Advanced Gas Dynamics
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Lecture – 36
Supersonic Flow past a 3D Cone at an angle of attack: Flow Visualization- 1

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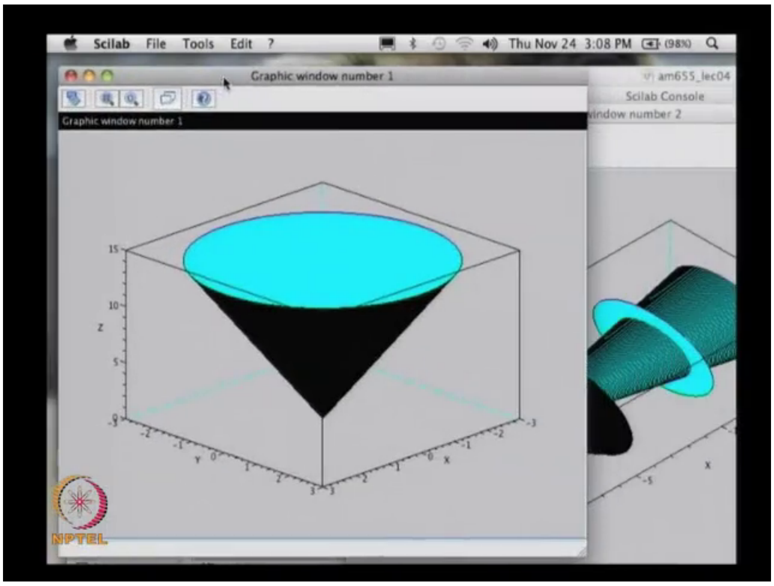

So, let us continue from the last lecture. So, let us go ahead and basically look at the axisymmetric flow properties and then will go over you know go over to the corne at angle of attack right. So, just to kind of remind us as what we did last time right. So, we were looking at the cone right. This is a cone surface right and so this is essentially a solid cone right.

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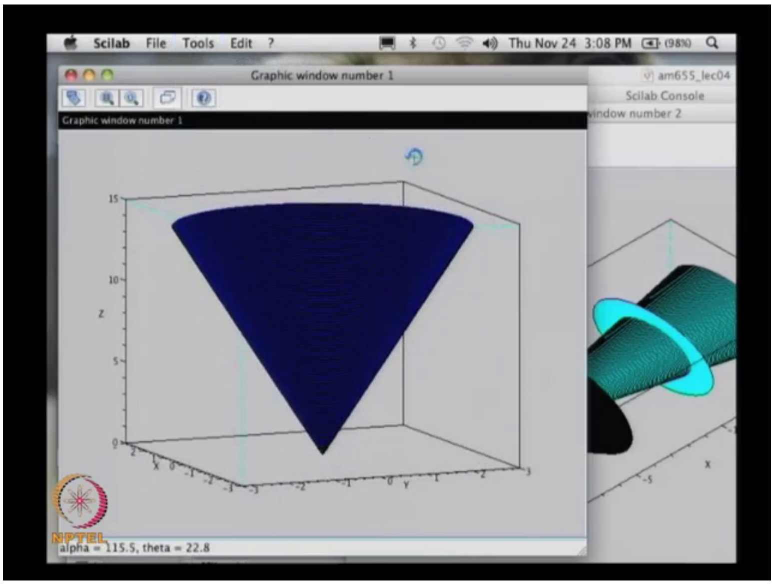

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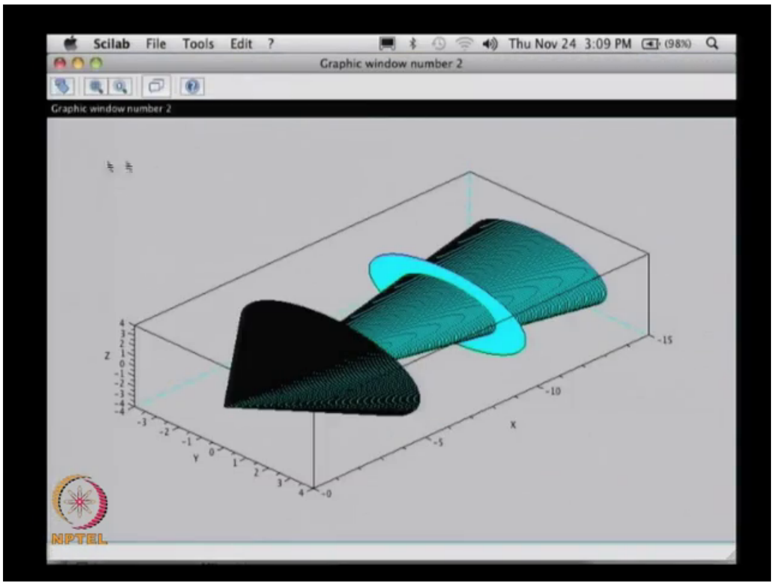

And then we looked at the shock structure if we have a cone if you have a supersonic flow coming onto the cone like that.

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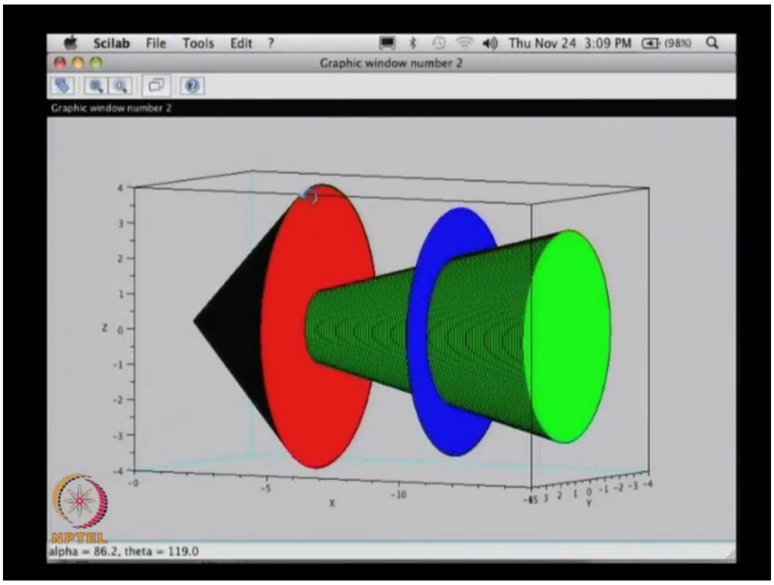

So, basically we try to understand what the geometry definition looks like. You know the geometry is important why because that is the geometry of the shock itself and its orientation with the free stream decides whether this is a 2D flow or a 3D flow. Because if you look at this here, if you look at this shock structure over here, now this is going to be pretty much the same even when it is at an angle of attack with the free stream. But then in that case, however we will consider the flow to be a 3D. In this case when it is the shock the cone and in line with the free stream then it is axisymmetric then it is a quasi 2D flow which we have studied conical flow right, that is the reason why it was important to kind of understand the geometry of it and so and so forth.

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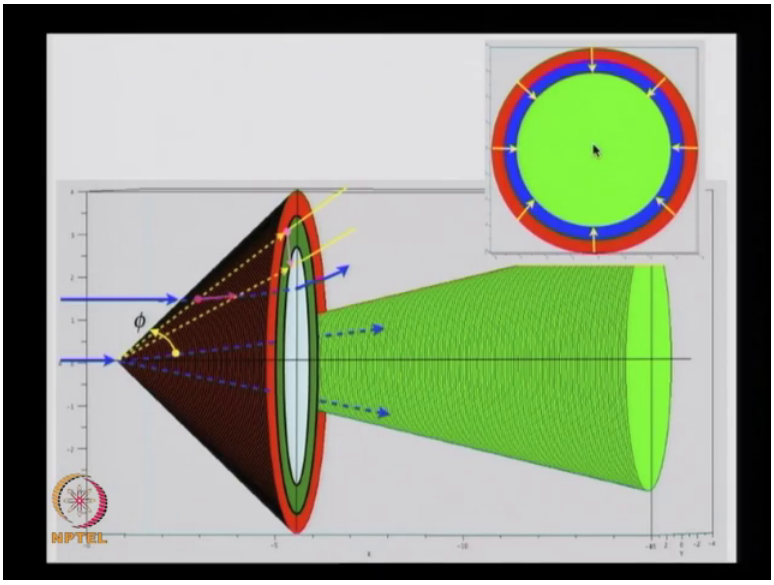

So, let us go ahead and look at the ok. So, we started with this. So, let us now go and look at the basically the shock structure. So, here we are. So, this green I think here is our cone, and the red one here is our shock which is which is happening both these have a same vertex, and this is the free stream that we are looking at this is the free stream. So, in this case, so say we have in a streamline like that it comes and it will deflect, we have something like this.

Now, what we will do is like we just did earlier we will drop. Now, we will basically look at cone you know another cone within this cone. So, these are all you know you know all these know when I draw this dark green stuff here, this is again another cone within this red one right. So, if I draw a radial line, so I am going to basically draw a radial line from there. So, what is I am actually doing here is that I am drawing this radial line and this dotted basically means that you know it is happening inside this you know red cone. So, basically so we have this surface of a cone like that right.

Now within this surface within this cone surface shown by the red, I take a small smaller circle which is shown by this dark green circle here. So, this is inside. So, I drop a radial line from the vertical line within the cone to you know to which intersects the in a streamline here that you can see. So, this is the part of the streamline which is going inside the cone surface again finally, deflects. So, then here basically I dropped this, this dotted yellow line, so that is essentially inside the cone surface which intersects the cone surface the streamline here. And it is actually on this you know green cone all right. So, we will draw drawn one more I will draw one more.

So, this is yet another cone, and I draw one more, I draw one more a line and then what we see here is again it crosses this the streamline at this point and then I see something look over here. Now, the reason we are doing this you know because as we did in the last lecture is that this is a 3D flow. So, the way you look at it. So, the plane in which you are looking at one should become comfortable with that because this is the 3D picture that we are looking at right.

Now, if I look at this thing if I look at this whole picture through the base of the cone, I will see something else when I look at the shock structure through the vertex through the vertex then it looks something else, but it is important to familiarize ourselves you know with the different views of this. So, now basically so these points are in 3 D space as you can see right.

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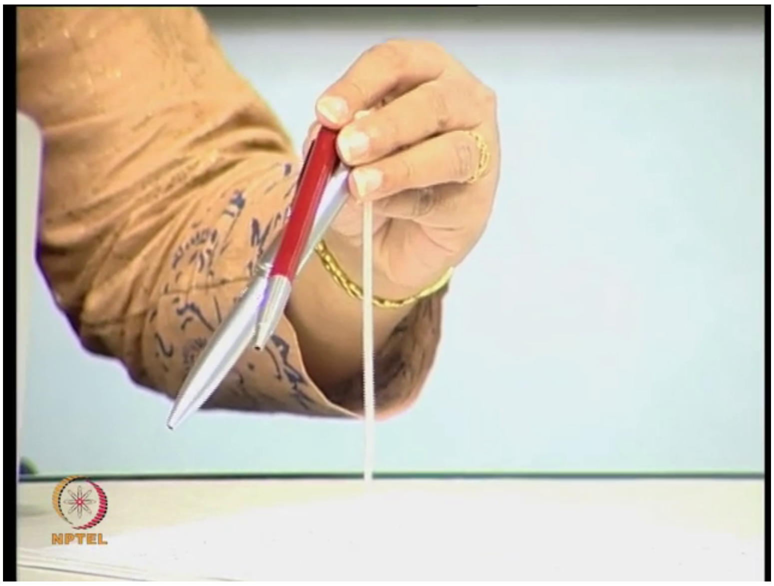

So, all I have done is I have basically you know this is my say in perpendicular about this I have my cone. So, I have essentially I have essentially dropped to radial lines, I have drop two radial line and not saying anything about the phi angle or whatever. So, I have dropped these two radial lines. And what I would like to see is if I were to look at the base of the cone here where these two points would lie. So, what I would do then these two points these two points on the streamline lie where on this base on this xy axis or in this plane surface xy plane. So, if I see here so if I can say basically this streamline is moving from here to here, so the streamline moves from this point you know shown by this filled circle to this arrow. So, the streamline moves in this direction. And in this movement or this positioning is translated onto this plane into this plane as such. So, in this plane is moving from here to here right.

So, similarly we can look at various other streamlines and we will see how that look now. So, what is important to see here is that we have these you know streamlines, which encounter this shock surface and then they deflect. Now, similarly we will also have you know flow or streamlines which will go on the surface of the cone. And if you remember right the only surface of the cone you know while you did the problem on the surface of the cone basically there is only the radial velocity v theta is 0, if you remember.

So, on the surface of the cone however so we will have these straight streamlines. So, essentially you can think of it that you know you have this cone you have this cone and you have flow coming in and it just you know goes out in all directions equally symmetrically about this particular point, you know about this surface. So, here is my you know cone here is my cone and I have flow coming in. So, I have a shockwave. So, some of my streamlines come here and deflect, and then I enter here and I encounter the shock and encounter the solid cone. So, then I just come there and move you know uniformly over the cone and move along its surface. So, basically these are all straight lines. So, we have you know straight streamlines which go on this surface of this cone.

Now let us look at the 2D picture of this. As you know here, so this part here, so this part here is the green part is the cone. And the red is basically the shockwave well blue is the section which we were using we not using that here. So, it is just a reference you basically this entire place is you know filled with the shockwave. So, now in here how would these lines look I am just trying to look at it is if I were to look from here what would these look like to me. So, if I look here, so if you these will be all translated in two lines like this on the 2D face here.

So, therefore, in this particular case, these two you know dotted yellow lines that I have these are basically lines of you know phi. And if you remember here basically about the vertex about the vertex I have a symmetric flow happening. So, if you remember from what we did regarding the conical flow right. So, there was not any change in the phi direction as far as the flow is concerned. So, therefore, you know that there were no changes in properties in the phi direction. And so properties were properties were constant along the along the radial along the radius here along the radial line, properties are constant over it and they vary only in the theta plane. So, if I change the theta this way, in this plane it will change. So, therefore, the only dependent variable out here is theta.

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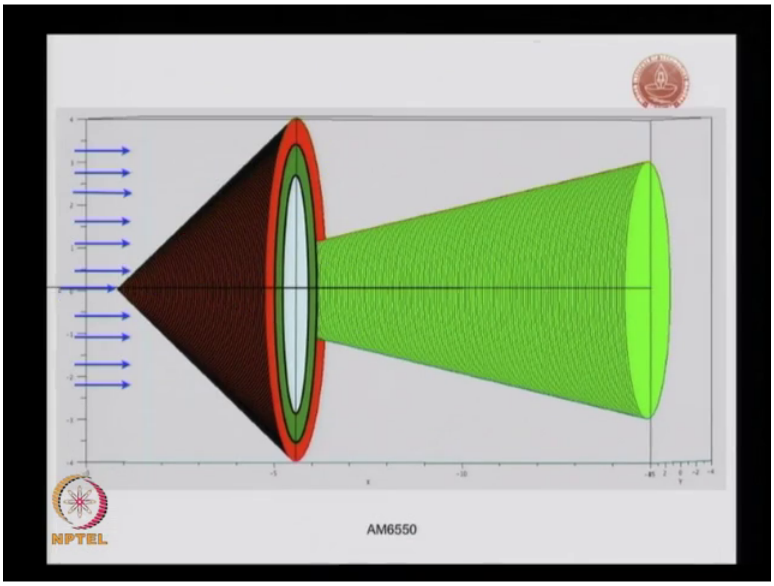

So, looking at this picture here; let us now look at what exactly is happening when we talk about the 3D flow or conical flow at an angle of attack. So, since essentially this is an axisymmetric case, this is an axisymmetric case. So, if I will just sort of remind you here, so you have you know streamlines like this, you have streamlines like this. And these are the streamlines on the surface of the cone. So, keep that in mind and let us go and look at this. So, what exactly is happening in this particular case? So, we have this you know this surface, this is a cone, this is a shock wave which is you know on the sitting on the cone.

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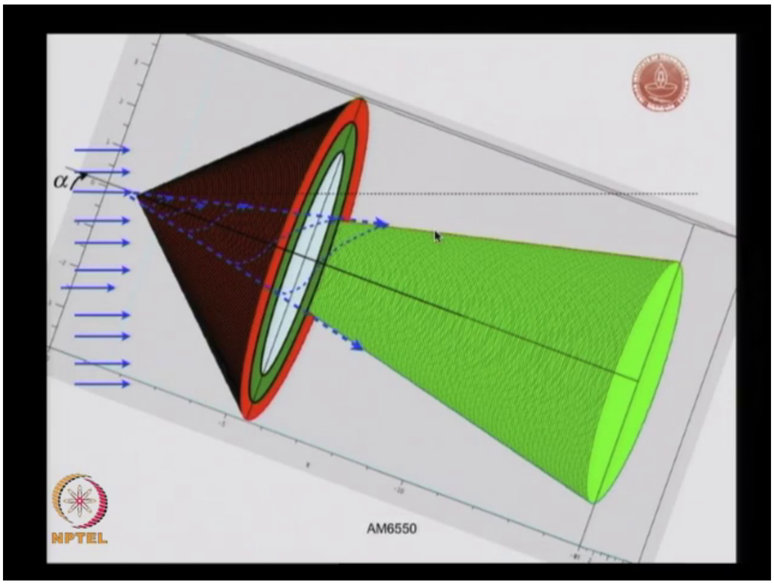

So, what is this. So, what we have actually here is that this now if you look at this, this is the free stream, this is the direction of the free stream, and you can see that the z-axis or the line of symmetry of this cone, I know a shock structure is making an angle of alpha with the free stream. So, this is essentially the angle of attack. So, this is the angle of attack. Now, so if you look at this picture here, so this dotted black line is essentially extending the free stream direction. Now, it is important to look here and find the difference between this, this and what we just saw which is the axisymmetric case.

So, clearly now if you look at this. Now, previously what we saw is that now free stream here now my access system would be to the free stream with a certain angle of attack, the lines of symmetry of the shock and the physical cone failed to coincide. So, what is important to note that you know the obviously, the geometry of the cone it is a physical cone that remains the same, but the flow around it is no more symmetric. So, the flow is a symmetric and due to the inclination of the cone. And what do you mean to say is now we do actually have one more parameter to take care which means that this is a you know another increase in dimension any other free stream.

Now, if you look at the free stream and about that try to see how this shock is located, you know how the shock looks like with respect to the free stream. So, look first of all for example, look at this red plane and the shock structure. And look at this conical shock with respect to the black dotted line here, you can see the a symmetry in it. And then just compare that and similarly and also compare it that, if in fact, the free stream was along this dark black line then what would the free stream be seen most importantly what would the free stream be seeing in that case. So, look what the free stream is seeing now and if the free stream was along this dark black line then what it would be seeing.

So, if you look here this, this dotted line. So, you can see that basically in here the symmetry is lost, the symmetry is lost. So, we do will not have flow coming here distributing equal in all directions and move along the surface of the cone, you never going to have that right we are not going to have that. So, we had a streamlines coming in you know straight streamlines coming in and deflecting and going past the shock structure here. Now, these are going to be curved here is that they are not going to be straight streamlines.

Now, let us look at this. So, here basically only for the so the way the cone is located only to streamlines right at the top of the cone and at the bottom of the cone, they will be straight sort of straight lines. And they are the ones which will guide along in a top and bottom surface of the cone. The rest of it now if you see look at if you think of the cone, so you have this you know, you have this cone here right. So, you have the free stream right free stream which comes; and then from the bottom of the cone, it moves to the it will curl around thee what it will do the flow will come here. And if you look at this free stream it will come here, it will meet the shock the solid cone here; it will curl around the bottom and come up to the top surface. So, what it will do is it if the free stream say comes here if I am looking at say this region here it comes here, it is curl around the bottom and move to the top of the cone.

So, now, this let us look at some of those streamline and see what I mean by that. So, basically what is happening is it is coming from the bottom curling up and coming here, come from the bottom and curling up and coming to the top, come from the bottom curling up and coming to the top. Let us do that again. Let us look at this again. So, this is my angle of attack etcetera. And then so we have these two streamlines will of course, be along the cone surface, and then you have the streamlines on the rest of the surface which will be curled streamlines.

Now, if you remember if you just try to declared what we did one slide back if this was axisymmetric, you know all these lines all these curved lines there would be no curling from the bottom of the top, they would not be they would all just be straight lines like this. So, in here, so I have a flow coming in I have a flow coming in right like that, but you have for the axisymmetric case, I encounter cone right here and I distributed you know I distributed equally in all sides and I started going this way. Instead here now I come here my cone is like this my cone is like this instead of being right in front of me my cone is not symmetric to me at all. So, I just move just for the top and bottom surfaces, there are one straight lines and then as I try to move I try to go I go from the bottom and curl up and go to the top that is what the streamlines are showing and well some just trade name.

So, this is the when it comes from this is a windward side. So, windward side, it goes from the windward side of the leeward side. So, well this is I guess you know you will probably call this from your geography definitions in back in school because the (Refer Time: 20:28) of the mountain was receives the wind that is called windward and the other set is called leeward right. So, here so basically you because of the angle of attack, so you have the cone this way because of the angle of attack you can see you have the free stream here. So, that will kind of reach the bottom part which is so the bottom part is receiving the you know free stream and then that curls run and goes to the top. So, this is just a training.

So, essentially if I were to look at this, so I think it is important now, so that we will kind of you know deliberate is to what exactly is going on here. So, now, if we look at this, now in the axisymmetric case, what are the velocities that we had. We had a v r, we had a redial velocity in the direction of the riddle direction, and we had a v theta right, we had that. We do not have any v phi, because it is axisymmetric. In this case, we have a v are we have a v theta and we have a v phi we have all of that. So, why do not we just sort of understand why do not I use the board and try and show you what I am trying to say over here.

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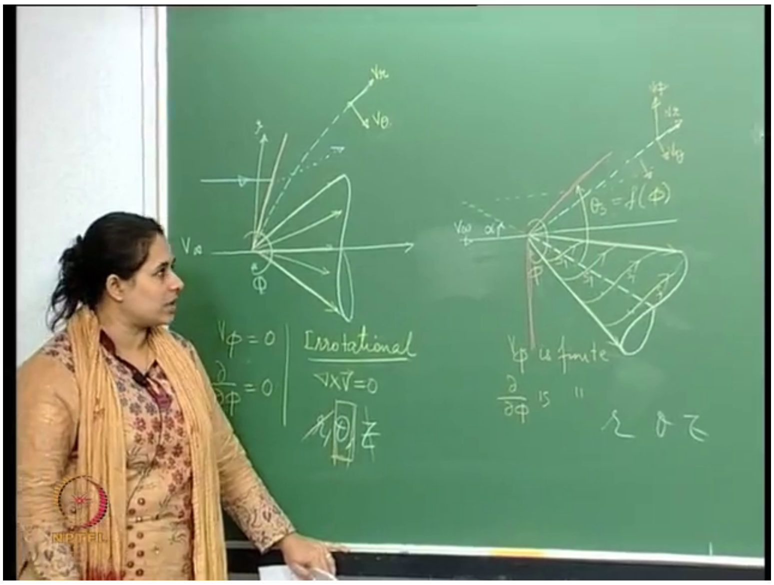

So, so this is something that we have done last time right. So, if I do this, so this is my basically this is my z direction and this is the radial direction, and this of course, is in the a phi. So, the phi here is of course out of the plane of the board this is it this time hopefully you understand what phi means. And theta is of course, in this plane if I look at it in a plane which is perpendicular to the z – the x and y axis. So, and this was our free stream, this was our free stream. So, in this particular case, let us look at so we will have a shock structure somewhere, we have a shock here. So, we have a shock here somewhere.

Now, let me just take a radial line let us just take a radial line. So, this is a radial line. So, what sort of velocities are we looking at and this here. So, what sort of velocities are we looking at, what we are looking at here is radial and that. So, basically this is the velocity that I am velocity component I am looking at when I am looking at the axisymmetric case. So, essentially now also and there is no property variation phi direction. Now, the properties remain constant along a radial line, however, change as we change our theta. So, this is something you know we do so. So, in this particular r and z are independent. So, we have only one dependent variable which is our theta.

So, now let us look at the same picture. So, let me look here, let me make this a little short. So, here let me look at the same picture. So, now let this is our free stream and now what we have here. So, what we have now this is the z. So, this is the z. So, if you look at this, so this is my z-axis. So, this is my angle of attack. So, this is how it looks like at this point of time. So, again I have a shock waves and so on and so forth.

Now, again I will again draw a radial line, I will draw again a radial line. So, if I look at this, say I will draw a same similar kind of a radial line. I do this then what sort of velocities are we looking at, we are of course looking at v r we are looking at v theta we are looking at that, but we are also looking at a v phi we are also looking at this is my phi. So, in this case you will have to see that I am also looking at a phi direction. So, in this particular case, v phi is not zero. So, essentially, if you look at this particular point, so I do have a change of properties if I take my radial line you know in if I give my radian line a deflection of some phi in this direction, in this phi direction.

So, earlier case if I remove the radial line about you know the center line, you know in like that I would not have I would not see any changes. V phi first of all here is 0, this is for the axisymmetric case. So, any property change there is no property change in the phi direction. Here of course, that is not so. So, here V phi is finite, this exists. And of course, so del del phi is also finite, del del phi is also finite. So, this does not go to 0. So, this is in terms of what sort of velocities we would be looking at.

Now, also we just seen right we have just seen that we will have you know straight streamlines over here for this particular case for the axisymmetric case straight streamlines on the cone surface as well as here within the shock and the cone. Whereas here we will not have that we had streamlines which were which we saw something like this. These were the streamlines which were moving like this and so on and so forth. So, this is also important, they were curling up from and moving and except we had these two streamlines which we are moving on top. Here on the other hand, if you look at this, so on the surface we would have something like this, would have something like this. So, that is basically which is giving us the you know slowly you can see the difference between the two cases.

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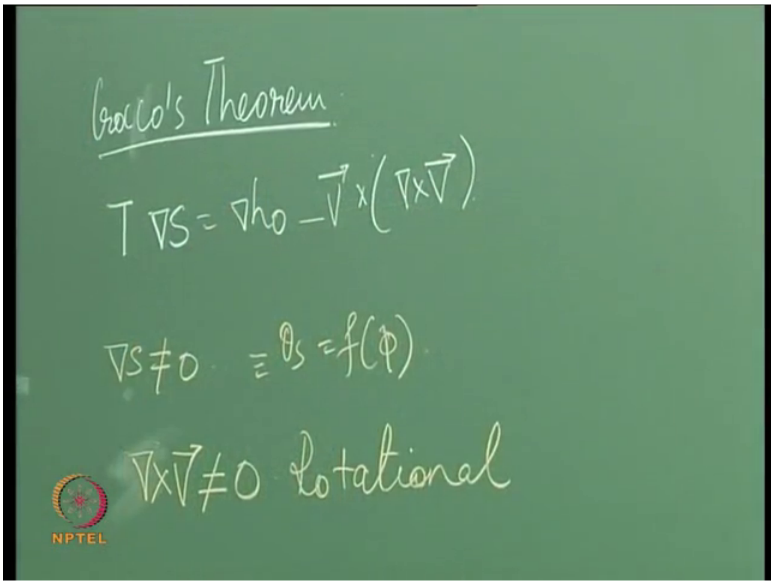

Now the next thing is if you remember the Taylor Mccon equation, let me sort of write that down. So, if I write this down not the Taylor Mcccon actually the Crocco's theorem. So, now if I have the Crocco's theorem, so now if this was our Crocco's theorem right and we considered adiabatic and inviscid, and as a result of which our delta s was you know isentropic. So, delta s was 0, and adiabatic and inviscid so our enthalpy was 0. So, and that meant that this was automatically irrotational. So, if which was in this particular case. So, in this particular case it this was irrotational.

Now, important thing is if you look at these two pictures here, does this flow out here does this flow out here look is rotational to you it does right. These two floors look very different, if you look at the two cone structures here, they look very different right now irrotational. Now, does this look a rotational to us or not. We use this of course, So, we have adiabatic inviscid etcetera, etcetera, so we got you know from here we were able to say since you know these two go to 0, so automatically it is irrotational.

So, this was a statement that for axisymmetric case here, the flow is irrotational. So, if you look here since in this case since v phi is 0, v phi is 0, so this del cross V in this case is 0. So, essentially what we are looking at is that the velocity in a plane which is perpendicular to the x y plane which is the cone surface in this particular case. So, in this plane whether del cross V is 0. So, a this is irrotational in it this particular case. But look at this here you know clearly this is this does not look like it is irrotational. So, the flow see the we do not have straight streamlines we are streamlines which are curling also streamlines which are within between the shock wave and the body there are no more straight lines there are also curved lines.

So, what exactly so how can we explain this, how can we explain this. Now, for a streamline now for a you know a streamline it has the same it has it has a constant entropy. So, streamlined will have a constant entropy, but in this particular case now in here ok. So, when we have the shock wave now look at this, this sort of in this picture here the shock wave what is the angle of the shock wave angle what is the shock wave angle how does it look like in this particular case

So, if I am a sort of go back to my the diagram that we were doing over here. Let us go back to that and this is the diagram that we had right. Now, if you look here this red this red plane here now it could have the in our point on this a point on this green plane or on this light blue plane here you know it can have a different phi. So, therefore, in inside this shockwave, now this is our if you look at this dark, you know black dotted line here that is the free stream this is the free stream. So, about this if you look the shock wave angle is varying as we make different phi angles.

So, if I make a different phi angle, so if I make a certain phi angle about the axis of this cone then my shock wave angle will also change which is not the case what we did for the axisymmetric case. So, in this case the shock wave angle is also a function of the phi angle is not it. So, we have a shock wave like this. Now, however, now since this shockwave right is not symmetric about the z-axis. And if you look at the free stream direction then every time you make a small change in phi, every time you try to do that, there is a change in the shock wave angle. The shock wave angle also changes every time you make a phi direction. So, therefore, phi, so theta s, theta s which is our common nomenclature, theta s is a function of phi.

Let us go back to the board do that. So, if I were to do this let me just say I will just draw say I am drawing this as a shock wave, I am drawing this is a shock wave. So, clearly this is a spherical surface this is a spherical surface I am just drawing it like that. So, therefore, in here if you were to look at this the shock wave angle if I were to use this. So, this shockwave anger although right this theta s, now that is a function of phi that is a function of phi.

Now, a single streamline, a single streamline has a particular value of entropy. Entropy is constant on a particular streamline. Nut in here when a streamline is going from one point to the other, where it is going from one point to the other then there is a certain increase in the entropy. There is a certain increase in the entropy because when it is going from one point to the other, there is a change in phi, which means that there is a change in the shock wave angle. And if the shock wave angle changes right which means that the nature of the shock changes is not it, is not that what we have learned. Whether a shock is very strong or weak and so on and so forth that depends on what the difference of the shock wave angle.

So, as the flow comes in here, it goes within the shark structure goes within shock structure then I have a streamline, it is within the shock structure is moving from one point to the other, the theta s is changing you understand what I am trying to say. For example, let me try to do this here. In this particular case, so this is my shock wave, and I have a streamline which is say coming here now as this moves as this moves within the within the shock wave. Now, there is no change in the phi direction there is no change in the phi direction which means that my radial lines that all my radial lines are equally spaced. There is no change in the angle in which those radial lines are located. So, if I take the vertex of the cone all my radial lines are all equally fanning out from this particular point.

So, when I have this streamline, so a streamline comes it moves in here the angle of the shock does not change; angle of the shock does not change. So, therefore, the constant entropy constant entropy, and hence we can use the Crocco's theorem del s 0 del s 0 and hence it is irrotational. Here though that is not the case, if you come here if you come over here so if you look at the if you come to the vertex, you come to the vertex then we have a shock structure, which is developed. And then I move from one point to the other when I move from one point to the other, since right this cone and the shock is not symmetric is not symmetric about the z-axis.

As I move from one place to the other, the angle of the shock with respect in the phi direction you know the phi that changes right which means the shock changes which is almost equal to saying that we encounter a different intensity of the shock, which is I think that would be fair to say. So, I encounter a different like I said a different shock, if I encounter a different shock then essentially I will have a different entropy and which means that I will have an increase in entropy as I move through the shock and the finite increase in this entropy non zero increase in entropy.

So, and that basically means phi direction few direction is so if when I have the streamline moving in the phi direction. So, therefore, in the direction perpendicular to this say yeah the streamline. So, if I have this is not a streamline this is a radial line sorry. So, whatever streamline I have a in a particular streamline like that. So, perpendicular to it I do have a finite increase in the entropy. So, in here is not zero. So, delta s is not 0. If you look at Crocco's theorem, there is a finite change in the entropy and that is the reason. Because as the streamline moves from one point to the other, there is a change in the azimuth angle, which is the phi which in turn means that we are looking at a different you know strength of shock with the theta s is I have a different you know entropy attached to it.

So, therefore, in here it is irrotational because here right. So, here this is not equal to zero it is finite, and therefore, we do have in this particular case entropy, we do it is irrotational, it is rotational and not irrotational. So, means that this is which means that this is in fact, rotational so unlike in the previous case, I think we sort of closing into this it will take with a while to kind of try to understand the geometry a little bit more try to visualize that, and hopefully this will start making a little bit sense it was you think about this a little more.

I think another suggestion would be if you can you know make you know like I try to do this using you know scilab and you know PPTs and all that. So, you can try to make a model yourself try to make a model yourself and then try to visualize as to what I am saying and look at this shock and cone in an axisymmetric sense as well as a asymmetric sense right. So, let us ask one more time if I look at these here. If I look at this cone if I look at this cone out here, what do you think will be for example, in here I have these streamlined. So, what sort of entropy am I looking at, what is the entropy from here to here the each streamline? Well each streamlined here each streamline will have say you know say s 1, s 1, s 1 each streamline will have the same entropy. On one particular streamline, there is no you know there is one value of entropy right.

Now, you move from this plane into another plane, which means you are changing your phi. So, this is also another way of looking at it is that in this particular case del del phi now when we go ahead and you know use this in our equations which we will do that hopefully another couple of lectures. So, then you will see because del del phi is not zero; in this particular case unlike here del del phi is 0. So, anyway when we start doing the equations these terms cancel out. So, here this is finite, this is finite. So, when we change the phi, when we change the phi then the change in the entropy is also finite the change in entropy is also not zero. So, that is another way of saying that, so that is another way of looking at the you know this entropy change in this particular case.

So, of course, two sort of wind up slowly, so in this particular case we have, so this we have an r and theta and z which actually we have these coordinates. We have this coordinates for both these cases. Now, if you look here for the irrotational, the only dependent variable here is the del theta, these cancel do not need that. So, we basically have one dependent variable, so that is all and then we will have this you two components v r and v theta. Here of course, we have two dependent variables theta and phi of course, so theta and phi unlike in the previous case. So, here of course, again the del del phi is finite, v phi is finite.

So, having a sort of come to that for chronicle flows, we use the Taylor Maccoll equation. So, Taylor Maccoll equation and we solved for it. So, we will see here how you know what we are going to do in this particular case when you have an axisymmetric case. But before that I want to sort of you know look at this geometry a little more try and understand as to you know what is well what is happening to the streamline once it goes inside the shockwave and why it happens and so on and so forth.

So, again if I were to look at for example, this picture here if I were to look at sort of this picture here. So, which is this was for the symmetric case, this was for the axisymmetric case. So, this is for the axisymmetric case. And what I am looking for is so this you can see that this is the now it is important to see this over here. So, you sort of move from phi. So, this picture looks like this is not it, this looks like it. So, if I look at the plane are you know region here, and I sort of what I am doing is super posing you know this you know these flows superposing all this on the cone surface onto this plane. If I keep a cone here as the flow coming flow is moving all around it, I am trying to look at it from this face, I am superposing all that flow. So, if I were to look at this face, this face like this I would basically see the point move from here to here. So, I would see things move you know like this.

So, now in this particular case again what happens? So, if you have something like this, so if I were to superpose this on to the play onto the plane then what would I see so that is something I would be interest interesting to see. So, let us see if we can do that in the next couple of lectures and we can sort of figure that out. So, we will do that a first try and understand the geometry because the geometry is very important over here, and then we will slowly go into you know looking at the equations. And hopefully by that time all of that should make sense all right, nothing else stop there.

Thank you.