Advanced Gas Dynamics
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Lecture - 32
Example Problems

So, let us sort of do a couple of problems, right. And see if we understand what we have been you know trying to learn. So, let us take this problem. So, say thrust equation for a rocket engine.

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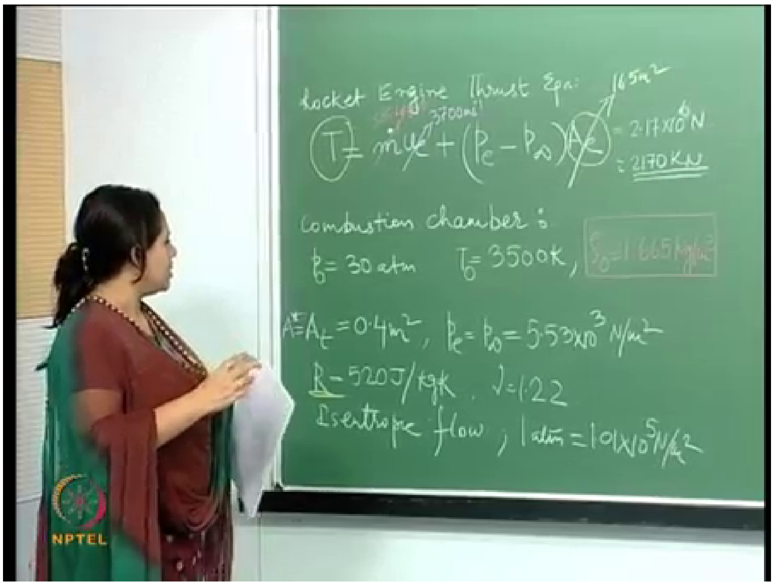

So, we have a rocket engine and I am going to write the thrust equation for that. So, this is thrust right. So, this is given. So, the thrust that is going to be generated by the rocket engine is given by this equation.

So, here this is the mass flow rate, m dot. This is e u e; which is the exit gas velocity. This pressure is the exit, e is for exit. Now pressure, p infinity is the ambient atmospheric pressure (Refer Time: 01:35) in pressure, and A e is the area of the nozzle at the exit. So, given that; so, we have a liquid hydrogen and oxygen which have burnt in combustion chamber producing a combustion gas pressure. So, in the combustion chamber so, like I said so, liquid hydrogen and oxygen are burnt in the combustion chamber; which results in a pressure of 30 atmospheres and a temperature of this.

Now, regarding the nozzles some other parameters, some other values are given right. So, if you remember. So, A t here, this is the area at the throat. So, this is given. This is given as 0.4-meter square, right. Pressure at the exit is essentially the ambient pressure; which is equal to that the universal gas constant is given joules per kg kelvin.

So, this is the parameters which I given. So, the coefficient of specific heats is given as 1.22 gamma that is; and assume isentropic flow, as the gas expands so, you basically have isentropic flow. And what we have to find out here is area at the exit here. So, what we need to find out is calculate this area at the exit and the total thrust. Area at the exit and the total thrust. So, these are the 2 things that we need to calculate here. So, in order do that, what we can see here first things first, that we need m dot we need m dot, we need u dot.

So, let us go ahead and proceed to calculate this. Let us go ahead how we will sort of go about this.

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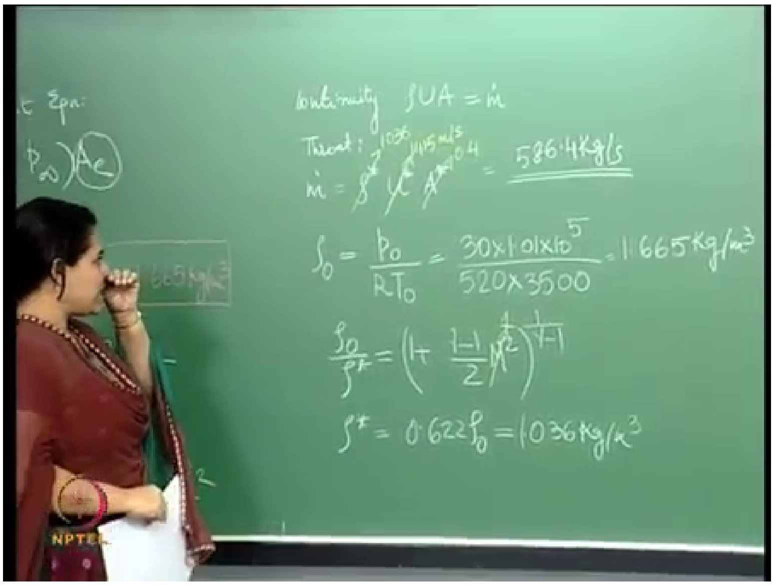

Now, so, let us start with the continuity equation. So, continuity equation tells us what? So, from continuity so right. So, what would be a convenient place to take the to consider this mass flow rate. Let say let take the throat as the convenient location, and if you remember right. So, we all the values at the throat of the nozzle is regarded with the superscript start. So, we will do that here.

So, let us say so, therefore, taking at other throat, right. At the throat convenient location. So, what we get is now to get this rho star to get this rho star. Now let us get the recevoir density, the recevoir density, right. Which I can basically get for my gas equation right. So, so this is my the not here corresponds to recevoir conditions, which here how do we calculate this rho naught, how do we get this out here. Is there anything here you think we should be able to use in this particular case? If I have to use the if I have to find out the recevoir density, right.

So, that is the density which we can in this here one we say that in the combustion chamber 2 gases have burnt which results in these. So, this is essentially the recevoir conditions. So, we can consider essentially this p and this t are the recevoir conditions, and R is given to us R is given to us here. So, we will use that. So, p naught now 30 atmospheres, and 1 atmospheres is how much? If you remember right. So, 1 atm 1 atmospheres is; in newton per meter square. So, we will got p naught and we got t naught and we got R. So, often kelvin's that kind of kelvin.

So, if I do that. So, essentially, I can write this as. So, this is the pressure in newton by meter square, this is the universal gas constant, and this is the temperature in the combustion chamber. And we get this to be; we should just cross check this you just do it in use your calculator just cross check that have got this correct you know, I might be wrong. So, next should I have got these calculations, right, alright. So, I get the density of the gas, right gasser of mixture in the combustion chamber out here we were given just pressure and temperatures now we have calculated the density. This is my recevoir conditions, right. In the combustion chamber which I have got as 1.665. So, why do not we, right that here also.

Since you have calculated this. So, since we have calculated values let me write that here. So, this is the calculated volume. Now then how to we get the density at the throat? So now, the what we will use here if you remember for a calorically perfect gas, right for a calorically perfect gas, right. This is an equation which wholes, this is an equation which wholes for a calorically perfect gas.

Now, rho naught is something that we have calculated. Now let us take rho star if I take rho star. So, if I consider the rho star, which is at the throat right. So, I basically get m star which is equal to 1. So, basically throat is the sonicson right. So, this goes to 1. So, we have left with this expression here right. So, therefore, if I calculate this. So, what I get out here, then I put gamma is equal to 1.22, right. And what I am able to get is this, rho star is this. And I calculate that to be; so, I get the density at the throat to the 0.6 times the density in the combustion chamber, which comes out to be this. We should cross check this. Just put in 1.22 here, and calculate this you know using your calculators or whatever. And make sure I have got these values, right.

So, this is the density in the; so, density is done. So, what we get over here is this is now done. So, what we get this is 1.036, rho star. Now having done that now the next thing is we will go ahead and calculate this u star, we will calculate u star.

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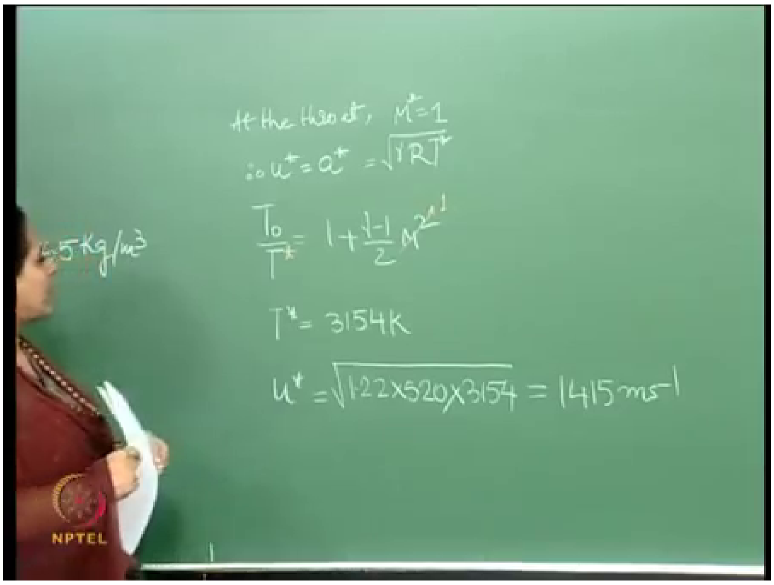

So, how do we do that? Now at the throat which is sonicson m star or mach number is 1. So, in at the throat so, therefore, u star is equal to A star, right. It is equal to the speed of sound right. So, which is again speed of sound using an equal to gamma R T, gamma R T and here because it is at the throat I can write this as T square, right. T star and a again gamma and R are constants which are known, I need to find out T star.

So, if your already not guessed we will again use the equation for calorically perfect gas, like we use for the density and the corresponding relationship is this, right. If you remember correctly. This is the relationship of the recevoir conditions and temperature. Recevoir conditions of and the temperature anywhere in the flow in the nozzle. So, in this case if I put this has T star, then this goes to what, right. If I do that. So, the once I do that so, therefore, I can write basically T star comes out to be; so, the temperature at the throat comes out to be 3154 kelvin, and if you remember the combustion chamber it is 300, 3500 kelvin this is around 3160.

So, temperature having got that. So, therefore, now we can get u star, isn’t it? So, therefore, we get now u star, u star so, therefore, we get that as 1.22, right. And this comes out to be. So, u star comes out to be 1415 meter per second. This is what we get here. So, therefore so, what we do you know over here? We have m star. So, rho star and u star. So, we got u star now, we got the velocity as per which is 1415 meter per second. I still need A star A star which is the here. Now A of the throat is actually given to us, A of throat is given to us.

This is nothing but A star, this A is nothing but A star. So, which is given to us as 0.4. Now this is given which is 0.4. So, if I do that then what I get as m star is essentially 586.4 kg’s per second. So, basically, we get this this is the mass flow rate. So, we get a mass flow rate of 596.4 kg per second. So, having got that.

Now, the next thing we definitely need is the also this is something we have got. Let us look how we can calculate the velocity at the exit u e at the exit how do we go about doing that, right. Now. So, let us try and find out the mach number at the exit. We have no clue has to what that is, right. And then we will see because you if you see here we have relationships between the densities and mach number. Change in density and mach number change in temperature and mach number. So, maybe we will go and use this sort of thing.

If we look here let us try to see what we can use. So now, what we know here is the recevoir pressure. If you look here from the information that is available to us we have the recevoir pressure here which is p naught. And we are also we also know the recevoir pressure at the exit which is this, right. Now this should this these should therefore, give us and relationship of with the exit mach number.

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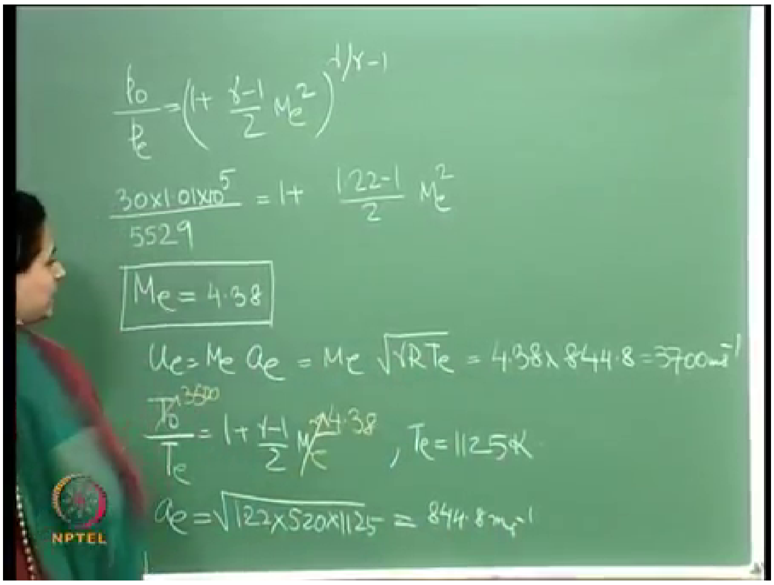

Because so, like we have density relationship. We also know that we have right. So, we have this relationship of the stagnation pressure, and pressure anywhere in the nozzle with the mach number. So, here if I use consider this as an exit. This is a corresponding exit mach number.

So, I am going to use this and find out the corresponding mach number. So, and this is a given this is given to us, this something this is also actually given to us p naught is also something that is given to us. So, a using that using here. So, A I will just write down the values over here right. So, what are the what are this of equation relationship that we get here. So, p naught p naught is essentially, right. This is given then this is p e. This is equal to 1 plus this is also given ok.

So, from here I can calculate the exit mach number. So, I get the exit mach number to be; so, I have an exit mach number just over 4. So, having down this therefore, how do we calculate the at the exit mach we have an exit mach number we do not have the exit speed of sound. So, let us do that. So, therefore, now u e is of course, M e into speed of sound at the exit. This we do not have that. Now let us do this again. Basically, we can write again you know, this we can write as gamma or T at the. So, I can write the speed of the sound of the exit in terms of this in terms of the temperature. Again, the temperature that exit something I haven't calculated here.

Let us see if I can do that. I am going to again use this equation here. Last time we put the throat instead of this time we put this as the exit. So, therefore so, basically, I have T is equal to right. So, in this case we have this as the exit, we have this is the exit. And by now we have calculated the mach number as 4.38. So, we have calculated this. And T e not which is the ambient recevoir conditions this is 3500, right. Having done that, we can now calculate T e from here. So, T e from here therefore, comes out to be T e from here comes out to be this, right. And hence corresponding A e and hence this comes out to be corresponding speed of sound comes out to be 1.22 520, right. This comes out to be 844.8. It is nearly 845 meters per second. So, this is the speed of sound at the exit ok.

So, once have we found that out will go back and put it in here. So, what we get here is basically this is, right. And this comes out to be; it is nearly 4000 meters per second. It is 3700 meters per second. So, velocity at the exit is 3700 meters per second. Let us go and look here. So, what to we have here? So, did we find out the mass flux here we did. So, this mass flux a is 586.4, this 586.4 kg per second, right. And the velocity at the exit is 37, 3700 meters per second. It is a put in high speed on to seek. And of course, if you look here that is all I need to calculate the thrust. Because the pressure at the exit is equal to the ambient pressure which is given to us.

So, therefore, this term will anyway go to 0 out here. If not then of course, if some the answers given then we will (Refer Time: 23:32) use this, but never be less we still to calculate the area at the exit we will do that. So, anyway we are done with the thrust those. So, thrust comes out to be let us, right. Out thrust first. So, thrust comes out to be around 2.2 or 22 kilo newtons actually. 2170 actually it is 2170 kilo newtons if you are a look at this.

So, this I can write as just to give a little more idea. So, basically what we have here is we have nearly 2200 kilo newton thrust been generated by the by the rocket engine, this is the total thrust. So now, let us look at what would be so, area at the throat is 0.4-meter square. Now what is the area at in the exit? So, let us go and see how we will do that.

So, in order to do that so now, again using the continuity. Continuity between the so, continuity so, at any point if I say at the throat conditions right.

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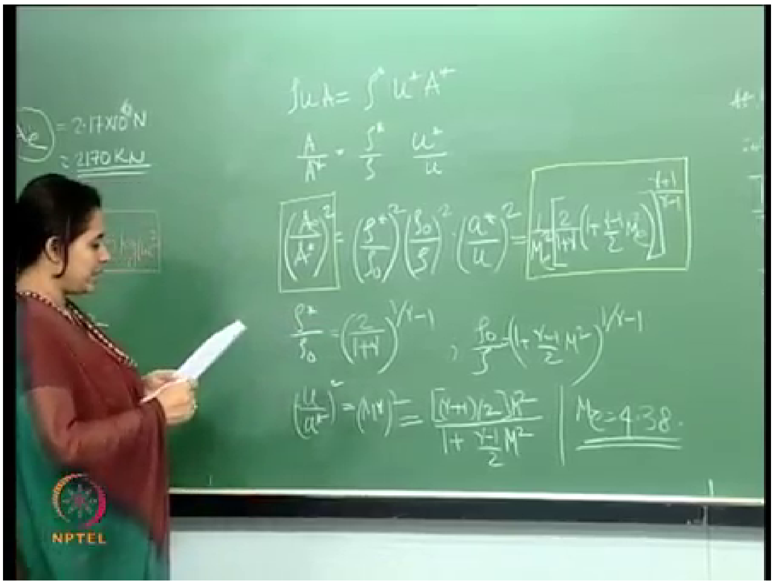

So, mass flow rate at any through any portion of the nozzle is equal to that through the throat right. So, from this I can write this equation as, right. This is equal to; so, now then I am going to write this now basically I am going to introduce the; I am going to introduce the recevoir conditions here, right. Because we have already calculated rho star by rho naught etcetera. So, let me write is so, therefore, I can write this square this in write it like this. Now this u star, u star is velocity at the throat. So, which is again equal to the speed of sound because mach is 1. So, then I can write this as a star by u square.

So, if I will write it like this now if you remember, right. Now this rho star by rho naught is essentially, cross check this. So, we have just we have just written out the relationship between the we just written out the relationships between the densities with mach number. So, if you write here with respect to the throat the mach number goes to one and this is the relationship. So, rho star by rho naught is something which is available here.

So, and this and of course, this is the relationship right. So, we will get value for this. So, we can write that it is a finite value from here. Now rho naught by rho will be in terms of the mach number. It will be in terms of the mach number. So, we can get that in there. And again, a star by u right. So, this is what? This is nothing but so, velocity, velocity by velocity sound at the throat right. So, this is nothing. So, essentially velocity by velocity at the throat (Refer Time: 28:51) speed of sound at the throat this is nothing but, right.

And this I can also write in terms of; so, this gamma plus 1 by to write this this is the, so, this is then you can I path with the mach number.

So, this is the relationship that we have. So, if you look here for this particular if you this particular expression here. So, A by A star, A by A stars if we look at this. So, what we will have here is that this is a finite value which we get from here now this is a term which we can write in terms of the mach number here. Again, this is something we will write in terms of the mach number over here mach over here right. So, if I do that. So, I will get an expression for this in terms of the mach number. So, let be write that out. So, if I write that out it comes out to be this right. So, basically what we have here is what we have done here, if we look at this if we look at this; that we have connected the area at the throat with the area and mach number anywhere in the nozzle that is what we have done.

If you see this is the area at the throat which is given to us and this is the area anywhere in the nozzle and the corresponding mach number at that point were at the location we considering this area. And the gamma is known to us. So, in this case of course, we need the area at the exit, right. And we have calculated the mach number at the exit. So, therefore, in this particular case if I have to write this. So, basically what I need is area at the exit, right. As so, then this becomes mach number at the exit. And mach number at the exit is something that we have calculated right. So, here 4.38 so, this so, essentially this values known to us now this is given to us. So, what we get as if I calculate this. So, A e therefore so, A at the exit right.

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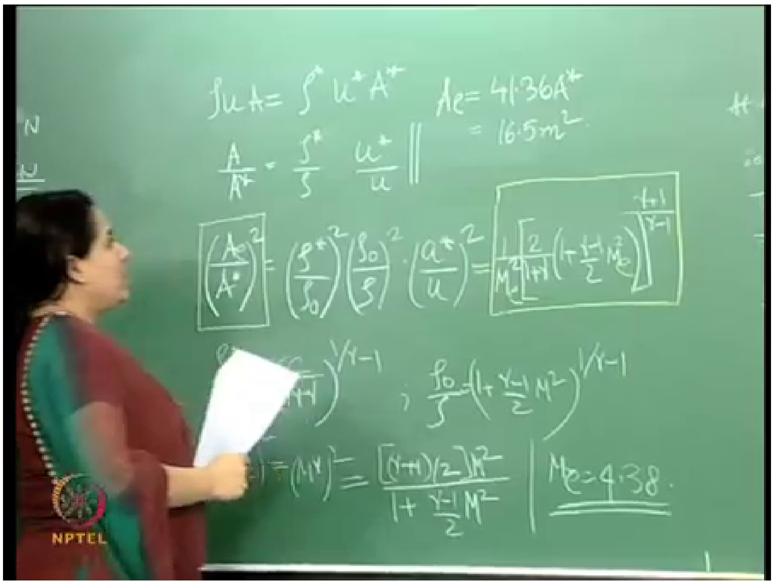

Now, A at the exit is comes out to be a little more than for 40 times the area at the so, area at the exit. So, area at the exit here comes out to be right. So, therefore, now we calculate the area at the exit, right. And this comes out to be nearly actually little more than 40 percent of the area at the at the throat right. So, let us just take step back and see what is happening here. So, we have a rocket engine rocket engine. So, we have a you know diverging nozzle, where the area at the throat where the area of the exit is nearly 40 times, or little more than 40 times the area at the throat right. So, we have this diverging nozzle.

So, the combustion chamber you know pressure temperature etcetera is given. So, we have an isentropic expansion of the flow over here. So, at the exit did we calculate yes, we have calculated I think right. So, at the exit the temperature is 1125. So, at the exit at the exits. So, let me just sort of write this down, this is this is the interesting part here.

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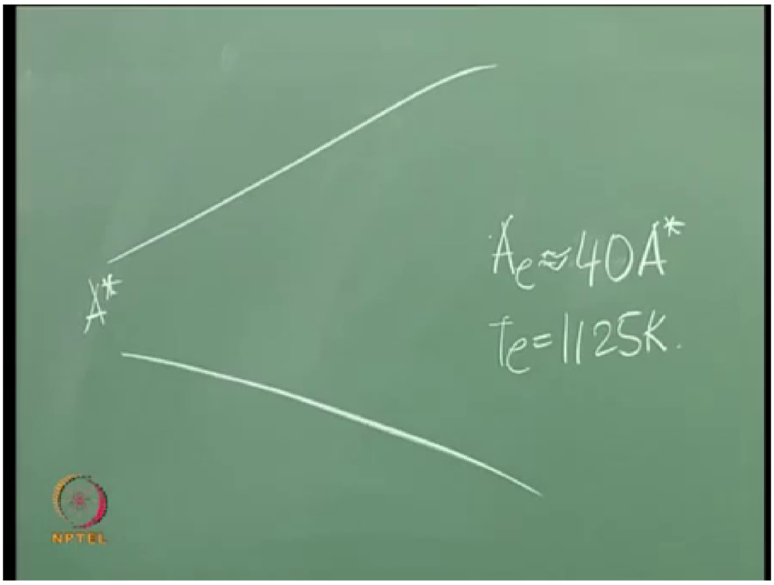

So now I have this say diverging nozzle. So, this is the area at the throat this is the throat region. And this area, area at the exit it is this area at the exit is a is actually is nearly 40 times the area at the; is 40 times the area of the throat that is how this expanding. So, then and the combustion chamber pressure, temperature combustion chamber pressure is 30 atmospheres, and temperature is 3500 kelvin. The temperature at the exit drops, right to a little over thousand kelvin. So, combustion chamber the pressure the temperature generated in the when the gas is produced, it is produce the 3500-kelvin due to the expansion at the x is drops down to nearly a little over thousand kelvin.

So, and this when I am able to do this. So, I generate thrust which is which is a nearly 2200 kilo newtons of thrust is generated when I have a rocket at the diverging nozzle like this. So, I was basically how you know we would use a differential area nozzle in order to generate a thrust. So, and the connection of area the change in area with the mach number and hence the velocity. So, that is that is one problem.

So, let us now go ahead and try another problem. So, this is like I said this is the flow properties connected with the area. And then important thing note here is that the area of change can be related to the corresponding mach number. And hence can be related to the pressure temperature density changes is well; which is what we just did over here. So, I guess we do not want to finish the next problem, but we will start it never the less.

We will start this and see you know see some interesting things from there as well. This is also very interesting problem when I thought that it would be you know interesting to sort of understand this. So, we have done, right. We have done basically normal shocks, oblique shocks, and we have also done in a shock interactions in so on and so forth.

So now, here what we have is basically we have a supersonic flow, which has a mach number 3. And we want to basically the simple thing is we have a mach 3 flow, and we want to slow it down to subsonic speeds, that is all.

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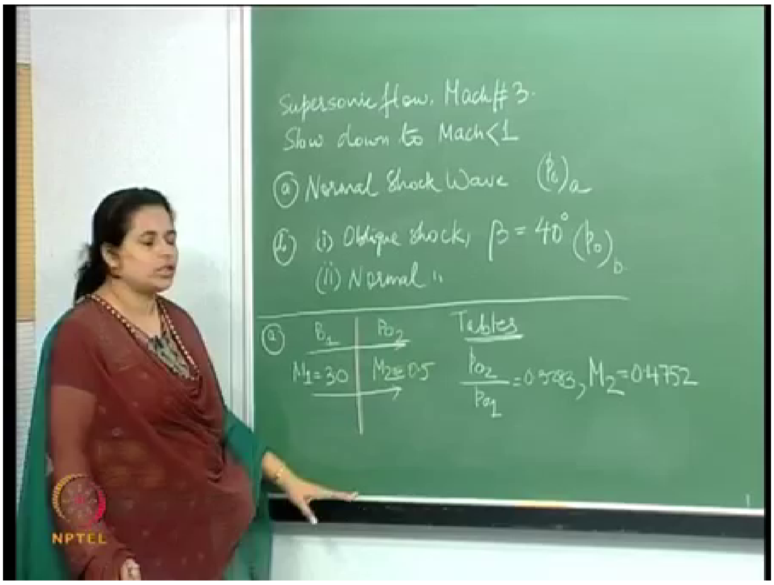

So, we have a supersonic flow, right. Which is of, right we want to slow this down to subsonic speeds, right. We want to slow this down to subsonic speeds.

Now, you know in what you think we should do, I mean just you know stop it you know turn the flow into itself. So, may be just you know if I want to do that, will go through a shock you know what we will do; however, look at this in 2 separate ways. So, apparently, we can do this you know less or more refreshently, will see how. So, let us try these 2 cases. One is will pass this directly through a normal shock wave, and normal shock wave. So, we will pass this through a we will pass this through a normal shock wave. And the second is first pass this through know oblique shock wave and then a normal shock wave ok.

So, first in this case first it goes to an oblique shock, right. Where the shock wave angle is given as 40 degrees. And next is through a normal shock wave. So, we will see where these will be different. So, and if these are different at all, which one should I choose? We should I choose this or should I choose this? And of course, why that? So, in order to do that so, also another these. So, calculate I think you know this kind of the answer kind of lies in the question, through if this these 2 are going to result in different, if these 2 are going to give us different results and how we going to then choose which one should be go about you know, using in order to do this at the answer kind of lies in question itself, but I will write it down anyways.

So, calculate the ratio of the final total pressure values for the 2 cases. So, basically in this case so, we can say the so, what we need to calculate here is the ratio of the total pressure values for the 2 cases and this significants of that. Well, the significants of that itself is actually going to give us clues have to which one we will need to choose in order to do something like this. So, what we will do here is go ahead and just do the problem first which by this time you should be you know it should not be a problem, but what we will look at is that you know the physical aspect of it.

So, let us do this. So, first things first let us go ahead and you know we will do this normal shock wave. So, if we look at this, so we have basically. So, this is our normal shock. And so, let us call this as total pressures here and this is this, so now so, this is basically for we just passing the flow through a normal shock wave. If I do that now from if I now start using the tables, right. If you go and look at the tables.

So, from the tables what we get is that, right; is equal to this and the mach number in here is; so, this is the mach number, which is let me say less than 0.5 actually right. So, all I do is just you know what is the big deal you know just slow it down to mach one. So, we will just pass it through a normal shock wave go to the tables this, look at you know corresponding to this we get ratio of the total pressures in front of the behind the normal shock wave, and the mach number behind it; which is subsonic you know why we going through this? Process you know, is the motivate we will see. So, that is all there is used to it.

So, we have subsonic speed you know of the flow behind the shock. So now, since you can have assuming there is more to it. So, let us go ahead and see this you know let us do this for the second one; which is the combination often oblique shock and a normal shock. Let us do that.

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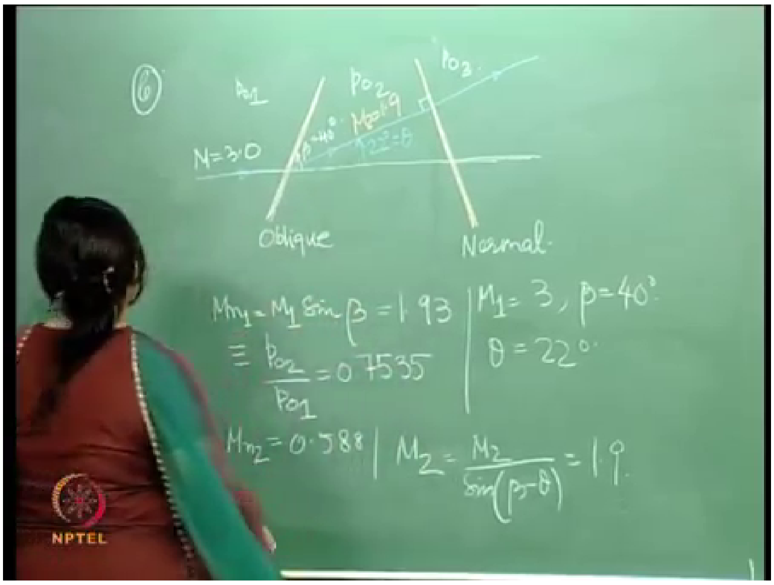

So, if I do for the second one right. So, this is the second one right.

So, essentially here. So, I have an oblique shock. So, I do have; and this is an oblique shock and this is a normal shock. So, essentially what we will therefore, have so, we have say if I look at the free stream here. So, if I look at this free stream here. So, I have this, and then this free stream is deflected, right. Free stream is deflected like that. And so, do this has to be a normal shock wave. So, what I am going to do is to a this keep my shock basically normal to this. So, what I have do here is keep my shock normal to it.

So, then here it does not deflect around it goes like that. So, that is essentially the so, you understand. Now what you have to if you look at this you will be like both these are total or inclined. Well, these are inclined with respect to the horizontal of course, these are, but whether we this is an oblique shock or a normal shock is dependent how the angular is making with the free stream.

So, in this case you look at this, this is the angle. So, this is the angle which is 40 degrees actually this beta is 40 degrees which is given. So, you see this is the free stream in this case, this is the free stream and the shock is making an angle you know, the acute angle is 40 degree if we look at this. So, this is the oblique shock. And here if you look at this

this shock is actually making 90 degrees with the incoming flow. So, here the flow comes. So, this my incoming flow which is mach 3, right. And then it goes and hits the hit then it gets and count. So, this is oblique shock and is deflected it comes here and I make it pass through another shock which is normal to it. So, there is no deflection it flows is path, but of course, there will difference in it is properties.

So, this is the normal shock. So, if I do this then. So, basically this I am going just call this as p naught, one this at; this is going through changes in we can consider the flow field as basically 3 regions compared with the 3 regions. So, if got this this in this. So now, the way let us look at the oblique shock first. Let us look at this first. So, when I do that. So, how do we go above this is the first things first that we do is calculate the component which is normal to this oblique shock, based on this now beta is given to us right.

So, therefore, what we calculate is M n 1 which is M 1 sin beta, right. And this comes out to be 1.93 which is nearly 2. So, then corresponding to this M n 1, corresponding to this we get a p naught 2 by p naught 1 to be to be this. And so, there the if you look at this ratios so, we had a point you know we had a like a 0.3. We can say 0.33 and 0.8. So, p naught 2 by p naught 1. So, p naught 2 is 0.3 times p naught 1. And p naught 2 here in this case is 0.8 times p naught 1. So, in this case p naught 2 is less. So, the total pressure behind the normal shock is actually less. Than the total pressure behind the oblique shock in this particular case. And of course, then we calculate the M n 2 which is again point it is around 0.6. And of course, then you know we have an M 1. So, corresponding to M 1 equal to 3 corresponding to a shock wave angle of 40 degrees will go to the theta beta m relationship, calculate the deflection angle, right. Calculate the deflection angle to be 22 degree.

So, this angle here, this is the deflection. So, this is the deflection angle. So, having done that. So, we should be able to calculate the mach number. Mach number behind this oblique shock which comes out to be; this is something that we have done which comes out to be 1.9. So, therefore, the mach number here is 1.9 which is nearly the which is nearly 2 ok.

So, here of course, you know it was 0.5 we directly got a so, so I passed through the normal shock I directly get a subsonic flow here of course, I haven’t got subsonic flow yet it is tilt slightly supersonic. So, therefore, we will now pass this through this normal shock and see what we get. So, we will stop here we will continue from here next class, and complete this.

Thank you.