Advanced Gas Dynamics

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Lecture - 34
Examples Problems

So, let us continue from last lecture. And we will go ahead and do another problem like we said supersonic flow encountering a flat plate, angle of attack. So, let us see what we are going to get over here.

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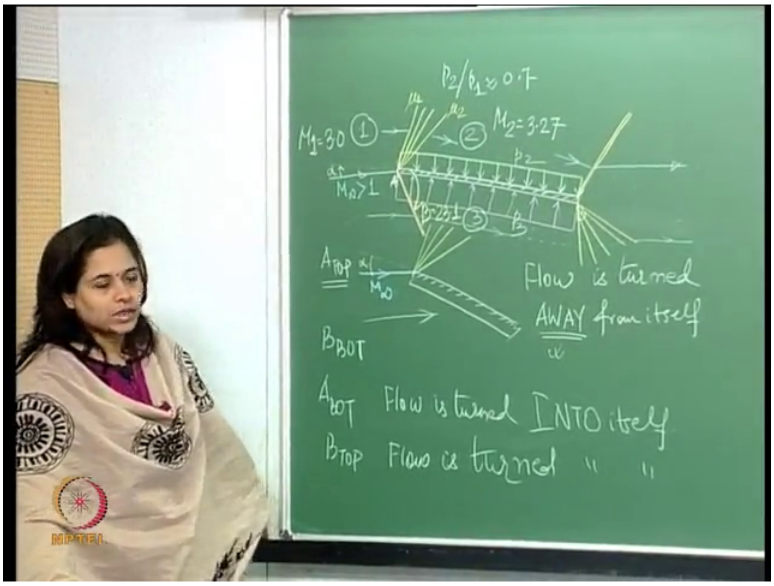

So, this is my flat plate. And as we saw in the wedge from last time and with this flat plate as well basically dealing with surfaces which have really straight surfaces. You know this is like one straight surface, the wedge yesterday that we did in the last lecture that we did basically consisted of three straight surfaces. And we were able to use you know without too much of complications, we were able to find out coefficient of lift and drag and you know talk about the properties in general.

So, if I have this and what we have here is essentially so we have a free stream. So, let us say this is a free stream, it is a supersonic free stream, and this is making an angle of attack of an alpha. We have this let us call this as say A and B locations. So, basically this is the flat plate you know it is got this is slightly thick here. So, we do have a pressure distribution on the top surface and the bottom surface. So, let us kind of draw that. So, on the top surface; so on the top surface we will have, so this is the top surface pressure distribution. And on the same way, we will have the same on the bottom; you will have the same at the bottom.

So, essentially we basically have you know the flat plate which is you know just a straight element straight segment, free stream, supersonic angle of attack alpha this is a kind of pressure distribution on the top and bottom surfaces. Now, when this encounters we have a supersonic speed when encounters this flat plate what do you think is going to happen. Now, look at this corner, look at this corner here at A and look at the top surface look at the top surface. Now, if you look at this top surface of really be exaggerated that. Let me exaggerate the top surface.

So, let us say this is A at the top surface. Therefore, here I have the free stream coming in. I am just going to draw a line for the top surface. Just look at this we are looking at this particular surface out here, this is just the top surface. I am exaggerating this. So, I have a free supersonic free stream. And at here at the top surface here, this is the angle of attack which the flat plate is making with the free stream. So, what you think is going to happen at this point, what does this point look like?

So, from what we have learned so far this is expanding over this corner, this is like an expansion corner here. So, what we would expect to see over here is an expansion span right. So, let me draw that right here. So, let me say draw it this way. What we expect to see over here is an expansion fan. And if I were to draw the same thing over here, so this is the expansion fan. And basically here if you remember right the mu 1 and mu 2. So, this is. So, let us in this. So, now, that we have an expansion fan over here. Let us call this region as 1 and this region as 2. Therefore, what I will do is called the Prandtl-Meyer function corresponding to this is mu 1 and this as mu 2. We have that.

Now let us look at the bottom surface at a. So, if I look at the bottom surface at a over here. So, what we can say is in this particular case if you just look here. So, flow is basically turned away from itself. So, this was the flow it is turned away from itself, so that gives us an expansion fan. Now, let us look at the bottom. If you look at that right over here itself, this is the flow then because of this the flat plate here, the flow is turned into itself is not it if you look at this the flow is turned into itself.

If you consider a free stream like that the flow is turned into itself. So, here what should that give us it should give us a shockwave. So, what we will have at the bottom actually is a shock wave, we will actually have a shock wave over here. So, we will basically have a shockwave in here. So, let us call this region again as this region as 3, which is behind the shockwave. So, this region is one right which is the region in front of the entire flat plate. We have the free stream over here. And at the top surface, it encounters this expansion fan up here. So, the region behind it let us call that is region 2. And at the bottom surface we have a shock wave coming in, so the region behind the shock wave is say region 3.

So, let us do that. Now, again so in here now that the flow has you know is expanding over here, flow is expanding over here. Now, let us look at the trailing edge B, look at the location of the B. Now, the flow came in here. Now, let us see what will happen at the trailing edge. Now, the flow came in here flow came in here it expanded. Now, in here we add the trailing edge, the flow still needs to merge with the free stream. The flow still needs to merge with the free stream. And when it comes over here, see the flow is moving along the say flat plate out here, now it has to go along the free stream over here, you cannot allow it to expand any more or move away from itself.

So, this sort of definition comes in handy you know when we trying to understand this. In a sense that when it came at this corner it had a surface to sort of guide it along, so it came in here it expanded but went along the surface. But now if when you come say towards the end of it when you come towards the end B, if you are still allowing it expand it, we will not have it moves smoothly away from B, it will not leave the edge B smoothly and it will not join the free stream. And we do not want that. So, in order that you know it follows the free stream smoothly at point B. So, this will have to be shocked so to speak. So, basically we will turn the flow into itself, so that it keeps following the free stream right and it when it leaves the top surface at the point B. So, what we will have here now is a shockwave.

So, let me sort of write that here. B at the top flow is turned into itself B at the top, so we again have a shockwave in here. So, what we will have over here is A so that if you now look at it flow and it expands comes over here, now you shocked this, so it with the shock it deflects and keeps a following it does not come and expand over here and leave it like this. It comes here, it reaches the shock and it deflects and keeps going according to the free stream that is what the shock does. And on similar veins if you look here if you look here basically now when it gets shocked, it gets deflected comes here it would have gone this way you do not want that at the bottom. You wanted to go along the free stream, so you are going to expand it along this corner, so that it follows the free stream.

So, therefore, so what we have at B, B bottom actually So, B at the bottom flow is turned away from itself. So, we have an expansion fan right. So, we will have an expansion fan over here. So, let me just sort of try to draw a free stream or something. So, if you have say a streamline which comes like that. So, it will move like this, and it will move along this will be my free stream, free stream values it comes like this now if when it comes over here. So, I have a free stream which is moving like that. Now, over the surface now if you look at this is not it. So, this is how it is moving. Now, I will clearly in even in encounter the shock what happens it deflects and starts moving this way. So, there you go. So, we have your free stream back.

So, similarly when we come here, so say we have this here right and it deflects. So, we get something like that, it comes over here. Now, look at this right and it continued it would have gone that way we do not want that we want to go along with the free stream. So, it goes like that and then basically follows the free steam to reflects and follows the free stream. My artwork is a little out of place here, so there we go a little better. So, there we go and that is how it flows. So, now this is all. So, in a simple sort of very innocent sort of problem, we just have a free stream which is supersonic and we have you know a flat plate with an angle of attack. We see a lot of interesting things going on here.

So, now let us go and find out basically the lift and drag coefficients for an angle of attack of 5 degrees and the Mach number 3, for a Mach 3 flow. So, how are we going to do this. So, now that we know this let us proceed here.

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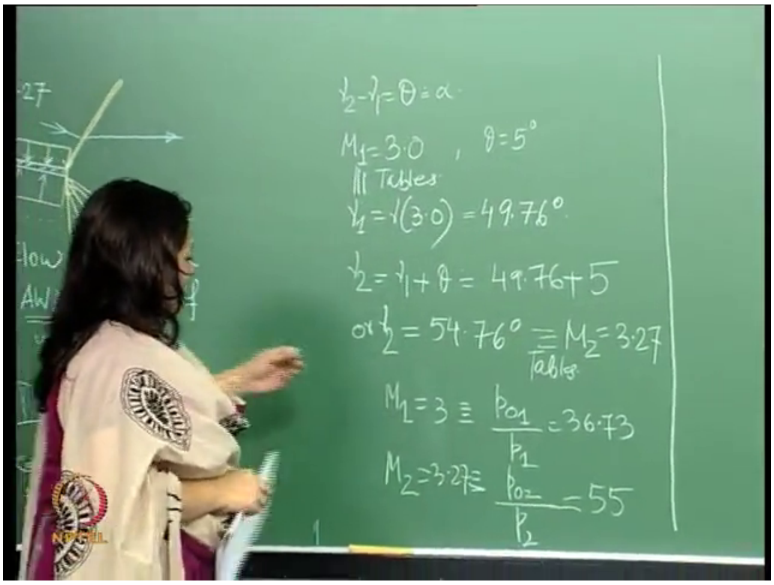

So, first things first, so here the deflection here is what deflection here if you see deflections here is, so basically Prandtl-Meyer function from Prandtl-Meyer function we know that theta. So, this is my same thing. So, from the Prandtl-Meyer function, we know that mu 2 minus mu 1 is theta-theta is a deflection which in this case is our alpha is not it. So, this is known to us now, which is 5 degrees. So, now this is here. So, I can calculate mu 1 because I know Mach 1. So, now, M 1 is given to us as 3 and theta is equal to 5 degrees. So, when we look up the tables, so from the Prandtl-Meyer function tables corresponding to M 1 equal to 1, M 1 equal to 3, we get mu 1, which is the Prandtl-Meyer function corresponding to Mach 3 and we get that as forty nine point. You should go and cross check this; you should go and cross check this and make sure I have got these numbers.

So, once we get that, so then we will calculate mu 2, mu 2 is equal to mu 1 plus theta. So, this is say it is 5. So, once we do that what we get here is so or 54.76 degrees. So, once we have that again corresponding to this mu 2 we can calculate the Mach number from the Prandtl-Meyer function tables. So, corresponding to this we get this from the tables to be 3.27. So, it is expanding. So, it is speeding up. So, this is given. So, we get these from tables, we get these from tables mu 1. Then I calculate this again this we get now we kind of go back and forth, back and forth. Here we knew the Mach number we calculate the corresponding Prandtl-Meyer function here we need the Prandtl-Meyer function, we found out of the corresponding Mach number. So, we keep going back and forth. So, this is from the table.

So, essentially what we see that we what we have found out now is the Mach number in this region 3.27. So, let us write that here. So, we have the Mach number in this region to be 3.27 and Mach 1 is 3. So, once we do that, so now, we can again find out from the isentropic tables. So, for now M 1, so let us find out. Now, for M 1 corresponding to this again we get this from the isentropic tables. So, M 1 equal to 3 from tables we get p naught 1 by p 1 and I get that as nearly 37 and corresponding to Mach 2 equal to 3.27 from the same tables, we get p naught 2 by p 2 equal to 55.

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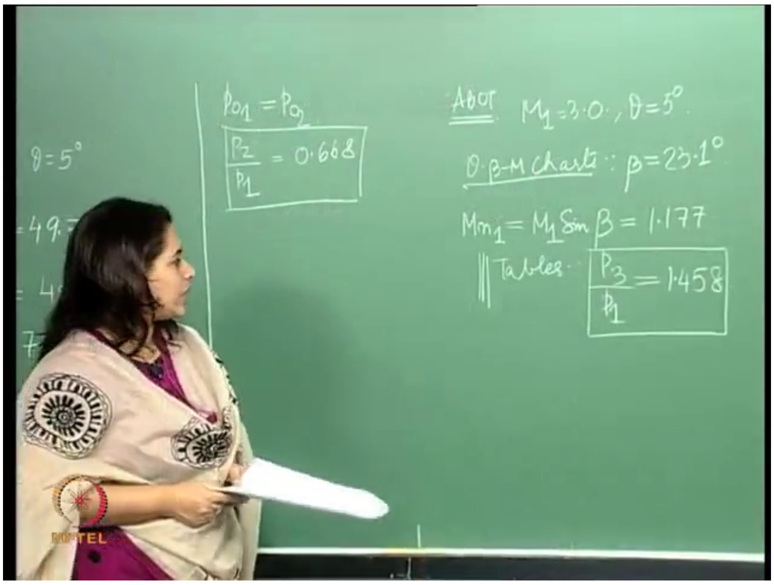

Now, so this is it. Now, p naught 1 the stagnation or the total pressures they remain same. So, p naught 1 since p naught 1 is equal to p naught 2. So, therefore, we can get a value for p 2 by p 1. So, what we will get here is p 2 by p 1 is equal to which is nearly 7. Now, all of this that we did this is really this is at; so all these calculations that we did. So, now what we have found out is so p 2 by p 1 is nearly 0.7. So, this is the relation between the pressures now. So, this was the expansion fan.

Now, let us go and look at the bottom surface. So, now, at the bottom surface, so we have a incoming Mach number of 3. So, we have a theta, which is the deflection which is 5 degrees alpha angle of attack, so this is also given. Therefore, from the theta beta m relationship we could find out the shock wave angle. So, from the theta beta m charts or plots or graphs whatever you want to call it. So, corresponding to that, we get a beta which is 23.1 degrees. So, this is the shock wave angle let us go back in here. So, basically what we get here is the so this is the shock wave angle, this is 23.5.

So, we get that here. So, now basically what we will do like we did last time you want we want to get a and you know the pressure, the pressure distribution and hence we will be able to calculate the total force and the lift and drag, and hence their coefficients. So, let us look at the pressure changes in this particular case when the flow is going across the shock the bottom surface. So, now that we have we have an oblique shock here. So, then you know the usual M n 1, so we can calculate that by M 1 sin beta that comes out to be 1.177, and this is the normal component, this is the normal component of the shock when across this the oblique shock.

So, once we have that then corresponding to this again from the normal shock tables what we will get is the static pressure ratios which in this case is p 3 by p 1. Remember this. So, this is the region 1. So, I am looking at pressures behind the shock in front of the shock this region is three. So, we will call that p 3 by p 1. So, we get that as around 1.46. So, we have a p 3 by p 1 we have this. So, now, let us go ahead now you know that is all basically we need out here.

So, we have an expression for p 2 by p 1, and p 3 by p 1 this is something that you know we have already learned done it. So, many times as long as you understand what the shock this flat plate is doing basically expanding the flow here shocking the flow here, shocking it back and expanding it back we are basically through. So, now just to calculate the; let us look at this here. So, what exactly how exactly will we calculate the (Refer Time: 25:46) series.

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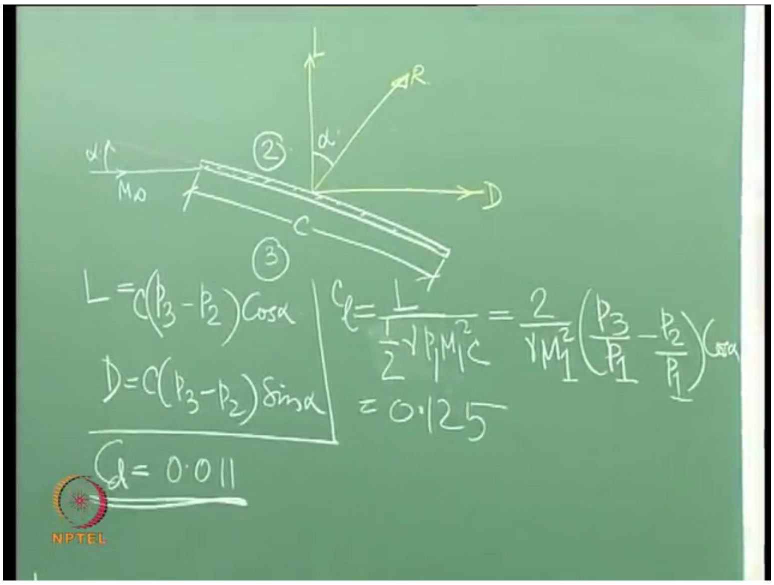

So, if you look at this, although I think you should be aware by now, but anyway let me write it and do this here let me draw this here. So, the this is my flat plate. So, this is my sort of exaggerating a little bit. So, this is my free stream and this is the alpha. So, in here therefore so we have pressure distribution on the pressure distribution on the bottom. So, then we have this say there is a resultant force. So, I take the components of the resultant force with basically this I mean in the direction of one is in the direction of the free stream, other is perpendicular to it. So, it is the free stream that we are taking as reference.

So, then this becomes my drag and this becomes my lift. So, as you can see over here, so this angle is basically alpha, this is basically alpha. The way to look at that, the way to look at this is that this flat plate if it was like this then I would have my R vertically like that. Now, I turn it here. So, I turn this resultant vector by an angle alpha which is the angle of attack. Therefore, these whole things transfer. So, then but we still take the component of the force in the direction perpendicular to the free stream as lift and in the direction as drag, it will be ok.

So, if I do that if I do this. Therefore, that is alpha. So, if you see from here and I think this length we will denote this as C, which is you know this is the nomenclature which we normally use the cord length or whatever. So, if you look at this, so lift now, so lift out here is. So, this is this was region 3, this was region 2. So, what is the total pressure, total pressure difference is basically what p 3 minus p 2. So, like we said if this was horizontal then the entire lift would be just p 3 minus p 2 - the pressure difference. Now, it is inclined. So, we want to take this. So, this becomes p 3 minus p 2 cos alpha and multiply it by the length you know. So, this is the total length, so that is all there is to it, so p 3 minus p 2 cos alpha that is it.

So, then you know c l and the drag of course, is you know in the other direction. So, again p 3 minus p 2 sin alpha c that is it that is all. So, now, again let us write this down. So, C l, so if I were to write here let me write it over here now or now here that of it. Now, C l, therefore, now this becomes lift by this is a dynamic pressure, so which if you remember right this is a half rho v square and all that. So, we wrote this differently you know last lecture. So, just remembering that, so the way we will write this is half gamma p 1 M 1 square by C this. So, the bottom basically becomes this gamma p 1 M 1 square C. So, l if you see over here we will put this l out here we will put p 3 minus p 1 here. So, what we get if we do this, so if I put this in over here what we will get is 2 by gamma M 1 square p 3 by p 1 minus p 2 p 1 cos alpha.

So, therefore, now we know M 1 given to us is 3, p 3 by p 1 and p 2 by p 1, p 3 by p 1 is what we have calculated p 2 by p 1 is what we have calculated. So, we know these values alpha is 5 degrees. So, we can calculate this and what I get from here is point around 0.12. So, coefficient and lift comes out to be 0.125. So, similarly if you do the drag, so again c coefficient of drag comes out to be coefficient of drag. So, coefficient of drag is the same expression except this is a sin alpha instead of cos alpha, so that comes out to be. So, which is a pretty good thing because you know the coefficient of drag is not very much, so that is it that is all we do. So, essentially this if we say this was our pressure p 2, this was our pressure p 3.

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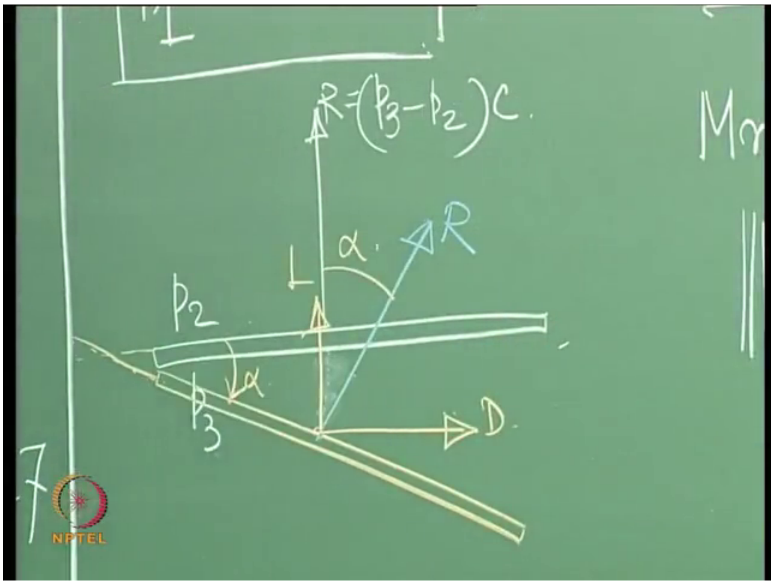

So, essentially, if I had the pressure p 2 and p 3 here, so my entire resultant force would be what would be p 3 minus p 2 into c, which is this distance. Now, what I do is I incline this I incline this and I incline it by alpha. So, then what happens is so this comes here. So, then this is still R, this is the same R, but we will need to take components like this, this is your lift this is your drag. So, as you can see now I rotate this whole thing. So, this becomes alpha, so that is it.

So, all we have basically done is used our knowledge of you know the expansion fan and oblique shocks, how to calculate proper is it, this is something which is not new. But what is interesting is how we were able to apply it somewhere here and calculate or a study properties across surfaces, which again here are basically made up of straight elements. So, like we saw yesterday, so we had more than one surfaces attach to each other and we were able to use you know our knowledge of shocks and expansion fans etcetera to calculate the lift coefficients of lift and drag across them as well.

So, this is like a simpler case when we have like one you know surface let us you know just look at you know one more small problem, and then we will cut a sort of move onto other things, so that is that. Now, instead of say something like this let me leave that in for a while let me leave that in for a while just to you know make a comparison. So, let me leave that. So, like I said we are basically talking about you know talking about you know surfaces which have several straight lines.

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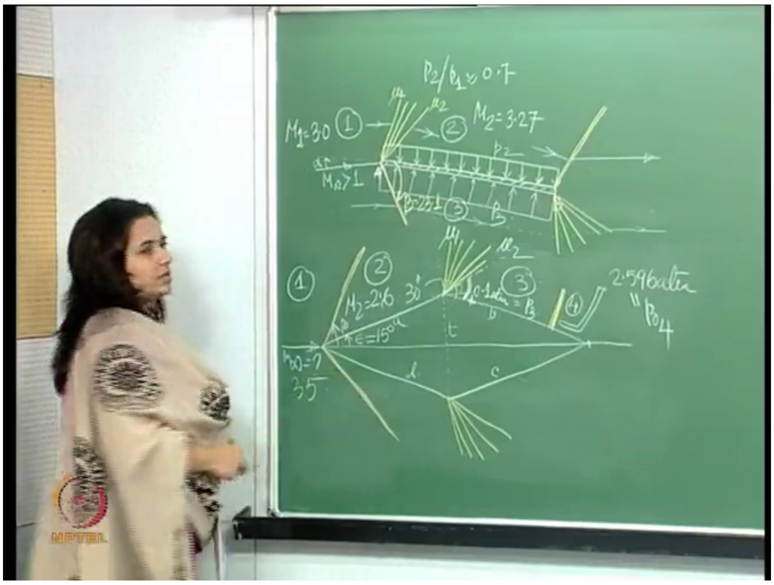

So, for example, let us look at this trying to be this. This is basically a diamond shaped wedge. You can think of this like yesterday what we did was just you know just this part, if you look at this you know just the top triangle we did something like that is not it, but now we are looking at this is like a diamond wedge. So, and of course, we have a free stream coming over here. So, this is my free stream. So, again if I have to calculate the coefficient of lift in pressure we are looking at each surface. So, we basically look at the pressure distribution there and take the net pressure force coming on resulting due to that.

So, now if you look at this, if you look at this thing what you think is happening in here. So, let us look at say let me call these that is ok I am not calling anything let us say here let me call these surfaces, let me call these surfaces as say a, b, c, d; and say this angle is epsilon, and this entire length, this length is say t. So, this and this is entire length is t. So, now, if you look at the announced let us start looking at this wedge like we did here right, for example, here we looked at the top part we look at the bottom part so and so forth. We look at there, so will do the same thing here.

So, if you look here if it makes more sense go and draw this separately and look at that. So, if you do this, so if you just look at this part, let me just you know draw up say this separately this point out here. So, I have the free stream coming in here, and I have this surface this surface coming in here. So, what do you think is going to happen, the flow is being turned into itself. So, we will have a shockwave, you have a shockwave. So, what we will do here is now what happens at the bottom what has happened to the bottom here same thing here to the flow if you look at this the flow if here just look at a streamline. So, if I have a streamlined here flow is turned into itself. What happens to a stream line here same thing it is turned into itself. So, we will have a shock wave at the bottom as well here. So, we will also have a shock wave at the bottom over here.

Now, then what happens here, when the flow comes here it gets deflected and then it comes here. And then if you look at this place here, what you think is going to happen we have this surface. So, the flow has it there is a free stream. So, I have a free stream which comes here, it gets deflected, comes here. What do you think is happening over here, it should not have gone that way it should not have gone that way, but what happens, it is turned away from itself right this surface is going away from itself here is not it this surfaces like that. So, it is like an expansion corner. So, what we will have there is an expansion fan, makes sense.

So, you can do this yourself you know that is it. So, essentially what is happening is that if you look at if you extend say this surface, this surface is may turned away from itself. So, therefore, we have a flow expansion over there right. So, this is the deflection angle over here right. So, in here this happens so similar similarly over here too here also we have the same thing the flow comes in here it gets you know turned away from itself. So, again we will have the same thing over here and so on and so forth. And again we will have at the trailing edge again we will use the same explanation that we did there right because now it has to join the free stream here it has expanded. So, it is moving this way you want it back in the free stream. So, you shocked it. So, we will basically have again two shocks here, so that it is you know goes into the free stream.

So, now our again job is so if you have something like a wedge shaped something like this, how do we calculate the c l and c d. So, again let us call this region as 1, I will call this region is 2, let me take that off. So, this region is 2 and this region is 3. So, essentially we will again have similar the way I have draw the pressure distribution over here. So, I will also have the same thing on this surface, and again the same thing on this surface. So, I think you know that by now. So, if I have to do this, so let me go ahead and sort of, so let us see.

Let us do a problem where this all these stuff is given here. So, this epsilon is given to be say 15 degrees. Now, at a particular point here, the pressure is given to be 0.1 atmosphere the pressure is given to be 1 atmospheres. And here suddenly we in insert a pitot tube, we insert a pitot tube because of that what will happen is it you know there is a you insert it there, there will be a shock wave there, is a normal shock actually here. So, if you have a normal shock here and so because the pitot tube is going to measure the static pressure. So, here if the flow comes here encounters this normal shock and let us call this region as 4 like which is just behind the normal shock and the reading of the pitot tube is given to be 2.596 atm.

So, what we have to find out is this Mach number this is not known. So, this free stream Mach number is something that we need to calculate and this is the information that is given to us. So, how do we calculate this? Now, the pitot tube, you know the pitot tube will basically calculate the total pressure is not it, total pressure behind the shock, so which is the region which we have called as 4. So, this region is 4. So, what we are getting here. So, this is actually nothing but total pressure, total pressure in the region 4. And this reading is nothing but the static pressure region in region 3. So, this I can write this is basically p 3 is not it, this is p 3.

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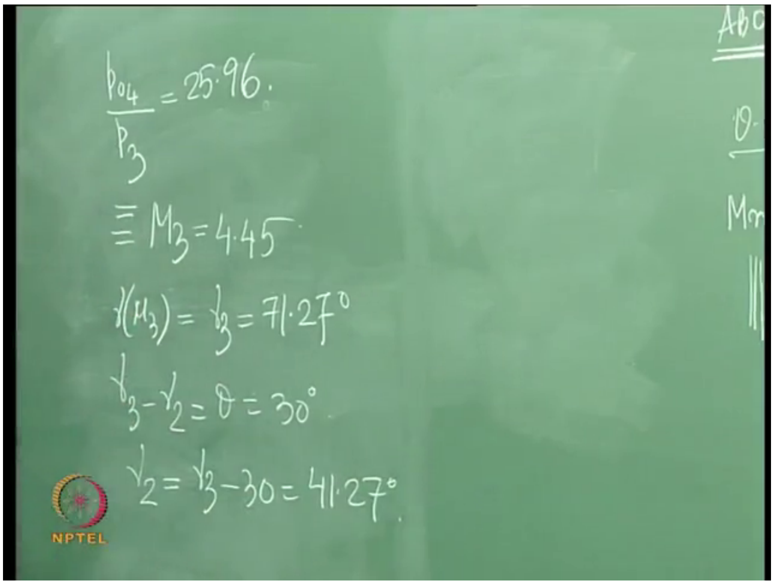

So, what is given to us is essentially p naught 4 by p 3 right and this is given to us. So, therefore, now using this right using this we can calculate the corresponding Mach number from the tables. So, using this that corresponding Mach number Mach which is basically the incoming Mach number into the shock is not it in region 3. So, this is the Mach number which we get as 4.45, you have to cross check this. So, what we get from here we will do a lot of these inverse, we kind of see we do not know anything here, we started from here the last time; here we do not know any anything other than this angle. So, we know pressures etcetera from here, we are almost kind of calculating it back. So, what we have got now is the Mach number behind the expansion wave.

Now, this of course, this you will remember this is mu 1 and mu 2 is not it. So, corresponding to this Mach number, we can calculate the Prandtl-Meyer function of the right end of this expansion fan. So, again corresponding so nu of M 3 we can get as basically nu 3 which is 71.27 degrees. So, therefore, again nu 3 minus nu 2 is the total deflection. So, if you look at this here, this angle is 15, and again this angle is also 15. So, this total deflection here between the two surfaces is 30 degrees. So, this is total deflection is 30 degrees.

So, then this comes this is 30 degree. So, what we can calculate is mu 2 which is mu 3 minus 30 which is. So, once we calculate mu 2, we can calculate the corresponding Mach number is not it. So, corresponding Mach number again M 2 comes out to be M 2 comes out to be 2.6. So, what we have calculated now is the Mach number here.

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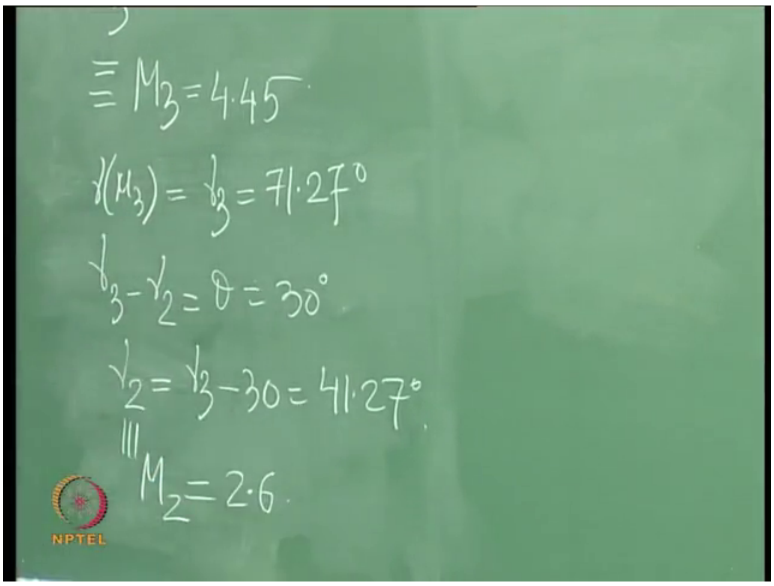

So, M 2 is 2.6. What we need to do is find out this incoming Mach number. Let us see what we have over here. So, let the shock wave angle be beta, let the shock wave angle, let the shock wave angle be beta.

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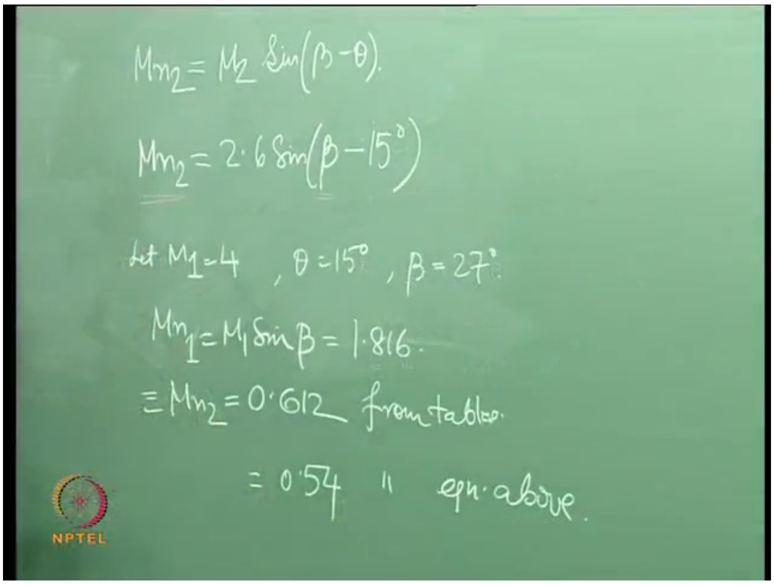

So, M 2 is this. So, we need this normal component which is equal to M 2 sin of beta minus theta is that right. So, therefore, M n 2 is this is 2.6 sin of beta I do not know and theta in this case is this. Now, if you look at this particular equation here, if you look at this particular equation, this is unknown M n 2. I do not know, and beta too also I do not know how do I calculate this well now simple way. So, this is just trial and error what we will have to do is just do trial and error.

So, I will just sort of give you one example of how we can do that. So, let us say let us like one case we can do. So, let M 1 be 4. If M 1 is 4 then theta is 15 degrees, we get a corresponding beta, we get a corresponding beta to be 27 degrees. Then we can calculate M n 1, which is M 1 sin beta and we get that as 1.816. Corresponding to this, we get corresponding to this from normal shock tables, corresponding to this, we get an M n 2 which we get to be here M n 2, I get this. Now, this is this is something this is what I get from the tables, isn't it.

But now if you put this beta into this and calculate it from this relationship from this equation then you get M n 2 as 0.54, this is from equation above the equation that we have written here. So, you can see there that they do not match. So, we have to go and try one more, so that is what I mean by trial and error. So, you have to go. So, next time say maybe try 4.5, M 1 is 4.5 and see what you get. So, more or less when you kind of you know conversion of value which is not too off then you are done.

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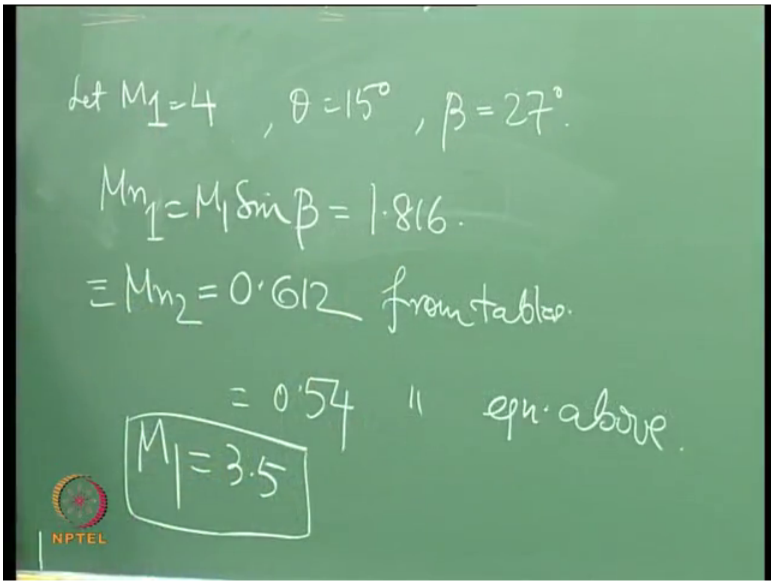

So, one possible solution M 1 is so using this you can you can try you can you know just try one more you can try four 4.5 and then and then 3.5 and see what you get, so that is it.

So basically, therefore in this particular case we sort of came all the way back and we are able to calculate the Mach number here. So, this free stream Mach number was 3.5, so that is sort of takes care of some problems that I wanted to do. What we will do now is trying to understand the 3D flow and like written asymmetric theory which is a quasi theory, but we will try and understand; what is the difference with fully 3D flow. We will try to do that.