Advanced Gas Dynamics
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Lecture – 33
Examples Problems

So, let us continue from the problem that we were doing in last class right. So, basically, we were trying to slow down, right a supersonic flow down to subsonic speed, I using 2 ways. One is just pass it straight down a normal shock, and other words is a combination of an oblique and a normal shock.

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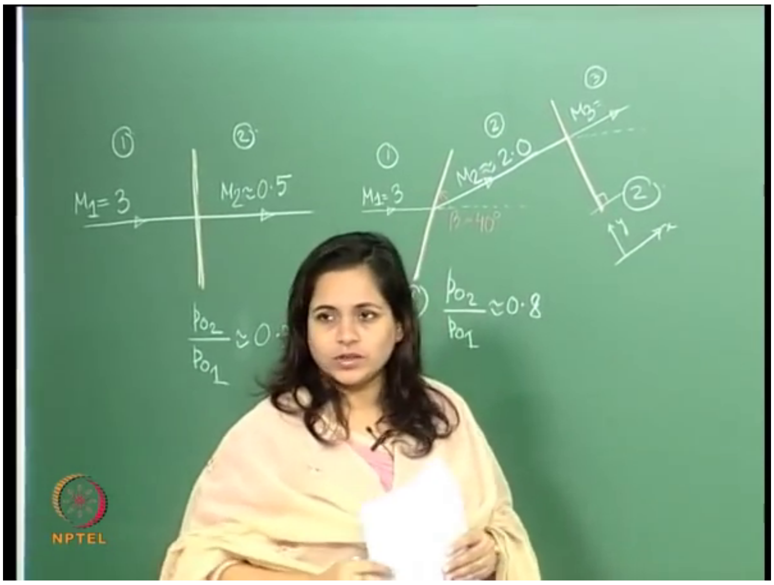

So, just took an a no I will just we started off. So, this was our normal shock right. So, here we go so, right. So, what we got was; so, we had a flow which was this, right. And what we got here was nearly 5, was nearly 0.5 mach we got that 0.4752 to be precise. And we got a ratio of the total pressures. So, basically this is region 2 and this is region 1.

We got this ratio to be 0.33 actually 3283. So, this is what we got let us say it is just around so 0.3. So, this is what we got using the normal just passing the flow through a normal shock. Next thing we did was we passed it through a combination of oblique and normal shock. So, first off, we had this. So, we have an oblique shock. So, then we had the incoming flow. So, again this is 3 this is region 1, then that deflects. And this is M 2, right. We did that, and then we pass this across a normal shock, and this is M 3.

So, what we were so, what we did here. So, this is region 2, and this is region 3 right. So, and this shock wave angle was given to us, right. This shockwave angle, this is the beta which was 40 degrees. And we calculated M 2 to be around 2, it is 1.9 precisely. So, let me just write it like this. What we are looking to do here we have slowed it down, but we haven't yet got to the subsonic you know speeds yet. So, and what we got here p naught 2 by p naught 1, here the total pressures is around 0.8.

So, clearly the drop-in pressure is much less here than it is here. But here it has been you know brought down to subsonic speeds and here it hasn't been subsonic speed. And I am just and we have passed this through a normal shock here, we have passes through an oblique shock here. So, we just we stopped kind of here we haven't done this part yet. So, let us go ahead and continue to do that. So now, what we are going to do is pass it again corresponding, but you know pass it through this normal shock and calculate the properties in region 3. This is our final you know region, and now we will find out the properties there.

So, let us do that let us go to this next space here. So, therefore, we so, for this normal shock now the incoming mach number is around is around 2 right. So, this is the normal shock. So, what it is seeing here? So, basically for you to look at this. Now if you look at this reference frame. This becomes a normal shocks, because now this makes this makes an angle 90 degrees. And the flow is perpendicular to it. So, therefore, now using this shock using the normal shock. So, let us so, this is the oblique shock let us call it say shock structure 1, and this is shock structure say 2.

So, this is oblique and 2 is normal. So, using so, therefore incoming mach number is around 1.9.

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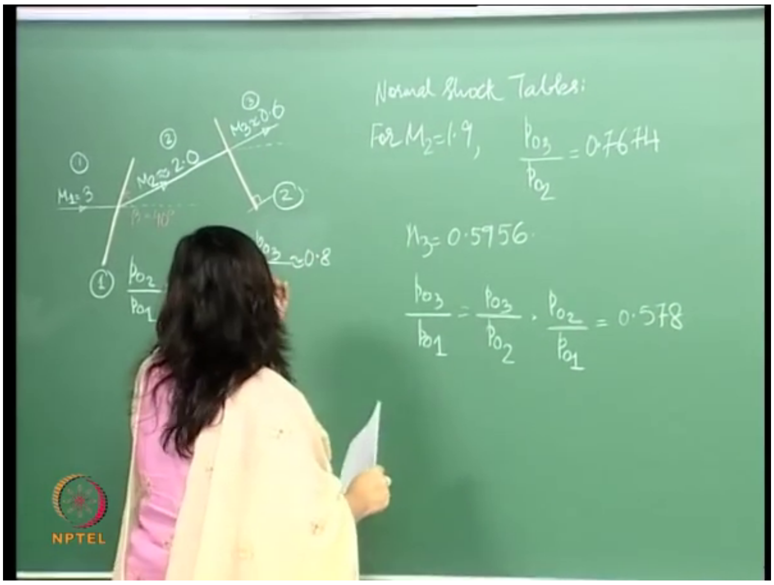

So, we will go to the a tables. So, using normal shock tables, from the tables using normal shock tables what we get is the stagnation is the yeah total pressures behind and in front of this normal shock. So, essentially, we get a ratio for p naught 3 by p naught 2 which comes out to be, right. Which comes out to be this. And it which is nearly 0.8 actually which is nearly 0.8, and M 3 comes out to be 0.5956 which is merely 0.6 nearly 0.6.

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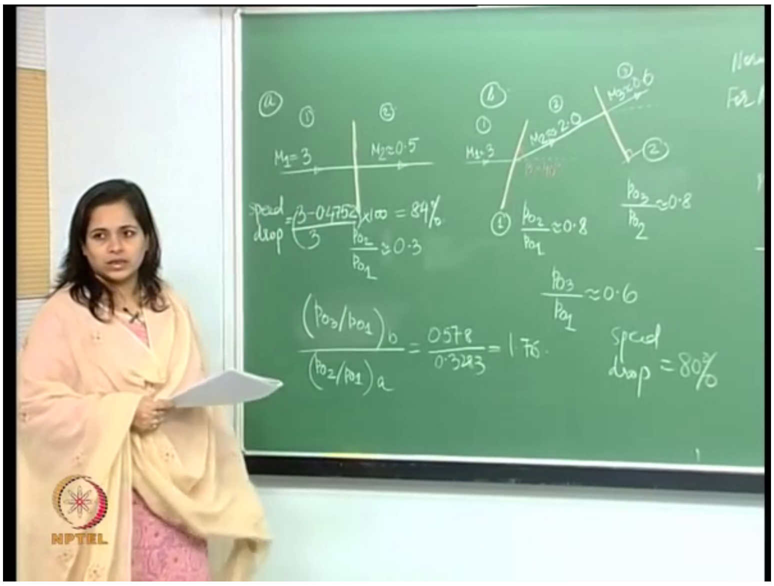

So, let us write that down over here. So, let us not congest this; So now so, what we have found is that p naught 3 by p naught 2, right is around, this also 0.8 and M 3 is around 0.6. So, this is around 0.6 which is and here we got around 0.5 here we got around 0.6. So, we can if nearly at the same speed. Now job is basically to find out whether this is more viable or this in why. So now, basically what we need in in this particular shock structure is what is the total loss in to; what is the what is the change in total pressures in 3 compared to 1, isn't it?

So, for this particular case. So, if I have to calculate p naught 3 by p naught 1, if I want to do this by this time you have should have some idea. So, what we will do is; just some elementary clever math. So, once you do that. So, we have got the values for both of these p naught 3 where p naught 2 is here, p naught 2 by p naught 1 is what we have calculated earlier, right. And this comes out to be which is around 0.6, right. Which is around 0.6. So, here so, therefore, what you can actually see is the here the pressure loss is almost double, that that of in this particular case. Whereas, the speeds are more or less the same, isn't it? If we look at it the subsonic speeds to wish this has been reduced, and this has been reduced is almost similar. Whereas, the pressure drop, the pressure drop in the process for just the single normal shock is almost double than that of this.

So, that is essentially the difference in both of these. So now, let us sort of compare the 2 if I were to compare this eco. So, this and this say so, let me compare this. So, if I call this say case a, right. And this is b right. So, although so, basically say p naught exit by p naught exit and this is entrance, or let me have I write this. This is region 1, this is for case b, right. And if I for case a right. So, this is what this turns out to be. So, what you see is that the total pressure, right. If you look at this the total pressure in you know, the total pressure in terms of in in case b the total pressure in case b, right. Behind you know, right. Here after it has gone through this you know system of shocks is basically 76 percent more than that of the pressure behind the normal shock here, in case a right.

So, what we can say here is that the total pressure. So, the total pressure in for this shock structure, right. After it has crossed both these you know shocks is if you look at this here, 76 percent more than what it is in what it is in this case which is case a, right. Now is that a good thing or a bad thing? Now what that what is that sort of mean. Well, what it means over here now this total pressure, this total pressure is basically it gives us an indication of the total useful work that this can do. And in this particular case. So, the whether weather this is more efficient or this is more efficient is basically going to be decided, depending on how much we are losing in terms of the total pressure. And what we can see over here is that we are losing less pressure when it comes to this; to this system of shocks, right.

So, the loss in total pressure basically gives us an idea about the efficiency of the fluid in the efficiency of the fluid system that you have. So, within this particular case, right. Since we are using less pressure and less total pressure when it comes to here; to this system this is more efficient than this. That system is more efficient than this single normal shock. So, yes also see from if you look at the tables, if you if go and sort of scrutinize the tables little more right. So, if you increase the upstream sort of mach number, right. The total pressure if you increase the upstream mach number, the total pressure decreases. If this mach number is increased, the total pressure decreases.

If you go and look at the normal shock tables. So, therefore, if you see here so, therefore, in this particular case, if you look at this normal shock, and if you look this at this normal shock. So, this is at a higher in the air flow the speed incoming, right. Is that mach 3 here the speed incoming is mach 2 right. So, therefore, if you look at this; this is going to have a higher-pressure decrease compared to this normal shock. So, what this oblique shock actually does is that, it decreases the it slows down rather the speed of the incoming mach number into this normal shock. So, that is what this oblique shock is actually doing right.

So, if I want to keep my you know total pressures in check, I do not want them to you know reduce too much. Then you know I have to work with that mach number. If I can decrease that mach number I will have a less reduction in the total pressure, which is what is happening in this particular case yeah. So, let us just make a percentage of how much you know a mach number is being reduced. So, if you see over here. So, here basically I mean we said we are going to just slow it down to subsonic speeds. So, in this particular case. So, the reduction in mach number speed reduction I would say. So, let us say let us write it over here. So, in this say particular case in this particular case. So, speed drop let me write it this way drop in speed is equal to basically it goes from 3 to 0.5. So, this is it, right. And we take a percentage of that. And that comes out to be 84 percent, and here it goes from 3 to points 5956. So, I do the same thing here. So, here the speed drop do the same thing here, and you get an 80 percent.

So, a speed so, what you essentially see is that for an almost the same, almost same slowing down or drop in speed, we have 76 percent higher total pressure in this case. Which is significant this pressure here is 76 percent you know, higher than the pressure drop in in total pressure in region p 2 right. So, clearly; obviously, this is a more efficient way of trying to sort of slow down here incoming flow. So, this therefore, this is more efficient. So, essentially what you know we kind of you know we have learnt you know, how to you know look up the charts what is happen to the total pressures, isentropic flows how to calculate the you know various properties so on and so forth.

Now, this is kind of an application. So, how you if you wanted you know how you would be actually. So, you can if you look at this you can actually have a system of shocks to use it you know if the way you want. So, this is a kind of using the same knowledge, but looking at it in a different way. So, let us do that for a little bit. Let us do that for a little bit. Let us look at you know another problem, and we will see how we will be able to use the simple knowledge which we have acquired so far you know. So, let us let us say you know and we have these systems, we have sort of we encounter these in lot of practical applications you know, nozzle entrances and you know inlet us several times is so, we are going to encounter things like this.

So, this sort of information comes in handy. So now, I will do something else, we will do another sort of problem, and see how you are going to go about it. So, it is a simple case of basically we have a wedge. We have a wedge, and it is encountering a shock. We want to calculate the drag it is causing. So, let us see how are we going to go about this.

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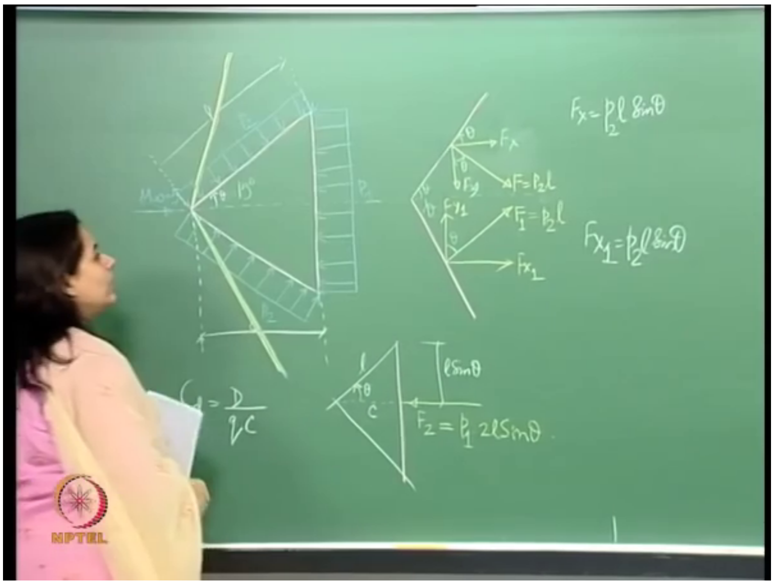

So, essentially if you have say and so, then this is theta. And so, I am going to call this length as l. And we have an incoming mach number, we have this. So, basically, we have you know a wedge like this. So, the mach number is given the theta is given what we need to calculate is the drag coefficient for this wedge, and something else is also given. So, this distance is c.

Now, this theta this theta is given as 15 degrees, and the mach is given as 5. So, we basically have this you know a wedge and we need to calculate the drag coefficient, what we need understand why drag coefficient. So, well if you would probably know this I will just write it. So, this is nothing but this is the drag force, this is the dynamic pressure, and this is the length. So, length or you know area whatever. So, this is your coefficient of drag. So, how do we sort of go about this we know about shocks, and normal shocks and oblique shocks and so on and so forth. What do we do? How do you go about this? So alright.

So, you can see here that, you know I have a mach 5 flow, and I have a wedge out here. So, I am going to have a shock. So, I have a super supersonic flow coming in when we have you know a wedge like this. So, we will have a shock. So, let us let us draw a shock structure over here. So, let us draw a shock structure over here. So, I think that it is not symmetric, but it should be. So, please bear with me. So, a basically I have a shock structure like this now in this case. So, basically what is going to happen now if you look at now it is important to look at this you know body out here. So, if you look at this body. So, this you know wedge out here it basically had the body I can I can define this by 3 straight lines. I got one straight line there, I got one straight line here and got one straight line here. And that kind of helps. We will see how.

So, what we will do is let us sort of you know. So, therefore, there is going to be a despression distribution, right. All these surfaces. So, let me draw that and let us see if that makes any sense to you. So, say so, say now because of the symmetry if you look at this surface and this surface. So, they are going to have a pressure you know just the same pressure distribution. So, if I look at this; so, for example, if I look at this just consider you know this this surface. Just consider this surface. Forget about this whole picture here just consider this surface. If you look at this, just look at this particular you know just forget about this whole thing. Forget about this whole thing just look at just look at this particular surface. And see that we have a shock wave coming over here it. So, there it will be a certain pressure distribution over here. If that is the case, I am going to I am basically going to just draw that pressure distribution like this. Just look at this, right. And let us call this as so, basically this region is 1, I am going to call this as say pressure distribution 2. And it will be the same here as well, it is going to be the same here as well.

Like I said you know it is a symmetric, and this is also going to be so, we are going to call let me call this also as so, this is also p 2. And at the same time, we are going to have a pressure over here. I am going to call this as a P 1, now the basically the point is that this is a body, right. This is a body, and I have a certain pressure distribution on it. So, if I can calculate this pressure distribution you know, it is sort of in a it is encountering a mach 5 flow. So, there is a certain pressure distribution on it, I do not know what that is I do not know how it should be, but you know this makes sense to me with or without the shock wave.

So, this has a certain pressure distribution, and I need to calculate the coefficient of drag. So, drag so, I can always calculate the drag if I can get a phi if you having to get some value for this pressure distribution. In this case; however, it is not that simple because we might have a shock wave here. So, let us go and see how that sort of comes into the play right. So, like I said so now, let us look at. So, if you look at say, I need to sort of emphasize this I am going to emphasize this these sides. So, if I have to emphasize this. So, just look at this here. So, this is essentially so, this is here.

Now, let us look at; let you know let us look at this side, if I were to look at this side. So, let me just say that the total force out here. Let me draw this separately, let me draw this separately. So, let us do that we have that. So, this is the surface this is the side, this is the side on which we have, this is a side on which we have a pressure distribution of p 2 the length is basically l, right. The length is l. So, what I am going to say here is that let me call this force, let me call this you know forces this right. So, this F it I can write this force I can write as p 2 into l, isn't that right? So, p 2 into l. So, if I were to do that, now if you look at this now let us get some angles in here, let us do the do the geometry.

So now we have this this theta is something that I know, right. This theta is something that I know. So, let me make this a little more clear. So, this is an F which is equal to p 2 into l, right. And this is a theta, right. If this is a theta. So, therefore, this is essentially here yeah, let us call this as F x and F y, you know makes more sense to me. Let us call it as F x and F y, if that is true see if that is true let us just write down here, what do you think F x is? If you look from here, right. F x is what F x is p 2 into l into sin theta right. So, p 2 into l into sin theta, right. Because right. So, F x is that right. So, basically. So, F x is p 2 l and sin theta alright. So, and similarly if I come at this here if I look at the; so, this is for this surface right.

So, this I am writing it as this force, this entire pressure force I write there as p 2 l, and I take this horizontal component which is F x comes out to be p 2 l sin theta make sense. We could concern about the drag which will be in this direction. So, that is why I am concerned about F x, I am not going anywhere near F y. So, this is now this surface this is now this surface if you look at this. So, then here to let us just draw that. So, here also let us call this as again F this is this is also going to be F and right. So, or and this is going to be F x 1 and F y 1 right, is that right? So, therefore, here again this is my theta this is my theta and this is my theta, is that right? Yeah, I think that is correct.

So, if this is this is true again. So, again this F 1, again if you look at this here. So, with this distance is l like this total distance is l and this is p 2. So, F 1 again F 1 again is equal to p 2 l. So, therefore, again here F x 1 is p 2 l sin theta. So, F x 1 is basically p 2 l sin theta. So, having done that, now let us look at this this side over here. So, if I look at that side this. So, this is the side. So, if you sort of look at this. So now, we will do a little bit of a little bit of geometry over here. So, here of course, there is no clear pranks. So, we will have this as the; let me call that this is I will be F 2.

Now, this is a side which is the length of the side is not known to us unlike l and l over here. So, we need to find that out, but that is where we will use this. So, this is essentially alright, this is essentially that. And this is our l. This is our theta. And this is our c, isn't it? This length to c this length is l and this is theta. So, therefore, we can find out this particular length, isn't it? We can find out this particular length, and that comes out to be you can see it from here. So, this is l sin theta, right. If that is. So, then F 2 becomes, F 2 again becomes P 1, right. Into 2 l sin theta, right. P 1 into 2 l sin theta ok.

So, that is all we have done all we did is look at the pressures on the 3 surfaces, and then we writing out the forces. If I do that now then what we will do here is get the drag. What, how do we write the drag?

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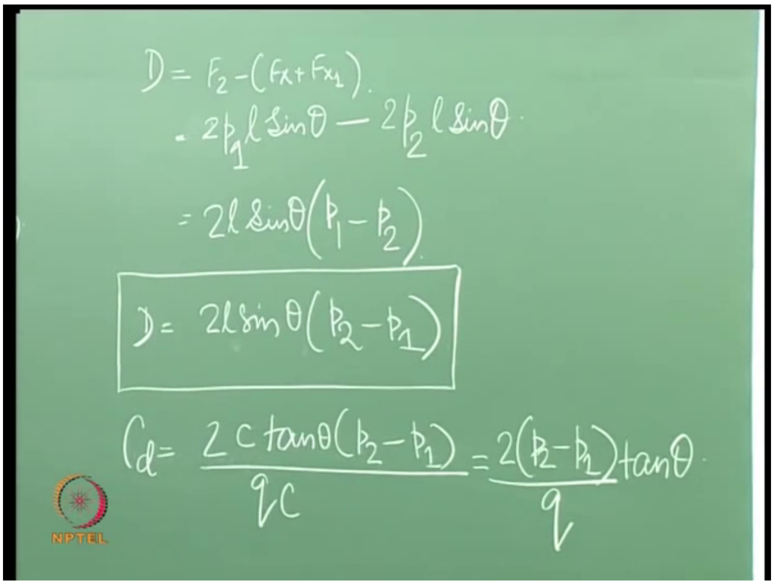

Now so, drag is basically the total forces in the horizontal direction the direction of the free stream. So, here now drag now that comes out to be so, if you look at, but it is important to see here, that this F these 2 this F x and F x 1 are in this direction F 2 is in the horizontal direction here, right. So, what it will be a you know, drag is basically opposite to the free stream. So, we will say that it is equal to basically F 2 minus F x and F x 1, isn't it?

So, which comes out here to be. So, 2 say P 1 l sin theta minus 2 p 2 l sin theta. So, this I can write as 2 l sin theta into this. So, and here what we have done is we have basically taken care of the direction. So, this is pointed in the this is pointed up this is basically facing the free stream. Or if you want to write if you want to be you know just you know, not do that and keep a like consider this as your positive direction.

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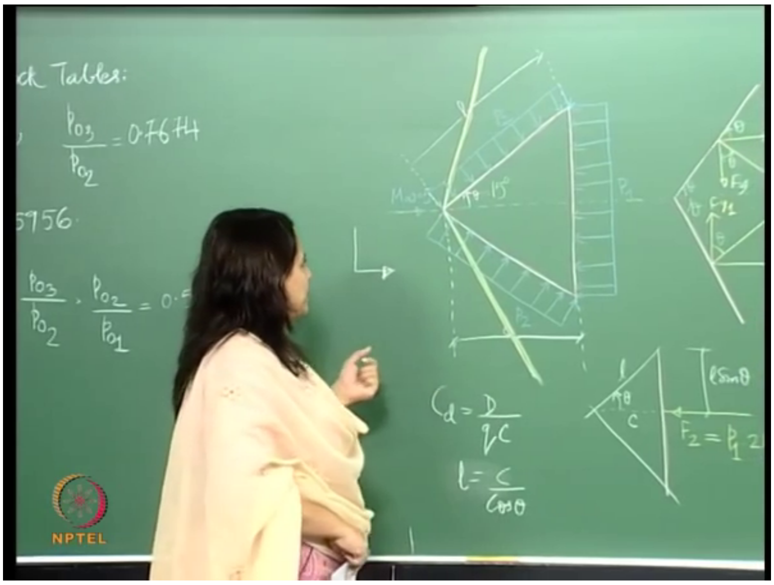

If you want to keep this as your positive direction, then we will just write this as you know this is just a way of writing. So, I will just write this as you know. So, that let us do this the direction should automatically get taken care of. We will see how let us not talk about the direction. Now let us keep it at this it is just the same thing I have just I just basically put a negative sign over here.

So, let us keep it this way and then we will come back and see whether you know this is this this relationship is telling us a direction in which the drag is acting. We kind of know that, but you know let us keep it this way. So now, basically we need to find out the coefficient of drag. So, this C d l is equal to c by cos theta, isn't it? Because c is l cos theta. So, l cos theta. So, l is equal to c by cos theta now c is the reason we are trying to use c over here because somewhere it is kind of in this particular case analogous to the cord length, if you would have an airfoil. And this theta is kind of acting as you know an angle of attack. And so, if I so, I am going to replace that this l out here. L is equal to c by cos theta.

So, if I do that here. So, so then C d becomes basically the drag by q into c q into c. So, what we get is 2 l is c by cos theta. So, then this becomes tan theta. You get that I replace this l by c by cos theta c comes here. So, then I get x cos theta that becomes tan theta. So, then we get this c and c instead of cancels out over here right. So, it does so, then what we get here is essentially this by q tan theta. So, get this alright. So now, q is basically half rho v square. So, this is the dynamic pressure.

So, let us just sort of you know work around this value q out here. Let us do some little math and see what we will get. So now, this q is a dynamic pressure which is half rho v square right.

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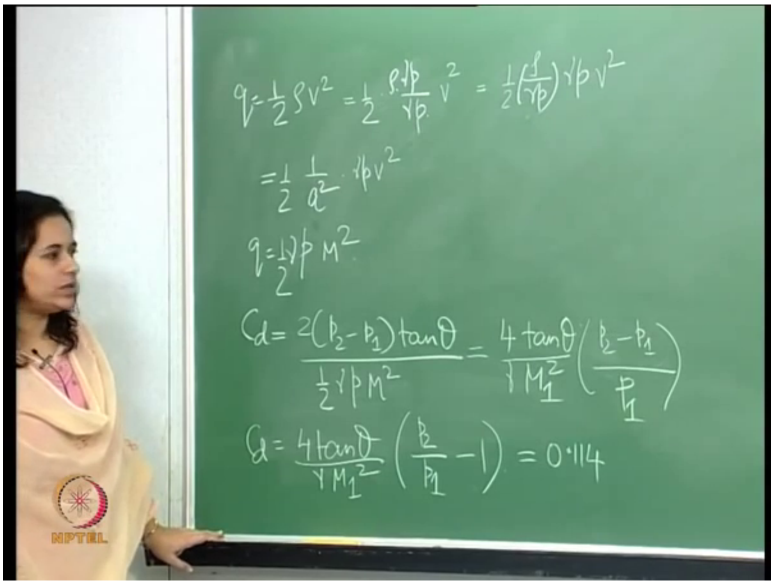

So, I am going to write this rho like. So, let me write this as half or say rho basically. So, I am basically multiplying the whole thing by gamma p. If I do that, then what I get over here, I take this rho by gamma p keep the other gamma p in there, v square. So now, do you recognize what rho by gamma p is you remember this one. So, then this is nothing but so, basically, we choose the speed of sound.

So, then and we have gamma p v square. And again, if you see this v by a this becomes a mach number right. So, therefore, q I can also write as gamma p sorry. So, half gamma p mach number, right. So, therefore, I will write my C d so, therefore, my C d is basically 2 into by q. So, by so, in this particular case. So, this is my q p over here. So, if I do that if I do that or in this particular case. So, then so, this comes out to be 4 4 tan theta this is 4 tan theta, then gamma m square now the question is; so, the skew corresponds to the gamma pressure and mach number.

So, if I was you this is the you know corresponding. So, let us say in this in this in here. So, when I say p 2 by P 1, let us go back and sort of look at this over here. So, this is my p 2 and this is my P 1. So, in this in this case. So, therefore, in here the dynamic coefficient of pressure is something that we are taking in this particular direction, in this particular direction. The dynamic pressure in this direction. So, what we will do; is we will consider this direction which is this P 1 and the corresponding mach number, you know what I mean.

So, therefore, here what in this particular case we will take this to be M 1 and P 1. So, if I do that then of course, you can see. So, therefore, what I will get here C d. So, C d comes out to be 4 tan theta by gamma M 1 square p 2 by p 1 minus 1. So, M 1 is something that we do not know. We are saying here is that the flow passes we have an incoming flow, right. It encounters this wedge out here, which means there is encountering these the shock waves. Then it goes past this and we have a pressure distribution like this. So, once this goes past this wedge this is here that we are looking to find out mach number, and the corresponding pressure.

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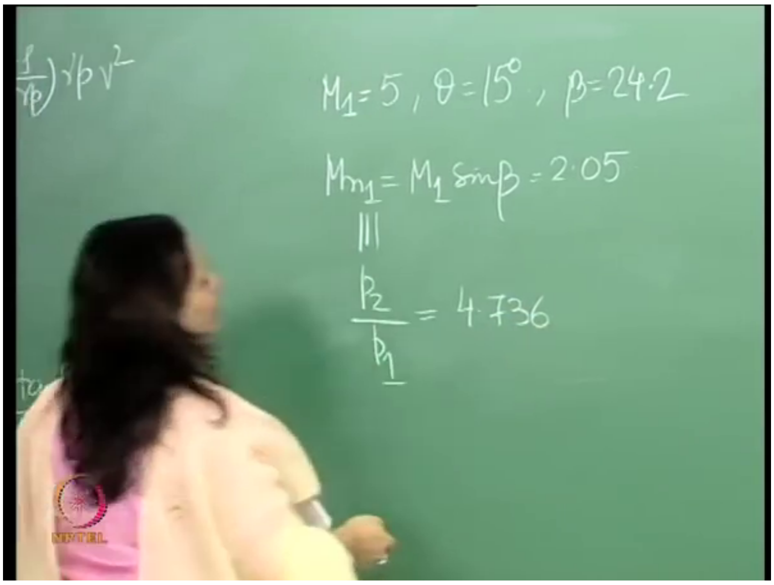

So, basically, we have a mach number. So, mach number is 5, and theta is equal to 15 degrees. So, using the charts theta beta m relationship, we get a beta in this case of 24.2. So, this is the incoming mach number, which we know theta is given. So, therefore, we find out the find out the shock wave angle.

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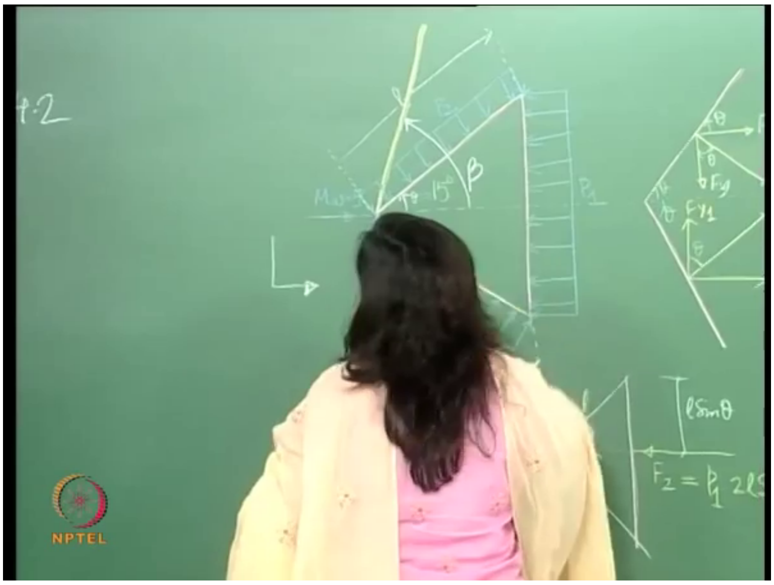

So, we find out the shock wave angle this beta to be 24.2 right. So, therefore, then we calculate the normal of course, right. Normal component which comes out to be 0.2 which is nearly 2. And again, from the soaked now once we get this, once you get this then we calculate corresponding to this mach number. So, corresponding to this we get p 2 by p1, right. Which comes out to be 4.736 which is nearly 5.

Now, we take that. So, we have here M 1 is 5 p 2 by P 1 is you know 4.736, and theta is 15. So, what we get from here is 0.114 right. So, there is no d Alembert's paradox here we have a finite a C d and then essentially call the wave drag ok. So, I think we will stop here. And we will actually do something a little another interesting problem. We will continue this and do another interesting problem, were basically we have a flat plate encountering a supersonic flow, and we will see how that goes, all right.

Thank you.