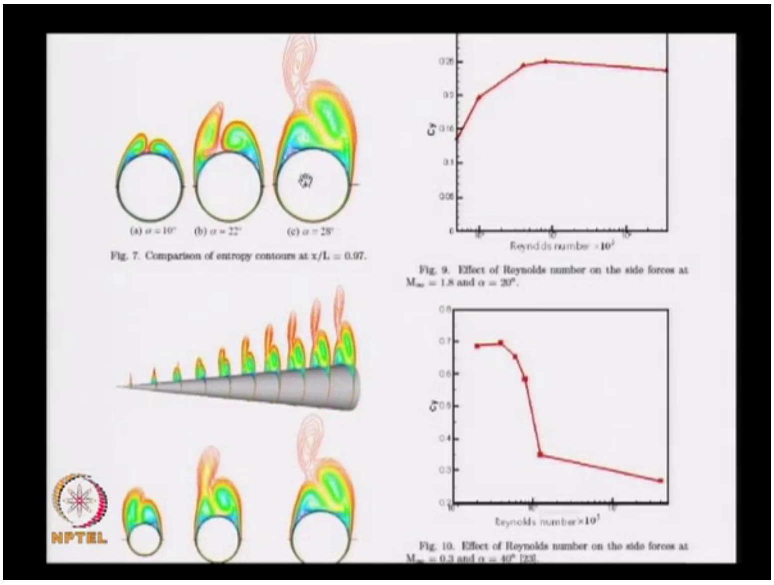

So, I am not going to go here let us go and look at some other some real nice. So, this is the one I think all of you like a comparison of entropy contours at x is equal to x by L equal to 0.97 ok and for different angles of attack.

So, it is a very interesting pictures is not it. So, if I want to look at this, let us see if I can bring this up here. So, if I do this. So, there we go. So, what we are looking at here can we come back there we go. So, now when we say x is equal x by L is equal to 0.97 what do we mean let us go back to this figure here. So, L is this whole length. So, if I take x along this. So, I am almost like to at the end of it I am always to the end of it.

So, it is nearly the at the edge that I am looking at. So, if I go down here. So, if I look at this picture here. So, this is the. So, now, what you can see here it is very interesting what you can see here is pretty interesting let us keep it at that. So, now, what you see here is that as you this is angle of attack is 10 right and this is the angle of attack is 22, angle of attack is 28. So, we really increase this a quite a large leap from 10 to 28 nearly 20 degrees leap and you can see the difference, you can really see the difference in the form of in the entropy contours over here and change of the entropy as we do that. And what is more important I think which we sort of discussing in the first couple of lectures when we started this is the how the vertical singularity you know, how that vertical singularity it moves away from the body as we increase the angle of attack beyond the theta c is not it. So, I think you can sort of see that into play over here. So, you have an angle of attack of 10 degrees, like I said I have not read through this papers I do not know whether this is less than the angle of attack, this is the less than theta c here, but you can see sort of you can sort of sync off you know this point still being attached to over here on the body and then you see this you come here increase the angle of attack and also you see a lot of asymmetry, the asymmetry is very very obvious this is the entropy and you see that here and then you come over here.

So, you can see these two sort of you know loops of this entropy here and then you see this point. So, as you increase the angle of attack. So, and then if I go down here. So, if I. So, what again. So, this is again. So, this the another nice one. So, this is again. So, the what we saw just now is at the same location and if you change the angle of attack.

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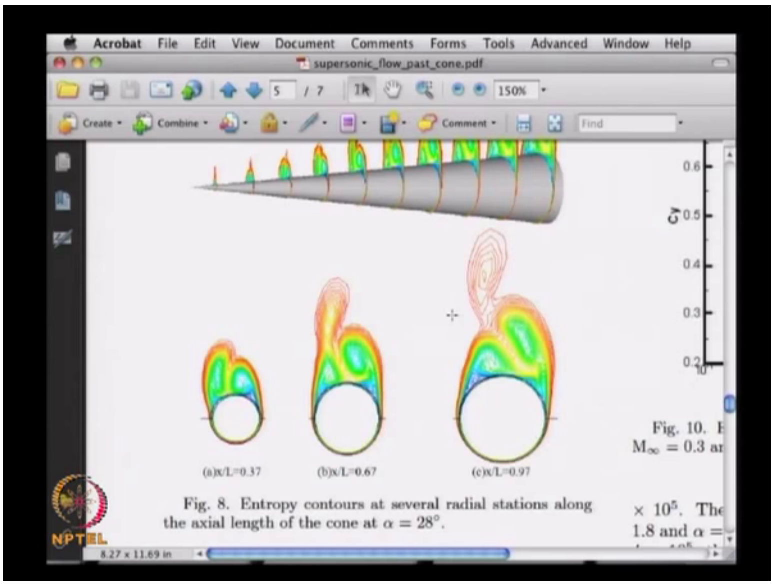

And here we keep the angle of attack the same this is the picture where we keep the angle of attack same angle of attack is 28 degree.

So, it is very highly skewed. So, to speak with the full stream. So, you have a angle of attack and we are looking at the entropy values moving along the length of the cone right which is what is shown over here. So, if you move along the length of the cone, you can see how the entropy is you know varying and how is it changing right. So, 3 little these sections are given here for further look at it.

So, I hope you understand. So, this is how I will look at it and these 3 pictures down here is if I look at it from here or from this side right. So, if I look it at that way. So, you can see here you know that as we move along as we go further downstream on this cone right the asymmetry increases and we have a larger asymmetry or larger changes in entropy ok.

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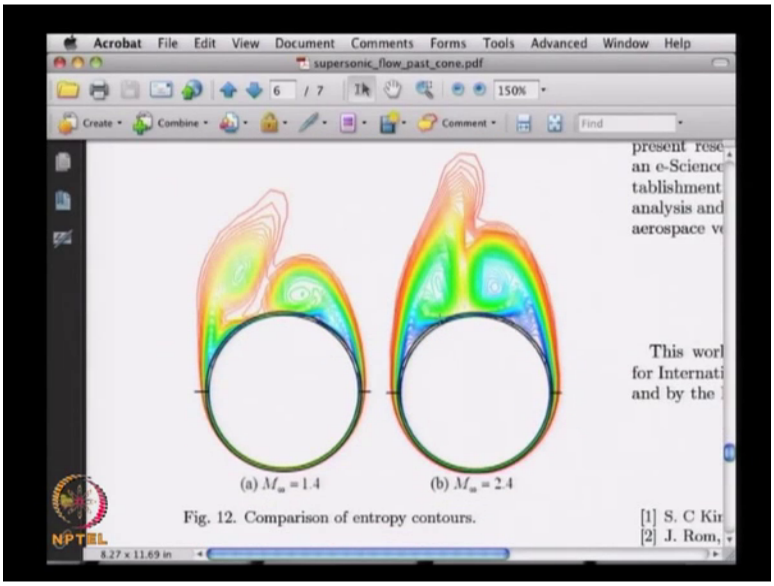

This is what we see over here and this is yet another one. So, we looked at the change in the you know length, now what would look at this as if we change the incoming mach number right you see. If I can see the same the latching is entropy if you change the mach number from 1.4 to 2.4 even you know that also a is a quite a important change here.

So, I think we will end on that note what will do next class is you know supersonic flow pasta plat body because that again will have it is own specialist we will talk about that in the next class. We will stop here and hopefully this gives you at least a startup on to you know what we are talking about in terms of 3 dimensional flow field ok.

Thank you very much.